This powerpoint file contains a summary repertory of the models available with the QNM solver QNMEig.

The models have been developped over time from 2013 by students and postdocs working at LP2N-CNRS lab in Bordeaux

If you develop your own models with the solver and wish to make them available, you may propose a summary with the reference to the original publication and your email address, so that potential users may contact you directly.
Please address your ppt summary sheet to
philippe.lalanne@institutoptique.fr

plasmonic nanocavities ........................................................ p 2
photonic microcavities & particles ........................................ p 10
gratings and crystals ............................................................ p 17
nonreciprocal resonators ...................................................... p 21

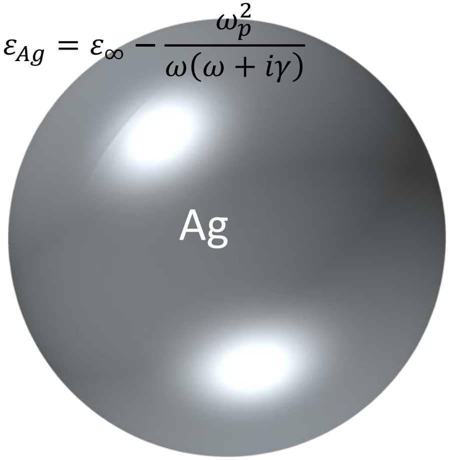
# plasmonic nanocavities

# Silver sphere in air

Contact: Wei Yan <yanwei@westlake.edu.cn>

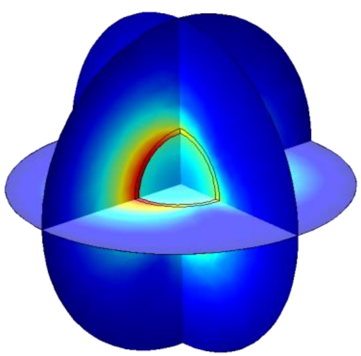
COMSOL model available on the website:
“QNMEig_Sphere.mph’’

$$\varepsilon_{Ag} = \varepsilon_{\infty} - \frac{\omega_p^2}{\omega(\omega + i\gamma)}$$

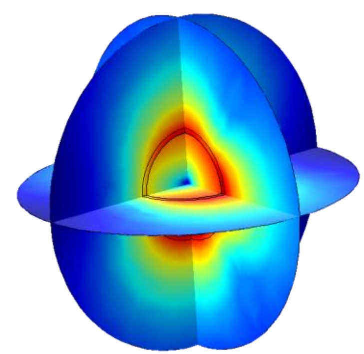



air

Eigenfrequency=9.2471E14+1.5213E14i Multislice: log(emw.normE)



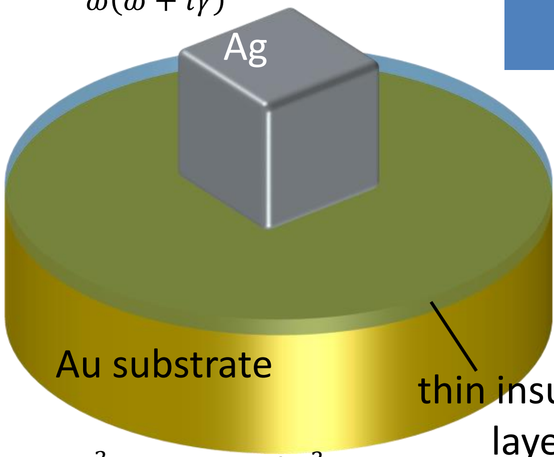
Eigenfrequency=1.2359E15+2.2070E13i Multislice: log(emw.normE)



W. Yan , R. Faggiani, P. Lalanne, Phys. Rev. B **97**, 205422 (2018).
"Rigorous modal analysis of plasmonic nanoresonators"

# Silver nanocube on a coated gold substrate

$$\varepsilon_{Ag} = \varepsilon_\infty - \frac{\omega_p^2}{\omega(\omega + i\gamma)}$$

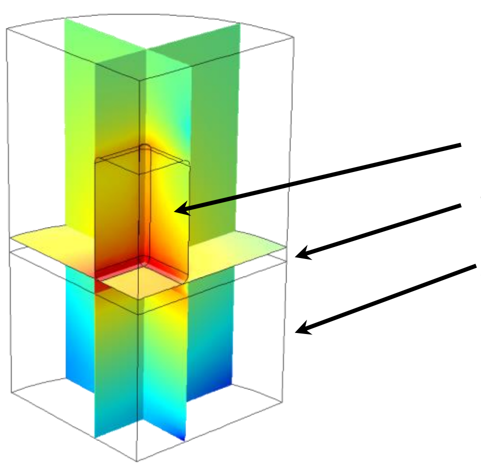



$$\varepsilon_{Au} = \varepsilon_\infty - \frac{\omega_p^2}{\omega(\omega + i\gamma)} - \frac{A\omega_1^2}{\omega^2 - \omega_1^2 + i\gamma_1}$$

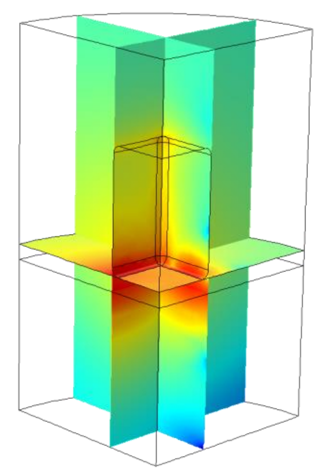
Contact: Wei Yan <yanwei@westlake.edu.cn>

COMSOL model available on the website:
“QNMEig_Cubesubstrate.mph’’

Eigenfrequency=4.5032E14+1.0479E13i Multislice: log(emw.normE)




Eigenfrequency=5.8505E14+3.9778E13i Multislice: log(emw.normE)

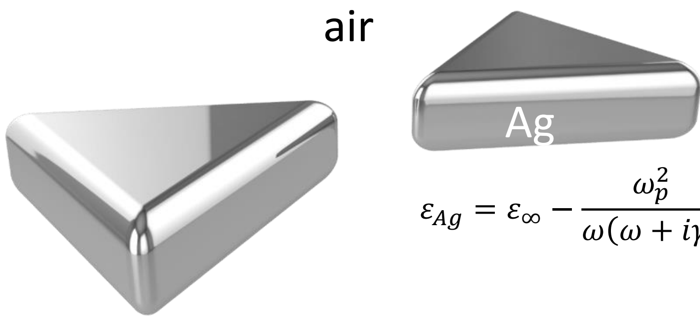


P. Lalanne, W. Yan, A. Gras, C. Sauvan, J.-P. Hugonin, M. Besbes, G. Demesy, M. D. Truong, B. Gralak, F. Zolla, A. Nicolet, F. Binkowski, L. Zschiedrich, S. Burger, J. Zimmerling, R. Remis, P. Urbach, H. T. Liu, T. Weiss, J. Opt. Soc. Am. A **36**, 686 (2019). "Quasinormal mode solvers for resonators with dispersive materials"

# Silver bowtie in air

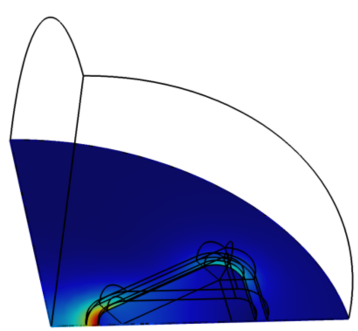
Contact: Wei Yan <yanwei@westlake.edu.cn>

COMSOL model available on the website:
“QNMEig_bowtie.mph’’

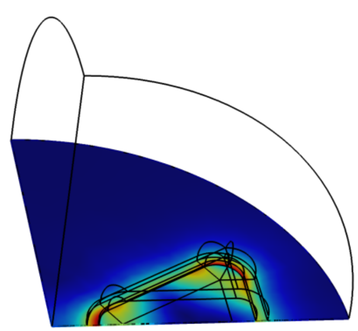



$$\varepsilon_{Ag} = \varepsilon_{\infty} - \frac{\omega_p^2}{\omega(\omega + i\gamma)}$$

Eigenfrequency=7.5277E14+2.947E13i Hz
Slice: Electric field norm (V/m)

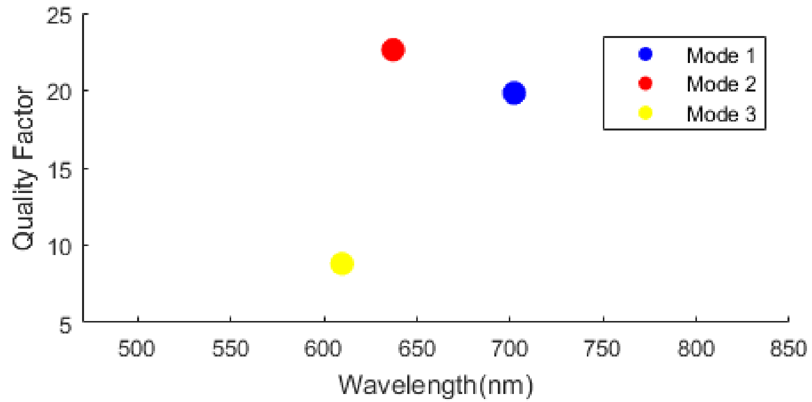


Eigenfrequency=1.0984E15+2.9362E12i Hz
Slice: Electric field norm (V/m)



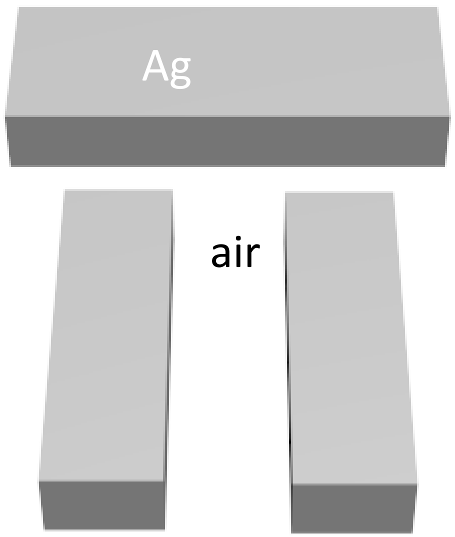
W. Yan , R. Faggiani, P. Lalanne, Phys. Rev. B **97**, 205422 (2018).
"Rigorous modal analysis of plasmonic nanoresonators"

# Dolmen nanoantenna



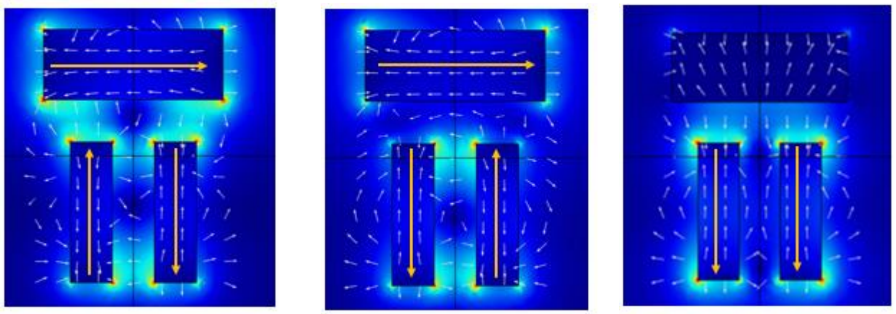

Contact: Tong WU [wutong1121@sina.com](mailto:wutong1121@sina.com)

COMSOL model available on the website:
“QNM_Dolmen.mph”
“QNM_Dolmen_sym.mph”
or associated matlab files




$$\varepsilon_{Ag} = \varepsilon_\infty - \frac{\omega_p^2}{\omega(\omega + i\gamma)}$$

Eigen electric field distributions



Mode1 Mode2 Mode3

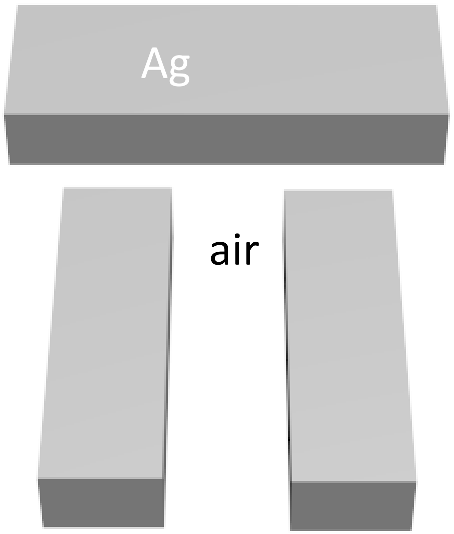
R. Faggiani, A. Losquin, J. Yang, E. Mårsell, A. Mikkelsen, P. Lalanne, ACS Photonics **4**, 897-904 (2017).
"Modal analysis of the ultrafast dynamics of optical nanoresonators"
T. Wu, A. Baron, P. Lalanne, K. Vynck, Phys. Rev. A **101**, 011803(R) (2020).
"Intrinsic multipolar contents of nanoresonators for tailored scattering"

# Multipolar decomposition of QNMs

Contact: Tong WU wutong1121@sina.com

COMSOL model available on the website:
“QNM_Dolmen.mph’’
“QNM_Dolmen_sym.mph’’
or associated matlab files

$$\varepsilon_{Ag} = \varepsilon_{\infty} - \frac{\omega_p^2}{\omega(\omega + i\gamma)}$$




Main features:

- Computation the multipolar decomposition in the vectorial spherical harmonics of a QNM at the complex frequency of the QNM
- Provide multipolar decomposition that is intrinsic (excitation-independent) to the the nanoparticle
- Avoid the traditional problem that a multipolar decomposition at real frequency  excitation dependent, as it depends on the frequency, polarisation, incidence angle of the illumination

$$\tilde{\mathbf{E}} = \frac{n_b^2\tilde{\omega}^2}{c^2}\sum_{n=1}^{\infty}\sum_{m=-n}^{n} E_{nm}\left[\tilde{a}_{nm}\tilde{\mathbf{N}}_{nm}^{(3)}(\mathbf{r}) + \tilde{b}_{nm}\tilde{\mathbf{M}}_{nm}^{(3)}(\mathbf{r})\right]$$

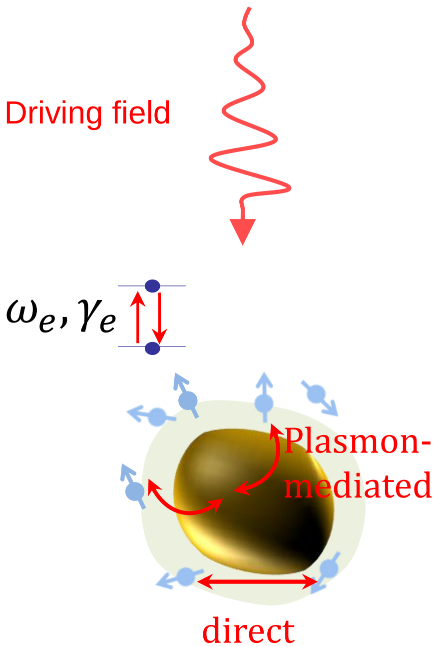
$$\left\{\begin{array}{l} \tilde{a}_{nm} = \dfrac{c^2\int \tilde{\mathbf{E}}\cdot\tilde{\mathbf{N}}_{nm}^{(3)}(R,\Omega)d\,\Omega}{n_b^2\tilde{\omega}^2E_{nm}\int\left|\tilde{\mathbf{N}}_{nm}^{(3)}(R,\Omega)\right|^2 d\,\Omega} \\ \tilde{b}_{nm} = \dfrac{c^2\int \tilde{\mathbf{E}}\cdot\tilde{\mathbf{M}}_{nm}^{(3)}(R,\Omega)d\,\Omega}{n_b^2\tilde{\omega}^2E_{nm}\int\left|\tilde{\mathbf{M}}_{nm}^{(3)}(R,\Omega)\right|^2 d\,\Omega} \end{array}\right. \Rightarrow \left\{\begin{array}{c} \tilde{p}_x \\ \tilde{m}_z \\ \tilde{Q}_{xy}^e \end{array}\right.$$

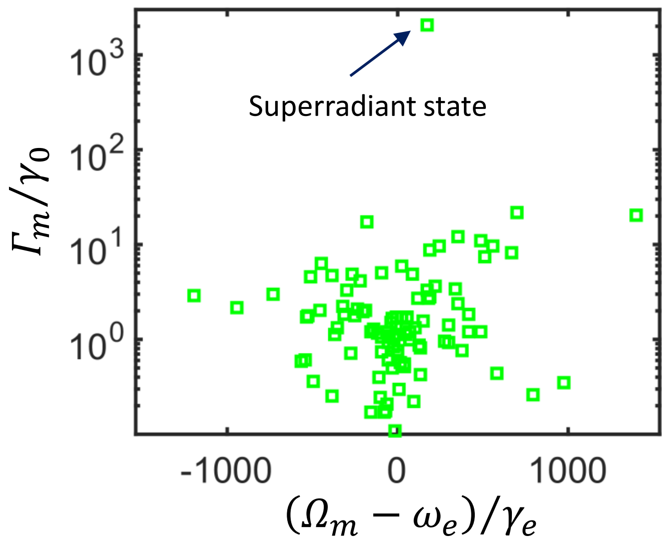

T. Wu, A. Baron, P. Lalanne, K. Vynck, Phys. Rev. A **101**, 011803(R) (2020).
"Intrinsic multipolar contents of nanoresonators for tailored scattering"

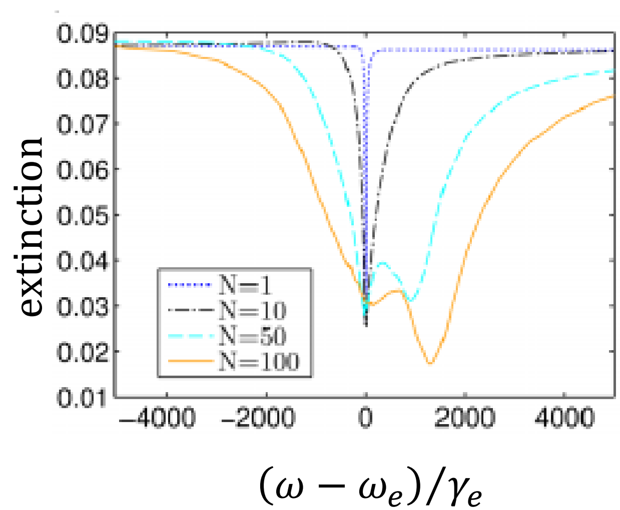
# Superradiance of disordered ensembles of two-level resonators coupled by a plasmonic resonator

Contact: Philippe Lalanne<philippe.lalanne@institutoptique.fr>

Program to compute the superradiant and subradiant states of the coupled ensemble is available in PRB 95, 195418 (2017).



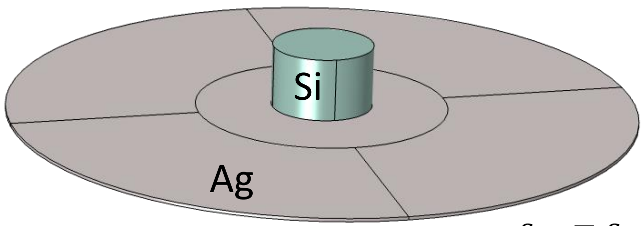




P. Fauché et al., Phys. Rev. B **95**, 195418 (2017).
"Collective scattering in hybrid nanostructures with many atomic oscillators coupled to an electromagnetic resonance"

COMSOL model available on the website:
"QNMEig_NanolettSi.mph''
"QNMpole_NanolettSi.mph''
November 2020

# Silicon cylinder above a thin Ag film

Contact: Tong WU wutong1121@sina.com

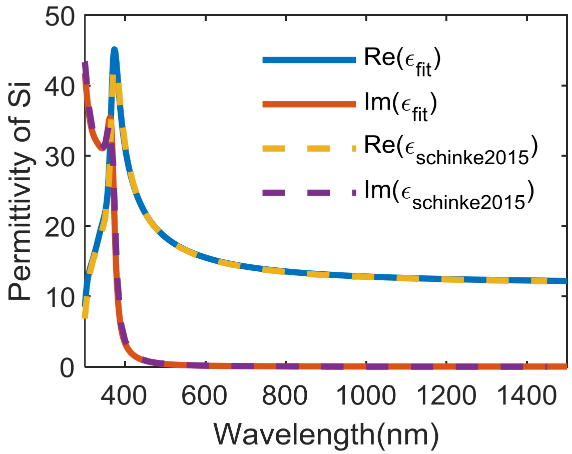



$$\varepsilon_{Ag} = \varepsilon_{\infty} - \frac{\omega_{pAg}^2}{\omega(\omega + i\gamma_{Ag})}$$

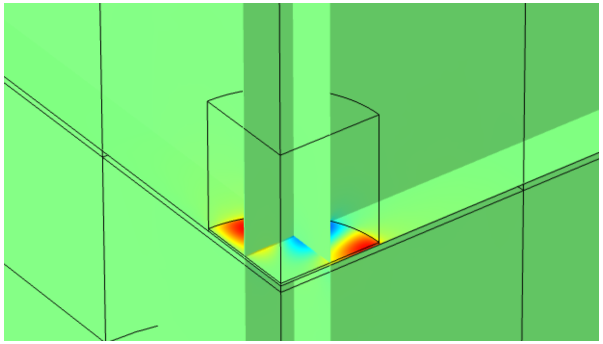
See Fig. 4 in Nano letters, **17**, 3238 (2017) for structure details.

The weak form and normalization factor are specific for "QNMEig_NanolettSi.mph'' in the Si domains owing to the specific dispersion of Si.

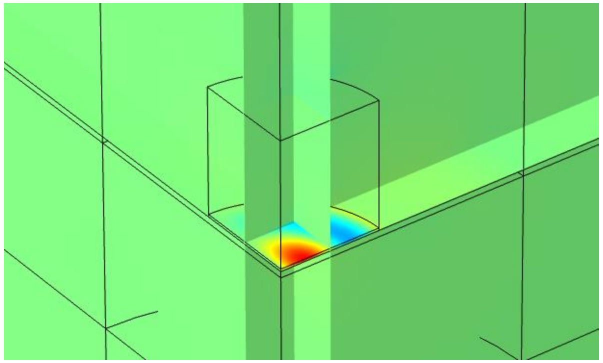
$$\varepsilon_{Si} = \varepsilon_{\infty} + \sum_{j=1}^{4} \frac{A_j}{\omega - \Omega_j} - \frac{A_j^*}{\omega + \Omega_j^*}$$




$\varepsilon_{Si}$ taken from Opt. Lett. **42**, 1145 (2017) is in perfect agreement with the experimental data in AIP Advances **5**, 67168 (2015).



Mode 1: Eigenfrequency 5.8384E14+4.0888E12i Hz



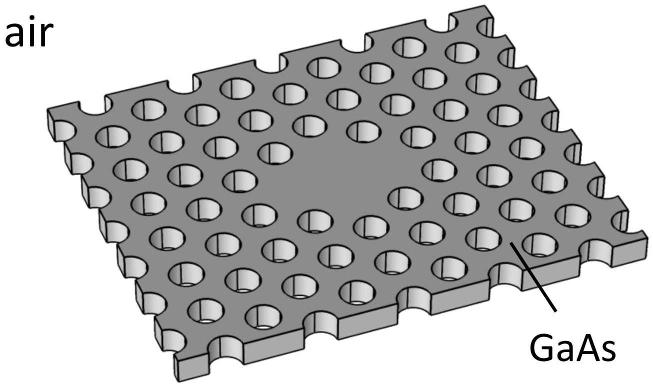
Mode 2: Eigenfrequency 5.8744E14+1.5474E13i Hz

# photonic microcavities and particles

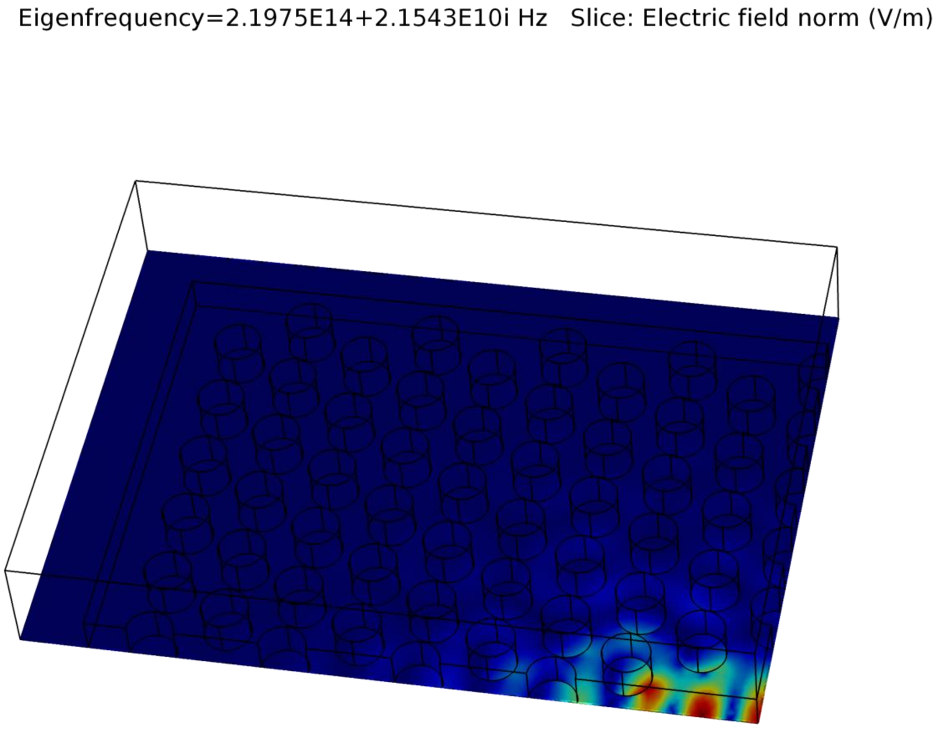
# GaAs photonic crystal membrane in air

Contact: Wei Yan <yanwei@westlake.edu.cn>

COMSOL model available on the website:
“QNMEig_GaAsPhc.mph’’




(no frequency dispersion)



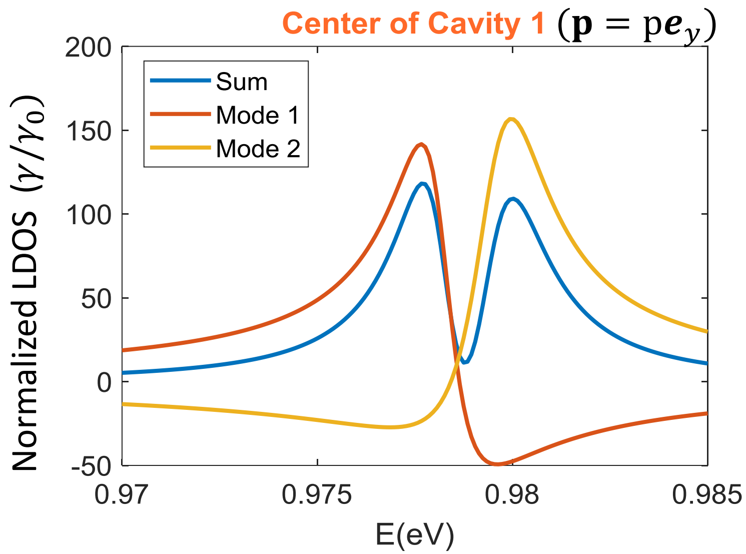

Complex mode volume $\tilde{V}_n(\mathbf{r}) = \left[2\varepsilon_o\varepsilon(\mathbf{r})\left(\tilde{\mathbf{E}}_n(\mathbf{r})\cdot\mathbf{u}\right)^2\right]^{-1}$

K. G. Cognée et al., Optica **6**, 269 (2019).
"[Mapping Complex Mode Volumes with Cavity Perturbation Theory]"

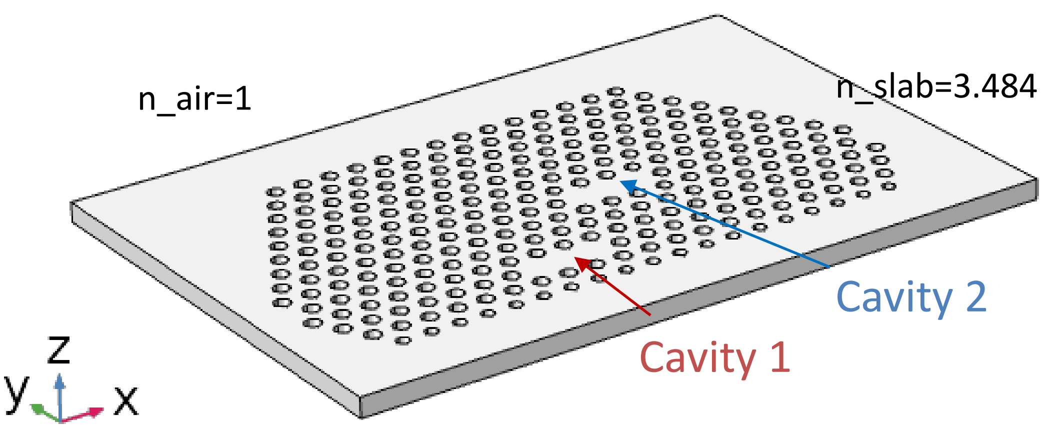
# Coupled photonic crystal cavities: non-Lorentzian LDOS

Contact: Tong WU wutong1121@sina.com

COMSOL model available on the website:
“QNMEig_PhCcoupled.mph’’
November 2019




$$\frac{\gamma}{\gamma_0} = \sum_n F_n \frac{\omega}{\Omega_n} \frac{(\Gamma_n/2)^2}{(\omega-\Omega_n)^2 + (\Gamma_n/2)^2}\left(1 + \frac{\mathrm{Im}(\tilde{V}_n)}{\mathrm{Re}(\tilde{V}_n)}\frac{\omega-\Omega_n}{\Gamma_n/2}\right)$$




$$F_n = \frac{3}{4\pi^2} Q_n \,\mathrm{Re}\left(\frac{(\lambda/n)^3}{\tilde{V}_n}\right) \text{ (Purcell factor)}$$

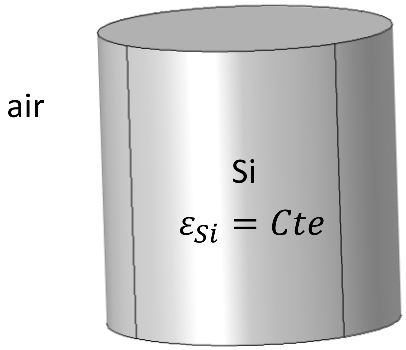
$$\tilde{V}_n(\mathbf{r}) = \left[2\varepsilon_o \varepsilon(\mathbf{r})\left(\tilde{\mathbf{E}}_n(\mathbf{r})\cdot\mathbf{u}\right)^2\right]^{-1}$$

$$Q_n = -\frac{Re(\tilde{\omega}_n)}{2Im(\tilde{\omega}_n)} = \frac{\Omega_n}{\Gamma_n}$$

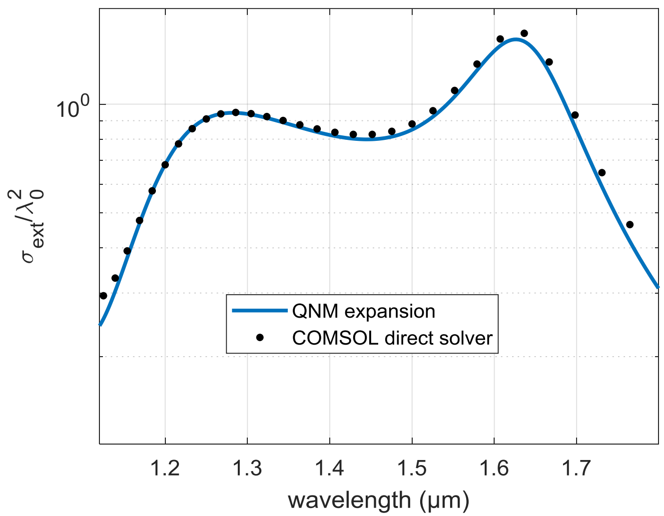
$$\gamma_0 = \frac{\omega^3 n}{3\pi\epsilon_0 c^3 \hbar}|\mathrm{p}|^2$$

D. Pellegrino et al., Phys. Rev. Lett. **124**, 123902 (2020)
"Non-Lorentzian Local Density of States in Coupled Photonic Crystal Cavities Probed by Near- and Far-Field Emission"

# Semiconductor nanorod in air

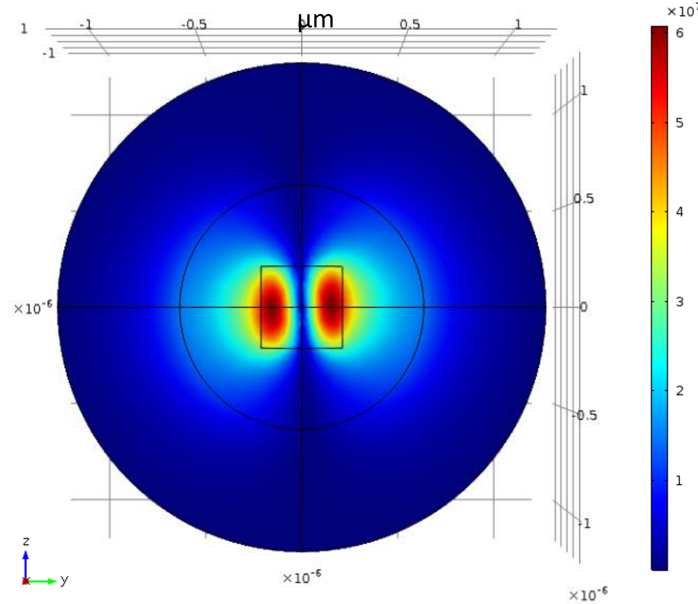



COMSOL model available on the website:
"QNMEig_Nanorod.mph"
January 2019




The extinction computed with the QNM-expansion is obtained with the toolbox **QNMEig_toolbox_1** for an incident plane wave that propagates parallel to the rod axis.

Electric-field norm of mode with $\tilde{\lambda}$ = 1.652-0.8211i

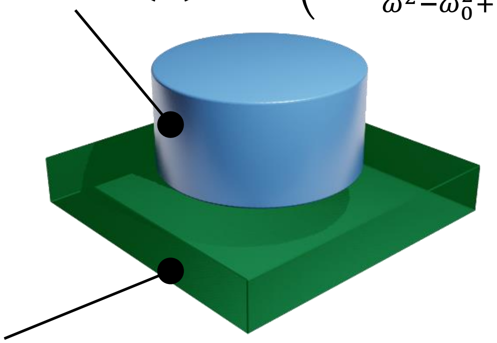


# Nonlinear generation in dielectric nanoparticles on substrate

Contact: Tong WU wutong1121@sina.com

COMSOL model available on the website:
“QNMEig_NLnanodisk.mph’’
March 2020

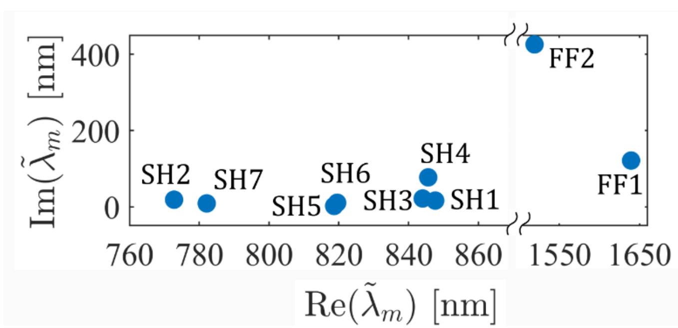
AlGaAs $\varepsilon(\omega) = \varepsilon_\infty \left(1 - \frac{\omega_p^2}{\omega^2 - \omega_0^2 + i\gamma\omega}\right)$

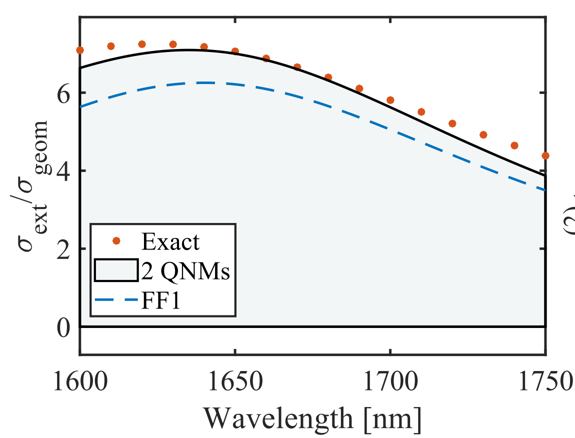


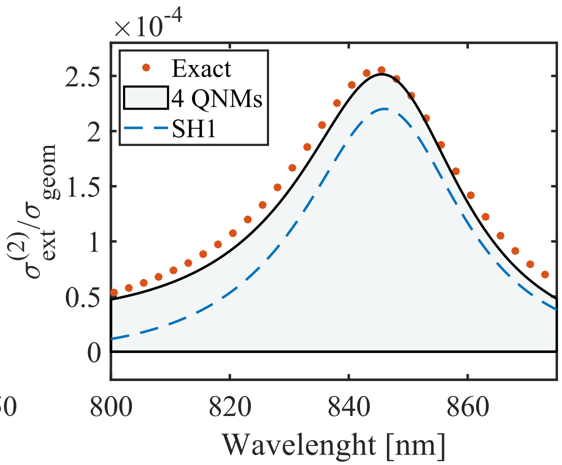
substrate AlOx $n_s = 1.6$

Main features:

- Computation of linear and nonlinear scattering efficiencies by a nanoresonator on a substrate
- Analysis of mode contribution to the scattering
- Study of nonlinear overlap integral and phase matching conditions



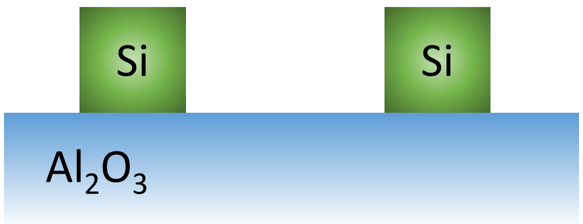

C. Gigli, T. Wu, G. Marino, A. Borne, G. Leo, and P. Lalanne, arXiv 1911.06373 (2019) & ACS Photonics (2020).
"Quasinormal-mode modeling and design in nonlinear nano-optics"

# Two silicon wires on sapphire: TE

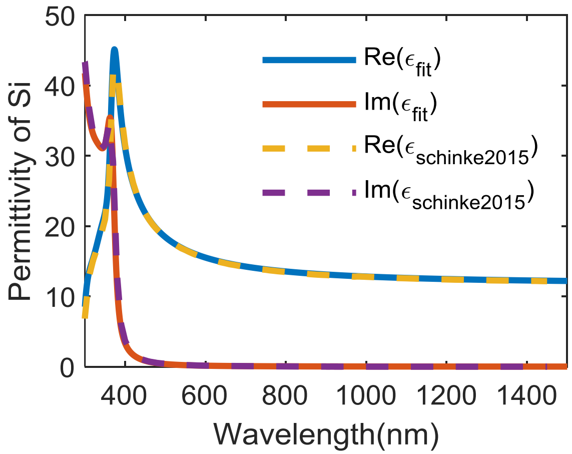
COMSOL model available on the website:
"QNMEig_2DSi_wire_TE.mph''
"QNMpole_2DSi_wire_TE.mph''
November 2020

Contact: Tong WU wutong1121@sina.com

- The "Script_QNM_web_multiobj2D_TE.m" should be used together with "QNMpole_2DSi_wire_TE.mph''
- For " QNMEig_2DSi_wire_TE.mph'', the weak form and normalization factor are changed for the Si domains

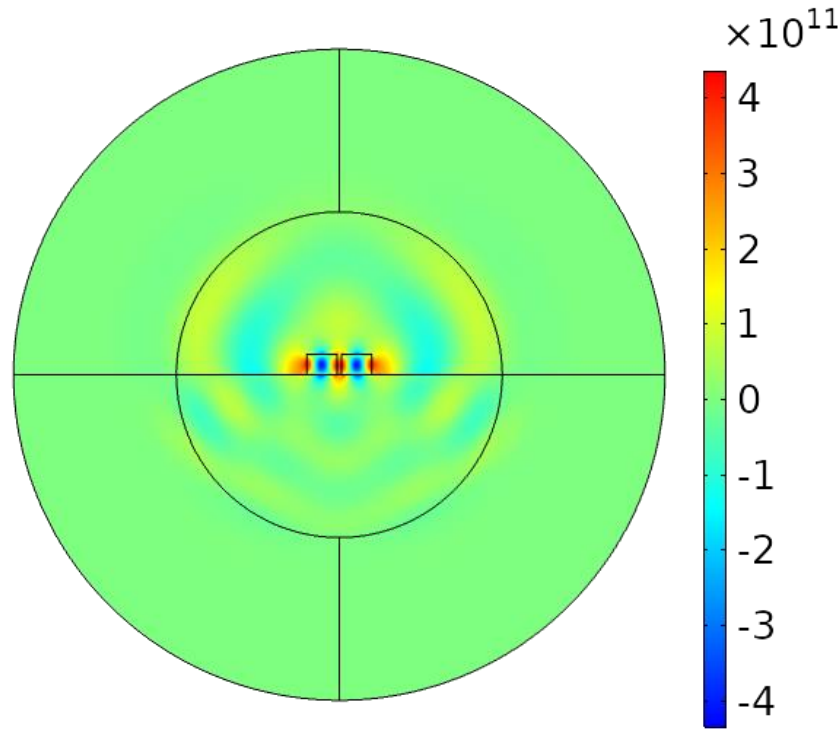



$$\varepsilon_{Si} = \varepsilon_\infty + \sum_{j=1}^{4} \frac{A_j}{\omega - \Omega_j} - \frac{A_j^*}{\omega + \Omega_j^*}$$




$\varepsilon_{Si}$ taken from Opt. Lett. **42**, 1145 (2017) is in perfect agreement with the experimental data in AIP Advances **5**, 67168 (2015).

TM-mode



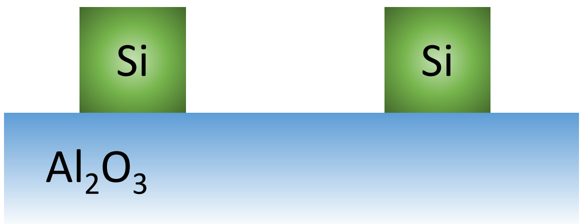

Mode : Eigenfrequency 5.7413E14+1.1172E13i Hz

# Two silicon wires on sapphire: TM

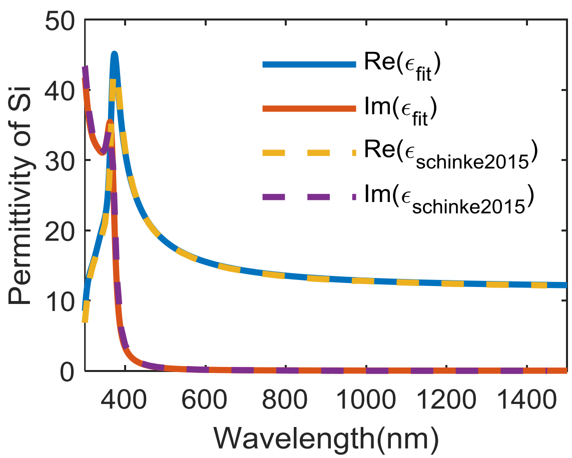
COMSOL model available on the website:
"QNMEig_2DSi_wire_TM.mph''
"QNMpole_2DSi_wire_TM.mph''
November 2020

Contact: Tong WU wutong1121@sina.com

- The "Script_QNM_web_multiobj2D_TM.m" should be used together with "QNMpole_2DSi_wire_TM.mph''
- For "QNMEig_2DSi_wire_TM.mph'', the weak form and normalization factor are changed for the Si domains

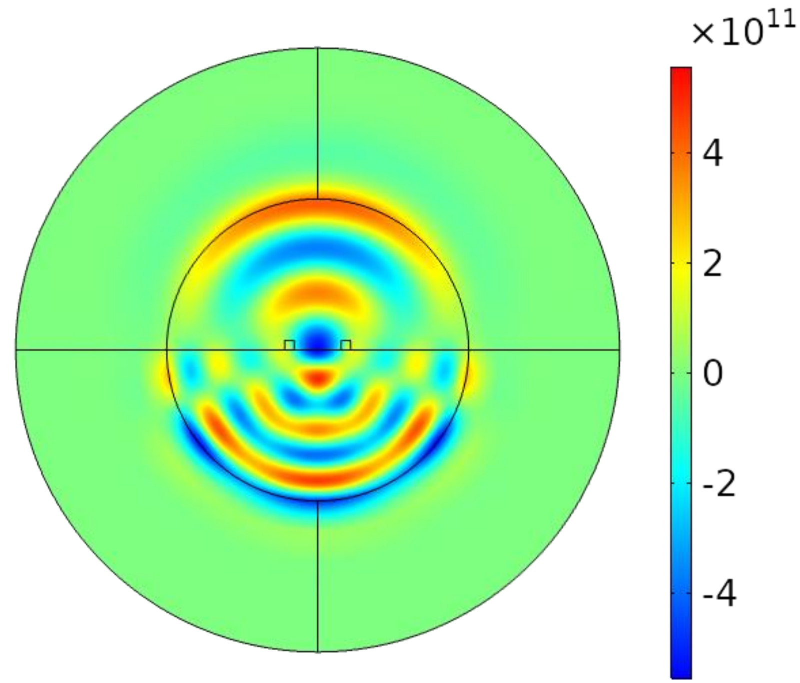



$$\varepsilon_{Si} = \varepsilon_\infty + \sum_{j=1}^{4} \frac{A_j}{\omega - \Omega_j} - \frac{A_j^*}{\omega + \Omega_j^*}$$




$\varepsilon_{Si}$ taken from Opt. Lett. **42**, 1145 (2017) is in perfect agreement with the experimental data in AIP Advances **5**, 67168 (2015).

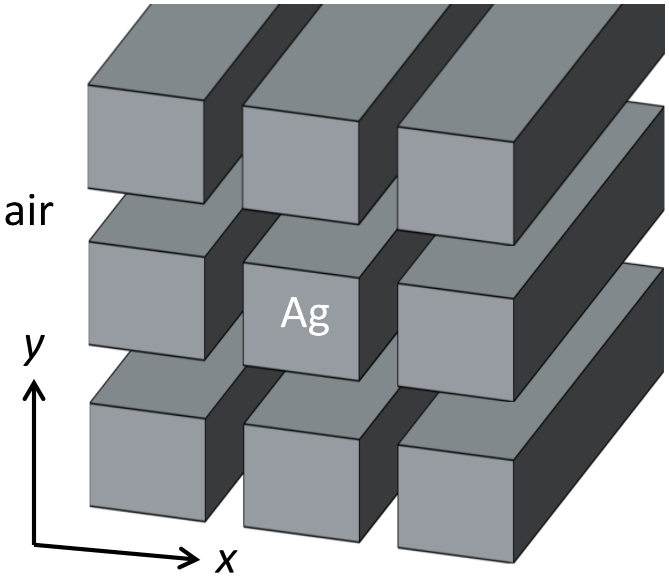
TE-mode




Eigenfrequency 6.5099E14+6.6392E13i Hz

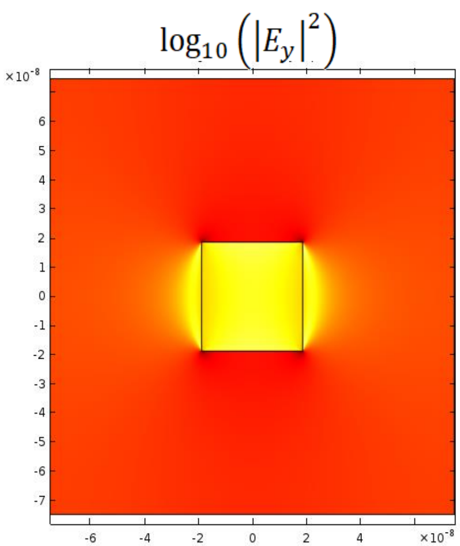
# gratings and crystals

# 2D plasmonic crystal

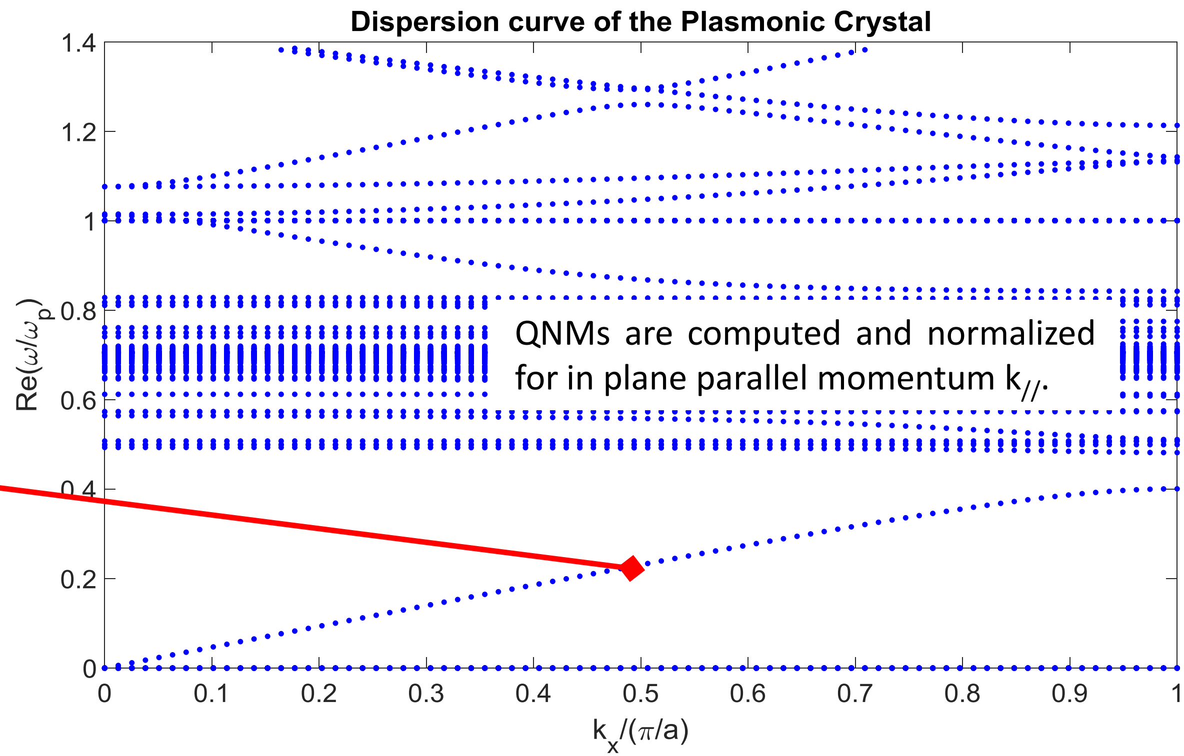



Contact: Wei Yan <yanwei@westlake.edu.cn>

COMSOL model available on the website:
“QNMEig_PlasmCrystal.mph’’




Eigenmode $k_x$=0.5

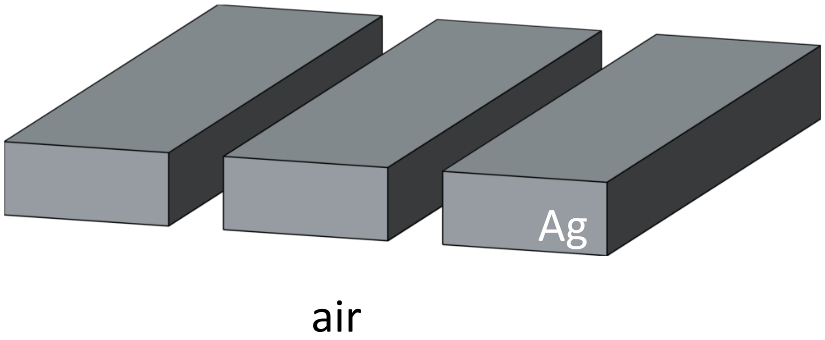


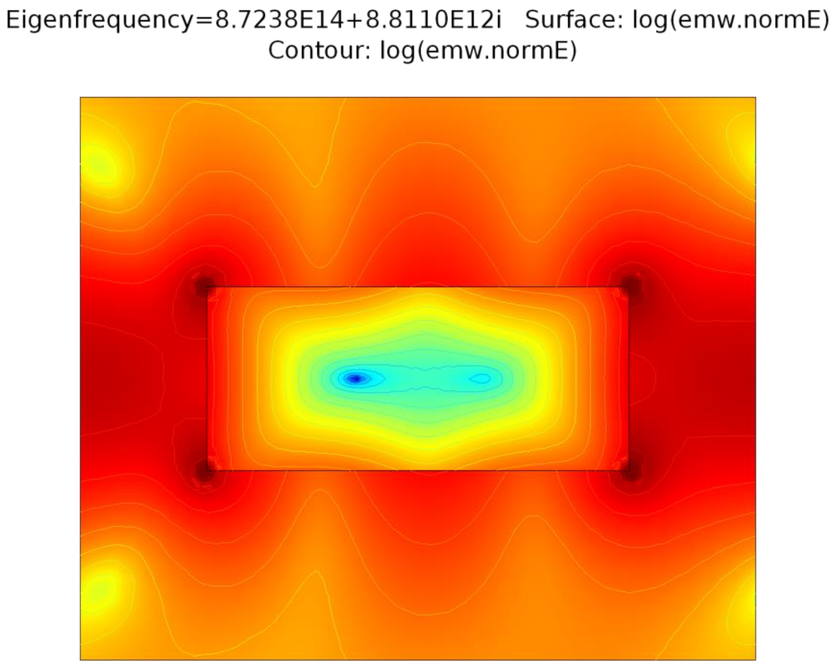

P. Lalanne, W. Yan, A. Gras, C. Sauvan, J.-P. Hugonin, M. Besbes, G. Demesy, M. D. Truong, B. Gralak, F. Zolla, A. Nicolet, F. Binkowski, L. Zschiedrich, S. Burger, J. Zimmerling, R. Remis, P. Urbach, H. T. Liu, T. Weiss, J. Opt. Soc. Am. A **36**, 686 (2019). "Quasinormal mode solvers for resonators with dispersive materials"

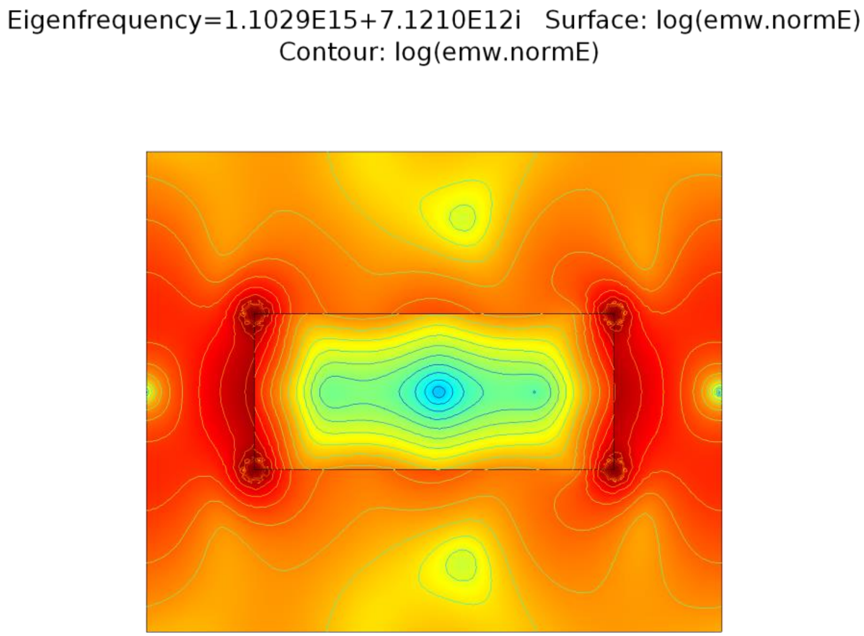
# 2D periodic slit array in a silver membrane in air

Contact: < philippe.lalanne @institutoptique.fr>

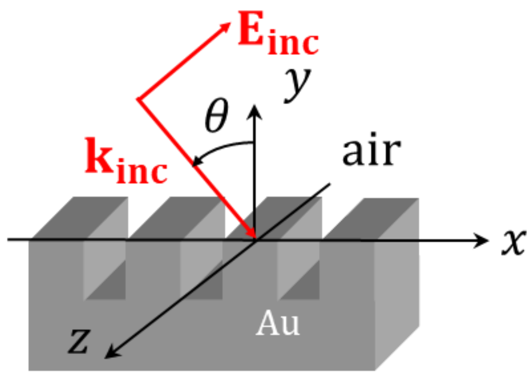
COMSOL model available on the website:
“QNMEig_1DGrating.mph’’




P. Lalanne, W. Yan, A. Gras, C. Sauvan, J.-P. Hugonin, M. Besbes, G. Demesy, M. D. Truong, B. Gralak, F. Zolla, A. Nicolet, F. Binkowski, L. Zschiedrich, S. Burger, J. Zimmerling, R. Remis, P. Urbach, H. T. Liu, T. Weiss, J. Opt. Soc. Am. A **36**, 686 (2019). "Quasinormal mode solvers for resonators with dispersive materials"

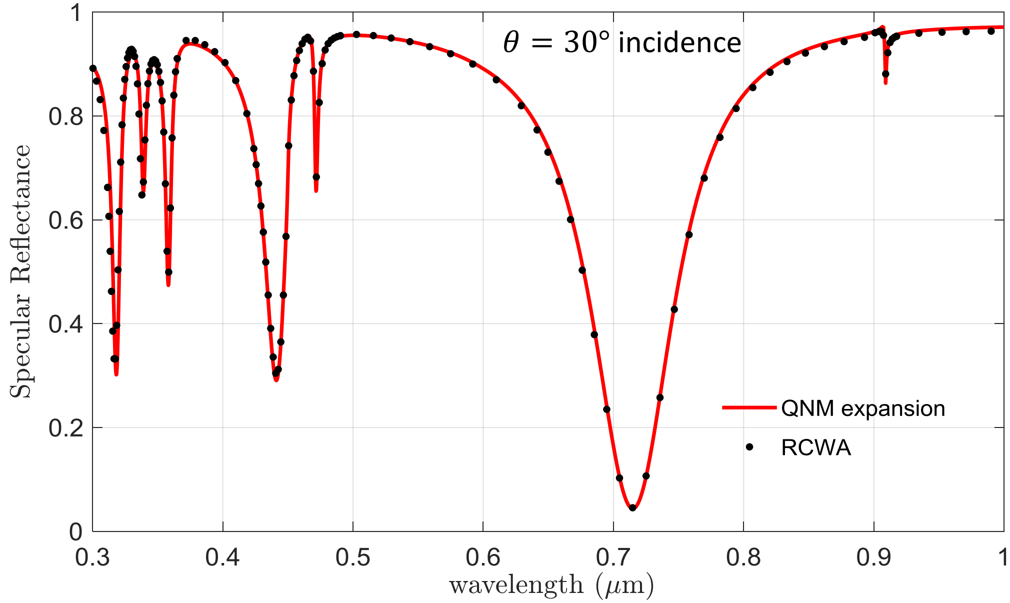



# 2D grating with grooves on a gold substrate with fixed incidence angle

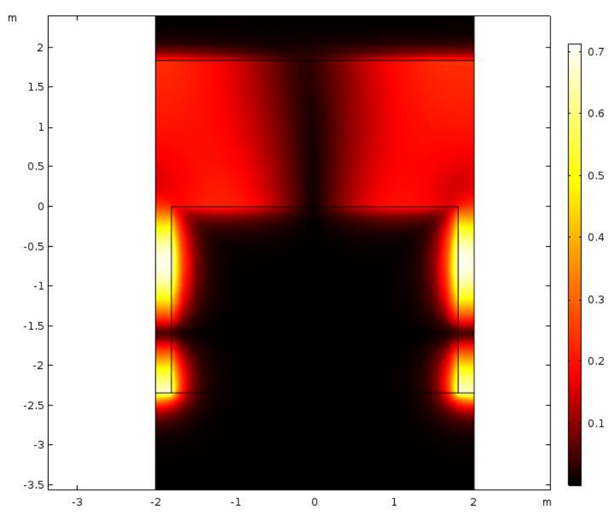
Contact: <philippe.lalanne@institutoptique.fr>

COMSOL model available on the website:
"QNMEig_grating_theta.mph"

The specificity of the model is that QNMs are computed and normalized for a fixed angle of incidence $\theta$, which is exactly what is happening in many experiment: the wavelength is scanned while $\theta$ is maintained constant.




Normalized $|\tilde{\mathrm{H}}_z|$ of mode at
$\tilde{\lambda}_m = 714.8 + 39.0i$ nm
for $\theta = 30°$



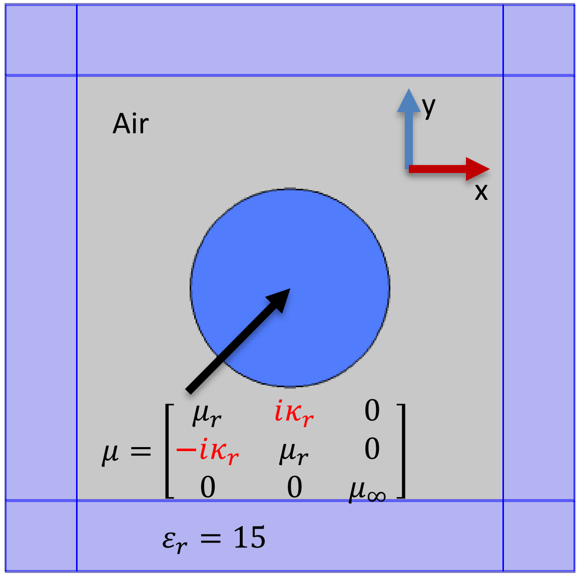
A. Gras, W. Yan and P. Lalanne, Opt. Lett. **44**, 3494 (2019).
"Quasinormal-mode analysis of grating spectra at fixed incidence angles"

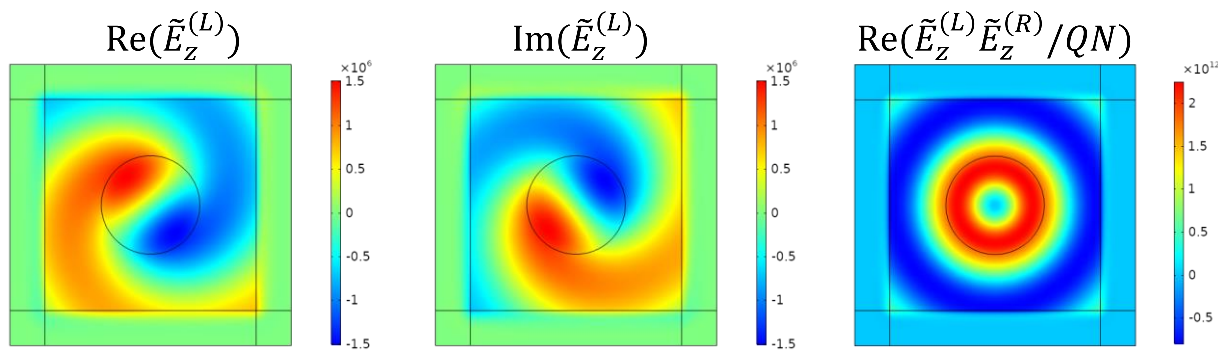
# Nonreciprocal resonator

COMSOL model available on the website:
"QNMEig_wire_YIG.mph''

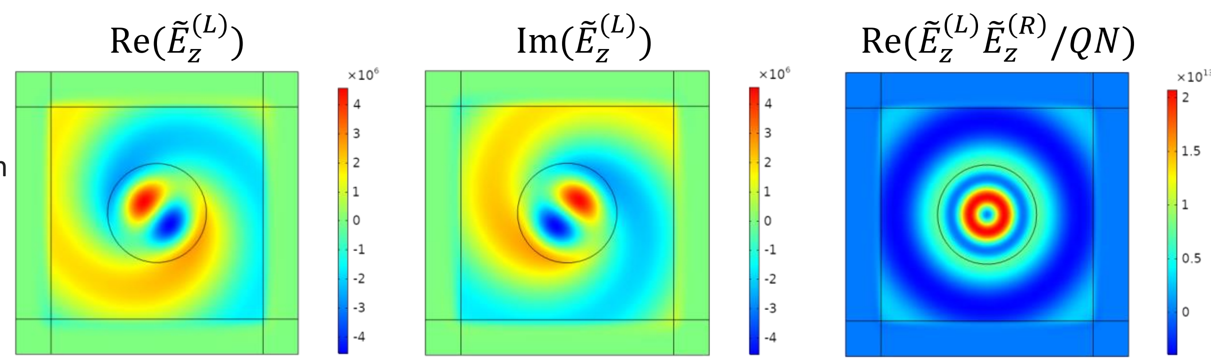
# Non-reciprocal YIG cylinder

Contact: Tong WU wutong1121@sina.com




Mode 1: Eigenfrequency 7.901+0.040627i GHz




Mode 2: Eigenfrequency 9.7908+0.16432i GHz

The permeability formula for the YIG is taken from [Phy. Rev. B **97**, 014419 (2018) ]

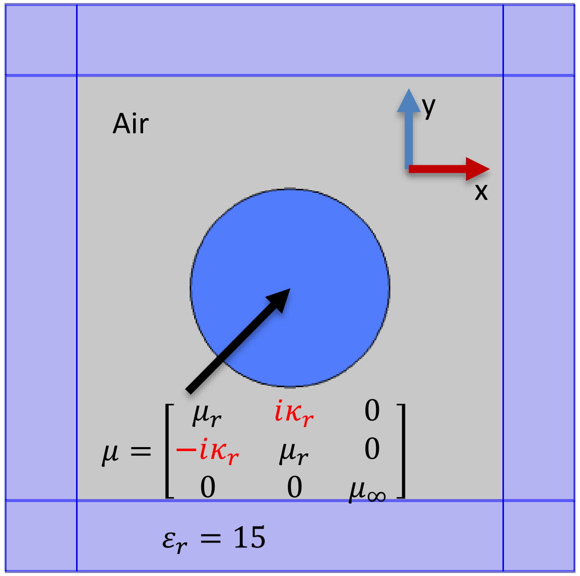
$$\mu_r = \mu_\infty\left[1 + \frac{(\omega_H - i\alpha\omega)\omega_M}{(\omega_H - i\alpha\omega)^2 - \omega^2}\right]$$

$$\kappa_r = \frac{\mu_\infty\omega\omega_M}{(\omega_H - i\alpha\omega)^2 - \omega^2}$$

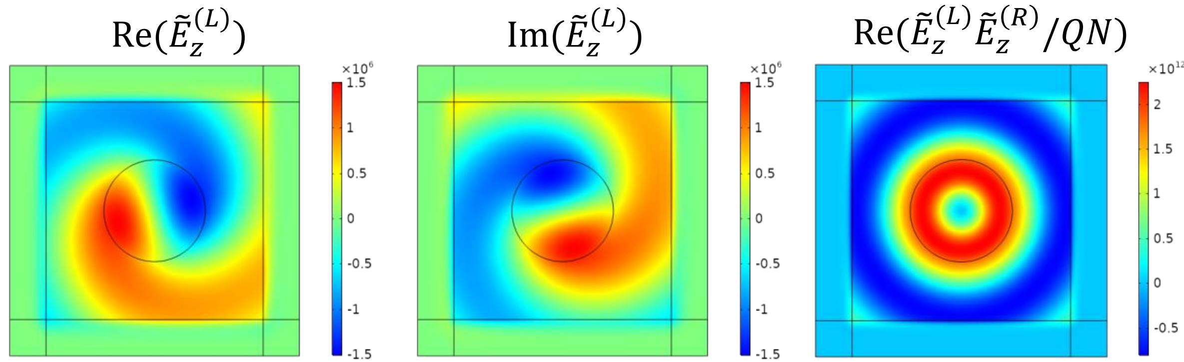
The weak form and normalization factor are changed for the YIG.

# Non-reciprocal YIG cylinder

COMSOL model available on the website: "QNMPole_wire_YIG.mph''
"QNMpole_unis.m" should be used for the computation.




Contact: Tong WU wutong1121@sina.com

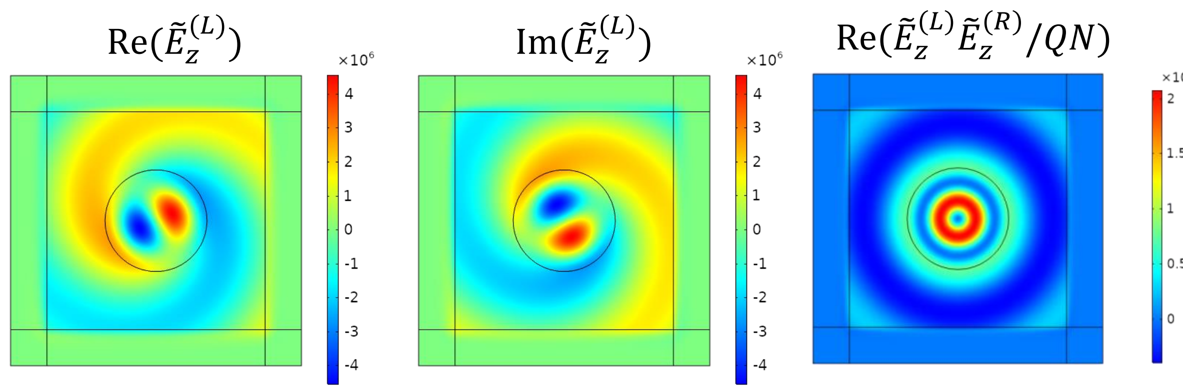



Mode 1: Eigenfrequency 7.901+0.040627i GHz
(exactly same with the result given by QNMEig)

The permeability formula for the YIG is taken from [Phy. Rev. B **97**, 014419 (2018) ]

$$\mu_r = \mu_\infty \left[ 1 + \frac{(\omega_H - i\alpha\omega)\omega_M}{(\omega_H - i\alpha\omega)^2 - \omega^2} \right]$$

$$\kappa_r = \frac{\mu_\infty \omega \omega_M}{(\omega_H - i\alpha\omega)^2 - \omega^2}$$




Mode 2: Eigenfrequency 9.7908+0.1643i GHz