Artificial Intelligence & Data Science (Sem VI)

ADC 601 : Data Analytics & Visualization

Module - 3 : Time Series
(7 Hours)

Instructor : Mrs. Lifna C S

Topics to be covered

- Definition of time series.
- Times series forecasting.
- Time series components
  - Decomposition – additive and multiplicative.
- Exponential smoothing
  - Holt winters method.
- Time Series Analysis
  - Box-Jenkins Methodology - ARIMA Model
  - Autocorrelation Function (ACF)
  - Partial Autocorrelation Function (PACF)
  - Autoregressive Models
  - Moving Average Models
  - ARMA Model
  - ARIMA Models
- Building and Evaluating an ARIMA Model.

Why Time Series Analysis ?

In this analysis, you just have one variable – **TIME**

You can analyse this **time series** data in order to extract meaningful statistics and other characteristics

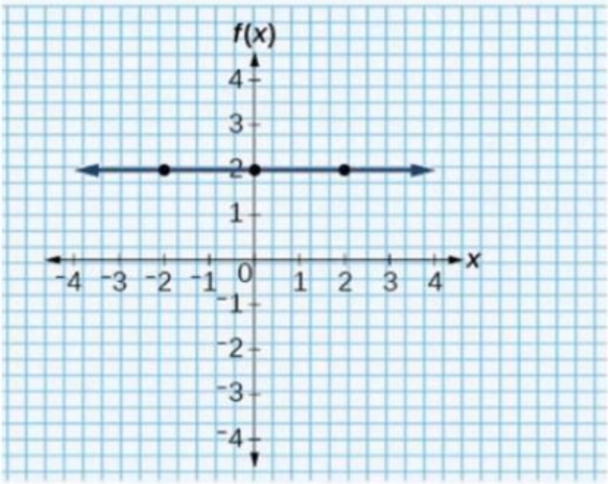
What is Time Series ?

- A time series is a set of observation taken at specified times usually at equal intervals
- It is used to predict the future values based on the previous observed values

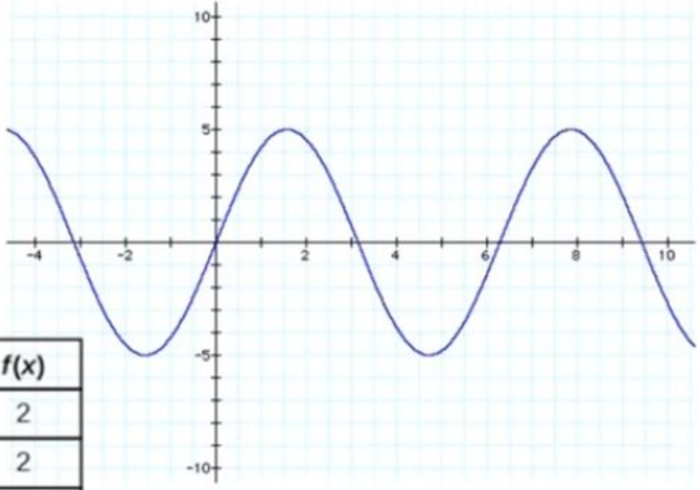
Courtesy : [Investopedia]

When not to use Time Series Analysis ?

1 Values are constant



2 Values in the form of functions



x	f(x)
−2	2
0	2
2	2

Components of Time Series

Courtesy : [Investopedia]

Components of Time Series

- **Trend :**
  - refers to the long-term movement in a time series.
  - It indicates whether the observation values are increasing or decreasing over time.
  - Ex : Steady increase in sales month over month or an annual decline of fatalities due to car accidents.
- **Seasonality :**
  - describes the fixed, periodic fluctuation in the observations over time.
  - often related to the calendar.
  - Ex : monthly retail sales can fluctuate over the year due to the weather and holidays.
- **Cyclic :**
  - refers to a periodic fluctuation, but one that is not as fixed as in the case of a seasonality component.
  - Ex : retails sales are influenced by the general state of the economy, i.e., retail sales time series often follow the lengthy boom-bust cycles of the economy.
- **Irregularity**
  - Noise is a random component,
  - The underlying structure to this component needs to be modeled to forecast future values of a given time series.

Courtesy : Investopedia

Decomposition of Time Series Components

- technique used to separate a time series into its constituent components,
  - typically trend, seasonality, and noise (or error).
- These components can be **additive or multiplicative**,
  - depends on the characteristics of the data.
  - If seasonality and trend remains constant regardless of the level of the series, additive decomposition
  - If seasonality and trend grows or shrinks with the level of the series, multiplicative decomposition
- Both methods are widely used in time series analysis and forecasting to better understand the underlying patterns and components within the data.

Decomposition of Time Series Components

1. **Additive Decomposition:**

In additive decomposition, the components are added together to reconstruct the time series. This method assumes that the seasonal variation and trend in the data remain relatively constant over time, regardless of the level of the series. Mathematically, an additive decomposition model can be represented as:

$$y_t = \text{Trend}_t + \text{Seasonality}_t + \text{Noise}_t$$

where:

- y_t is the observed value at time t.
- Trend_t represents the long-term progression or direction of the data.
- Seasonality_t captures the periodic fluctuations within the data.
- Noise_t represents random fluctuations or errors.

Courtesy : Investopedia

Decomposition of Time Series Components

2. **Multiplicative Decomposition:**

In multiplicative decomposition, the components are multiplied together to reconstruct the time series. This method assumes that the seasonal variation and trend in the data vary in proportion to the level of the series. Mathematically, a multiplicative decomposition model can be represented as:

$$y_t = \text{Trend}_t \times \text{Seasonality}_t \times \text{Noise}_t$$

where the components have a multiplicative relationship.

- y_t is the observed value at time t.
- Trend_t represents the long-term progression or direction of the data.
- Seasonality_t captures the periodic fluctuations within the data.
- Noise_t represents random fluctuations or errors.

Box-Jenkins Methodology

- also known as the **ARIMA (AutoRegressive Integrated Moving Average) modeling approach**,
- is a statistical technique <u>used for time series forecasting and analysis.</u>
- developed by **George Box and Gwilym Jenkins** in the early 1970s.
- Involves three main steps: **identification, estimation, and diagnostic checking**.

1. **Identification:**
   a. <u>Stationarity:</u>
      - Ensure that the time series data is stationary by checking for trends or seasonality.
      - If needed, apply differencing to make the series stationary.

   b. <u>Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF):</u>
      - to identify the order of autoregressive (AR) and moving average (MA) components in the model.
        - ACF : correlation between a variable and its lag,
        - PACF : only the direct effects.

Box-Jenkins Methodology

2. **Estimation:**
   a. Fit the ARIMA model:
   - Estimate the parameters of the model using methods like the maximum likelihood estimation (MLE).

   b. Diagnostics:
   - Check the residuals (the differences between predicted and actual values) for randomness and lack of patterns.
   - Use *statistical tests to ensure that the model assumptions are met.*
3. **Diagnostic Checking:**
   a. Residual Analysis:
   - Examine the residuals to ensure that they are white noise, indicating that the model captures the underlying patterns in the data.

   b. Model Adequacy:
   - Evaluate the overall adequacy of the model via Akaike Information Criterion (AIC) or Bayesian Information Criterion (BIC). Lower values indicate a better-fitting model.

Courtesy : Investopedia

Stationary Time Series

- First step of the Box-Jenkins methodology, it is necessary to remove any trends or seasonality in the time series.
- To achieve a time series with certain properties to which autoregressive and moving average models can be applied. Such a time series is known as a stationary time series.
- The stationary Time series follow the below 3 conditions :

  - (a)The expected value (mean) of y_t is a constant for all values of t.
  - (b)The variance of y_t is finite.
  - (c)The covariance of y_t and y_{t+h} depends only on the value of $h = 0, 1, 2, \ldots$ for all t.

Stationary Time Series

The covariance of y_t and y_{t+h} is a measure of how the two variables, y_t and y_{t+h}, vary together. It is expressed in Equation 8.1.

$$8.1 \quad cov\left(y_t, y_{t+h}\right) = E[(y_t - \mu_t)(y_{t+h} - \mu_{t+h})]$$

If two variables are independent of each other, their covariance is zero. If the variables change together in the same direction, the variables have a positive covariance. Conversely, if the variables change together in the opposite direction, the variables have a negative covariance.

For a stationary time series, by condition (a), the mean is a constant, say μ. So, for a given stationary sequence, y_t, the covariance notation can be simplified to what's shown in Equation 8.2.

$$8.2 \quad cov(h) = E[(y_t - \mu)(y_{t+h} - \mu)]$$

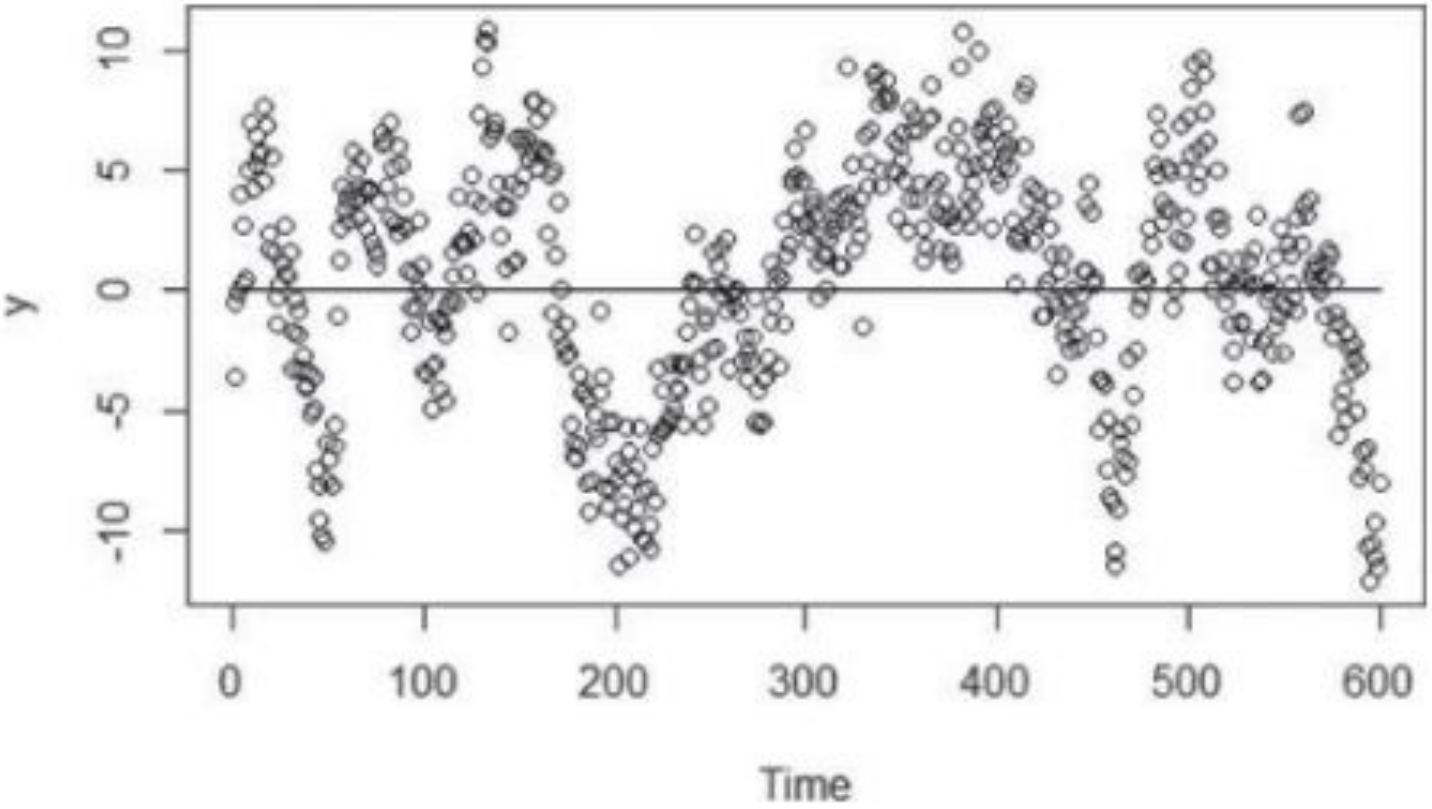
Stationary Time Series

By part (c), the covariance between two points in the time series can be nonzero, as long as the value of the covariance is only a function of h. Equation 8.3 is an example for $h=3$.

8.3 $$cov(3) = cov(y_1, y_4) = cov(y_2, y_5) = \ldots$$

It is important to note that for $h=0$, the $cov(0) = cov(y_t, y_t) = var(y_t)$ for all t. Because the $var(y_t) < \infty$, by condition (b), the variance of y_t is a constant for all t. So the constant variance coupled with part (a), $E[y_t] = \mu$, for all t and some constant μ, suggests that a stationary time series can look like Figure 8.2. In this plot, the points appear to be centered about a fixed constant, zero, and the variance appears to be somewhat constant over time.

Stationary Time Series - Sample Plot



Tests to Check Stationarity

1 Rolling Statistics

Plot the moving average or moving variance and see if it varies with time. More of a visual technique.

2 ADCF Test

Null hypothesis is that the TS is non-stationary. The test results comprise of a *Test Statistic* and some Critical values.

ACF - Autocorrelation Function

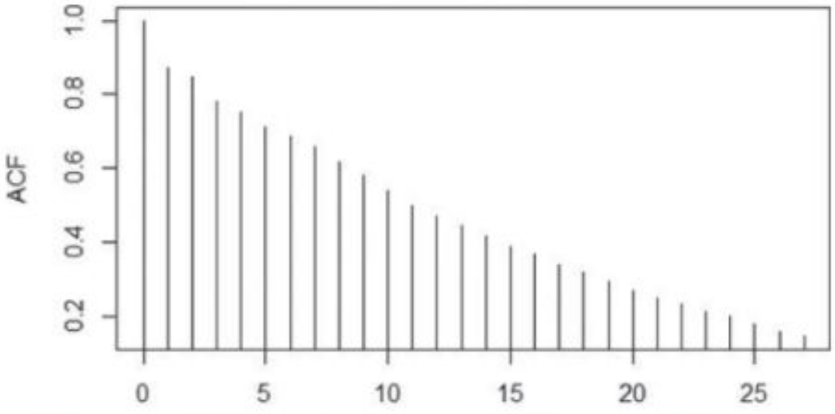
For a stationary time series, the ACF is defined as shown in Equation 8.4.

$$ACF(h) = \frac{cov(y_t, y_{t+h})}{\sqrt{cov(y_t, y_t)\,cov(y_{t+h}, y_{t+h})}} = \frac{cov(h)}{cov(0)} \quad (8.4)$$

Because the cov(0) is the variance, the ACF is analogous to the correlation function of two variables, $corr(y_t, y_{t+h})$, and the value of the ACF falls between –1 and 1. Thus, the closer the absolute value of ACF(h) is to 1, the more useful y_t can be as a predictor of y_{t+h}.

ACF - Autocorrelation Function

h : lag, the difference between the time points t and t + h.



the time points t and t + h. At lag 0, the ACF provides the correlation of every point with itself. So ACF(0) always equals 1. According to the ACF plot, at lag 1 the correlation between y_t and y_{t-1} is approximately 0.9, which is very close to 1. So y_{t-1} appears to be a good predictor of the value of y_t. Because ACF(2) is around 0.8, y_{t-2} also appears to be a good predictor of the value of y_t. A similar argument could be made for lag 3 to lag 8. (All the autocorrelations are greater than 0.6.) In other words, a model can be considered that would express y_t as a linear sum of its previous 8 terms. Such a model is known as an autoregressive model of order 8.

PACF - Partial Autocorrelation Function

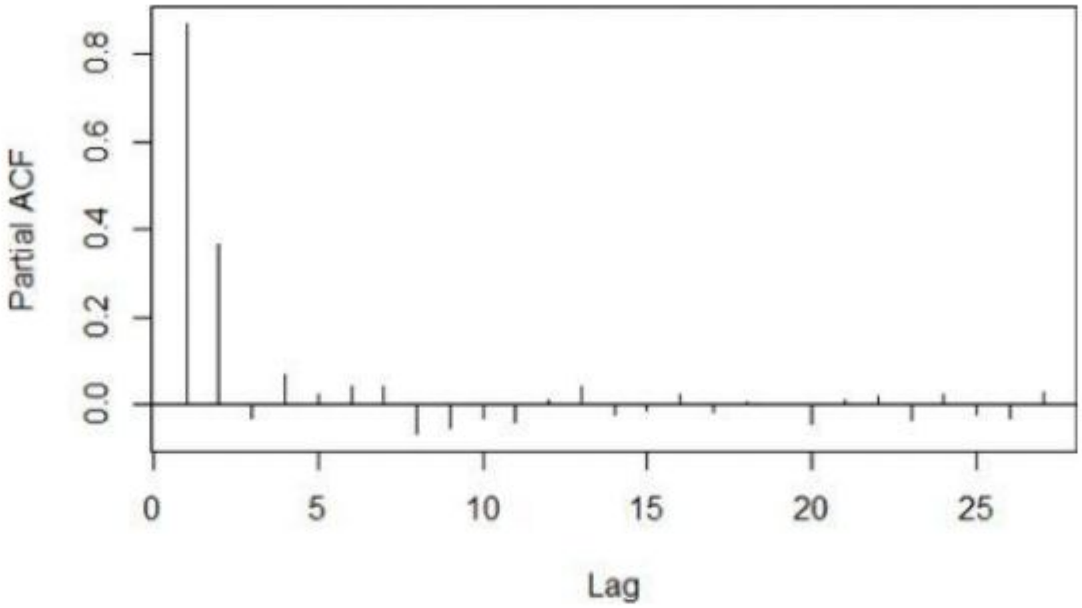
$$PACF(h) = corr(y_t - y_t^*, y_{t+h} - y_{t+h}^*) \text{ for } h \geq 2$$
$$= corr(y_t, y_{t+1}) \qquad \text{for } h = 1$$

where

1. $y_t^* = \beta_1 y_{t+1} + \beta_2 y_{t+2} \cdots + \beta_{h-1} y_{t+h-1}$,
   $y_{t+h}^* = \beta_1 y_{t+h-1} + \beta_2 y_{t+h-2} \cdots + \beta_{h-1} y_{t+1}$, and
2. the $h-1$ values of the βs are based on linear regression.

In other words, after linear regression is used to remove the effect of the variables between y_t and y_{t+h} on y_t and y_{t+h}, the PACF is the correlation of what remains. For $h=1$, there are no variables between y_t and y_{t+1}. So the PACF(1) equals ACF(1). Although the computation of the PACF is somewhat complex, many software tools hide this complexity from the analyst.

PACF - Partial Autocorrelation Function



Because the ACF and PACF are based on correlations, negative and positive values are possible. Thus, the magnitudes of the functions at the various lags should be considered in terms of absolute values.

Exponential Smoothing

- technique used for **forecasting time series data by assigning exponentially decreasing weights to past observations.**
- useful **when there is no clear trend or seasonality** in the data, or **when the data exhibit some level of randomness.**
- **Basic idea** :
    - To give more weight to recent observations while gradually decreasing the weight assigned to older observations.
    - Achieved through the use of smoothing parameters, denoted as α, β, and γ, depending on the specific type of exponential smoothing being used.

Exponential Smoothing

1. **Simple Exponential Smoothing (SES):**

Simple Exponential Smoothing is the most basic form of exponential smoothing. It updates the forecast using a weighted average of past observations, with the weights decreasing exponentially as observations become older. The forecast $\hat{y}_{t+1}$ at time $t + 1$ is given by:

$$\boxed{\hat{y}_{t+1} = \alpha \cdot y_t + (1 - \alpha) \cdot \hat{y}_t}$$

where:

- y_t is the observed value at time t.
- $\hat{y}_t$ is the forecasted value at time t.
- α is the smoothing parameter ($0 < \alpha < 1$) which determines the weight given to the most recent observation.

Exponential Smoothing

2. **Double Exponential Smoothing (Holt's Method):**

   Double Exponential Smoothing extends simple exponential smoothing to handle time series data with trend. In addition to the level smoothing parameter α, it introduces a trend smoothing parameter β. The forecast $\hat{y}_{t+1}$ and trend b_{t+1} at time $t+1$ are updated using the following equations:

$$\hat{y}_{t+1} = l_t + b_t$$
$$b_{t+1} = \beta \cdot (l_t - l_{t-1}) + (1-\beta) \cdot b_t$$
$$l_{t+1} = \alpha \cdot y_t + (1-\alpha) \cdot (l_t + b_t)$$

where l_t represents the level at time t.

Exponential Smoothing

3. **Triple Exponential Smoothing (Holt-Winters Method):**

Triple Exponential Smoothing further extends double exponential smoothing to handle time series data with both trend and seasonality. In addition to the level l_t and trend b_t, it introduces a seasonal component s_t. The forecast $\hat{y}_{t+h}$ at time $t + h$ is updated using the following equation:

$$\boxed{\hat{y}_{t+h} = l_t + hb_t + s_{t-m+h_{(m)}}}$$

where m is the length of the seasonal cycle, and $s_{t-m+h_{(m)}}$ is the seasonal component adjusted for the hth period into the future.

Box-Jenkins Methodology

- also known as the **ARIMA (AutoRegressive Integrated Moving Average) modeling approach**,
- is a statistical technique <u>used for time series forecasting and analysis.</u>
- developed by **George Box and Gwilym Jenkins** in the early 1970s.
- Involves three main steps: **identification, estimation, and diagnostic checking**.

1. **Identification:**
   a. <u>Stationarity:</u>
      - Ensure that the time series data is stationary by checking for trends or seasonality.
      - If needed, apply differencing to make the series stationary.

   b. <u>Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF):</u>
      - to identify the order of autoregressive (AR) and moving average (MA) components in the model.
        - ACF : correlation between a variable and its lag,
        - PACF : only the direct effects.

Box-Jenkins Methodology

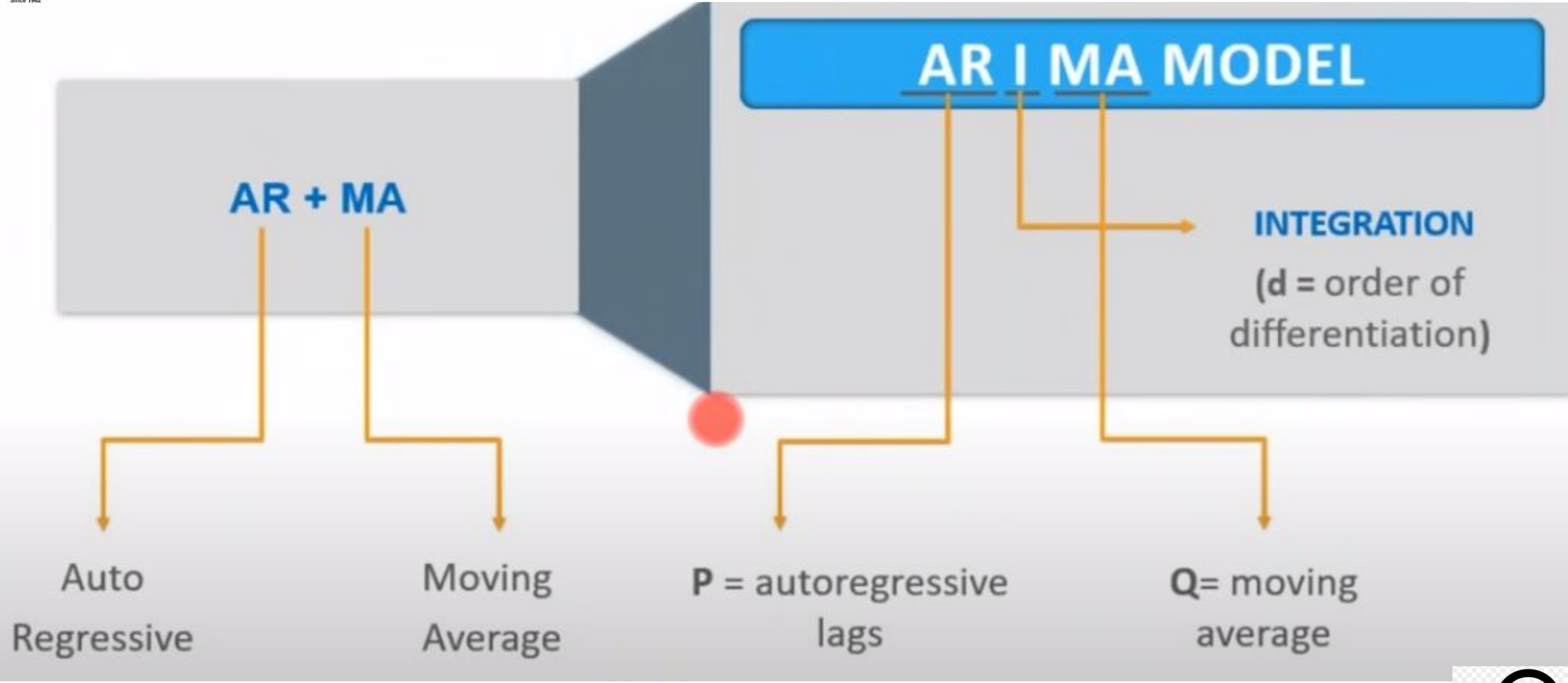
2. **Estimation:**
   a. Fit the ARIMA model:
   - Estimate the parameters of the model using methods like the maximum likelihood estimation (MLE).

   b. Diagnostics:
   - Check the residuals (the differences between predicted and actual values) for randomness and lack of patterns.
   - Use *statistical tests to ensure that the model assumptions are met.*

3. **Diagnostic Checking:**
   a. Residual Analysis:
   - Examine the residuals to ensure that they are white noise, indicating that the model captures the underlying patterns in the data.

   b. Model Adequacy:
   - Evaluate the overall adequacy of the model via Akaike Information Criterion (AIC) or Bayesian Information Criterion (BIC). Lower values indicate a better-fitting model.

ARIMA - Autoregressive Integrated Moving Average Model



AR - Autoregressive Model

For a stationary time series, $y_t, t = 1, 2, 3, \ldots$, an ***autoregressive model of order p***, denoted AR(p), is expressed as shown in Equation 8.5:

$$8.5 \quad y_t = \delta + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \varepsilon_t$$

where

1. δ is a constant for a nonzero-centered time series:
2. ϕ_j is a constant for j = 1, 2, …, p
3. y_{t-j} is the value of the time series at time $t-j$
4. $\phi_p \neq 0$
5. $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ for all t

Thus, a particular point in the time series can be expressed as a linear combination of the prior p values, y_{t-j} for j = 1, 2, …p, of the time series plus a random error term, ε_t. In this definition, the ε_t time series is often called a ***white noise process*** and is used to represent random, independent fluctuations that are part of the time series.

AR - Autoregressive Model

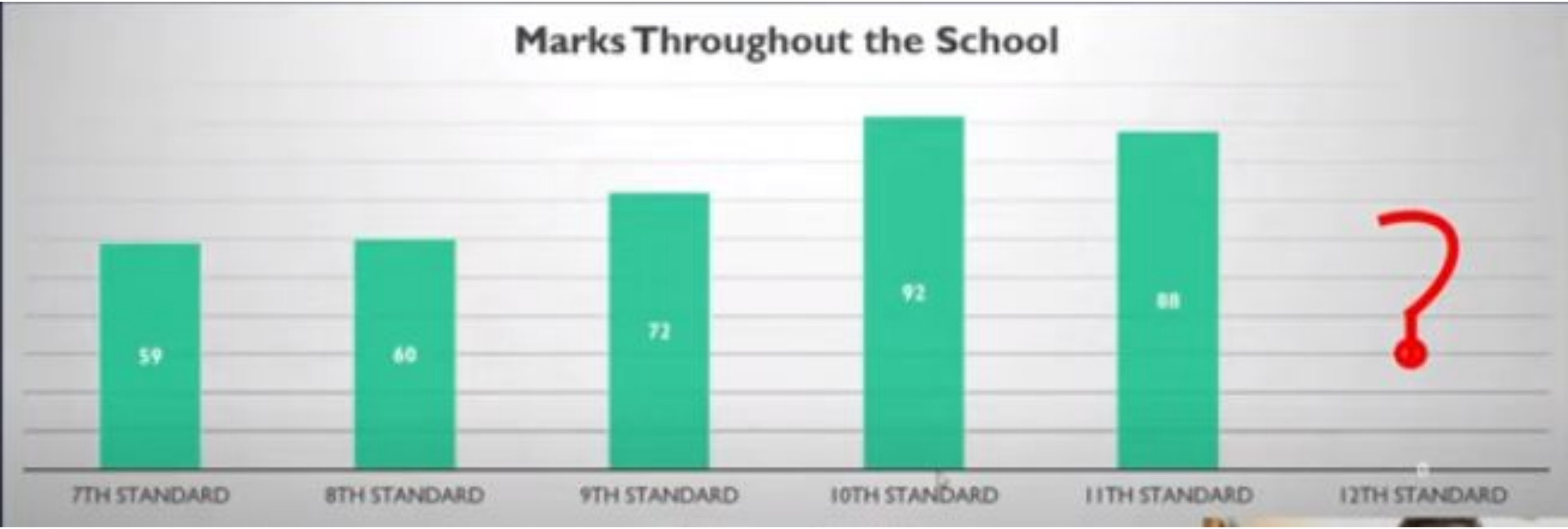
Regression : Used to predict continuous value of an item based on certain parameters

Auto : Uses its own past values to predict future values

$$y_t = C_1 y_{t-1} + C_2 \longrightarrow$$ AR(1) : 1st Order Auto Regression

$$y_t = C_1 + C_2 y_{t-1} + C_3 y_{t-2} \longrightarrow$$ AR(2) : 2nd Order Auto Regression

AR - Autoregressive Model : Example



12^{th} Marks = C1 + C2*(11^{th} Marks) + C3*(10^{th} Marks) +Error

Courtesy : [Investopedia]

AR - Autoregressive Model : ACF Vs PACF Plots

Correlation: An indicator of relationship between two variables

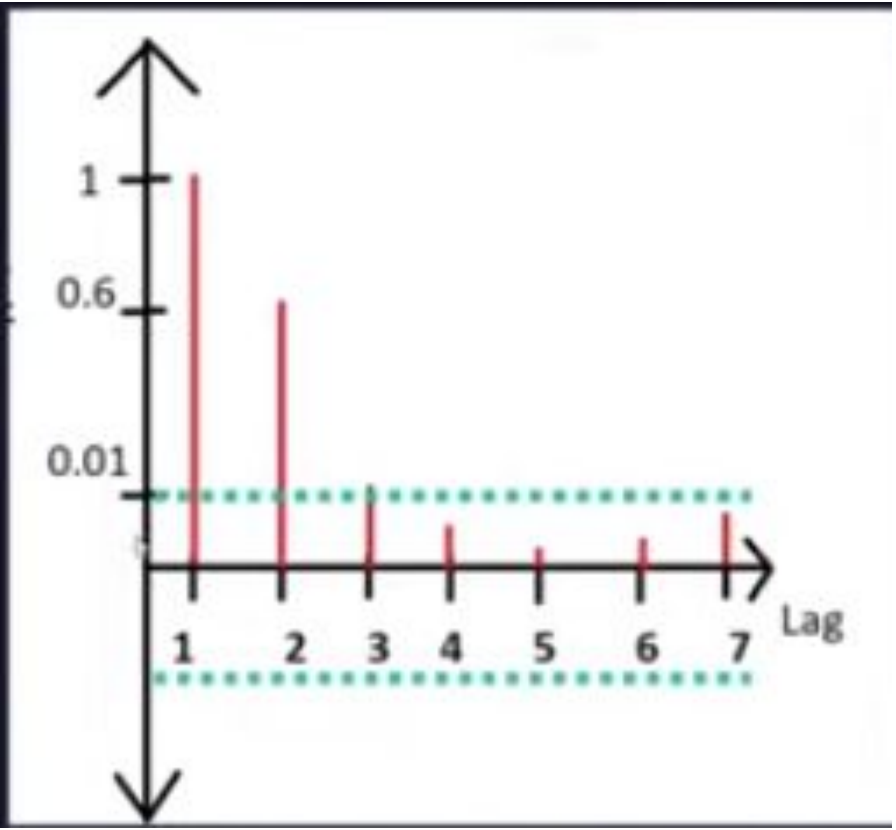
Auto-Correlation: Relationship of a variable with its previous time period values (Lags)

Pearson's Correlation Coefficient: [-1,1]

Auto-Correlation Function → Direct and Indirect effect of Values in Previous time Lags

Partial Auto-Correlation Function → Only Direct effect of Values in Previous time Lags

AR - Autoregressive Model : Sample PACF plot



Second order Correlation - AR(2)

$$y_t = C_1 + C_2 y_{t-1} + C_3 y_{t-2}$$

Courtesy : [Investopedia]

MA - Moving Average Model

For a time series, y_t, centered at zero, a ***moving average model of order q***, denoted MA(q),

$$y_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \dots + \theta_q \varepsilon_{t-q}$$

where

1. θ_k is a constant for k = 1, 2, …, q
2. $\theta_q \neq 0$
3. $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ for all t

In an MA(q) model, the value of a time series is a linear combination of the current white noise term and the prior q white noise terms. So earlier random shocks directly affect the current value of the time series. For MA(q) models, the behavior of the ACF and PACF plots are somewhat swapped from the behavior of these plots for AR(p) models

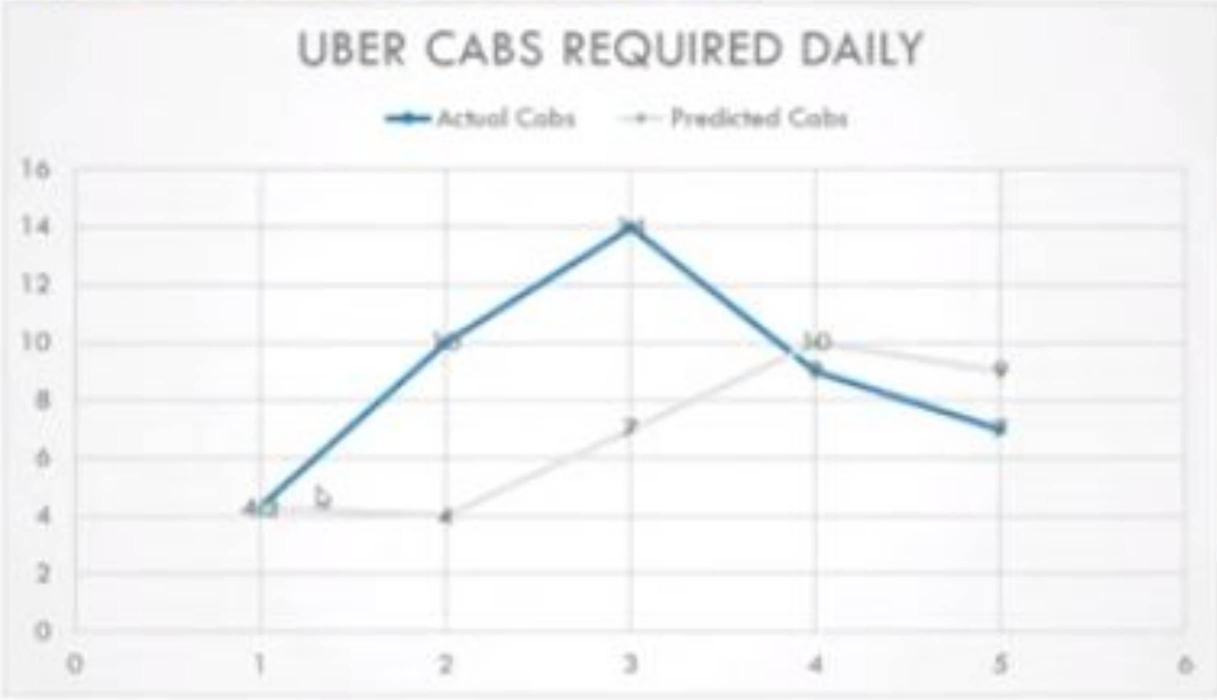
MA - Moving Average Model

Models that predict future values of a time series using the past errors

Auto Regression : $y_t = C_1 y_{t-1} + C_2 + \varepsilon_t \longrightarrow$ AR(1)

Moving Average : $y_t = \mu + C_1 \varepsilon_{t-1} + \varepsilon_t \longrightarrow$ MA(1)

MA - Moving Average Model : Example



C1=-0.5, Mean=4

Day	Prediction	Actual Value	Error
1	-	4	-
2	4	10	-6
3	4+3=7	14	-7
4	4+3.5=7.5	9	-1.5

MA - Moving Average Model

Ma(2) Model : $y_t = u + C_1\varepsilon_{t-1} + C_2\varepsilon_{t-2} + \varepsilon_t$

Ma(q) Model : $y_t = u + C_1\varepsilon_{t-1} + \cdots + C_q\varepsilon_{t-q} + \varepsilon_t$

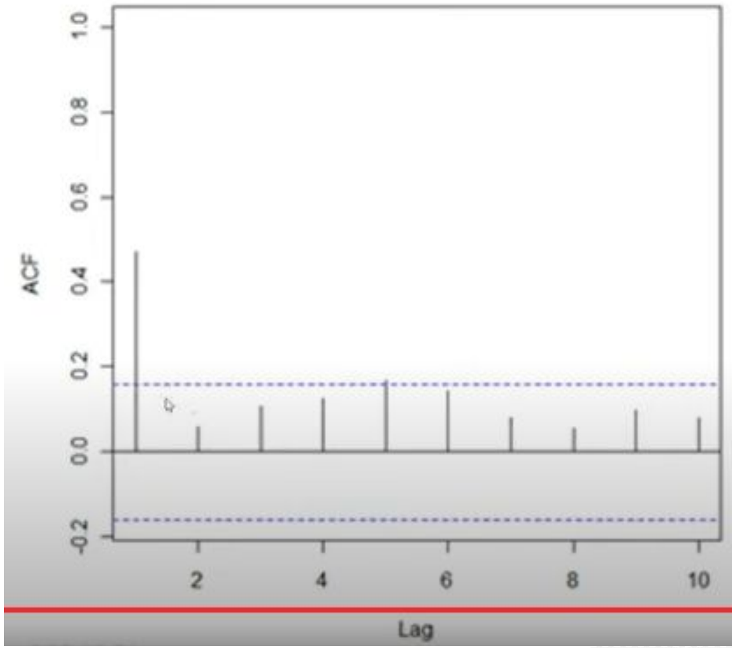
Courtesy : Investopedia

ACF - Autocorrelation Function

Measure the correlation between current time period and previous time lags

Takes into account direct and indirect effect

Between [-1,1] for Pearson Correlation coefficient



ARMA - Autoregressive Moving Average Model

$$y_t = \delta + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p}$$
$$+ \varepsilon_t + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q}$$

1. where δ is a constant for a nonzero-centered time series
2. ϕ_j is a constant for j = 1, 2, …, p
3. $\phi_p \neq 0$
4. θ_k is a constant for k = 1, 2, …, q
5. $\theta_q \neq 0$
6. $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ for all t

If $p = 0 \text{ and } q \neq 0$, then the ARMA(p,q) model is simply an AR(p) model. Similarly, if $p = 0 \text{ and } q \neq 0$, then the ARMA(p,q) model is an MA(q) model.

ARMA - Autoregressive Moving Average Model

- A simple combination of Auto regression and Moving Average model
- Auto Regression : Uses Past Values to make a prediction

$$y_t = \beta_0 + \beta_1 y_{t-1} + \varepsilon_t \longrightarrow \boxed{\text{AR(1)}}$$

- Moving Average : Uses Past errors to make a prediction

$$y_t = \beta_0 + \theta_1 \varepsilon_{t-1} + \varepsilon_t \longrightarrow \boxed{\text{MA(1)}}$$

- ARMA model : $y_t = B_0 + B_1 y_{t-1} + \theta \varepsilon_{t-1} + \varepsilon_t \longrightarrow \boxed{\text{ARMA(1,1)}}$

ARMA - Autoregressive Moving Average Model

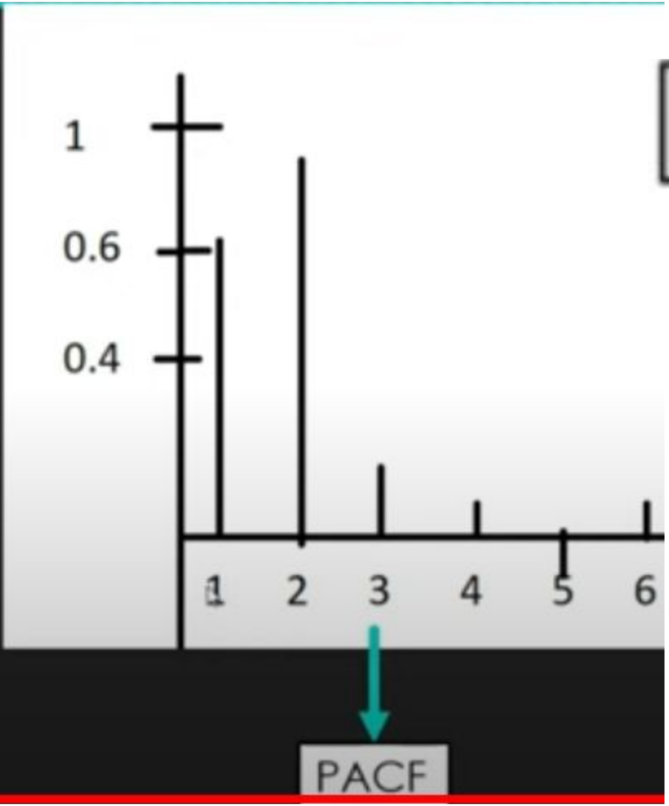
- ACF And PACF plots
- Measure the correlation between current time period and previous time lags
- Auto Correlation Function

  -Measure direct and indirect effect of previous time lags on current value

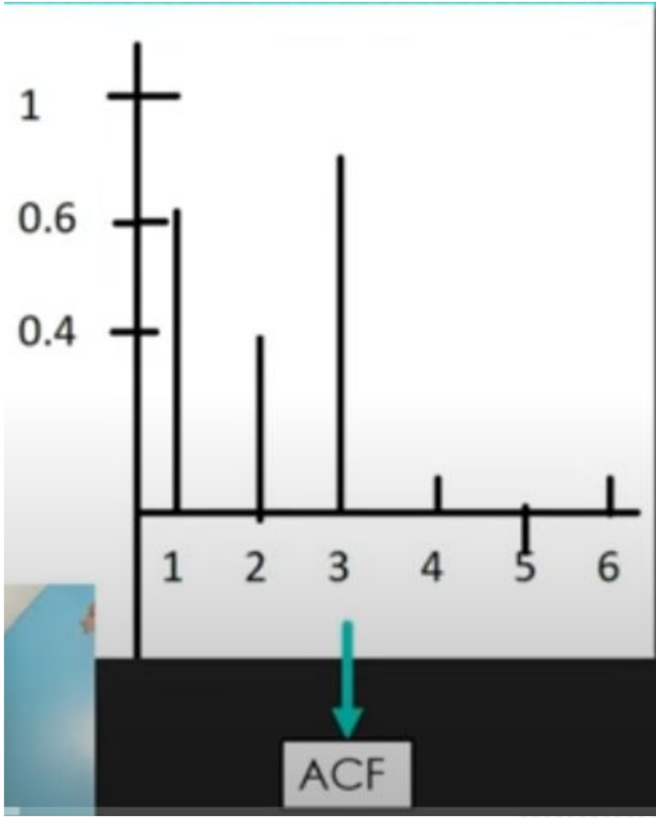
- Partial Auto Correlation Function

  -Measure only direct effect of previous time lags on current value
  -Used to find order of Auto

ARMA - Autoregressive Moving Average Model

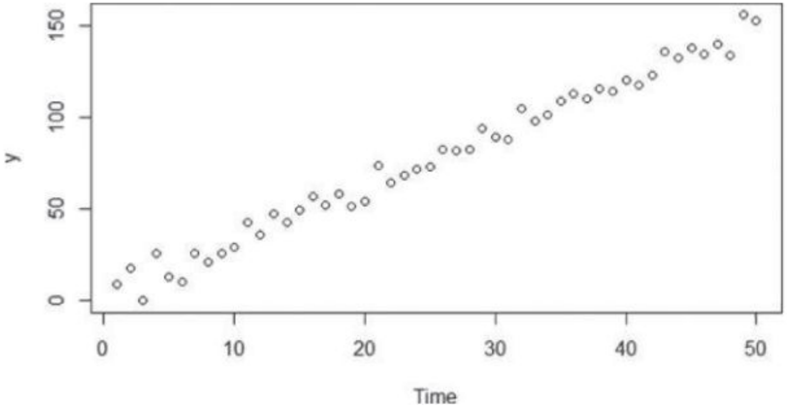


ARMA(2,3)



ARIMA - Autoregressive Integrated Moving Average Model

To apply an ARMA model properly, the time series must be a stationary one. However, many time series exhibit some trend over time. Figure 8.7 illustrates a time series with an increasing linear trend over time. Since such a time series does not meet the requirement of a constant expected value (mean), the data needs to be adjusted to remove the trend. One transformation option is to perform a regression analysis on the time series and then to subtract the value of the fitted regression line from each observed y-value.



ARIMA - Autoregressive Integrated Moving Average Model

If detrending using a linear or higher order regression model does not provide a stationary series, a second option is to compute the difference between successive y-values. This is known as **differencing**.

$$d_t = y_t - y_{t-1} \quad \text{for } t = 2, 3, \ldots, n$$

The mean of the time series plotted in Figure 8.8 is certainly not a constant. Applying differencing to the time series results in the plot in Figure 8.9. This plot illustrates a time series with a constant mean and a fairly constant variance over time.

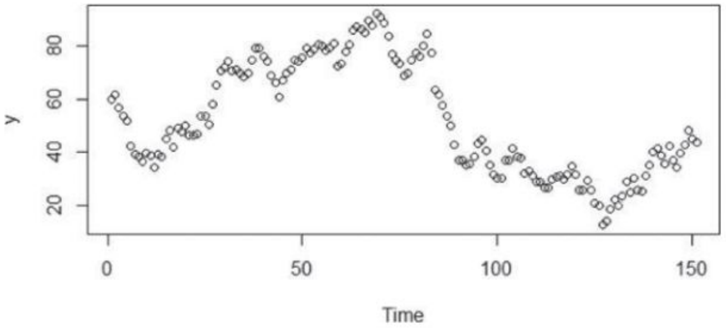


Figure 8.8 Time series for differencing example

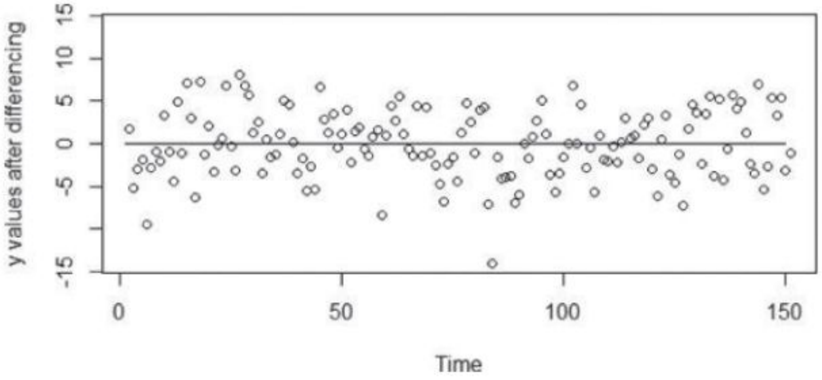


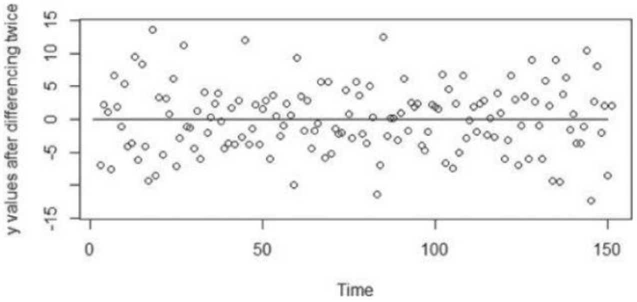
Figure 8.9 Detrended time series using differencing

ARIMA - Autoregressive Integrated Moving Average Model

If the differenced series is not reasonably stationary, applying differencing additional times may help. Equation 8.17 provides the twice differenced time series for t = 3, 4, …n.

$$
\begin{aligned}
d_{t-1} - d_{t-2} &= (y_t - y_{t-1}) - (y_{t-1} - y_{t-2}) \\
&= y_t - 2y_{t-1} + y_{t-2}
\end{aligned}
\tag{8.17}
$$

Successive differencing can be applied, but over-differencing should be avoided. One reason is that over-differencing may unnecessarily increase the variance. The increased variance can be detected by plotting the possibly over-differenced values and observing that the spread of the values is much larger, as seen in Figure 8.10 after differencing the values of y twice.



ARIMA - Autoregressive Integrated Moving Average Model

Because the need to make a time series stationary is common, the differencing can be included (integrated) into the ARMA model definition by defining the Autoregressive Integrated Moving Average model, denoted ARIMA(p,d,q).

appropriately adjusted. An alternative is to use a ***seasonal autoregressive integrated moving average model***, denoted ARIMA(p,d,q) $\times$ $(P,D,Q)_s$ where:

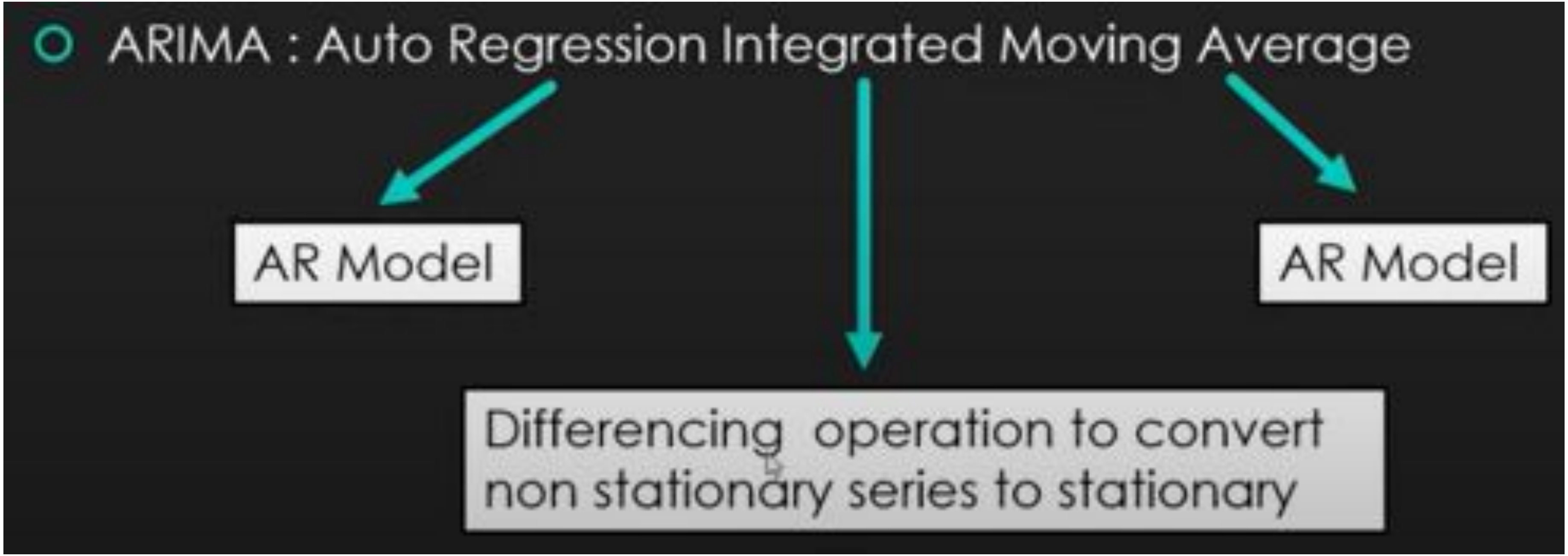
- p, d, and q are the same as defined previously.
- s denotes the seasonal period.
- P is the number of terms in the AR model across the s periods.
- D is the number of differences applied across the s periods.
- Q is the number of terms in the MA model across the s periods.

Typical values of s

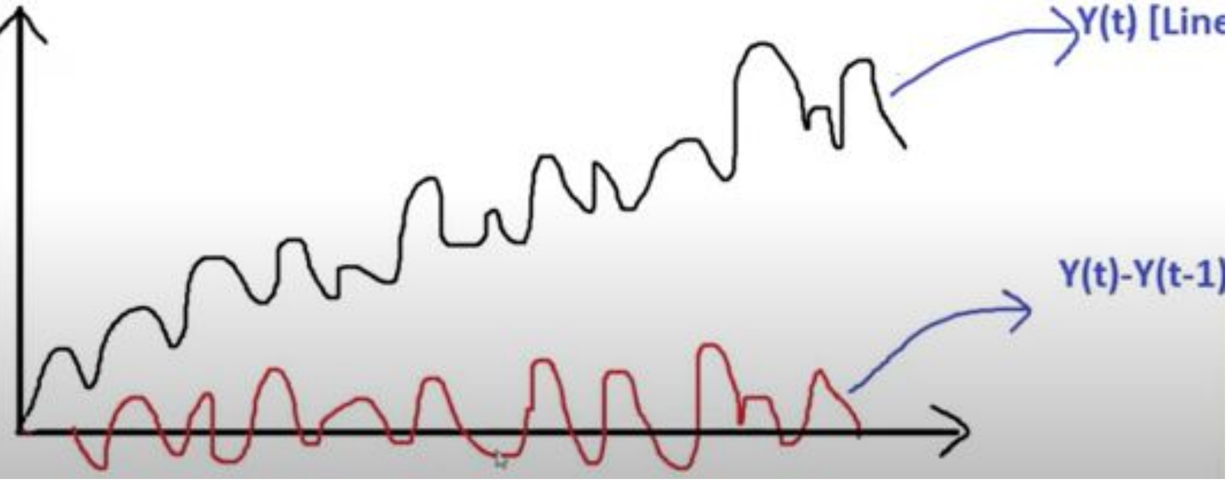
- 52 for weekly data
- 12 for monthly data
- 7 for daily data

ARIMA - Autoregressive Integrated Moving Average Model

- ARIMA is almost ARMA , Just does one simple operation of converting Non stationary series to stationary series before



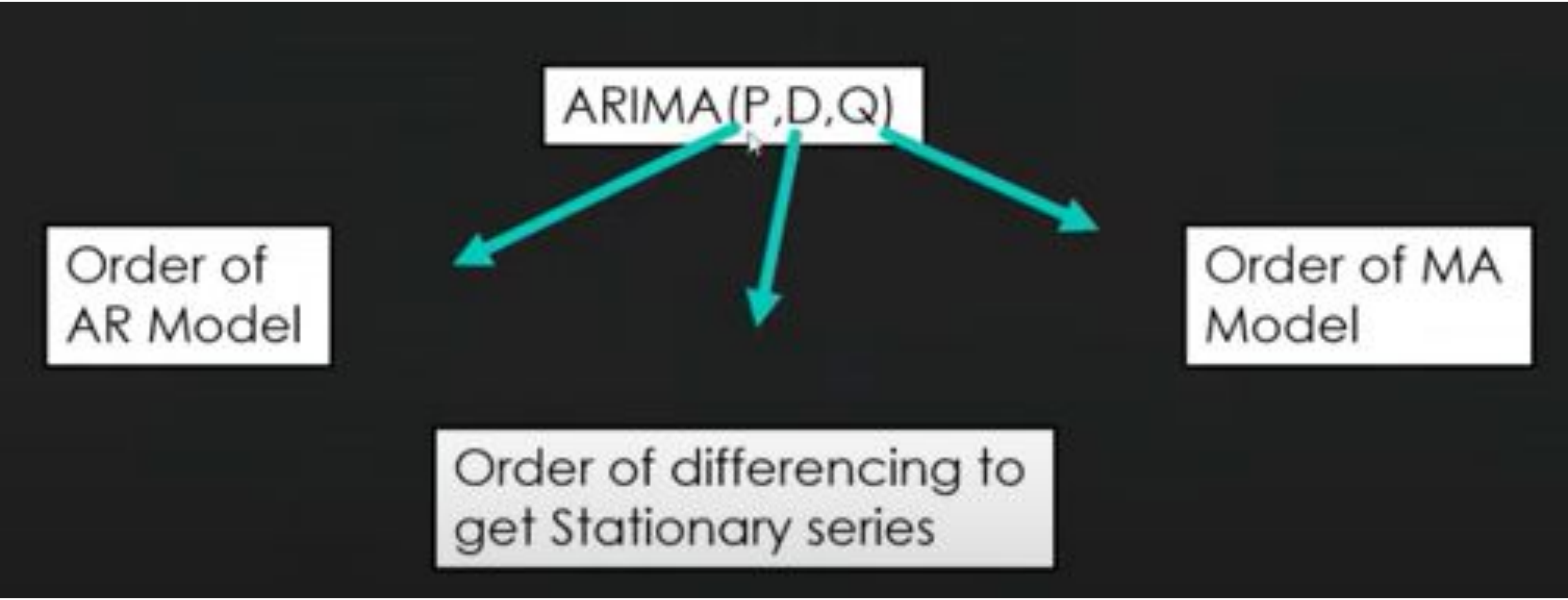
ARIMA - Autoregressive Integrated Moving Average Model



Y(t) : Linear Trend upwards.

- After differentiation, this is removed.
- We are able to make the Time Series a stationary one.

ARIMA - Autoregressive Integrated Moving Average Model



Evaluating Time Series Models

The `arima()` function in R uses Maximum Likelihood Estimation (MLE) to estimate the model coefficients. In the R output for an ARIMA model, the log-likelihood (□) value is provided. The values of the model coefficients are determined such that the value of the log likelihood function is maximized. Based on the □ value, the R output provides several measures that are useful for comparing the appropriateness of one fitted model against another fitted model. These measures follow:

- AIC (Akaike Information Criterion)
- AICc (Akaike Information Criterion, corrected)
- BIC (Bayesian Information Criterion)

Time Series - Problem Statement - Colab Implementation

1 Build a model to forecast the demand (passenger traffic) in Airplanes . The data is classified in date/ time and the passengers travelling per month.

Month	#Passengers
1949-01-01	112
1949-02-01	118
1949-03-01	132
1949-04-01	129
1949-05-01	121
1949-06-01	135

Additional Time Series Methods

- **Autoregressive Moving Average with Exogenous inputs (ARMAX)** is used to analyze a time series that is dependent on another time series. For example, retail demand for products can be modeled based on the previous demand combined with a weather-related time series such as temperature or rainfall.
- **Spectral analysis** is commonly used for signal processing and other engineering applications. Speech recognition software uses such techniques to separate the signal for the spoken words from the overall signal that may include some noise.
- **Generalized Autoregressive Conditionally Heteroscedastic (GARCH)** is a useful model for addressing time series with nonconstant variance or volatility. GARCH is used for modeling stock market activity and price fluctuations.

Additional Time Series Methods

- **Kalman filtering** is useful for analyzing real-time inputs about a system that can exist in certain states. Typically, there is an underlying model of how the various components of the system interact and affect each other. A Kalman filter processes the various inputs, attempts to identify the errors in the input, and predicts the current state. For example, a Kalman filter in a vehicle navigation system can process various inputs, such as speed and direction, and update the estimate of the current location.
- **Multivariate time series analysis** examines multiple time series and their effect on each other. Vector ARIMA (VARIMA) extends ARIMA by considering a vector of several time series at a particular time, t. VARIMA can be used in marketing analyses that examine the time series related to a company's price and sales volume as well as related time series for the competitors.