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Image Processing And Computer Vision-I

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Module I :-

Digital Image Fundamentals.

→ Each image is stored as a matrix (every pixel). Image is considered as 2D signal. Every value of matrix represents grey level at that point.

→ Digitization is achieved by sampling and quantization.

→ Each element of a matrix is called as a pixel. Each pixel is considered as a sample. More the pixels (i.e. increase in size), more are the samples and higher is the sampling rate which increases spatial resolution.

→ If 'A' rows and 'B' columns are used to represent image, size of the image is 'A×B'.

→ If we increase no. of pixels, size of the image also increases.

→ The grey levels at each pixel can be represented in terms of 0's and 1's.

→ If we have 'two' bits then we get 2^2 i.e. '4' combinations (grey levels). If we have '3' bits then we get 2^3 i.e. '8' combinations possible.

→ So as we increase no. of bits, no. of grey levels increases which increases tonal resolution which improves image clarity which is a part of Quantization.

→ If we use 'c' bits to represent image, size of the image becomes A×B×c. For 'd' such images, size becomes A×B×c×d.

→ As we increase no. of pixels, spatial resolution is more and as we increase no. of bits, tonal resolution is more but it increases storage space also.

* Statement:- No. of pixels And no. of grey levels decide quality of image. Justify.

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Classification of Images :-

I). Monochromatic Image :- (Binary) ~~Image~~) :-

→ Each pixel is stored as a single bit '0' or '1' where 0 represents black and 1 represents white. In this we get pure black and white image and no other shades of grey.

→ This is also called as Bit Mapped Image.

II). Grey-Scale Image :-

→ Each pixel is stored as a byte which is equivalent to 8-bits. Hence, we get 2^8 i.e. '256' shades of grey where '0' is black & '255' is white.

III). Color Image :-

→ Each pixel is represented by 24 bits [8 bits R | 8 bits G | 8 bits B i.e. R, G, B each having 8 bits] hence we get 2^{24} different colour combinations possible.

IV]. Half Toned Image :-

Image File Formats :-

tiff
jpeg
gif
png
jpg
bmp
hevc

→ It has two parts : header and image data.
→ Header is used to store information like size of the image, image type, image format (i.e. order in which pixels are stored) and compression ~~#~~ type.

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Module 2:

Image Enhancement :-

→ Image Enhancement is subjective and a cosmetic procedure.
→ No additional information is added to main image
→ It just improves subjective quality of the image
→ Image Enhancement is divided into

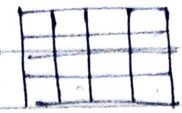
Image Enhancement
- SD (Spatial Domain).
  - Point processing
  - Neighbourhood processing. (not only looks at one point but also looks at its neighbours)
- FD (Frequency Domain)

☆ Image Enhancement in Spatial Domain :-

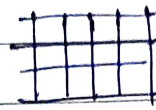
→ That is, working with pixel values or working with raw data.

changing each pixel value (pixelwise change)

I). Point Processing :-
→ It is working with single pixel.
→ Common examples of point processing are :-
1). Digital Negative
2). Contrast Stretching
3). Thresholding
4). Grey level slicing (Intensity Slicing)
5). Bit plane slicing
6). Power law transformation.
7). Dynamic range compression.

* Input Image $f(x,y) \rightarrow$ 

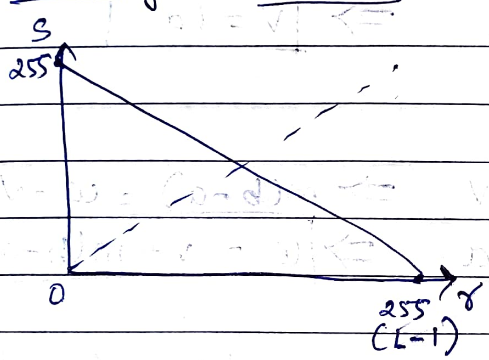
$\downarrow$ operator(T)

O/P or modified Image $g(x,y) \rightarrow$ 

$$\boxed{g(x,y) = T[f(x,y)]}$$

→ 'r' : input grey level
→ 's' : output grey level

*). Identity Transformation i.e. $s = r$



L = no. of grey levels
8 bit ⇒ L = 256.

1). Digital Negative :-

→ Inverting the grey levels i.e. black in original one will now look white & vice-versa.

→ Functions for digital negative can be given by

$s = 255 - r$

$s = (L-1) - r$

where, L = no. of grey levels.

→ If 'r' is zero, $s = 255$ and if $r = 255$, $s = 0$.

→ Applications of digital negative is X-ray measures

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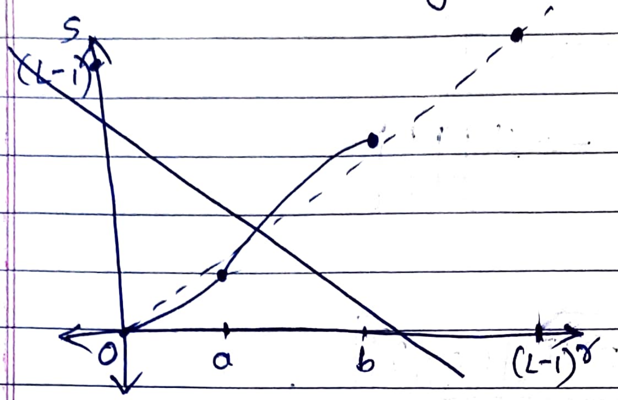
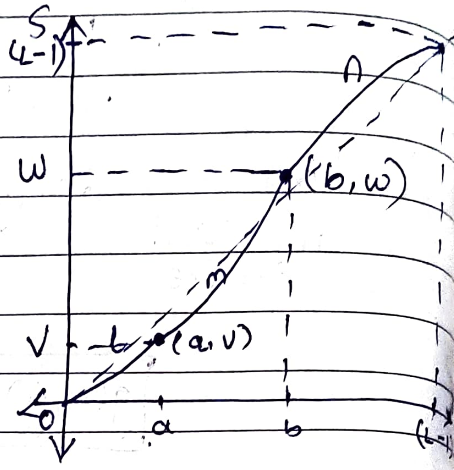
2). Contrast stretching :-





$$s = lr \quad ; \quad 0 < r \leq a$$
$$= m(r-a) + v \quad ; \quad a < r \leq b$$
$$= n(r-b) + w \quad ; \quad b < r \leq (L-1)$$

$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$; $l = \frac{v - 0}{a - 0} = \frac{v}{a} \Rightarrow \boxed{v = la}$

~~w = w~~ $m = \frac{w - v}{b - a} \Rightarrow m(b-a) = w - v$

$\Rightarrow \boxed{w = v + m(b-a)}$

* l, m and n are the slopes where l and n values are assigned less than 1 and m value is assigned greater than 1 so that dynamic range of the image can be increased.

which level our image is spread — from which level to

→ We get low contrast images due to poor illumination or wrong setting of lens aperture.

→ Contrast can be increased by making dark portions darker and bright portions brighter.

→ It increases dynamic range of the image.

only two grey levels (0 & 1)

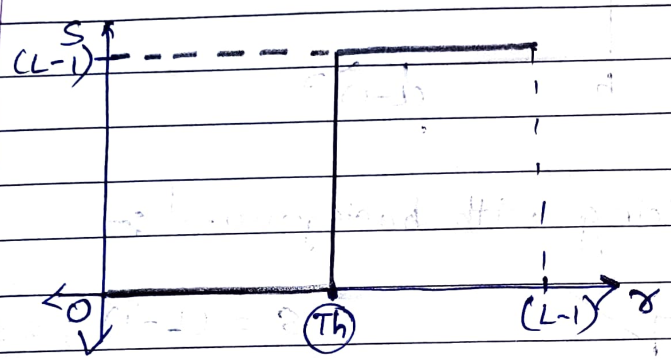
3). Thresholding :-

we achieve

→ ~~In~~ thresholding by making first and last slope of contrast stretching equal to zero and centre slope is increased.

→ This is also called as Extreme Contrast Stretching.

→ Maximum contrast is achieved in thresholding as it has only two grey levels i.e. black & white.



$s = 0 \; ; r \leq Th$ (Threshold)
$= (L-1) \; ; \text{else}$.

→ acc. to I graph

→ can enhance required band of image (eg. X-ray).

4). Grey level Slicing (Intensity Slicing) :-

→ Thresholding splits the grey levels in two parts.

→ Sometimes we need to highlight specific range of grey levels.

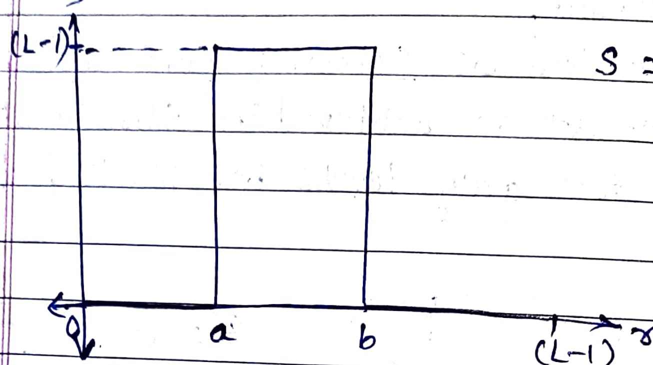
for eg. enhancing defect in CT image.

In such cases, we use a transformation known as grey level slicing, without background which looks similar to thresholding accept that we select a band of grey levels.

→ In some applications, we not only need to enhance particular band of grey levels but also need to retain the background.

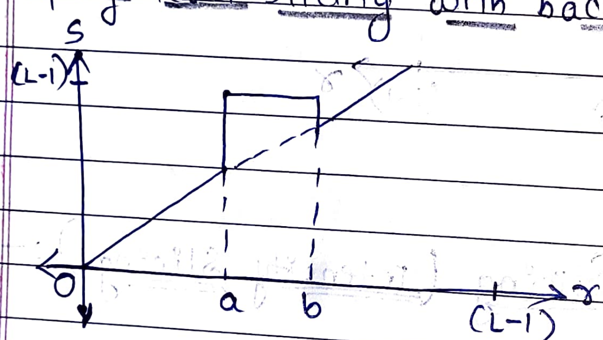
→ This technique is called as grey level slicing with background.

I). Grey level slicing without background :-



$$s = (L-1) \; ; \; a \le r \le b$$
$$= 0 \; ; \; \text{else}$$

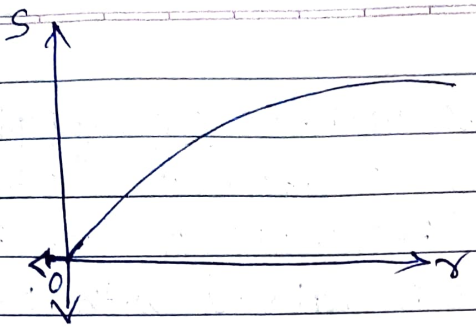
II). Grey level slicing with background :-



$$s = (L-1) \; ; \; a \le r \le b$$
$$= r \; ; \; \text{else}$$

7. Dynamic Range Compression :-

→ Also called as log transformation.
→ Sometimes, dynamic range of the image exceeds the capacity of display device.
→ Technique to compressing dynamic range is called as dynamic range compression.
→ Log is excellent compression factor.
→ Dynamic range compression can be achieved using
$\boxed{s = c \log(1 + |r|)}$ where ; 'c' : Normalisation factor



5). Bit Plane Slicing :-

→ In this technique, we find out contribution made by each bit to the final image.

→ Higher order bits contain visually significant data.

→ Bit Plane Slicing can be used in compression where we can use only higher Order bits and remove lower order bits.

→ Consider 3bit image

0	7	2
2	3	2
1	5	4

⇓

000	111	010
010	011	010
001	101	100

0	1	0
0	0	0
0	1	1

MSB image

0	1	1
1	1	1
0	0	0

Middle image

0	1	0
0	1	0
1	1	0

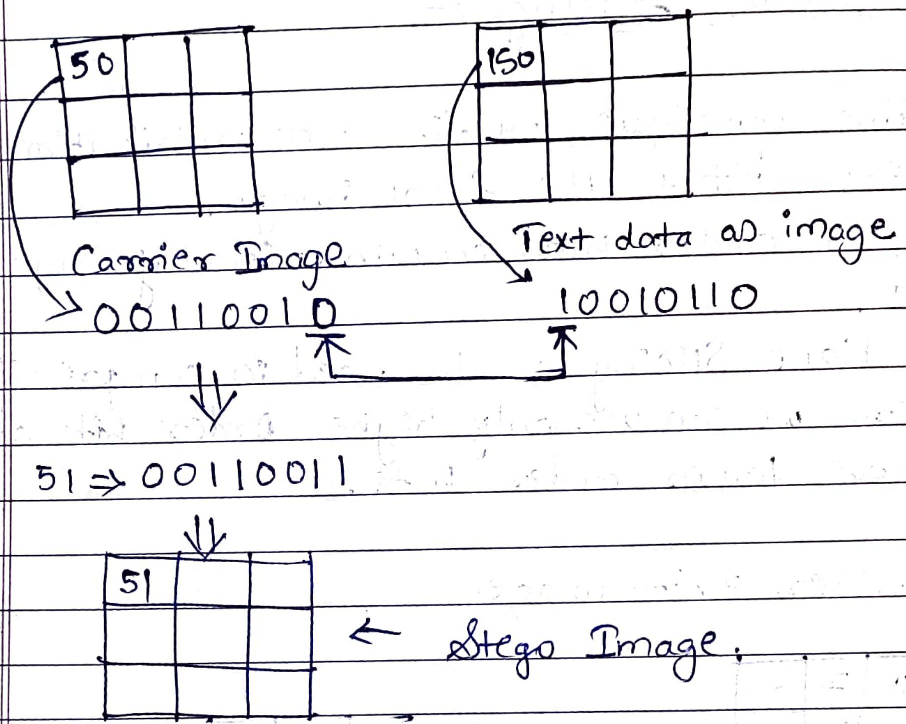
LSB image

(2) 0 1 0 → change LSB → 011 (3)
↓ change MSB
(6) 1 1 0

{Change is higher when MSB is changed}

* Bit Plane Slicing can be used for stegnography which is art of hiding the information where secret data is hidden inside the carrier image.

→ LSB of carrier image is replaced with MSB of secret data.



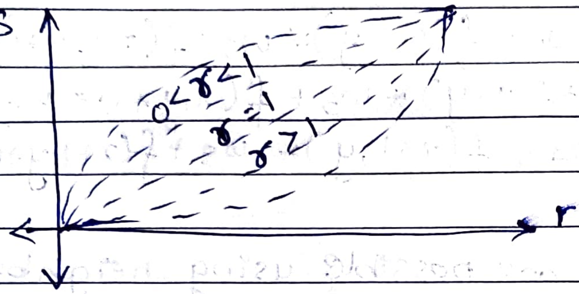
→ Stego Image looks visually same as the carrier image. If we replace more than 2 LSB's with MSB's we get superimposed image called as watermark.

6). Power Law Transformation :-

→ Non-linearities encountered during image capturing can be corrected using power law transformation which is also called as Gamma correction.

* It can be also used to increase dynamic range of image by changing the gamma value we get series of transformation curves.

* $s = c r^{\gamma \text{ (gamma)}}$; c = constant



2) * **Neighbourhood processing** :-

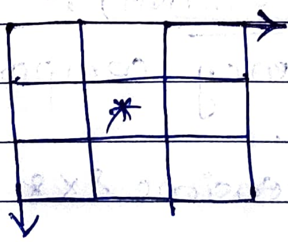
→ This is enhancement in spatial domain. In this, we consider pixel and its neighbours. We change the value of pixel based on its neighbours. Normally, 3x3 mask is used.

→ Operation involved for filtering is called convolution.

→ Input image = $f(x,y)$

	$y-1$	y	$y+1$
$x-1$	$f(x-1,y-1)$	$f(x-1,y)$	$f(x-1,y+1)$
x	$f(x,y-1)$	$f(x,y)$	$f(x,y+1)$
$x+1$	$f(x+1,y-1)$	$f(x+1,y)$	$f(x+1,y+1)$

→ Output image = $g(x,y)$



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* Mask / filter function :-

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

= W

* $g(x,y) = f(x-1,y-1) * w_1 + f(x-1,y) * w_2 + f(x-1,y+1) * w_3 + f(x,y-1) * w_4 + f(x,y) * w_5 + f(x,y+1) * w_6 + f(x+1,y-1) * w_7 + f(x+1,y) * w_8 + f(x+1,y+1) * w_9$

⇒ Many applications are possible using neighbourhood processing which are not possible using point processing.

⇒ We can have a mask of 3x3, 5x5, 7x7, etc.
Multiply each component of mask with corresponding value of image, add them up and place the value at the center. Shift mask to the right hand side and then downwards, so as to cover the entire image.

⇒ High frequency regions are where gray levels change a lot like edges, noise, whereas, low frequency regions are where gray levels stay similar like background.

⇒ Low pass filter or smoothing when applied to noise or edges, it blurs them. High pass filter when applied to background, removes it and enhances the noise or edges.

* LOW PASS FILTERING (SMOOTHING) :-

→ It removes high frequency component, normally used to blur the noise.

→ Consider image of dimensions 8x8, where the first 4 rows are (10) and the last four rows are (50).

* f(x,y) :-
I/P image

10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50

* Filter :- $w = \frac{1}{9}\begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$

* On convolution of f(x,y) and 'w' we get ;

	10	10	10	10	10	10	
	10	10	10	10	10	10	
	70/3 23.33	23.33	23.33	23.33	23.33	23.33	
	110/3 36.67	36.67	36.67	36.67	36.67	36.67	
	50	50	50	50	50	50	
	50	50	50	50	50	50	

→ The image has Gaussian noise, it can be removed using low pass averaging filter. Considering this example, low frequency regions have remained unchanged. Sharp edge between 10 and 50 has become blurred, i.e. from grey level 10 to 23.33 to 36.67 to 50.

→ We can have low pass averaging mask of order 5x5. Bigger the mask, more is the blurring.

→ Low pass averaging filters example :-

General (common) = $\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Other examples = $\frac{1}{6}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

→ Bigger the mask, more will be the blurring.

⊛. Low Pass Median Filtering :-

→ To remove salt and pepper noise, we work with non-linear filter like median filter, which is also called as 'order statistic filter'. There is no mask, we consider empty mask, put that on image and arrange the pixels in ascending or descending order, so as to calculate median value which is used in the output image.

→ Consider 8x8 image.

$$\begin{matrix} 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 \\ 10 & 250 & 10 & 10 & 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 \\ 50 & 50 & 50 & 50 & 250 & 50 & 50 & 50 \\ 50 & 50 & 50 & 50 & 50 & 50 & 50 & 50 \\ 50 & 50 & 50 & 50 & 50 & 50 & 50 & 50 \\ 50 & 50 & 50 & 50 & 50 & 50 & 50 & 50 \end{matrix} \Rightarrow \begin{matrix} 10 & 10 & 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 & 10 & 10 \\ 50 & 50 & 50 & 50 & 50 & 50 \\ 50 & 50 & 50 & 50 & 50 & 50 \\ 50 & 50 & 50 & 50 & 50 & 50 \end{matrix}$$

→ '250' as noise got removed using median filter. If we have image with salt and pepper noise and we apply averaging filter, noise will get spreaded over the entire region, which will detoriate an image.

→ Averaging filter is used to remove gaussian noise (Question → Statement).

→ Eg. above salt-pepper image with averaging filter.

36.67	36.67	10	10	10	10
36.67	36.67	10	10	10	10
50	50	45.56	45.56	45.56	23.33
36.67	36.67	58.89	58.89	58.89	36.67
50	50	72.22	72.22	72.22	50
50	50	50	50	50	50

(*) High Pass Filtering (Sharpening):-

→ It is used to enhance the edges and to remove the background.

→ Consider 8x8 image having 4 rows of 10 & 4 rows of 100,

10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

→ Mask

-1	-1	-1
-1	8	-1
-1	-1	-1

(High pass filter
(Addit^n of all pixels to be zero).

0	0	0	0	0	0
0	0	0	0	0	0
-270	-270	-270	-270	-270	-270
270	270	270	270	270	270
0	0	0	0	0	0
0	0	0	0	0	0

⇐ o/p image.

→ We can consider 1/9 with the mask, so that values are not out of the gray level range (here, 0 to 255).

→ Pixel cannot be negative, but mod is not possible, hence, -270 is like dark as black i.e. 0. We notice that edge between 10 and 100 is enhanced in output.

→ 1/9 is just the scaling factor.

(*) HIGH BOOST FILTER :-

→ In some applications, we want to enhance the edges as well as retain the background. So as to pass some part of background, we multiply image with multiplicative factor (A).

→ Show that if A=0, high Boost filter is same as High Pass filter.

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$HP = O - LP$

$HB = AO - LP$

$= AO - O + O - LP$

$HB = O(A-1) + HP$

$\therefore$ if $A = 0$,

$\boxed{HB = HP}$

and if $A > 1$, say 1.1,

-1	-1	-1
-1	X	-1
-1	-1	-1

$X = 9A - 1$

$X = 8.9$

$\Rightarrow$

-1	-1	-1
-1	8.9	-1
-1	-1	-1

→ Consider same image as High Pass filtering with above mask.

9	9	9	9	9	9
9	9	9	9	9	9
-261	-261	-261	-261	-261	-261
360	360	360	360	360	360
90	90	90	90	90	90
90	90	90	90	90	90

← O/P Image

→ We can see that background is not completely removed. Some part of the background is retained as we get values like 9 and 90 in the output image.

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Q. Find digital negative of given image which is represented using 8 bits.

121	205	217	156	151
139	127	157	117	125
252	117	236	138	142
227	182	178	197	242

* Solution :-

Given that the image is represented using 8 bits.

$\therefore$ Grey levels $0 - 255$

$\therefore (L-1) = 255$.

* We know that,

$S = (L-1) - r = 255 - r$.

* O/P image :-

134	50	38	99	104
116	128	98	138	130
3	138	19	117	113
28	73	77	58	13

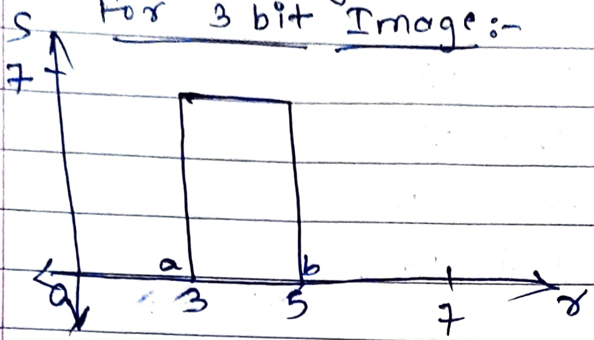
Q. Perform Intensity slicing for 3 bit image where $a = 3$, $b = 5$. Draw modified image with and without background.

2	1	2	2	1
2	3	4	5	2
6	2	7	6	0
2	6	6	5	1
0	3	2	2	1

* Solution:-

I) Without background:-

For 3 bit Image:-



$s = (L-1) ; a \le r \le b$

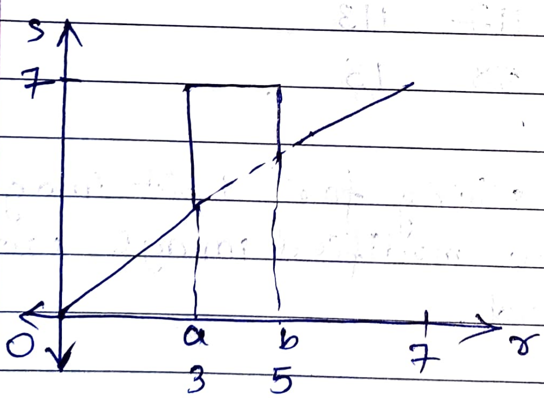
$= 0 ; else.$

* O/P Image:-

0	0	0	0	0
0	7	7	7	0
0	0	0	0	0
0	0	0	7	0
0	7	0	0	0

II). With Background:-

→ For 3-bit image:-



$s = (L-1) ; a \le r \le b$

$= r ; else.$

O/P Image:-

2	1	2	2	1
2	7	7	7	2
6	2	7	6	0
2	6	6	7	1
0	7	2	2	1

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Q. What would happen to the dynamic range if all the slopes in contrast stretch algorithm are less than 1 considering original image has grey levels b/w 0 to 10 and $a = 4$ and $b = 8$.

* Let $l = 0.2$ } Supposition.
$m = 0.5$
$n = 0.2$

* I $s = lr$; $0 \le r \le 4$
II $s = m(r-a) + v$; $5 \le r \le 8$
III $s = n(r-b) + w$; $r > 8$

* $v = la$

⇒ From I eqⁿ :-

r	s
0	$lr = (0.2)(0) = 0$
1	0.2
2	0.4
3	0.6
4	0.8

$v = la = (0.2)(4) = 0.8$

$w = v + m(b-a) = 0.8 + (0.5)(8-4)$
$= 0.8 + (0.5)(4)$
$= 2.8$

From II eqⁿ :-

r	$s[m(r-a)+v]$
5	$0.5(5-4) + 0.8 = 1.3$
6	1.8
7	2.3
8	2.8

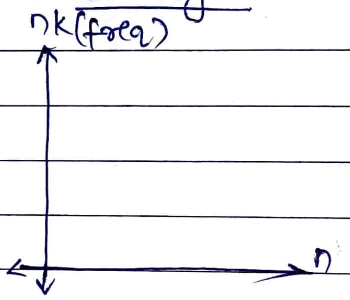
From III eq^n :-

r	$s[n(r-b)+w]$
9	3
10	3.2

* Input grey level : r = [0 – 10]
  Output grey level : s = [0 – 3.2].

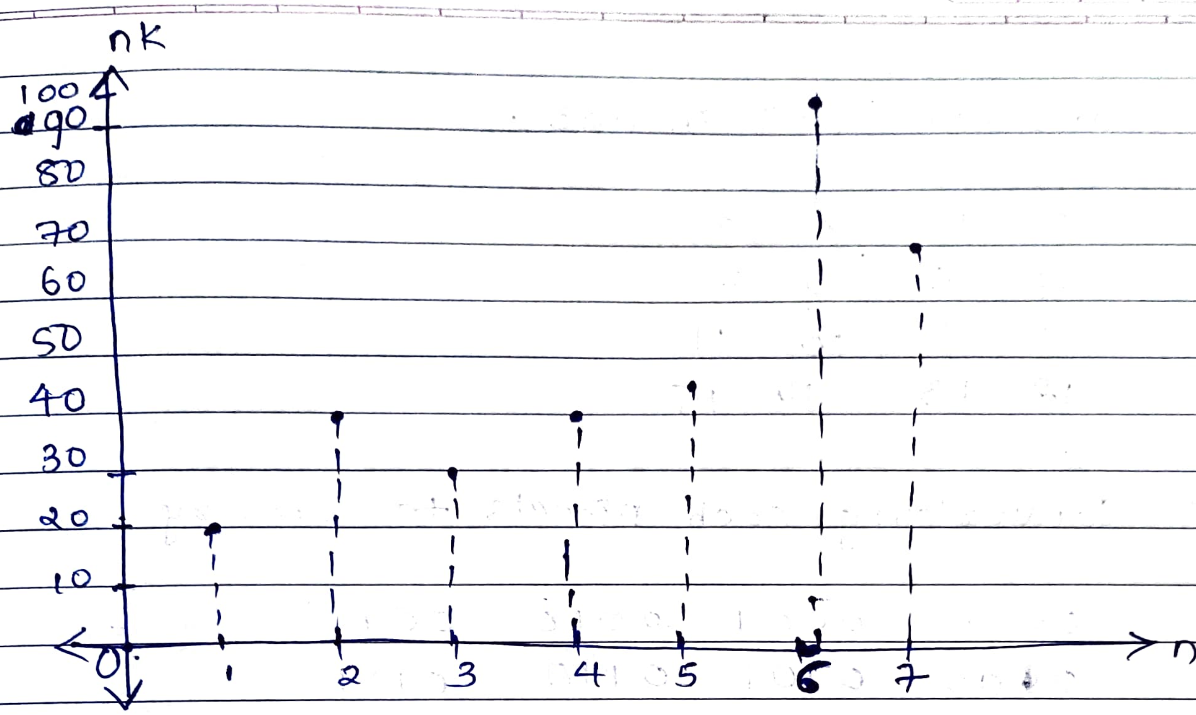
* We noticed that dynamic range becomes 0 to
  for the modified image.
  Hence, dynamic range decreases which should not happen ideally.

☆ Histogram Modelling :-

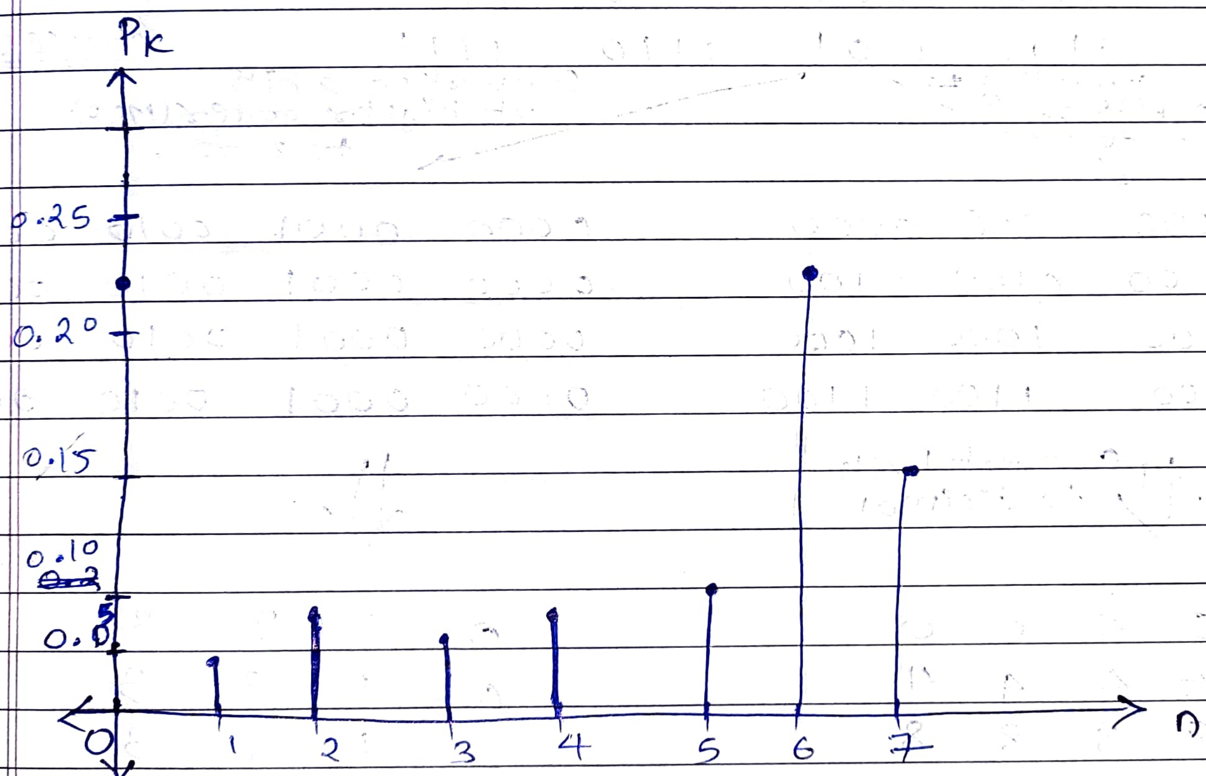


n	nK	$P_K = nK/\Sigma$
0	100	100/445 = 0.2247
1	20	20/445 = 0.0449
2	40	0.0898
3	30	0.0674
4	40	0.0898
5	45	0.1011
6	100	0.2247
7	70	0.1573
Σ	445	

I).



II).



Q. What would be the effect of setting zero lower order bits on histogram of image and what would be the effect if we set higher order bits to zero ?

→ Consider 4x4 image.

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

→ Converting each pixels to binary.

0000	0001	0010	0011
0100	0101	0110	0111
1000	1001	1010	1011
1100	1101	1110	1111

Case I: Converting 2 bits of lower order to 0.

Case II: Converting 2 bits of higher order (MSB) to zero.

Case I:

0000	0000	0000	0000
0100	0100	0100	0100
1000	1000	1000	1000
1100	1100	1100	1100

Case II:

0000	0001	0010	0011
0000	0001	0010	0011
0000	0001	0010	0011
0000	0001	0010	0011

⇓ Convert back to decimal

0	0	0	0
4	4	4	4
8	8	8	8
12	12	12	12

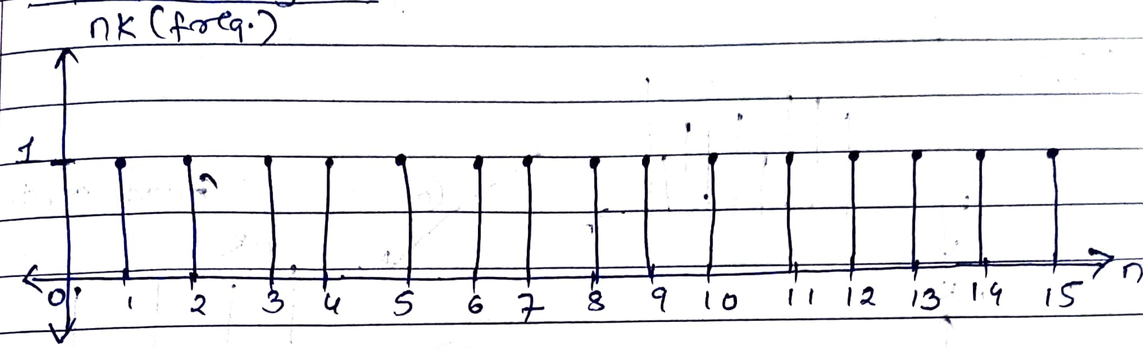
Modified image 1

⇓

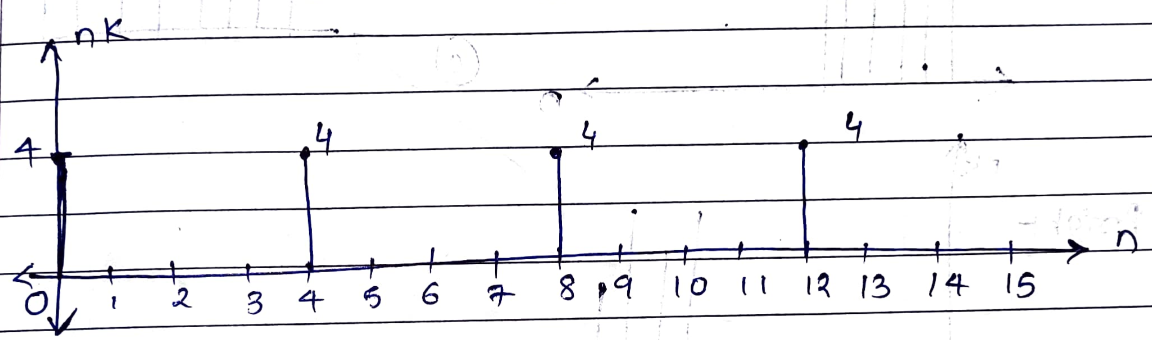
0	1	2	3
0	1	2	3
0	1	2	3
0	1	2	3

Modified image 2.

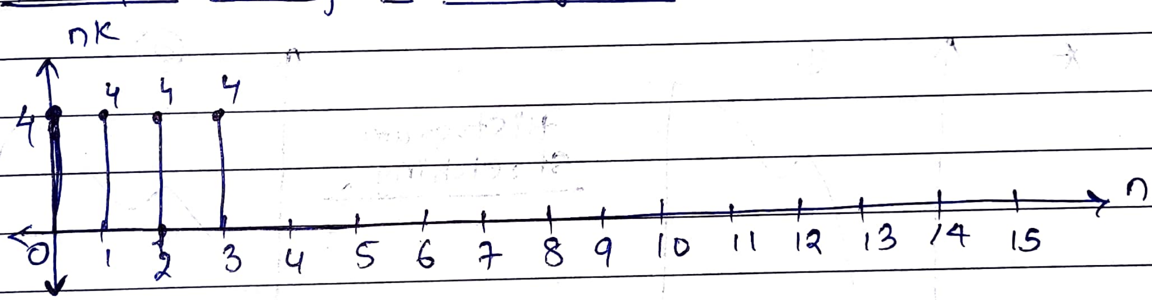
* Input histogram :-



* Modified image 1 Histogram :-

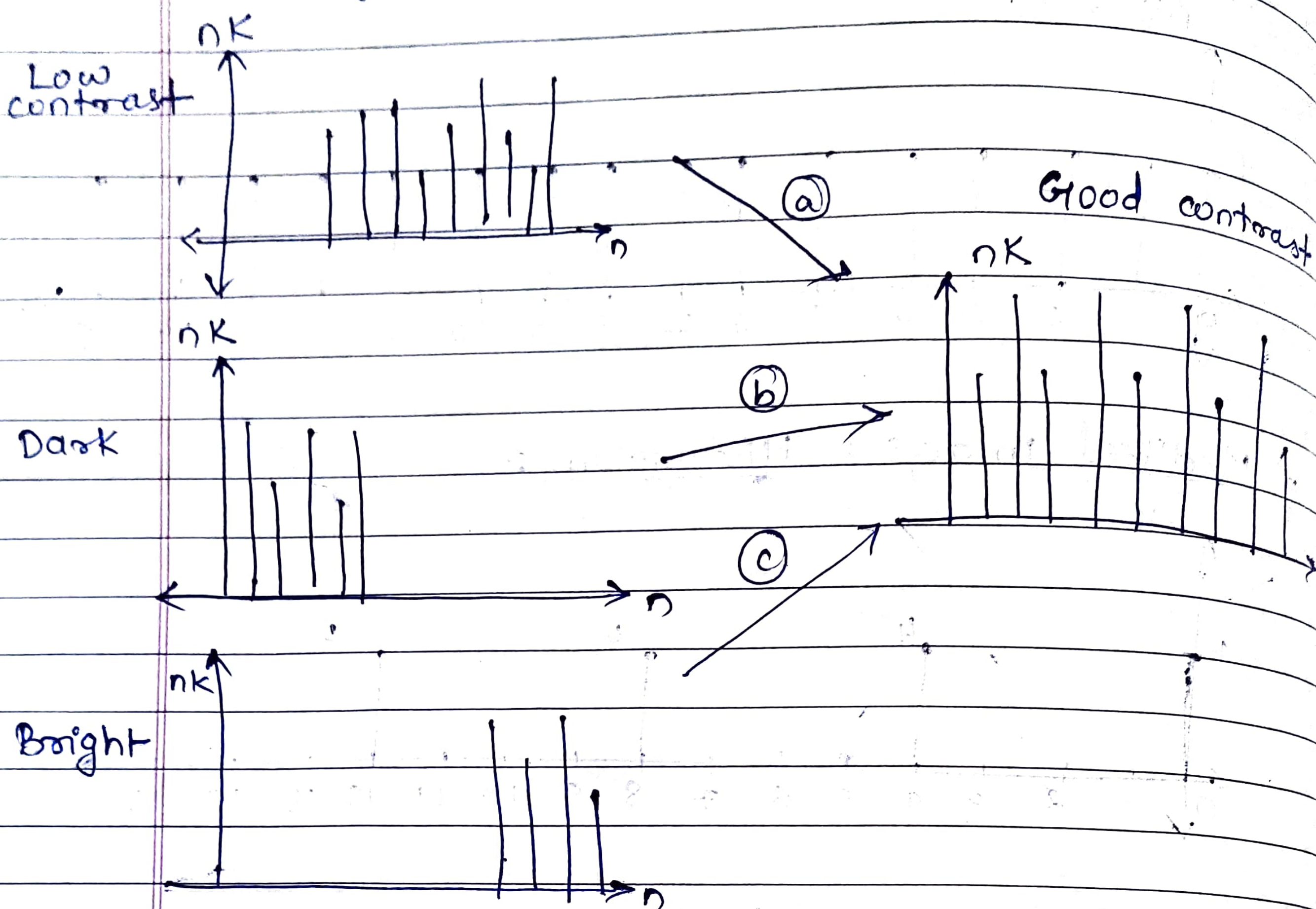


* Modified image 2 Histogram :-

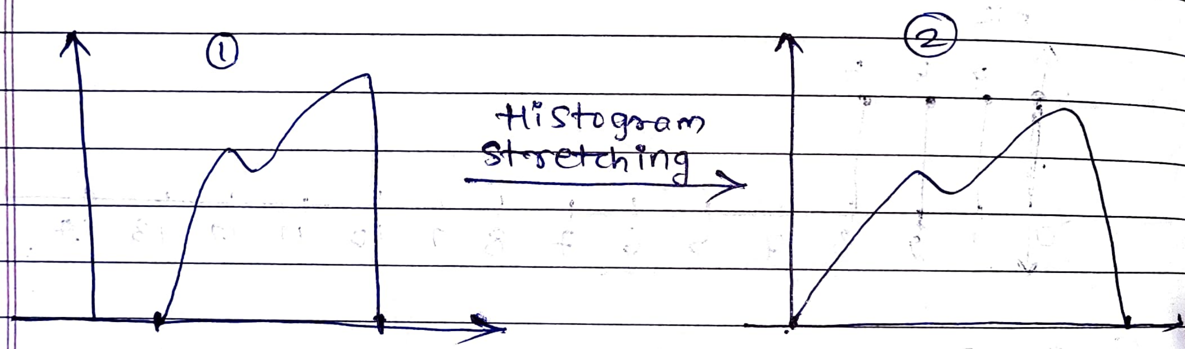


* For I modified histogram, variation in grey levels is less. No. of grey levels are reduced. Frequency of the grey level is higher.
* With respect to modified histogram 2, grey levels are reduced, frequency of grey levels is higher. At the same time, it is darker compared to input histogram.

* Histogram Stretching :-



*



* $s = T(r)$

* Linear Stretching :-

$$s = \frac{S_{max} - S_{min}}{r_{max} - r_{min}} (r - r_{min}) + S_{min}$$

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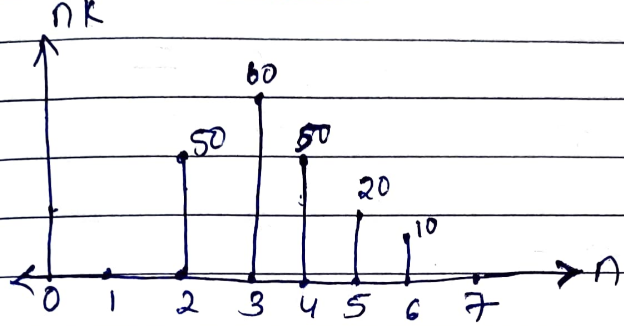
Q. Perform histogram stretching for given image

r	0	1	2	3	4	5	6	7
nk	0	0	50	60	50	20	10	0

* Input Histogram :-



$S_{min} = 0$, $S_{max} = 7$

$r_{min} = 2$, $r_{max} = 6$

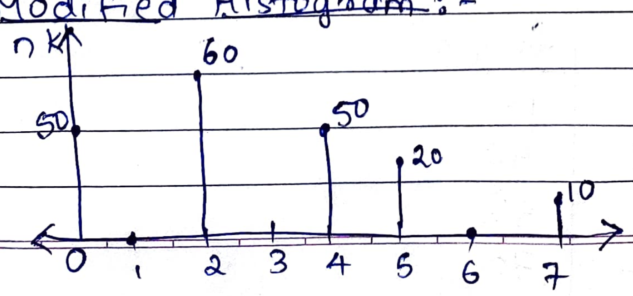
$$S = \frac{7}{4}(r-2) + 0$$

r	S		freq (r)
2	0	≈ 0	50
3	1.75	≈ 2	60
4	3.5	≈ 4	50
5	5.25	≈ 5	20
6	7	≈ 7	10

⇓

new S	0	1	2	3	4	5	6	7
nk	50	0	60	0	50	20	0	10

* Modified Histogram :-

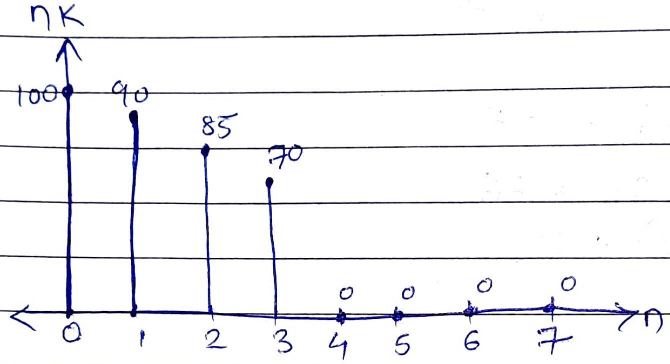


→ Dynamic range is increased and shape of histogram is same and frequency values also remains same.

Q. Perform ~~Confirm~~ histogram stretching.

n	0	1	2	3	4	5	6	7
nk	100	90	85	70	0	0	0	0

⇒ Input Histogram :-



* Formula :-

$$s = \frac{s_{max} - s_{min}}{r_{max} - r_{min}} (r - r_{min}) + s_{min}$$

o/p gray level

i/p gray level

* We can set o/p gray levels i.e. $s_{min} = 0$, $s_{max} = 7$.

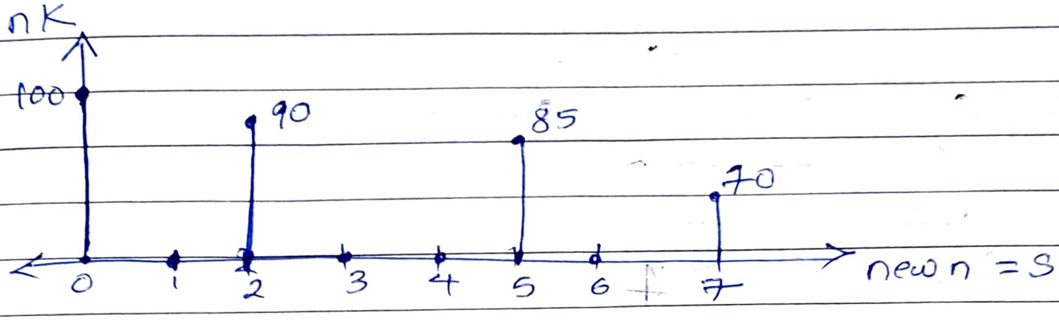
$\therefore r_{min} = 0$, $r_{max} = 3$

$$\therefore s = \frac{7}{3}(r - 0) + 0 = \frac{7}{3} r .$$

r	s = 7/3 r		freq(r)
0	0	≈ 0	100
1	2.3333	≈ 2	90
2	4.6666	≈ 5	85
3	7	7	70

* S	0	1	2	3	4	5	6	7
nk	100	0	90	[illegible]	0	85	0	70

* Modified Histogram :-



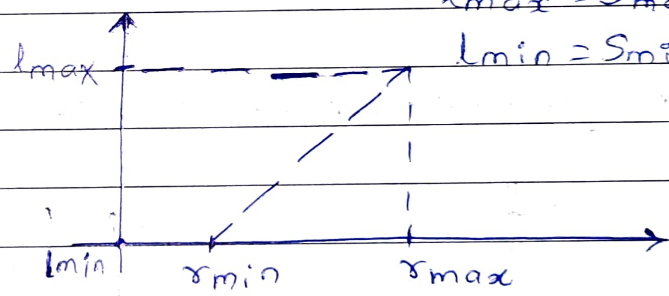
* We notice that after linear stretching (histogram stretching), dark histogram is converted into evenly spaced histogram.

Q. For the given slope transformation and image, draw frequency table. Also sketch input & output histogram. Also draw modified image.

2	3	4	2
5	5	2	4
3	6	3	5
5	3	5	5

$l_{max} = S_{max} = 7$

$l_{min} = S_{min} = 0.$



$r_{min} = 2$
$r_{max} = 6$ } from image. min = 2, max = 6.

$l_{max} = S_{max} = 7$

$l_{min} = S_{min} = 0$

* $S = \frac{S_{max} - S_{min}}{r_{max} - r_{min}} (r - r_{min}) + S_{min}$

$\therefore \; S = \frac{7}{4}(r-2)$

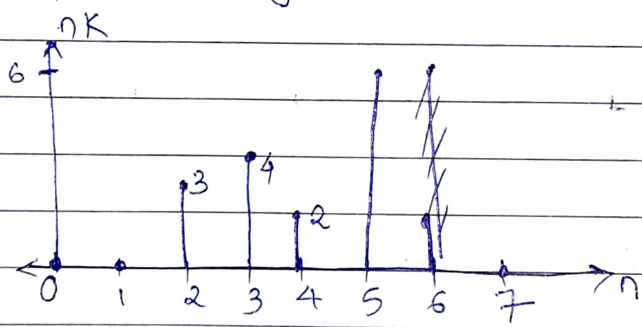
r	$S = 7/4\,(r-2)$	freq(r)
2	$0 \longrightarrow 0$	3
3	$1.75 \longrightarrow 2$	4
4	$\boxed{3.5 \longrightarrow 4}$	2
5	$5.25 \longrightarrow 5$	6
6	$7 \longrightarrow 7$	1

↑ new gray levels

* Frequency table:-

n	0	1	2	3	4	5	6	7
nK	0	0	3	4	2	6	1	0

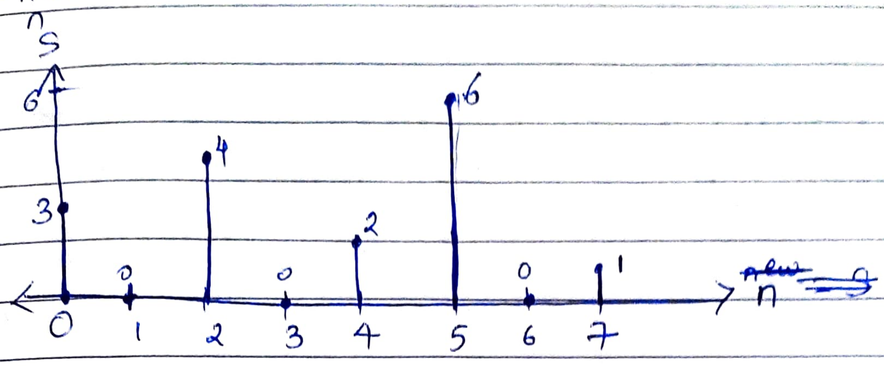
* Input Histogram:-



* New Table :-

new S	0	1	2	3	4	5	6	7
nK	3	0	4	0	2	6	0	1

* O/P Histogram :- new



* Modified Image :-

$r = 2$, new gray level corresponding to 2 is 0.
So, replace every 2 by 0 and henceforth.

0	2	4	0
5	5	0	4
2	7	2	5
5	2	5	5

$2 \to 0$
$3 \to 2$
$4 \to 4$
$5 \to 5$
$6 \to 7$

Q. Consider bright histogram and ~~for~~ show how histogram stretching helps to improve dynamic range.

⇒ Assuming a bright histogram distribution with frequency as follows :-

n	0	1	2	3	4	5	6	7
nk								

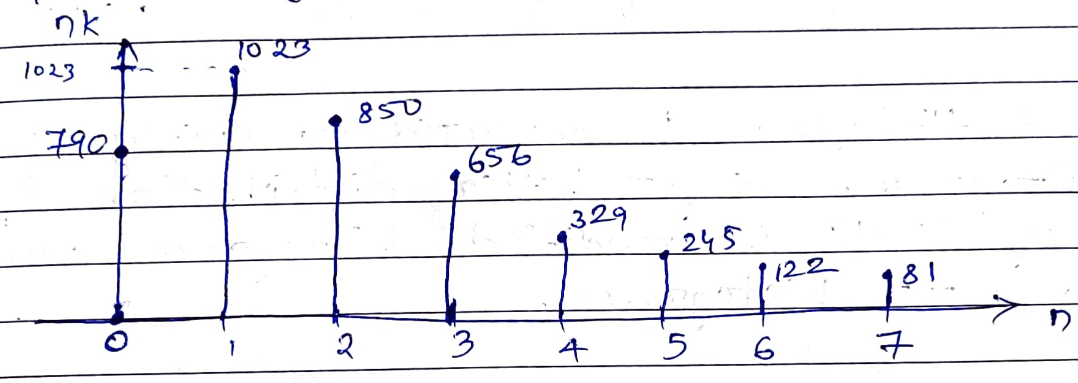
23/02/2023

* Histogram Equalization :-

Q. Perform histogram equalisation.

n	0	1	2	3	4	5	6	7
nK	790	1023	850	656	329	245	122	81

* Input histogram :-

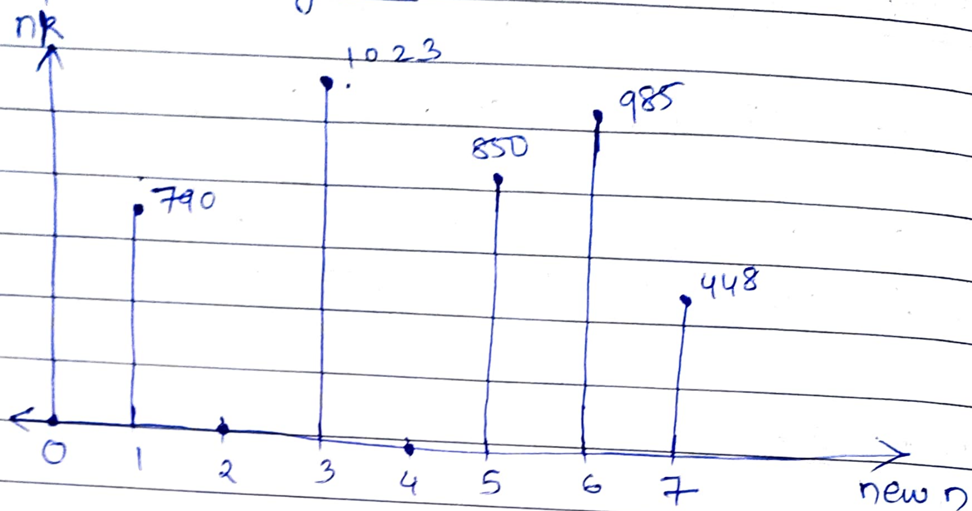


* n	nK	PDF nK/Σ	CDF	(L-1)CDF (7)CDF	Round up (new n)	freq.
0	790	0.193	$\rightarrow$ 0.193	1.351	1	790
1	1023	0.2458	0.193+0.245 0.438	3.08	3	1023
2	850	0.2075	0.438+0.2075 0.65	4.55	5	850
3	656	0.1601	0.81	5.67	6	985 (656+329)
4	329	0.08	0.89	6.23	6	
5	245	0.06	0.95	6.65	7	448 (245+122+81)
6	122	0.03	0.98	6.86	7	
7	81	0.02	1	7	7	

$\Sigma = 4096$

* new n	0	1	2	3	4	5	6	7
nR	0	790	0	1023	0	850	985	448

* Output Histogram :-

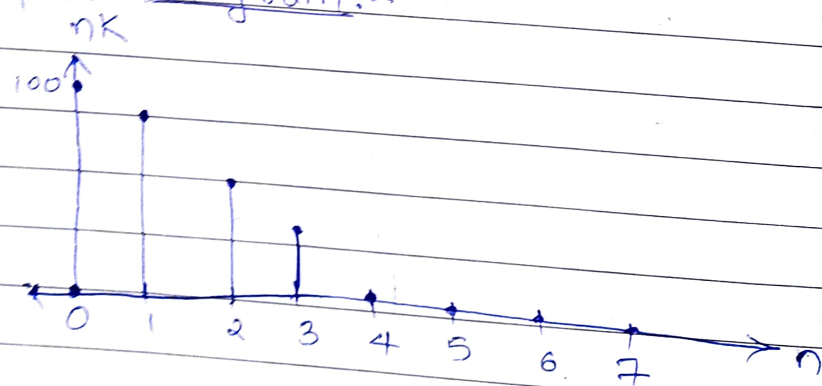


* Dark Histogram is converted to evenly spaced histogram. We get better distribution of the gray levels in modified histogram compared to input histogram.

Q. Perform histogram equalisation.

n	0	1	2	3	4	5	6	7
nK	100	90	50	20	0	0	0	0

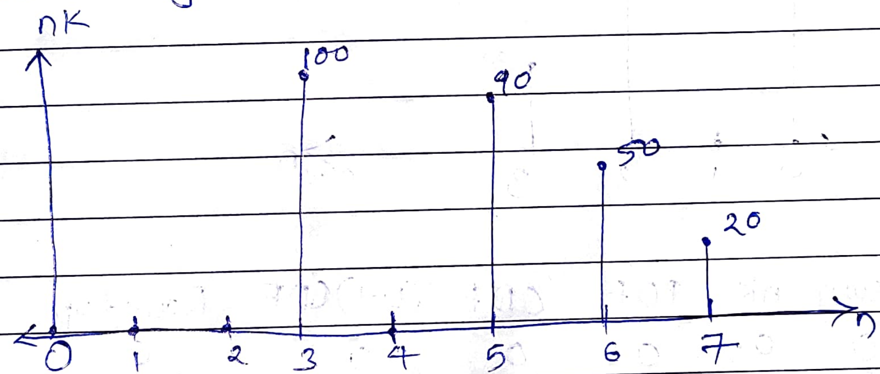
* Input histogram :-



* n	nK	PDF	CDF	(7)(CDF) (L-1)(CDF)	Round up	Freq.
0	100	0.38	0.38	2.66	3	100
1	90	0.35	0.73	5.11	5	90
2	50	0.19	0.92	6.44	6	50
3	20	0.08	1	7	7	20
4	0	0	1	7	7	
5	0	0	1	7	7	
6	0	0	1	7	7	
7	0	0	1	7	7	
	Σ260					

* new n	0	1	2	3	4	5	6	7
nK	0	0	0	100	0	90	50	20

* O/P Histogram :-

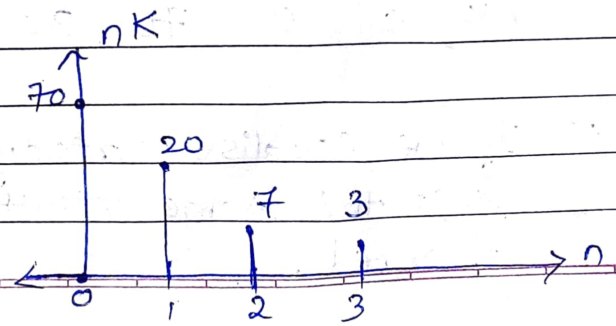


Q. What happens if we equalise already equalised histogram ?.

(1)

n	0	1	2	3
nK	70	20	7	3

* I/P Histogram :-



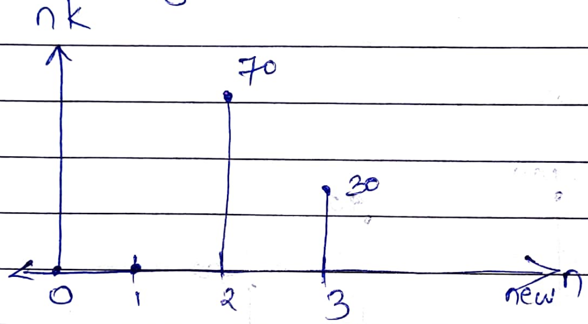
n	nK	PDF	CDF	(3)(CDF) (L-1) CDF	Round up	Freq
0	70	0.7	0.7	2.1	2	70
1	20	0.2	0.9	2.7	3	30
2	7	0.07	0.97	2.91	3	
3	3	0.03	1	3	3	
	Σ100					

* ~~o/p Histogram :~~

* (2).

new n	0	1	2	3
nK	0	0	70	30

* O/P Histogram :-



new n	nK	PDF	CDF	(L-1)CDF	Round up	Freq
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	70	0.7	0.7	2.1	2	70
3	30	0.3	1	3	3	30
	Σ100					

n	0	1	2	3
nK	0	0	70	30

* If we equalise already equalised histogram, it gives the same result and there is no change observed.

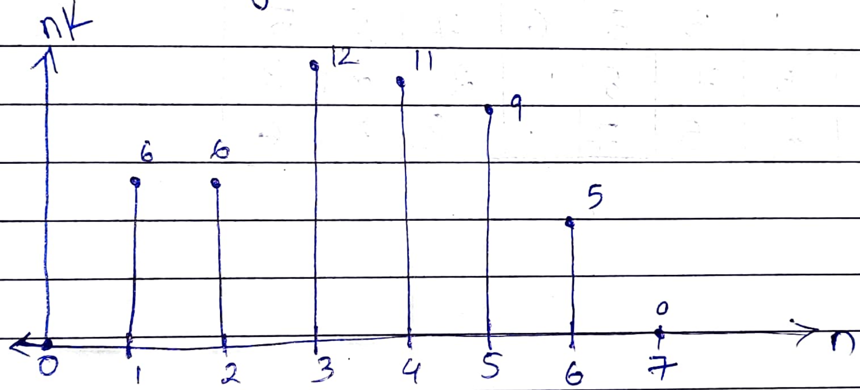
Q. Plot histogram of given image. Perform histogram equalisation. Plot modified histogram and draw Histogram equalised image.

1	1	3	6	4	3	1
5	6	3	4	5	5	3
3	4	3	2	4	3	5
5	5	4	1	3	2	3
1	3	4	5	6	5	4
4	6	4	1	2	2	3
2	4	6	3	2	4	5

* Table :-

n	0	1	2	3	4	5	6	7
nk	0	6	6	12	11	9	5	0

* Input Histogram :-

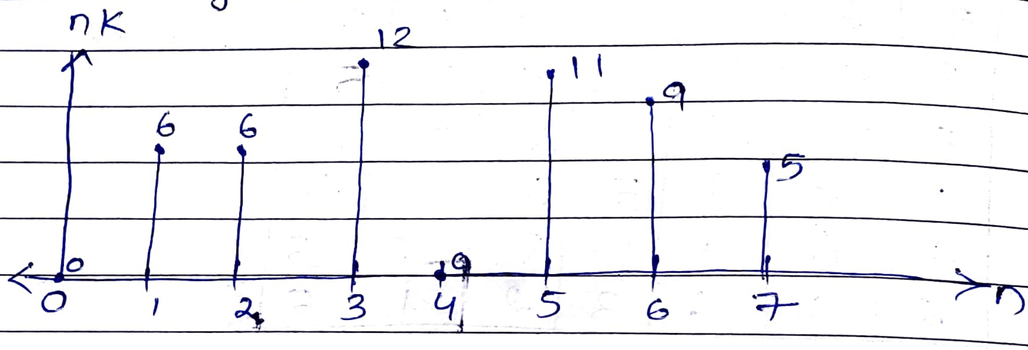


n	nk	PDF	CDF	(L-1) (CDF)	Roundup	Frequency
0	0	0	0	0	0	0
1	6	0.12	0.12	0.84	1	6
2	6	0.12	0.24	1.68	2	6
3	12	0.24	0.48	3.36	3	12
4	11	0.22	0.7	4.9	5	11
5	9	0.18	0.88	6.16	6	9
6	5	0.10	0.98	6.86	7	5
7	0	0	0.98	6.86	7	
	Σ 49					

*

new n	0	1	2	3	4	5	6	7
nk	0	6	6	12	0	11	9	5

* O/P Histogram :-



* Histogram Equalised Image :-

1	1	3	7	5	3	1
6	7	3	5	6	6	3
3	5	3	2	5	3	6
6	6	5	1	3	2	3
1	3	5	6	7	6	5
5	7	5	1	2	2	3
2	5	7	3	2	5	6

$4 \to 5$
$5 \to 6$
$6 \to 7$
Rest gray levels will remain same

28/02/2023.

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* Images are 2D, for images we need 2-D DFT (Discrete fourier transform) for which 1D DFT formula can be extended.

* $x(n) \xrightarrow{DFT} X[K]$
TD FD

* $f(x) \xrightarrow{DFT} F(u)$
TD FD

* Definition of 1D DFT is given by :-

$$F(u) = \sum_{x=0}^{N-1} f(x) e^{-j2\pi ux/N}$$

where, $u = 0, 1, 2, \ldots, (N-1)$.

* To calculate 1D DFT, we can use

$\boxed{F(u) = W \cdot f(x)}$

where, W = ~~quidat~~ Twiddle matrix
f(x) is signal
F(u) is DFT of signal

* If $f(x) = \{1, 2, 3, 4\}$ then

$$F(u) = W \cdot f(x)$$
$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$F(u) = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} \begin{array}{l} u=0 \\ u=1 \\ u=2 \\ u=3 \end{array}$$

* Inverse DFT (I-DFT) formula is given by:-

$$f(x) = \frac{1}{N} \sum_{u=0}^{N-1} F(u)\, e^{j2\pi ux/N} \quad ; \quad x = 0, 1, 2, \ldots, (N-$$

* DFT formula for 2D is given by :-

$$\cancel{F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi ux} \left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

$$\underset{\uparrow FD}{F(u,v)} = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \underset{\uparrow TD}{f(x,y)}\, e^{-j2\pi \left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

where, $M \times N$ is the size of image.

* I-DFT for 2D :-

$$f(x,y) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} F(u,v)\, e^{j2\pi \left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

* Separability Property :-

$$\Rightarrow F(u,v) = \sum_{x=0}^{M-1} e^{-j2\pi \frac{ux}{M}} \underbrace{\sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi vy/N}}_{F(x,v)}$$

$$\Rightarrow \quad = \sum_{x=0}^{M-1} F(x,v)\, e^{-j2\pi \frac{ux}{M}}$$

$$\Rightarrow F(u,v)$$

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Q. For given image, find '2D' DFT

0	1	2	1
1	2	3	2
2	3	4	3
1	2	3	2

* Solution :-

$X[K] = W x(n)$

$x(n) = \{0, 1, 2, 1\}$.

$$X[K] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 0 \\ -2 \end{bmatrix}$$

→ DFT {row 2} = DFT {1, 2, 3, 2}

$$X[K] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & +1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ -2 \\ 0 \\ -2 \end{bmatrix}$$

→ DFT {row 3} = DFT {2, 3, 4, 3}

$$X[K] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 12 \\ -2 \\ 0 \\ -2 \end{bmatrix}$$

→ DFT {row 4} = DFT {1, 2, 3, 2} $= \begin{bmatrix} 8 \\ -2 \\ 0 \\ -2 \end{bmatrix}$

$$
\begin{array}{cccc}
\text{col 1} & \text{col 2} & & \\
\downarrow & \downarrow & & \\
\end{array}
$$

* $\begin{bmatrix} 4 & -2 & 0 & -2 \\ 8 & -2 & 0 & -2 \\ 12 & -2 & 0 & -2 \\ 8 & -2 & 0 & -2 \end{bmatrix}$ ← Intermediate image.

* $DFT\{col\ 1\} = DFT\{4, 8, 12, 8\}$.

$$X[K] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 4 \\ 8 \\ 12 \\ 8 \end{bmatrix} = \begin{bmatrix} 32 \\ -8 \\ 0 \\ -8 \end{bmatrix}$$

* $DFT\{col\ 2\} = DFT\{-2, -2, -2, -2\}$

$$X[K] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} -2 \\ -2 \\ -2 \\ -2 \end{bmatrix} = \begin{bmatrix} -8 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

* $DFT\{col\ 3\} = DFT\{0, 0, 0, 0\}$

$$X[K] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & +1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

* $DFT\{col\ 4\} = DFT\{-2, -2, -2, -2\} = \begin{bmatrix} -8 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

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*
$$\begin{bmatrix} 32 & -8 & 0 & -8 \\ -8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -8 & 0 & 0 & 0 \end{bmatrix} \Leftarrow \text{Final Ans.}$$

* Translation Property (Shifting property):-

→ If $f(x,y)$ is multiplied by an exponential $e^{j2\pi\left[\frac{u_0x+v_0y}{N}\right]}$ then, the original fourier transform, $F(u,v)$ gets shifted by $F(u-u_0, v-v_0)$.

* $\cancel{f(x,y)}$ $f(x,y) \xrightarrow{expo} e^{j2\pi\left[\frac{u_0x+v_0y}{N}\right]}$; $F(u,v) \Rightarrow F(u-u_0, v-v_0)$.

* $F(u,v) = \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux+vy}{N}\right)}$

N×N image is used

$$= \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y) e^{j2\pi\left(\frac{u_0x+v_0y}{N}\right)} e^{-j2\pi\left(\frac{ux+vy}{N}\right)}$$

$$= \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left[\frac{(u-u_0)x+(v-v_0)x}{N}\right]}$$

$$= F(u-u_0, v-v_0)$$

* Substituting $u_0 = v_0 = \frac{N}{2}$.

$$e^{j2\pi\left[\frac{u_0x+v_0y}{N}\right]}$$

$$= e^{j2\pi\left[\frac{\frac{N}{2}x+\frac{N}{2}y}{N}\right]} = e^{j\pi(x+y)} = (-1)^{x+y}.$$

* $f(x,y) \xrightarrow{\text{2D DFT}} F(u,v)$
* $(-1)^{x+y} f(x,y) \xrightarrow{\text{2D DFT}} F\left(u-\frac{N}{2}, v-\frac{N}{2}\right)$

* For filtering in the frequency domain, first we have to multiply image with $(-1)^{x+y}$ then we should take 2D DFT. In case of situation where dynamic range is out of range we need to take log of the value.

TD ⇒ $\underset{\text{image}}{f(x,y)} * \underset{\text{filter } f^n}{h(x,y)} = g(x,y)$

↓ DFT(2D)

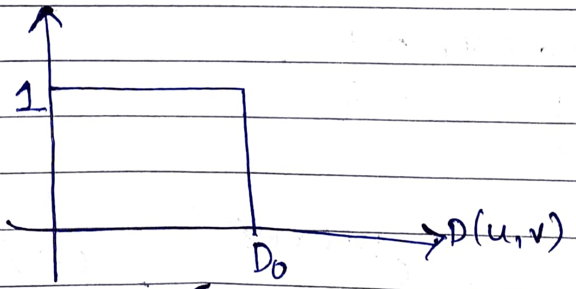
FD ⇒ $F(u,v) \cdot \underset{\text{DFT \{filter } f^n\}}{H(u,v)} = G(u,v)$

* Steps (Block diagram) for filtering in F.D:

$\boxed{f(x,y)} \rightarrow \boxed{(-1)^{x+y} * f(x,y)} \xrightarrow{\text{2D DFT}} \boxed{F(u,v)} \rightarrow \boxed{\times} \rightarrow \boxed{G(u,v)} \xrightarrow{\text{2D IDFT}} \boxed{g(x,y)}$

$\uparrow H(u,v)$ (into $\times$)

* Ideal LPF (ILPF) :-



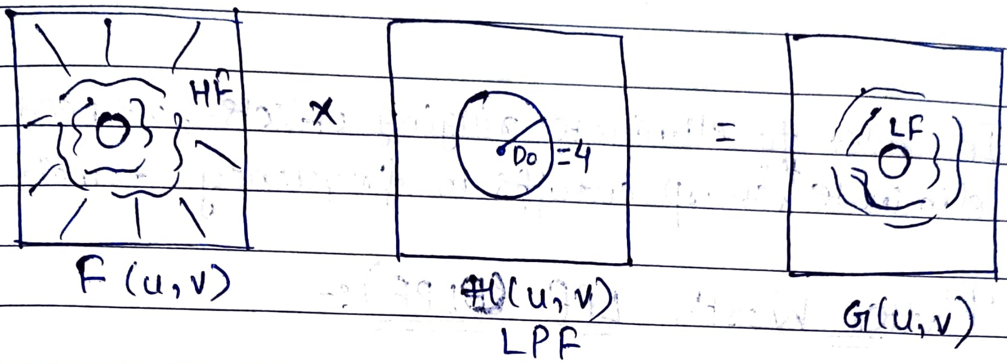
$H(u,v) = 1 \; ; D(u,v) \le D_0$
$= 0 \; ; \text{else}$

where,

$$D(u,v) = \left[\left(u-\frac{M}{2}\right)^2 + \left(v-\frac{N}{2}\right)^2\right]^{1/2}$$

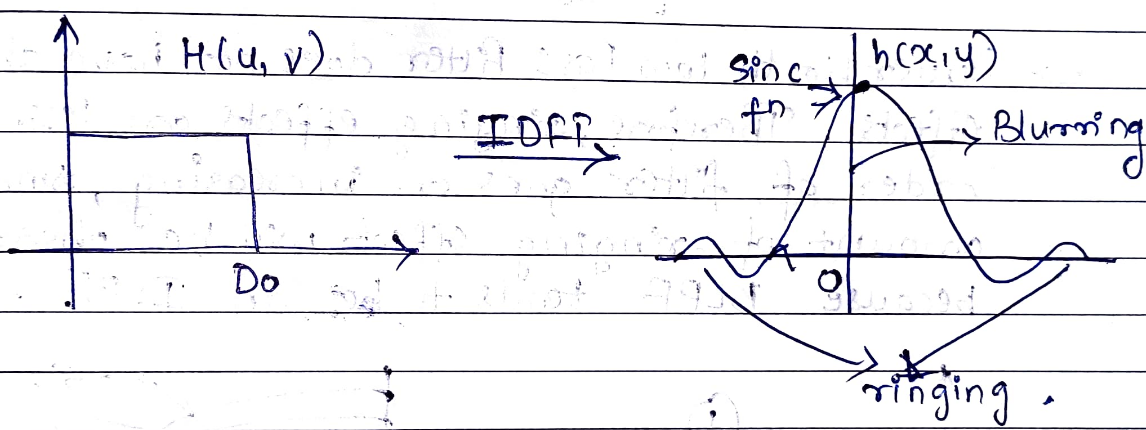
→ $D(u,v)$ gives concentric rings having a fixed value depending on value of D_0, white circle becomes larger or smaller.

* Suppose, $D_0 = 4$



→ It is called as Ideal Filter because it removes everything beyond D_0.

*



→ wrt $h(x,y)$, it has a dominant component at origin that is the center part which is responsible for blurring effect. It also has concentric rings which are responsible for ringing effect. ~~If we~~

→ If we increase D_0, Sinc f^n becomes narrow hence, we get more ringing and blurring effect in O/P image.

→ This is a disadvantage of a ideal LPF.

02/03/2023.

After centering, $\begin{bmatrix} HF & & HF \\ & LF & \\ HF & & HF \end{bmatrix}$ i.e. higher frequencies at edges & lower at center.

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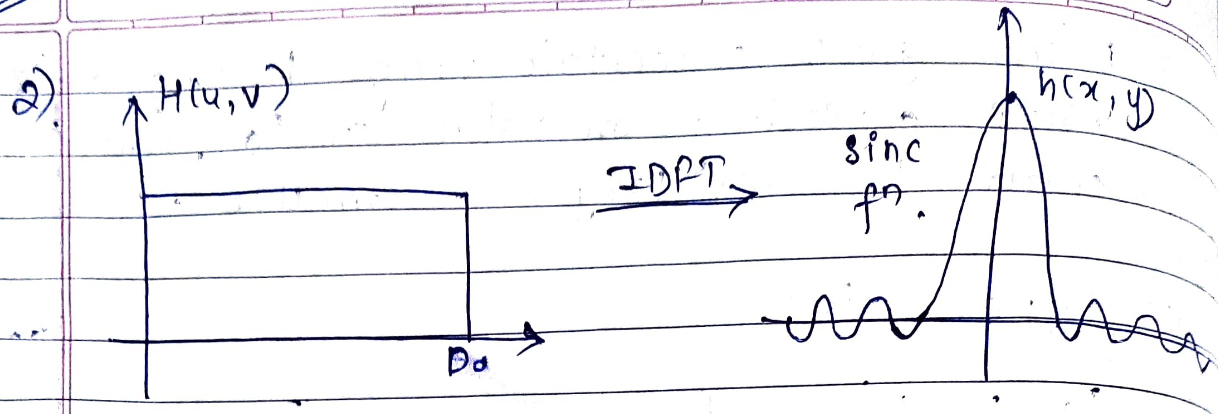
2)

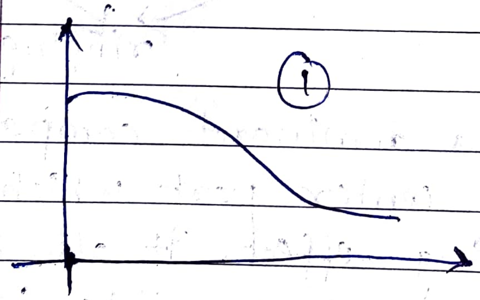


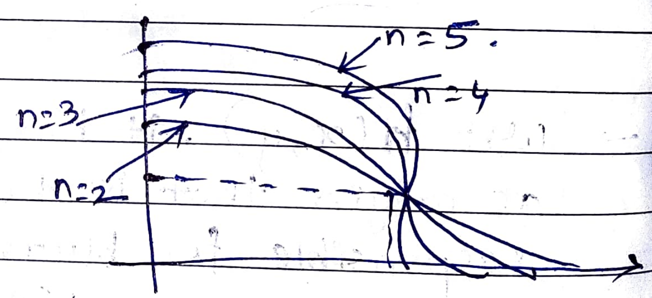
* In order to eliminate ringing effects, we need to eliminate the sharp cutoffs in frequency domain.

★ Butter Worth LPF (BLPF):-

⇒ $H(u,v) = \dfrac{1}{1 + \left[\dfrac{D(u,v)}{D_0}\right]^{2n}}$; n: order.

⇒ Butter Worth Low Pass Filter does not have sharp effects, Therefore, ringing effects are less. As order of filter goes on increasing, small amount of ringing effect will be observed because BLPF tends to be an ILPF.





⇒ Therefore, to get good results, order should be less than or equal to 5 i.e. $n \leq 5$.

LPF	HPF
→ edges blurred	→ edges enhanced
→	→

★ Gaussian LPF (GLPF) :-

⇒ $H(u,v) = e^{-D^2(u,v)/2D_0^2}$

⇒ Response of GLPF is similar to response of BLPF and there are no ringing effects in GLPF.

* GLPF is preferred over ILPF (Justify statement).

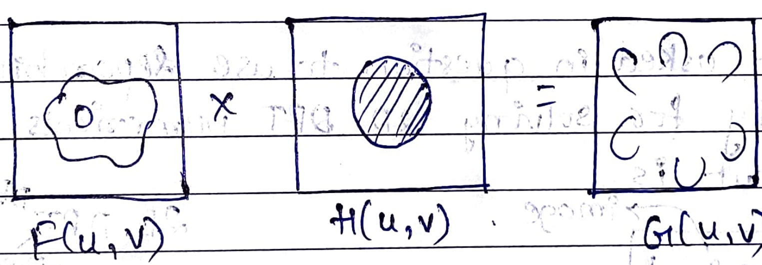
→ Write H(u,v) eqⁿ for both.
Show diagram of ILPF and its disadvantage.

* BLPF is preferred over ILPF (Justify).

★ High Pass Filter :-

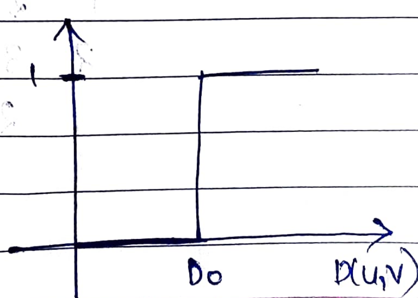
I). ~~Ideal HPF:-~~

HPF = 1 − LPF



1). IHPF :-

$H(u,v) = 0 \quad ; D(u,v) \le D_0$
$\quad\quad\;\; = 1 \quad ; \text{else}$



2). BHPF:-

$$\Rightarrow H(u,v) = \frac{1}{1 + \left[\frac{D_0}{D(u,v)}\right]^{2n}}$$

3). GHPF (Gaussian High Pass Filter):-

$$\Rightarrow H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

* Possible Questions:-

1). Separability Property
2). Translation Property
3). Block Diagram for filtering in F.D.
4). Explain any two or three filters
5). Justify / Contradict Past questions.
6). Problems on 2D DFT Image.

* If not asked in question to use separability property for solving 2D DFT numericals, shortcut is

2D DFT of image → $\boxed{F = T f T^{1}}$ → image

2D DFT: DFT rowwise (ans → rowwise); DFT columnwise (ans → columnwise)

$$f = \begin{matrix} 0 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 2 & 3 & 4 & 3 \\ 1 & 2 & 3 & 2 \end{matrix}$$

$$F = \underset{T}{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}} \underset{f}{\begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 2 & 3 & 4 & 3 \\ 1 & 2 & 3 & 2 \end{bmatrix}} \underset{T'}{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}}$$

$$f = \begin{bmatrix} 4 & 8 & 12 & 8 \\ -2 & -2 & -2 & -2 \\ 0 & 0 & 0 & 0 \\ -2 & -2 & -2 & -2 \end{bmatrix} \begin{bmatrix} 4 & -2 & 0 & -2 \\ 8 & -2 & 0 & -2 \\ 12 & -2 & 0 & -2 \\ 8 & -2 & 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$F = \begin{bmatrix} 32 & -8 & 0 & -8 \\ -8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -8 & 0 & 0 & 0 \end{bmatrix}$$

08/03/2023

* → In addition to 2D-DFT, few more transformations prove to be useful.
→ Image transform refers to a class of unitary matrices used for representing images.
→ Images can be extended as discrete set of basis arrays called as 'basis images'.

* Walsh-Hadamard Transform :-

→ This transform is based on Hadamard matrix which is a square array having entries +1, -1.
→ Hadamard matrix of order '2' is given by

$$H[2] = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \text{ or } \begin{bmatrix} + & + \\ + & - \end{bmatrix}$$

→ Hadamard matrices of order 2^n can be derived using Kronecker product which is given by -

$$H(2^n) = H(2) \cdot H(2^{n-1})$$

gives sign (+, +, +, -).

eg. $H(2^2) = H(4) = \boxed{H(2) \cdot H(2)}$

sequence (denotes sign change)

$$H(4) = \begin{array}{c} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}_{4\times4} \end{array} \begin{array}{l} 0 \\ 3 \\ 1 \\ 2 \end{array}$$

0 {all +ve, no sign change
3 (1+1+1 ⇒ pos to neg, neg to pos, pos to neg

* → Derive Walsh-Hadamard matrix for N=8.

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$H(8) = H(2^3) = H(2) \cdot H(4)$

$$
= \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1
\end{bmatrix}_{8 \times 8}
\begin{array}{c} \text{Sequency} \\ 0 \\ 7 \\ 3 \\ 4 \\ 1 \\ 6 \\ 2 \\ 5 \end{array}
$$

+ + + −

→ Compute Hadamard transform for given sequence $\{1, 2, 0, 3\}$

⇒ $x(n) = \{1, 2, 0, 3\}$

$$
X[K] = \underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}}_{H(4)} \underbrace{\begin{bmatrix} 1 \\ 2 \\ 0 \\ 3 \end{bmatrix}}_{x(n)} = \begin{bmatrix} 6 \\ -4 \\ 0 \\ 2 \end{bmatrix}
$$

$X[K]$ → Hadamard transform of $x(n)$.

$H(4)$ → since 4 values in sequence

Q. For given image, find Hadamard transform.

→
$$
\begin{matrix} 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 2 & 3 & 4 & 3 \\ 1 & 2 & 3 & 2 \end{matrix} = f
$$

→ Hadamard transform of f i.e.

$$\boxed{F = TfT'}$$

$$F = H(4)\, f\, H'(4)$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & +1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 2 & 3 & 4 & 3 \\ 1 & 2 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$$

$$\therefore F = \begin{bmatrix} 6 & 8 & 12 & 8 \\ 2 & 0 & 0 & 0 \\ 0 & -2 & -2 & -2 \\ 0 & -2 & -2 & -2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

$$\therefore F = \begin{bmatrix} 34 & 2 & -6 & -6 \\ 2 & 2 & 2 & 2 \\ -6 & 2 & 2 & 2 \\ -6 & 2 & 2 & 2 \end{bmatrix}$$

* Walsh Transform :-

→ Walsh Transform matrix is obtained from Hadamard matrix by rearranging rows in increasing sign change order.

$$H(4) = \begin{array}{cc} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} & \begin{matrix} 0 \\ 3 \\ 1 \\ 2 \end{matrix} \end{array} \longrightarrow W(4) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

Q. Derive Walsh Matrix for $N = 8$.

~~W(8) =~~

$$H(8) = \begin{array}{c} \left[\begin{array}{cccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{array}\right] \end{array} \begin{array}{c} 0 \\ 7 \\ 3 \\ 4 \\ 1 \\ 6 \\ 2 \\ 5 \end{array}$$

$$\downarrow$$

$$W(8) = \left[\begin{array}{cccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{array}\right] \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array}$$

Q. For given image, find Walsh Transform.

$$\begin{array}{cccc} 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 2 & 3 & 4 & 3 \\ 1 & 2 & 3 & 2 \end{array} = f$$

$\Rightarrow F = T f T'$

$F = W(4) . f \; W'(4)$

$$\therefore F = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 2 & 3 & 4 & 3 \\ 1 & 2 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

$$\therefore F = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

$$\therefore F = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$$

14/03/2023

* Image Morphology :-

⇒ Dialation :-

Let $A(x,y)$ be a binary image, B as structuring element, then the resultant element $C(x,y)$ i.e. O/P image.

$$C(x,y) = Max\{A(x,y) * B\}$$

$$C = A \oplus B$$

A: 1 0 0 0 1 0 0 0

B: 1 1 1

C: 1 0 1

A: 16 17 16 18

B: 1 1 1

C: • 17 18

⇒ Erosion :-

$$C = A \ominus B$$

$$C(x,y) = Min\{A(x,y) * B\}$$

Q. Consider 10x10 image. Perform dialation using structuring element

B =

1	1
1	1

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C = A \oplus B = \max\{A(x,y) * B\}.$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & \boxed{0} & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

1's boundary ↑es, no. of zeros (black part) ↓es.

last row & column :- copy from i/p image or keep it empty.

* Dialation adds pixels to the boundaries and it helps in enlarging or expanding the boundary.
* Size of black box at the centre has reduced and white boundary has increased in size.

Q. Perform Erosion on 10x10 image with same structuring element i.e. $B = \begin{array}{|c|c|} \hline 1 & 1 \\ \hline 1 & 1 \\ \hline \end{array}$ and same image (i/p).

$\Rightarrow$ Erosion :- $C = A \ominus B$

$$c(x, y) = \min \{A(x, y) * B\}.$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

* Size of white boundary has decreased. Black box or gap increased in the size. So, Erosion shrinks the boundaries and enlarges the gaps.

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* Opening :-

→ It is erosion followed by dilation.
→ Consider 'A' as image and 'B' as structuring element.

* $Open(A,B) = A \circ B = [A \ominus B] \oplus B$

* Closing :-

→ It is dialation followed by erosion.

* $Close(A,B) = A \bullet B = [A \oplus B] \ominus B$

Q. Perform opening on given image.

$$A = \begin{array}{cccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}$$

$$B = \begin{array}{|c|} \hline 1 \\ \hline 1 \\ \hline \end{array}$$

$$A \ominus B = \begin{array}{cccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}$$

copy ~~last row~~ from i/p image

$$[A \ominus B] \oplus B = \begin{array}{cccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}$$

* Original image had two white blocks which were connected by thin white strip.
* Opening of this image got rid of this strip.
* Size of white blocks remains unchanged.
* Opening breaks down narrow ridges or isolates objects which are just touching one another.

Q. Perform closing operation.

The marked zeroes are creating gap in block of 1's.

$$A = \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

$$B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

* $$A \oplus B = \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

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$[(A \oplus B)] \ominus B =$

0	0	0	0	0	0	0	0	0	0
1	1	1	1	0	1	1	1	1	1
1	1	1	1	0	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1
1	1	1	1	0	1	1	1	1	1
1	1	1	1	0	1	1	1	1	1
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Gaps are filled here by '1''s.

* Closing operation tends to fuse narrow breaks.
* Boundary extraction can be achieved by $A - \underbrace{(A \ominus B)}_{\text{erosion}}$.

★ Hit Or Miss Transformation :-

→ Small odd size mask (structuring element) normally 3X3 is scanned over entire image (binary).

→ If binary pattern of structuring element matches the pixels of an image then it is HIT condition. In this case, output pixel in spatial correspondence the centre pixel is set to desired binary state normally '1'.

→ If binary pattern does not match with the pixels, it is MISS condition. Output pixel is set to opposite binary state normally '0'.

Q. Given 10x10 image. Apply Hit or Miss Transformation.

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B1 = \begin{matrix} X & 1 & X \\ 0 & 1 & 1 \\ 0 & 0 & X \end{matrix}$$

First output. I_1 :-

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

$$B_2 : \boxed{\begin{matrix} X & 1 & X \\ 1 & 1 & 0 \\ X & 0 & 0 \end{matrix}}, \quad B_3 = \boxed{\begin{matrix} X & 0 & 0 \\ 1 & 1 & 0 \\ X & 1 & X \end{matrix}}, \quad B_4 = \begin{matrix} 0 & 0 & X \\ 0 & 1 & 1 \\ X & 1 & X \end{matrix}$$

$$I_2 = \begin{matrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{matrix}$$

$$I_3 = \begin{matrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{matrix}$$

$$I_4 = \begin{matrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{matrix}$$

$I = I_1 + I_2 + I_3 + I_4 \Rightarrow$ ORing

$$I = \begin{array}{c|cccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

* Another Method.

(I/P image) $\xrightarrow{\text{apply}}$ B1 $\Rightarrow$ ans 1
$(\text{I/P image})^c$ $\xrightarrow{\text{apply}}$ B2 $\Rightarrow$ ans 2
Take AND of ans1 and ans 2.

Q.