



# **Computer Engineering - Sem IV**

# NCMPC 41: Design and Analysis of Algorithms

Module - 3 : String Matching Algorithms (03 Hours)

Instructor: Mrs. Lifna C S



#### Topics to be covered

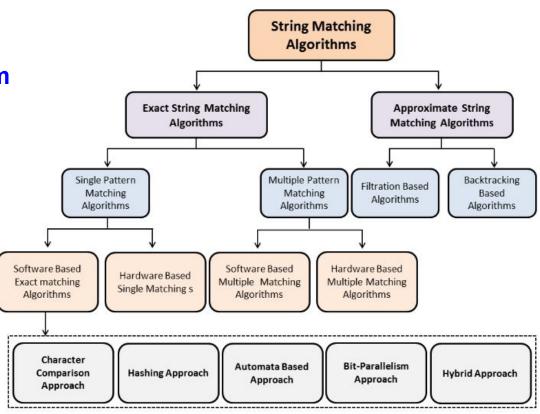


#### String Matching Algorithms

- Naïve string-matching algorithm
- Rabin Karp algorithm
- Knuth-Morris-Pratt algorithm

#### Extra Topics

- Finite State Automata
- Applications of String Matching





## Naïve string-matching algorithm

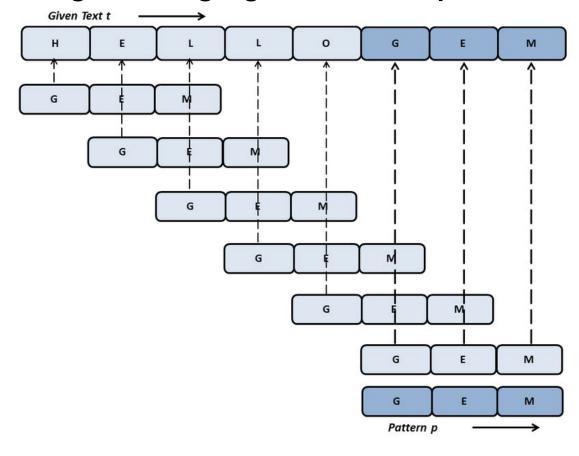


- Simplest approach for searching a pattern within a given text.
- **Logic**: Checking for the pattern at every possible position in the text, one by one.
- Algorithm :
  - Start with the first character of the text.
  - Compare it with the first character of the pattern.
  - If it matches, compare the next character, and continue checking until:
    - The entire pattern matches (successful match).
    - A mismatch occurs (shift to the next position in the text).
  - 4. Repeat the process for all possible positions in the text until the end.



## Naïve string-matching algorithm - Example







#### Naïve string-matching algorithm - Example



#### Given:

- Text: "ABABABCABABABCABABABC"
- Pattern: "ABABC"

#### Steps:

- Compare "ABABC" with the first 5 characters of the text: "ABABA" → mismatch at index 4.
- Shift by 1 and compare "ABABC" with "BABAB" → mismatch.
- 3. Continue shifting until a full match is found.
- 4. Match found at index 2, 9, and 16.



#### Naïve string-matching algorithm - Analysis



- Best Case: O(n) → When a mismatch occurs early in comparisons.
- Worst Case: O(n m) → When the pattern and text have many repeated characters, causing unnecessary comparisons.

#### Where:

- n = length of the Text
- m = length of the Pattern

#### Advantages & Disadvantages

- Simple to implement
- No preprocessing required
- X Slow for large texts and patterns
- X Inefficient for repeated substrings



### Topics to be covered

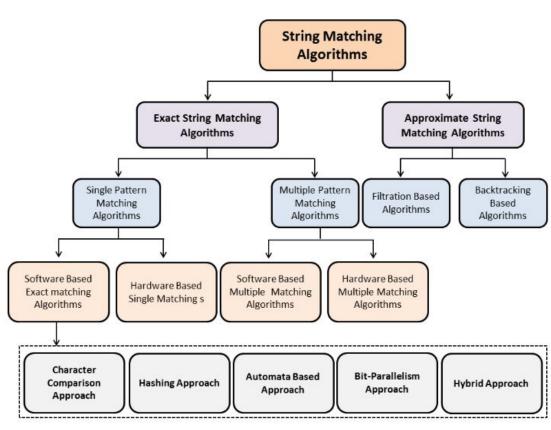


#### **String Matching Algorithms**

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## **Rabin Karp algorithm**



- string-searching algorithm that uses **hashing** to find a pattern in a given text efficiently.
- Calculates the hash value of the pattern and compares it with the hash values of substrings in the text.
- If a match is found ⇒ perform a character-by-character check to confirm the match.

#### **Steps of the Algorithm**

- 1. Compute the hash value of the pattern.
- 2. Compute the hash value of the first substring (of the same length as the pattern) in the text.
- 3. Compare the hash values:
  - If they match, perform a character-by-character comparison to confirm.
  - If they do not match, slide the window one position to the right and compute the new hash value efficiently.
- 4. Repeat the process until the end of the text.





- Given T = 31415926535 and P = 26
- We choose q = 11
- P mod q = 26 mod 11 = 4





As we can see, when a match is found, further testing is done to insure that a match has indeed been found.





#### Step 1: Define Parameters

- Pattern: "cdd"
- Text: "abccddaefg"
- Base (Prime Number): 101
- Modulus (to prevent overflow): 10^9 + 7 (optional for large numbers)

We use the hash function:

$$\operatorname{hash}(s) = \sum (\operatorname{ASCII} \text{ value of character} \times \operatorname{base}^{\operatorname{position}})$$

Rolling hash update formula:

new hash = (old hash - outgoing char 
$$\times$$
 base<sup>m-1</sup>)  $\times$  base + incoming char

where m = 3 (length of "cdd").





#### Step 2: Compute Initial Hash of the Pattern (cdd)

Using ASCII values:

Pattern: "cdd"

Text: "abccddaefg"

$$\begin{aligned} \text{hash}("cdd") &= (99 \times 101^2) + (100 \times 101^1) + (100 \times 101^0) \\ &= (99 \times 10201) + (100 \times 101) + (100) \\ &= 1009899 + 10100 + 100 = \textbf{1020099} \end{aligned}$$





#### Step 3: Compute Initial Hash of First Window (abc)

Using ASCII values:

- 'a' = 97
- 'b' = 98
- 'c' = 99

$$\begin{aligned} \text{hash("}abc") &= (97 \times 101^2) + (98 \times 101^1) + (99 \times 101^0) \\ &= (97 \times 10201) + (98 \times 101) + (99) \\ &= 989497 + 9898 + 99 = \mathbf{999494} \end{aligned}$$

Since 999494 ≠ 1020099 , slide the window.

Pattern: "cdd"

Text: "abccddaefg"







## Step 4: Rolling Hash Calculation

Text: "abccddaefg"

Using:

new hash = (old hash - outgoing char  $\times$  101<sup>m-1</sup>)  $\times$  101 + incoming char

Slide Window: "abc" → "bcc"

- Outgoing char: 'a' = 97
- Incoming char: 'c' = 99

$$\begin{aligned} \text{hash("}bcc") &= (999494 - (97 \times 10201)) \times 101 + 99 \\ &= (999494 - 989497) \times 101 + 99 \\ &= 9997 \times 101 + 99 = \textbf{1000696} \end{aligned}$$

No match, continue sliding.





Pattern: "cdd"

Text: "abccddaefg"

Slide Window: "bcc" → "ccd"

- Outgoing char: 'b' = 98
- Incoming char: 'd' = 100

$$hash("ccd") = (1000696 - (98 \times 10201)) \times 101 + 100$$
  
=  $(1000696 - 999798) \times 101 + 100$   
=  $898 \times 101 + 100 = 99898$ 

No match, continue sliding.





Slide Window: "ccd" → "cdd"

- Outgoing char: 'c' = 99
- Incoming char: 'd' = 100

Pattern: "cdd"

Text: "abccddaefg"

$$hash("cdd") = (99898 - (99 \times 10201)) \times 101 + 100$$
  
=  $(99898 - 1009899) \times 101 + 100$   
=  $1020099$ 

Match found at index 3! Perform a character-by-character check to confirm.





#### Window "dda"

- Outgoing char: 'c' (ASCII 99)
- Incoming char: 'a' (ASCII 97)

Pattern: "cdd"

Text: "abccddaefg"

new hash = 
$$(1020099 - (99 \times 10201)) \times 101 + 97$$
  
=  $(1020099 - 1009899) \times 101 + 97$   
=  $(10200) \times 101 + 97 = 1030197$ 

No match.





## Window "dae"

- Outgoing char: 'd' (ASCII 100)
- Incoming char: 'e' (ASCII 101)

new hash = 
$$(1030197 - (100 \times 10201)) \times 101 + 101$$
  
=  $(1030197 - 1020100) \times 101 + 101$   
=  $(10097) \times 101 + 101 = 1020298$ 

Pattern: "cdd"

Text: "abccddaefg"

No match.



Pattern: "cdd"



Window "aef"

Text: "abccddaefg"

- Outgoing char: 'd' (ASCII 100)
- Incoming char: 'f' (ASCII 102)

new hash = 
$$(1020298 - (100 \times 10201)) \times 101 + 102$$
  
=  $(1020298 - 1020100) \times 101 + 102$   
=  $(198) \times 101 + 102 = 20000$ 

No match.



Pattern: "cdd"

Text: "abccddaefg"



Window "efg"

- Outgoing char: 'a' (ASCII 97)
- Incoming char: 'g' (ASCII 103)

new hash = 
$$(20000 - (97 \times 10201)) \times 101 + 103$$
  
=  $(20000 - 989497) \times 101 + 103$   
=  $(-969497) \times 101 + 103$ 

Negative hash, adjusted using modulus (not needed here).

No match.

We found "cdd" at index 3 in "abccddaefg".



#### Rabin Karp algorithm - Analysis



- Best Case: O(n+m) (when no hash collisions occur)
- Worst Case: O(nm) (when hash collisions frequently occur, requiring full character comparison)
- Average Case: O(n+m)

Here, n = is the length of the text,

m = length of the pattern.

#### - Rolling hash function

- ensures that computing the next substring's hash is done in constant time, making Rabin-Karp efficient when searching multiple patterns.

#### **Applications of Rabin Karp**

- Efficient for multiple pattern searches (e.g., plagiarism detection).
- Best for cases where <u>hash collisions are minimal</u> (e.g., large alphabets with a good hash function).
- Less effective for single pattern matching if hash collisions are frequent.



## Rabin Karp algorithm - Problems



<u>Text</u>

<u>Pattern</u>

1. A B C C D D A E F G

C D D

2. ABCDEF

**CAB** 

**AABC** 

CDE

**DAACABCDBA** 

4. AAAABCAEAAABCBDDAAAABC

Courtesy : Programiz



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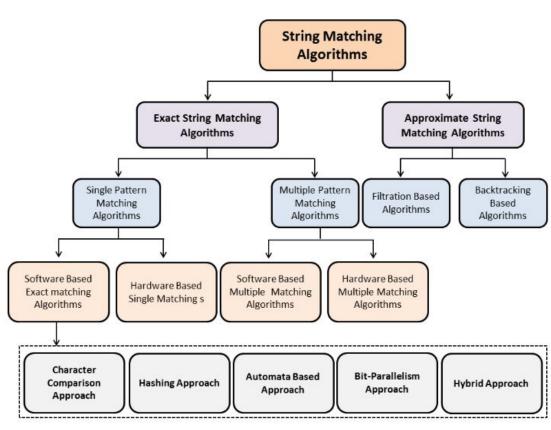


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Text: A A B A A C A A D A A B A A B A

Pattern:

A A B A A C A A D A A B A A B A

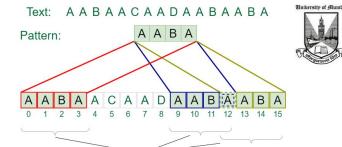
Pattern found at index 0, 9, 12

- Efficient string-searching (substring search) algorithm
- Finds occurrences of a pattern P within a text in O(n + m) time,
   where:
  - n = length of the text
  - m = length of the pattern
- KMP avoids unnecessary comparisons by using information gathered in a preprocessing step.

#### **Key Advantages of KMP**

- Avoids redundant comparisons using LPS (Longest Prefix Suffix)
- Works <u>efficiently for large texts and patterns</u>.
- Best suited for <u>cases where multiple searches with the same pattern</u> are needed.





Pattern found at index 0, 9, 12

KMP consists of two main steps:

- 1. Preprocessing the pattern (Computing the LPS array)
- Helps us skip unnecessary comparisons in the search phase.
- LPS[i] stores, the <u>length of the longest proper prefix</u> of the pattern = <u>Suffix of the pattern up to index i.</u>
- If a mismatch occurs, the LPS array tells us how much the pattern can be shifted while preserving already matched characters.

#### 2. Searching the pattern in the text using LPS

- Compare characters of P with T one by one.
- If a <u>mismatch occurs</u> ⇒ <u>use the LPS array to determine the next position.</u>
- This ensures that we do not compare already matched characters again.



AABAACAADAABAABA AABA

LPS is the **Longest Proper Prefix** which is also a **Suffix**.

- A proper prefix is a prefix that doesn't include **whole** string.
  - Eq: Given String "abc"
  - prefixes: "", "a", "ab" and "abc"
  - proper prefixes: "", "a" and "ab" only.
  - Suffixes: "", "c", "bc", and "abc".
- Each value, **lps[i]** = length of longest proper prefix of **pat[0..i]** = a suffix of **pat[0..i]**.



#### Knuth Morris Pratt algorithm - Algorithm for Constructing the LPS Array



Given a pattern pat [0...m-1], the LPS array is constructed as follows:

- Initialize variables:
  - lps[0] = 0 (First character has no proper prefix which is also a suffix)
  - length = 0 (Tracks length of the previous longest prefix suffix)
  - i = 1 (Start from second character)
- Iterate through the pattern while i < m:</li>
  - o If pat[i] == pat[length]: // Case 1 : Match found
    - Increment length (length++)
    - Assign lps[i] = length // Extend the LPS at the previous index
    - Move to the next index (i++)
  - o Else (pat[i] != pat[length]):
    - - Assign lps[i] = 0
         // Also, the current characters are also not matching ⇒ lps[i] = 0
      - Move to the next index (i++)
    - Else // Case 3 : There may be a smaller prefix that matches the suffix ending at i.
      - Update length = lps[length 1] // Go back to previous prefix suffix



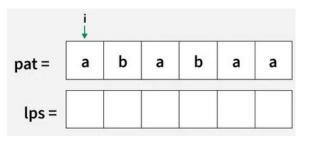
## Knuth Morris Pratt algorithm - Example for computing LPS Array



Initialize : len = 0

$$i = 0$$

lps[0] = 0 //No proper prefix for single char
i++



	len i ↓↓					
pat =	а	b	а	b	а	а
lps =	0					

	len ↓	i				
pat =	а	b	а	b	а	а
lps =	0	0				

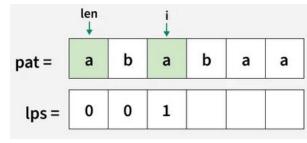
len	i
0	0
0	1
0	2

Courtesy : <u>Geeks For Geeks</u>

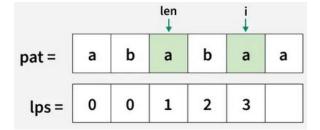


### Knuth Morris Pratt algorithm - Example for computing LPS Array





-		len ↓		ļ		
pat =	а	b	а	b	а	а
lps =	0	0	1	2		



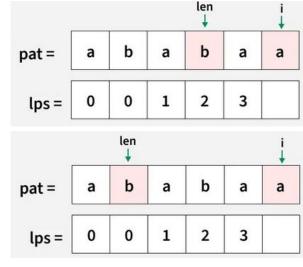
len	i
0	2
1	3
2	4
3	5

Courtesy : Geeks For Geeks



#### Knuth Morris Pratt algorithm - Example for computing LPS Array





	len ↓					i ↓
pat =	а	b	a	b	a	а
lps =	0	0	1	2	3	1

len	i
3	5
1	5
0	5
1	6

Courtesy : Geeks For Geeks

len++

j++

lps[i] = len



## **Knuth Morris Pratt algorithm - Practice**



Pattern	LPS []
AAAA	[0, 1, 2, 3]
ABCDE	[0, 0, 0, 0, 0]
AABAACAABAA	[0, 1, 0, 1, 2, 0, 1, 2, 3, 4, 5]
AAACAAAAAC	[0, 1, 2, 0, 1, 2, 3, 3, 3, 4]
AAABAAA	[0, 1, 2, 0, 1, 2, 3]
ABABCABAB	[0, 0, 1, 2, 0, 1, 2, 3, 2]



## Knuth Morris Pratt algorithm - Practice

Text (T): "ABABDABACDABABCABAB"

Embersity of Alumbai

Pattern (P): "ABABCABAB"

#### Step 1: Compute the LPS Array

Index i	Pattern Prefix	LPS Value
0	Α	0
1	AB	0
2	ABA	1
3	ABAB	2
4	ABABC	0
5	ABABCA	1
6	ABABCAB	2
7	ABABCABA	3
8	ABABCABAB	4

So, LPS = [0, 0, 1, 2, 0, 1, 2, 3, 4]





#### Step 2: Pattern Searching in Text using LPS

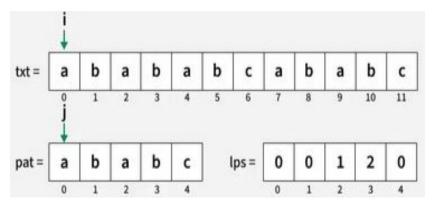
We align **P** with **T** and start matching characters:

- If characters match, move both i (for text) and j (for pattern) forward.
- If a mismatch occurs:
  - If j = 0, move i forward.
  - Otherwise, use LPS[j-1] to shift j and continue.

After applying this, the pattern "ABABCABAB" is found at index 10 in the text.







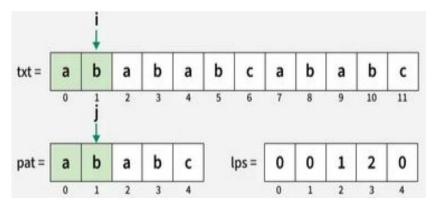
i	j
0	0
1	1
1	1

	+											
txt =	a	b	a	b	а	b	c	а	b	a	b	c
	ĵ	1	2	3	4	5	6	7	8	9	10	11
1	+					1	1					
17.00	_	b	a	b	С	1	ps=	0	0	1	2	0
pat =	a		u						-	_	_	-

Courtesy : <u>Geeks For Geeks</u>







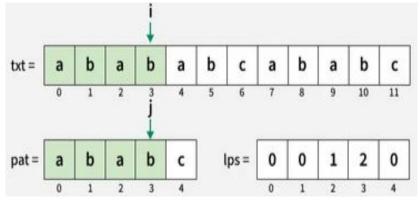
i	j
1	1
2	2
3	3

			i							C		
txt =	a	b	a	b	a	b	с	a	b	a	b	c
l	0	1	j	3	4	5	6	7	8	9	10	11
pat =	a	b	a	b	с		lps=	0	0	1	2	0
	0	1	2	3	4			0	1	2	3	4

Courtesy: Geeks For Geeks







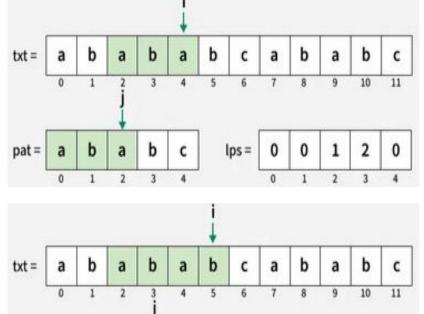
i	j
3	3
4	4
4	2

txt[i] != pat[j] j = lps[j-1] = lps[3] j = 2

					ļ			a :				
txt =	a	b	a	b	a	b	c	a	b	a	b	С
·	0	1	2	3	<sup>4</sup> j	5	6	7	8	9	10	11
pat =	a	b	a	b	С	lps =		0	0	1	2	0
	0	1	2	3	4			0	1	2	3	4







lps =

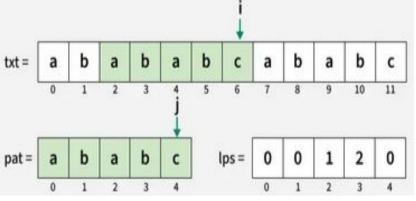
i	j
4	2
5	3
6	4

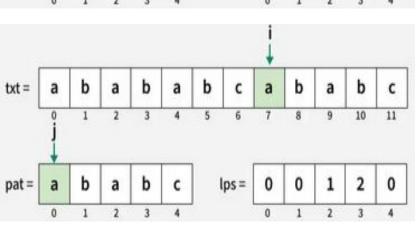
pat =

b









NCMPC41: DAA (2024-25)

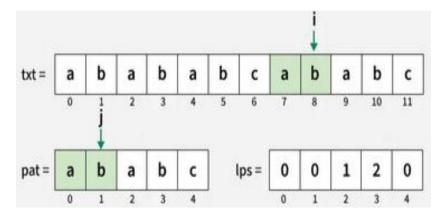
i	j
6	4
7	5
7	0

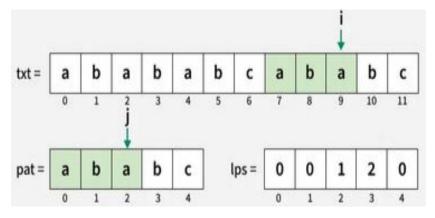
The Complete pattern is matched.

- 1. Update result[] with starting index
  - a. Starting Index (i-j) = (7-5) = 2
  - b. Result =[2]
- 2. Update j = lps[j-1] = lps[5-1] = lps[4] = 0







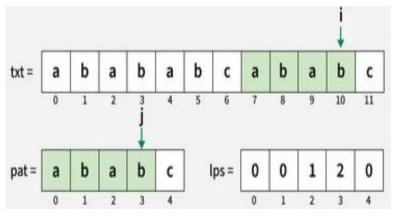


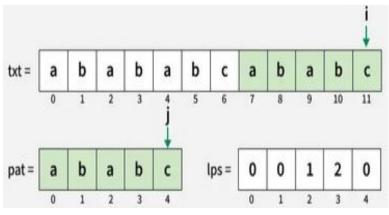
i	j
7	0
8	1
9	2
10	3

txt[i]	==	pat[j]
j++		
j++		









i	j		
10	3		
11	4		
12	5		
12	0		

The Complete pattern is matched.

- 1. Update result[] with starting index
  - a. Starting Index (i-j) = (12-5) = 7
  - b. Result =[2,7]
- 2. Update j = lps[j-1] = lps[5-1] = lps[4] = 0





### Preprocessing LPS Array:

- Each character of the pattern is processed once.
- Time Complexity: O(m)

### 2. Searching Phase:

- Each character of the text is processed at most once.
- Time Complexity: O(n)

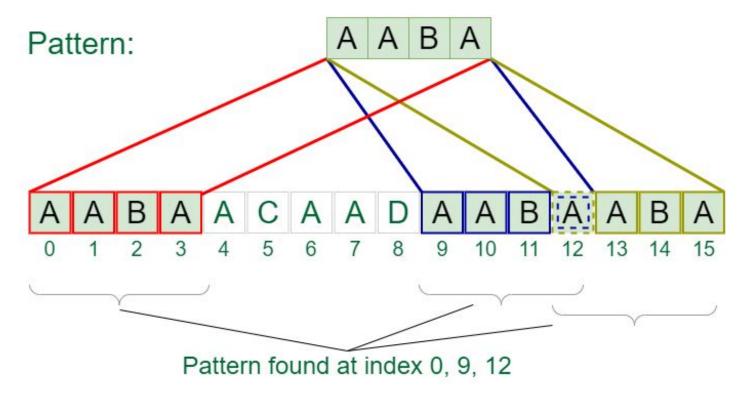
Thus, the total time complexity of KMP is O(n+m).



## **Knuth Morris Pratt algorithm - Example**



Text: AABAACAADAABAABA





## Topics to be covered

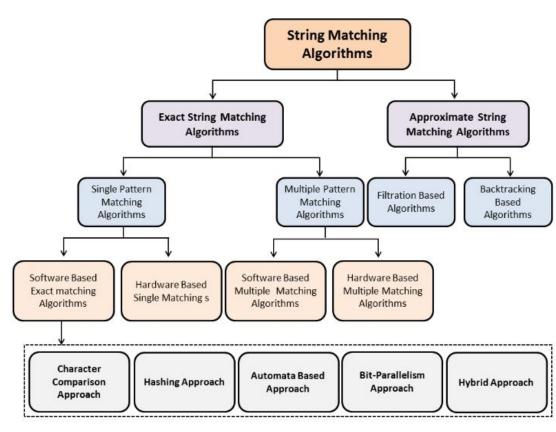


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- Naïve string-matching algorithm
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- Applications of String Matching





### **Finite State Automata**



- efficient pattern matching
- Involves two phases:
  - 1. **Preprocessing Phase**: Construct a Finite State Automaton (FSA) for the given pattern.
  - 2. **Searching Phase**: Use the FSA to scan the text in a single pass.
- pattern is recognized with O(m) preprocessing time and O(n) search time.
- Components of finite automata include:
  - <u>States</u>: represent different configurations or situations of the system being modeled
  - <u>Transitions</u> define how the automaton moves from one state to another in response to input symbols.
  - Start State: state from which the automaton begins its operation.
  - Accepting (or Final) States: These states indicate successful or valid outcomes.



### **Finite State Automata - Definition**



• Defined by tuple  $M = \{\Sigma, Q, q_0, F, \delta\}$ , where

Q = Set of states in finite automata

 $\Sigma$  = Set of input symbols

 $q_0 = Initial state$ 

F = Set of final states

 $\delta$  = Transition function defined as  $\delta$ : Q x  $\Sigma \rightarrow$  Q

- Working of Finite State Automata
  - Starts with input state q<sub>0</sub>
  - Reads the input string character by character and changes the state according to transition function.
  - It accepts the string ⇒ if a finite automaton ends up in one of the final / accepting states.
  - It rejects the string ⇒ if a finite automaton does not end up in final state.



# **Finite State Automata - Example**

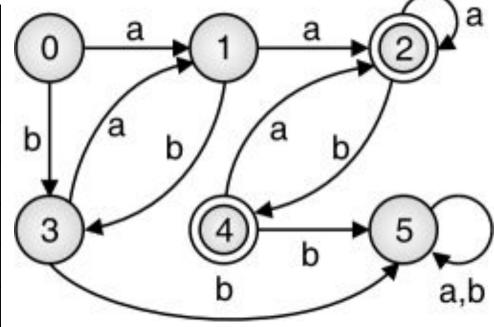


### • Example :

 $\Sigma = \{a, b\},\$ 

Q = 
$$\{0, 1, 2, 3, 4, 5\},\$$
  
 $q_0 = 0,\$   
F =  $\{2, 3\},\$ 

Transition function, δ					
Input State a b					
0	1	3			
1	2	3			
2	2	4			
3	1	5			
4	2	5			
5	5	5			





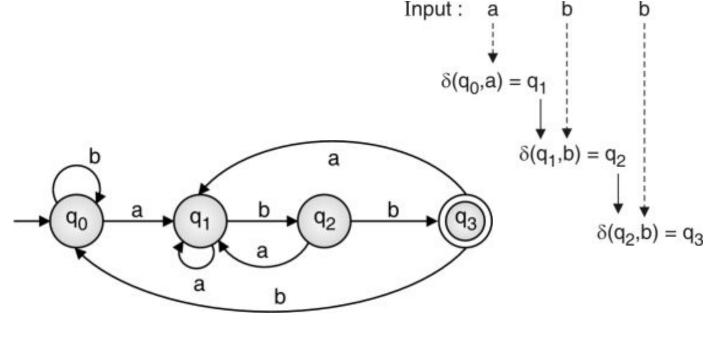
### **Finite State Automata - Practice**



Construct and show simulation of finite automata for matching the pattern **P** = **abb** for **text T** = **ababbaababba** 

Q = 
$$\{q_0, q_1, q_2, q_3\}$$
,  
 $q_0 = q_0$ ,  
F =  $\{q_3\}$ ,  
 $\Sigma = \{a, b\}$ ,

Transition function, δ		
Input State	а	b
$q_0$	q <sub>1</sub>	$q_0$
$q_1$	q <sub>1</sub>	$q_2$
$q_2$	$q_1$	$q_3$
$q_3$	$q_1$	$q_0$

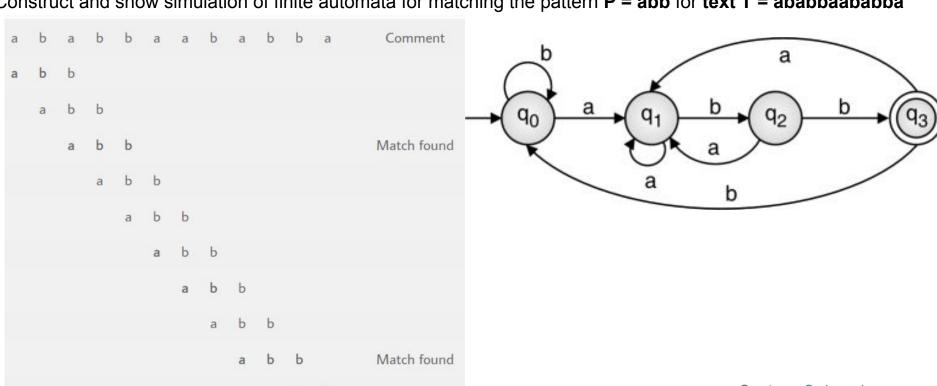




### **Finite State Automata - Practice**



### Construct and show simulation of finite automata for matching the pattern **P** = **abb** for **text T** = **ababbaababba**



Courtesy: Codecrucks.com



# **Finite State Automata - Complexity Analysis**



Phase	Time Complexity		
Preprocessing (Building FSA)	$O(m \cdot \Sigma)$		
Searching (Scanning text)	O(n)		

#### Where:

- m is the length of the pattern.
- n is the length of the text.
- $\Sigma$  is the alphabet size.

Since searching is **O(n)** and preprocessing is **O(m)**, this makes the FSA-based string matching very efficient for repeated searches on large text databases.



### Topics to be covered

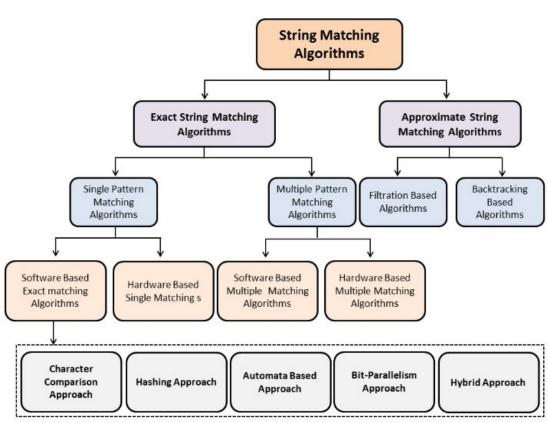


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#### 1. Text Processing & Search Engines

- **Keyword Search in Documents** (e.g., <u>searching for a word</u> in Microsoft Word, Google Docs).
- Plagiarism Detection Algorithms compare documents to <u>detect copied text.</u>
- Auto-Suggestions in Search Engines Google, Bing, and DuckDuckGo (suggest relevant queries).
- Spell Checkers & Auto-Correct Checks words against a dictionary using approximate string matching.
- Ex:
  - The **KMP Algorithm** is used in <u>grep</u>, a command-line tool for searching text.

#### 2. Bioinformatics & DNA Sequence Analysis

- DNA & Protein Sequence Matching identifying genetic patterns, mutations, or evolutionary relationships.
- **Gene Prediction** Finds specific gene sequences in large DNA databases.
- **Drug Discovery** <u>Identifies protein structures</u> for new medicines.
- Ex:
  - o Aho-Corasick algorithm is used to match multiple DNA sequences at once.





#### 3. Cybersecurity & Intrusion Detection

- Malware & Virus Detection Antivirus software scans files for known virus signatures.
- Intrusion Detection Systems (IDS) Identifies suspicious network patterns to detect cyber attacks.
- **Spam Filtering** Email services use string matching to detect spam messages.
- Ex:
  - o **Boyer-Moore Algorithm** is used in <u>intrusion detection</u> for efficient pattern searching.

#### 4. Natural Language Processing (NLP) & Chatbots

- Named Entity Recognition (NER) Identifies entities like <u>names</u>, <u>locations</u>, and <u>organizations</u> in text.
- Sentiment Analysis Detects <u>keywords related to emotions</u> in user reviews.
- Chatbots & Al Assistants Recognizes predefined commands or responses in chat applications.
- Ex:
  - Trie-based Aho-Corasick Algorithm is used in fast text classification.





#### 5. Data Compression & Encoding

- File Compression (e.g., ZIP, GZIP) Lempel-Ziv (LZ) algorithms use pattern matching to reduce file size.
- Data Deduplication <u>Identifies and removes duplicate data</u> in storage systems.
- Text Prediction & Autocomplete Used in mobile keyboards like SwiftKey and Gboard.
- Ex:
  - Lempel-Ziv-Welch (LZW) Compression is widely used in GIF image compression.

#### 6. E-Commerce & Recommendation Systems

- Product Search & Filters Online stores like Amazon use string matching to recommend products.
- Price Comparison Websites Compare product names and specifications across different sellers.
- Review Analysis <u>Identifies patterns in user reviews</u> for sentiment analysis.
- Ex:
  - Fuzzy Matching Algorithms help in typo correction in e-commerce search.





#### 7. Digital Forensics & Legal Applications

- **Identifying Fake Documents** <u>Detects manipulated or plagiarized</u> documents.
- Forensic Text Analysis Examines emails, messages, and legal documents for fraud detection.
- Log File Analysis <u>Detects unusual patterns in system logs</u>.
- Ex:
  - KMP Algorithm is used to efficiently search through large forensic data logs.

#### 8. Database & Big Data Search

- Indexing & Query Optimization String matching helps optimize database searches.
- Pattern-Based Data Extraction Used in log analysis, error detection, and report generation.
- Big Data Processing (e.g., Hadoop, Elasticsearch) Efficiently searches large datasets.
- Ex:
  - o Suffix Trees & Suffix Arrays are used in high-speed database indexing.





#### 9. Speech & Handwriting Recognition

- Speech-to-Text Systems <u>Identifies phonetic patterns in speech and converts them to text.</u>
- Handwriting Recognition Used in OCR (Optical Character Recognition) to extract text from images.
- Automatic Transcription <u>Converts spoken words into written text.</u>
- Ex:
  - Levenshtein Distance Algorithm is used to <u>correct OCR errors</u>.

#### 10. Gaming & Computer Vision

- Cheat Detection in Online Games Compares player actions with known cheating patterns.
- Al-based Character Recognition <u>Detects text in game environments.</u>
- Captcha Verification Ensures human interactions in online forms.
- Ex:
  - o Regular Expressions & Trie Structures are used to match game chat logs.



# **Compare String Matching Algorithms**



Algorithm	Preprocessing Time	Searching Time	Space Complexity	Best Case	Worst Case
Naïve Approach	O(1)	O(nm)	O(1)	O(n)	O(nm)
Knuth-Morris- Pratt (KMP)	O(m)	O(n)	O(m)	O(n)	O(n)
Rabin-Karp	O(m)	O(n) (single match) O(nm) (multiple matches)	O(1)	O(n)	O(nm) (worst case hash collision)
Boyer-Moore	O(m + Σ)	O(n/m)	Ο(Σ)	O(n/ m)	O(nm) (worst case)
Finite State Automaton (FSA)	O(m * Σ)	O(n)	O(m * Σ)	O(n)	O(n)