2014 first attempt

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1. State the value of the Chandrasekhar limit and explain why there are no white dwarfs with a mass that exceeds it. [4]
   State what the 'Schwarzschild radius' is, and give an alternative name for it. [3]

Chandrasekhar limit is equal to 1.4 solar masses
If a star has a mass greater than this, when it dies the force of gravity will be stronger than the repulsive force between atoms (electron degeneracy pressure)
When the mass is larger than this limit protons and electrons form to produce a mass made up only of nutrons - a nutron star

Schwarzchild radius - If a mass is confined to this radius, the escape velocity at the radius is greater than the speed of light, and so a black hole is formed:

$$r_{sch} = \frac{2GM}{c^2}$$

Also known as gravitational radius?

2. Briefly explain what 'Cepheid variables' are, and why they are useful in determining distances to galaxies. [6]

Cephid variables are stars which pulsate. They pulsate at a consistent rate based on their size/luminosity
If we know the period of the pulse of the cephid, we can know its luminosity.
If we know its actual luminosity, we can determine its distance from us (measured brightness is inversely proportional to the square of the distance)
This allows us to measure the distance of galaxies to us.
We can also measure the shift in wave length from other stars near the chephid variables, which can be used to determine the velocity of recession from us.
Knowing the distance and velocity allows us to make calcualtions for hubbles consstan, and so the age of the universe.

3. Give a brief account of the origin of the cosmic microwave background radiation in the early Universe and explain how it verifies the Cosmological Principle. [7]

Soon after the big bang the universe was incredibly hot. This high temperature meant that electrons and protons had too much energy to form atoms.
This meant the universe was filled with charged particles
This meant it was opaque to light.
When the universe cooled down, electrons and protons could bind. This is known as recombination.
When they had bound, the universe was transparent to light. Any photons around at this time could travel without being stopped.
We can still detect some of these photons as CMBR.

4. A street lamp emits light with a power output of 80 W at a wave-length of 589nm. What is the energy of a single photon of emitted light? How many photons are emitted per second by this lamp? [6]
   Consider a light source which emits photons with half the energy of the photons given off by the street lamp. Would this light be visible to the human eye? [2]

A)

$$E = \frac{hc}{\lambda} \rightarrow E = 3.38 * 10^{-19} J$$

$$\frac{Photons}{s} = \frac{80\frac{J}{s}}{3.38 * 10^{-19} J/photon} = 2.37 * 10^{20}$$

B)

$$E = \frac{3.38}{2} * 10^{-19} \rightarrow \lambda = \frac{hc}{E} = 1178nm$$

This is infra-red therefore not visible

5. Describe an experiment which implies that light travels as a wave, giving a short but clear argument for why the observations support a wave interpretation. Describe a second experiment which implies that light has particle characteristics. How is what is observed in this experiment incompatible with the classical wave theory of light? **[6]**

Youngs double slit experiment - If we let a coherent sorce of light split into 2, the measured light coming from the slits forms an interference patern. This supports the idea of light being a wave, with the bright and dark fringes on the screen being a result of constructive and destructive interference at the superposition on the screen.

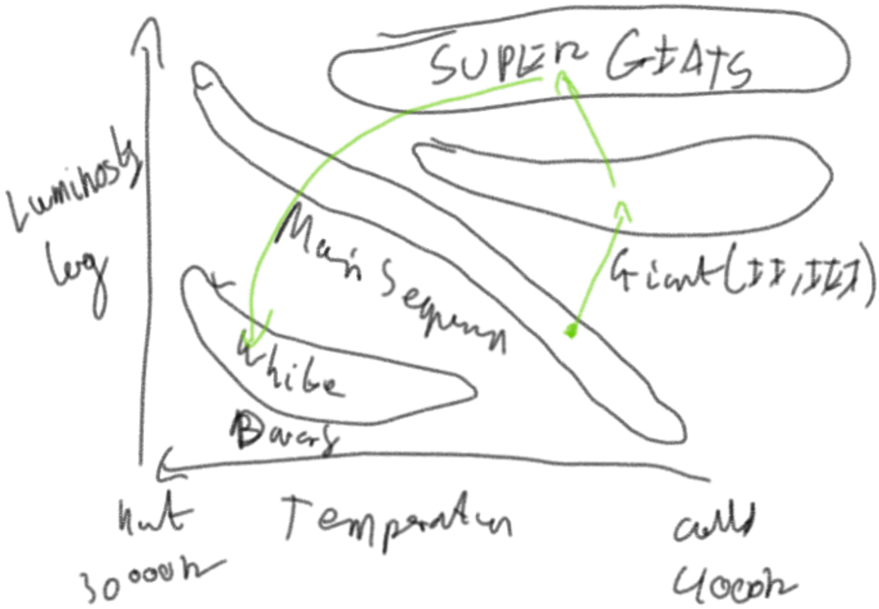
Photoelectric effect - if light of a high enough frequency is incident on a metal surface, electrons are emmitted.
This occurs as the electrons absorb photons of light.
This only occurs if the frequency (ie the colour) of the light reaches a thershold value. If a higher intesnity of light below this thershold is used, no photo-electrons are produced.
This fact is incompatible with weave theory of light. If light were only a wave, a higher energy from higher intesnity would produce photoelectrons.

6. The Higgs boson has received a lot of attention in the news this year. State another example of a boson and an example of a fermion. What is the Pauli exclusion principle? Give one example where the Pauli exclusion principle plays an important role in atomic physics. **[6]**

Boson - photon
Fermion - electron
Pauli exclusion principle - no two fermions can have the same quantum state.
This also means no 2 fermions (protnos, nutrons) can exist in the same state.
At high pressures, forcing two fermions produces a resistive pressure.
This occurs in white dwarfs close to the point of turing into nutron stars
If too much pressure exists, the electron degeneracy pressure is overcome and protons and electrons fuse to become nutrons.

7. (a) Draw a labelled Hertzsprung–Russell diagram. Show on it the evolutionary track followed by a $1\text{-M}_\odot$ star, starting from the main sequence. Mark the principal stages of the star's evolution on your sketch, and describe them in a few sentences. **[13]**

(b) Calculate the main sequence lifetime (in years) of a $10\text{-M}_\odot$ star if it has a luminosity of $10^4\ L_\odot$ and 10% of its mass will be converted from hydrogen to helium in the core. Compare this main sequence lifetime to that of the Sun. What will be the end state of the $10\text{-M}_\odot$ star? **[7]**



Planatary Nebular, ,

Planatory Nebula,
Proto star - gas clumps together -
friction produced heat
U
Main sequence - high enough pressure & temp for fussion.

Red giant - Stops fusing H, stars burning outer core - grows,
Star collapse - into white dwarf.

B) $m = 4 \times 1.0078 - 4.0026$

$= 0.0286$ amu

$= 4.7476 \times 10^{-29}$ kg

$L = 10^4 \times 3.8 \times 10^{26}$

$= 3.8 \times 10^{30}$ J/s

$10M \times 10\% = M_\odot = 2 \times 10^{30}$ kg

$E = mc^2$

$= 4.273 \times 10^{-12}$ J / nuclei

$\div 4 \times 1.0078 \times 1.66 \times 10^{-27}$

$E = mc^2$

$= 4.273 \times 10^{-12}$ J / nucleon

$\div\ 4 \times 1.0078 \times 1.66 \times 10^{-27}$

$= 6.39 \times 10^{14}$ J/kg

$$\frac{3.8 \times 10^{30} \text{ J/s}}{6.39 \times 10^{14} \text{ J/kg}} = 5.95 \times 10^{15} \text{ kg/s}$$

$$\frac{2 \times 10^{30}}{5.95 \times 10^{15}} = 3.36 \times 10^{14} \text{ s}$$

$\rightarrow\ \div (60 \times 60 \times 24 \times 365)$

$= 1.07 \times 10^{7}$ years

$\rightarrow$ 10 million years

Our Sun will live for 9×10^{9} years

$\rightarrow$ end state = Neutron star

($> 8 M_\odot$)

$(>8 M_\odot)$

8. (a) Sketch a diagram of the 'rotation curve' for our Milky Way Galaxy. Comment on the shape of the rotation curve, and the main inference to be drawn from it. **[6]**

(b) Suppose that the Sun is in a circular orbit of radius 8.0 kpc about the Galactic Centre, with an orbital velocity of 220 km s^{-1}. Determine the number of orbits completed by the Sun since the birth of the solar system. Based on this result, discuss the nature of the spiral arms in galaxies. **[9]**

(c) In a galaxy, at what redshift is the Lyman$-\alpha$ line (rest wavelength 121.6 nm) observed at 141.1 nm? Calculate the distance to the galaxy in units of Mpc. **[5]**

a)

The curve doesn't fit the expected kepler curve -
this means there is more mass than is visible.
→ dark matter.

b)

distan per orbit =

$2 \times \pi \times 8 \times 10^3 \times 3.1 \times 10^{16} m$

$= 1.56 \times 10^{21} m$

$\frac{1.56 \times 10^{21} m}{220 \times 10^3 m/s} = 7.1 \times 10^{15} s^{-1}$

$\frac{7.1 \times 10^{15}}{60 \times 60 \times 24 \times 365} = 2.25 \times 10^8 year^{-1}$

age = 4.5×10^9 year

→ 20 orbits

$\rightarrow$ 20 orbits

Spiral arms can't be star movement — if so after 20 orbits they would be wound too tight to be visible.

Instead they are density waves — high density areas of gas that don't move much — high density $\rightarrow$ more star formation $\rightarrow$ brighter.

c) $$\left(\frac{\lambda - \lambda_0}{\lambda_0}\right) = \frac{v}{c}$$

red shift $= \dfrac{141.1 - 121.6}{121.6}$

$= 0.16$

$\rightarrow v = 0.16 \times c = 4.8 \times 10^7 \, m/s$

Hubble Law:

$$v = H_0 d$$

$$d = \frac{v}{H_0}$$

$v = 4.8 \times 10^7 \, m/s$

$= 4.8 \times 10^4 \, km/s$

$$d = \frac{4.8 \times 10^4 \, km/s}{75 \, km/s/Mpc}$$

$= 640 \, Mpc$

9. (a) Name an experiment in which electrons exhibit wave-like properties, describe its setup and what is observed. [3]

(b) What is the de Broglie wavelength of i) an electron travelling at 1 metre per second? ii) a football (weighing 0.2 kg) travelling at 1 metre per second? [3]

(c) Why do we not see wave-like behaviour in macroscopic objects such as a football? [2]

(d) Consider a wavefunction $\psi(x)$ associated with a quantum particle in one-dimension. Which of the following conditions is fulfilled when a wavefunction $\psi(x)$ is normalised? [1]

i) $\int_{-\infty}^{\infty} \psi(x)dx = 1$

ii) $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$

or iii) $\int_{-\infty}^{\infty} |\psi(x)| dx = 1$.

(e) Describe one further mathematical condition (in addition to normalisation) which a physical wavefunction must satisfy. [1]

(f) Consider a particle in a well with the wavefunction:

$$\psi(x) = \begin{cases} A(9-x^2) & -3 \leq x \leq 3 \\ 0 & \text{elsewhere} \end{cases}$$

Sketch this wavefunction and calculate (to 2 significant figures) the value of A which normalises $\psi(x)$. [5]

(g) What is the probability of finding the particle between $x = 0$ and $x = 2$? [3]

(h) By solving an appropriate integral or via another clear argument show that the expectation value for the position of this particle is $\langle x \rangle = 0$. [2]

A) Young double slit experiment

Cohernt light is split by 2 small slits to produce 2 sources of coherent light with no phase difference.

A screen is placed a distance away, such that the distance from the middle of the slits to the screen is far larger than the distance between the two slits.

On the screen, bright and dark fringes are seen.

This is a result of interference between the two sources of light which have travelled different distances and so have different phase differences.

B)

$$\lambda = \frac{h}{p}$$

$$h = 6.63 * 10^{-34}$$

$i)$

$$p = mv = 9.11 * 10^{-31} kg * 1 = 9.11 * 10^{-31}$$

$$\lambda = 7.28 * 10^{-4} m$$

$ii)$

$$\lambda = 3.32 * 10^{-33} m$$

C) The wave length is far too small, the wavelength is smaller than that of visible light so we see no difference.

For wave like behaviour to be seen we need a similar order of magnitude

D)

Normalised:

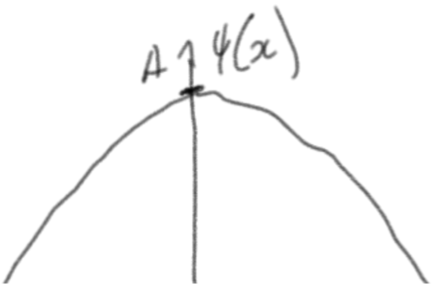
$$\rightarrow \int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$$

$\therefore ii$

E)

Wave form, and its first derivative, must be continuous

F)



$$\int_{-3}^{3} A^2(9-x^2)^2 \, dx = N$$

$$\int_{-3}^{3} 81 - 18x^2 + x^4 = \frac{1}{A^2}$$

$$\left[81x - 6x^3 + \frac{x^5}{5}\right]_{-3}^{3} = \frac{1}{A^2}$$

$$\frac{648}{5} - -\frac{648}{5} = 259.2$$

$$\frac{1}{A^2} = 259.2$$

$$A = \frac{1}{\sqrt{259.2}}$$

$$= 0.062$$

g) $$A^2 \int_0^2 (9-x^2)^2 \, dx = P$$

$$\frac{1}{259.2}\left[81x - 6x^3 + \frac{x^5}{5}\right]_0^2 = P$$

$$P = 0.46$$

h) $$\langle x \rangle = \int_{-\infty}^{\infty} |\varphi(x)|^2 x \, dx$$

$$= A^2 \int_{-3}^{3} 81x - 18x^3 + x^5 \, dx$$

$$= A^2 \left[\frac{81}{2}x^2 - \frac{9}{2}x^4 + \frac{x^6}{6} \right]_{-3}^{3}$$

$$= 0$$

Also -> symmetric about $x=0$

$\therefore \langle x \rangle = 0$

10. (a) Describe Rutherford's planetary model for the hydrogen atom. Describe two ways in which this model is inconsistent with experimentally observed properties of hydrogen. **[4]**

(b) Write down two of the additional postulates that Niels Bohr added to the planetary model to define his model of the atom. **[2]**

(c) Electron orbits in Bohr's model have energy:

$$E_n = \frac{-13.6}{n^2}\text{eV}$$

where n is a positive integer. Show that this is consistent with the Rydberg formula for the wavelengths λ of spectral lines of the hydrogen atom **[6]**

$$\frac{1}{\lambda} = R_H \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$$

(where n and m are positive integers) and hence derive the value of Rydberg's constant R_H in SI units.

(d) Describe one way in which the Bohr model is inconsistent with experimentally observed properties of hydrogen. **[1]**

(e) In a fully quantum mechanical treatment of the hydrogen atom, the wavefunction solutions to the time independent Schrödinger equation for the hydrogen Atom are indexed by three integer *quantum numbers* n, l and m, where

- n is any positive integer, e.g. $1, 2, 3, \ldots$.
- l is any non-negative integer less than n, e.g. $0, 1, 2, \ldots, n-1$.
- m is any integer such that $|m| \leq l$, e.g. $-l, -l+1, \ldots, -1, 0, 1, \ldots, l-1, l$.

Which combinations of quantum numbers correspond to wavefunctions with energy $E = -3.4$eV? **[4]**

(f) At which energy are there 9 distinct quantum numbers? Write out the valid combinations of quantum numbers for this energy. **[3]**

A) Dense nucleus of positive charge

Electrons orbit the nucleus

This is inconsistent with hydrogen -

Elements were known to have unique signatures of emmited light - this can't be explained simply by orbits

If this model were true, electrons would spiral into nucleus - this doesn't happen

B)

Only specific orbits were allowed

Only specific angular momentum for electrons allowed - allows model to be quantised

C)

$$E_n = \frac{-13.6}{n^2}\,eV$$

$$E_m - E_n = -13.6\,eV\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

$$\lambda = \frac{hc}{E}$$

$$\Rightarrow \frac{1}{\lambda} = \frac{E}{hc}$$

$$\frac{1}{\lambda} = \frac{-13.6\,eV}{hc}\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

$$\Rightarrow R_H = \frac{-13.6\,eV}{hc}$$

$$= \frac{-13.6 \times 1.6\times10^{-19}}{6.63\times10^{-34} \times 3\times10^{8}}$$

$$= 1.1\times10^{7}\ m^{-1}$$

d) Finer spectra seen in modern experiment explain-

& Angular momentum of H in ground state: 0

e) ?

11. The time-independent Schrödinger equation (TISE) in one-dimension is:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x).$$

where m is the mass of the particle, E its energy, $\psi(x)$ the wavefunction and $V(x)$ the potential.

(a) Consider a particle in a region of constant potential, $V(x) = V_0$. Consider the following wavefunctions: **[7]**

$$\psi_1(x) = A\sin(kx)$$
$$\psi_2(x) = Be^{-Kx}$$

Which of these wavefunctions is a solution of the TISE with this potential if $E < V_0$? Which is a solution if $E > V_0$? Determine the relationships between k and E, V_0, m and $\hbar$, and between K and E, V_0, m and $\hbar$.

(b) Without performing any further calculations, describe the process of quantum tunnelling. Include a sketch of a relevant potential and describe how quantum predictions differ from those of classical physics. **[4]**

(c) The probability that a quantum particle with mass m and energy E will tunnel through a square barrier of width L and height U is approximately equal to:

$$P = \exp[-2CL]$$

where

$$C = \frac{\sqrt{2m(U-E)}}{\hbar}.$$

In a Scanning Tunnelling Microscope (STM) electrons tunnel from an electrode across a potential barrier to a surface, completing a circuit whose current I is proportional to the tunnelling probability. Draw a labelled diagram of an STM indicating the path of an electron through the device and how the current is read out. **[4]**

(d) If L represents the distance between electrode and surface in the STM, the current I passing through the device will be given by the expression

$$I \propto \exp[-2CL].$$

Imagine you want to design an STM where a change in L by 1 Angstrom causes the current to change by a factor of 10. What value of constant C will achieve this? If the energy of incident electrons is 1eV, what barrier height U corresponds to the value of C you have just calculated?

$$-\frac{\hbar}{2m}\frac{d^2\psi(x)}{dx^2} = (E - V_0)\,\psi(x)$$

$$\frac{d\psi_1(x)}{dx} = Ak\cos(kx)$$

$$\frac{d^2\psi_1(x)}{dx^2} = -Ak^2\psi(x)$$

$$\frac{d\psi_2(x)}{dx} = -BKe^{-kx}$$

$$\frac{d^2\psi_2(x)}{dx^2} = Bk^2\psi(x)$$

$\psi_1(x)$:

$$-\frac{\hbar}{2m}(-Ak^2) = E - V_0$$

$$\rightarrow E - V > 0$$

$$\therefore \psi_1 \rightarrow E > V_0$$

$$\psi_2 \rightarrow E < V_0$$

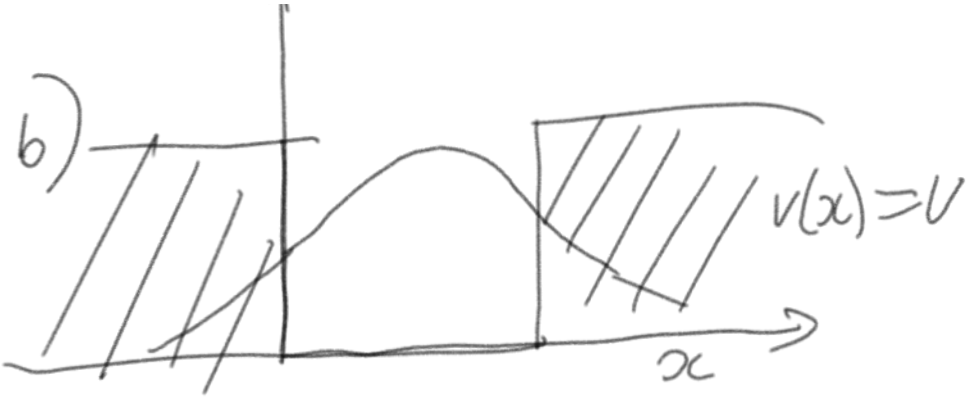
$\psi_1(x)$: $k^2 = \frac{(E - V_0)2m}{\hbar A}$

$\psi_2(x)$

$$\frac{-\hbar}{2m}Bk^2 = E - V_0$$

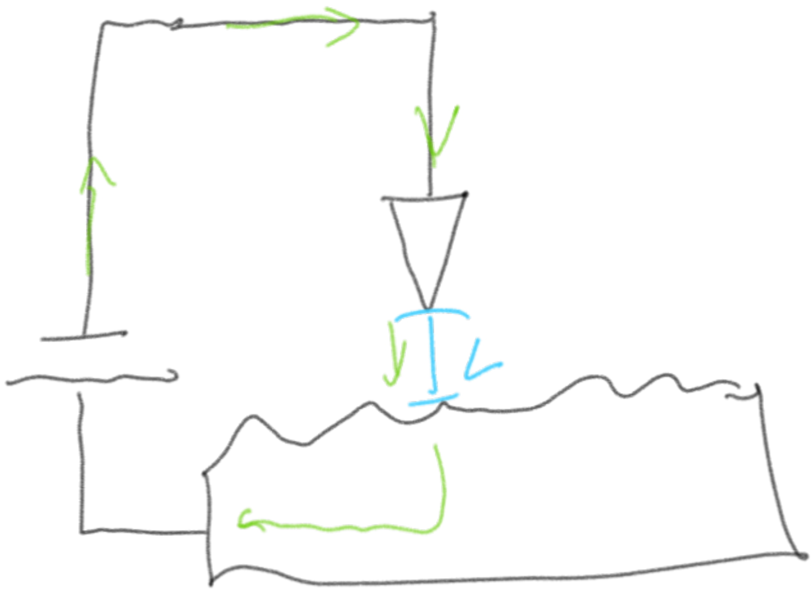
$$K^2 = \frac{V_0 - E}{\hbar B}2m$$

b)



Finite square well -
wave function must be
continuous.
If wave function meets
boundary with $V(x) > E$, the
wave must continue ∴
there is a probability of
of particle tunnelling through
potential.

c)



d) $I_0 = k e^{-2CL}$

$-c(L - 10^{-10})$

d)

$$I_0 = k e^{-2cL}$$

$$I_1 = k e^{-2c(L - 10^{-10})}$$

$$= k e^{-2cL} e^{2c \times 10^{-10}}$$

$$= I_0 e^{2c \times 10^{-10}}$$

$$I_1 = 10 I_0$$

$$10 I_0 = I_0 e^{2c \times 10^{-10}}$$

$$10 = e^{2c \times 10^{-10}}$$

$$\frac{\ln(10)}{2 \times 10^{-10}} = c$$

$$= 1.15 \times 10^{10}\ m^{-1}$$

$$1.15 \times 10^{10} = \frac{\sqrt{2 m_e (U - E)}}{\hbar}$$

$$\frac{(1.15 \times 10^{10} \hbar)^2}{2 m_e} + E = U$$

$$\frac{1.15 \times 10^{10} \times 1.05 \times 10^{-34}}{2 \times 9.11 \times 10^{-31}} + 1.6 \times 10^{-19} = U$$

$$U = 8.61 \times 10^{-19}\ J$$

$\div 1.6 \times 10^{-19} = 6eV$