ASU CHEATSHEET:

Common Equations:

$P = \sigma A T^4 = 4\pi r^2 \sigma T^4 = L$

$\lambda_{max} T = 2.9 \times 10^{-3}\,mK$

$E = hf \quad p = E/c = h/\lambda$

$I = 2ckT/\lambda^4$

$\frac{r}{r_\odot} = \left(\frac{T_\odot}{T}\right)^2 \sqrt{\frac{L}{L_\odot}}$

$R_{sch} = 2GM/c^2$

$v = H_0 d$

$\Delta\lambda/\lambda = v/c$

$T_0 = 1/H_0$

Thomson: e^- oscillate at same freq as incident waves and emit photons at this frequency.

Compton: $\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta)$ Deflection

Wave	Particle
- YDS Exp	- Compton
- Mach-Zehnder	- BB Radiation

Thomson Plum Pudding
↓ Geiger + Marsden Exp
Rutherford Model (Planetary)
- No spectral lines
- No stability of matter
↓
Bohr Model: $L = \hbar n = mvr$
$r = a_0 n^2 = \frac{1}{Z} n^2 \times 10^{-10}$
$E = -13.6\,eV/n^2$
↓
QM Model (shells):
- Pauli Exclusion gives each shell certain # of electrons with fixed Energy
- H-like model for for-out electrons.

Quantum #s:

Principal # = $n = 1, 2, 3, 4 \ldots$
Orbital # = $n - 1$
Magnetic # = $n - 2$

Rydberg:

$\frac{1}{\lambda} = R_H\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$

LYMAN: m = 1 UV
BALMER: m = 2 VIS
PASCHEN: m = 3 IR
BRACKET: m = 4
PFUND: m = 5

Pros	Cons
- Stable Atoms	- Only for H
- Predicted Rydberg + R_H = constant	- Ad-hoc
	- No QM

Wavefunctions:

$\int_a^b |\psi(x)|^2 dx = Prob(a \le x \le b)$

$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = N \neq 1$

So normalise: $\psi_{norm}(x) = \frac{\psi(x)}{\sqrt{N}}$

$\psi(x)$ and $\psi'(x)$ must be continuous and finite

$\langle x \rangle = \int_{-\infty}^{\infty} x|\psi(x)|^2 dx$

$\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2|\psi(x)|^2 dx$

$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$

$\Delta p \Delta x \ge \hbar/2$ (Uncertainty).

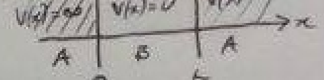
TISE:

$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$ (Solve for $\psi(x)$ and E)

Free Particle: $V(x) = 0$, $E = KE = p^2/2m$

Infinite Square Wells:

$V(x) \begin{cases} 0 & 0 \le x \le L \\ \infty & \text{otherwise} \end{cases}$

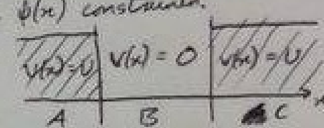


$\psi(x) = A\sin\left(\frac{n\pi x}{L}\right) \quad E = \frac{h^2 n^2}{8mL^2} \quad n = 1, 2, 3 \ldots$

So Energy quantised, $\psi(x)$ constrained.

Finite Square Wells:

$V(x) \begin{cases} 0 & 0 \le x \le L \\ U & \text{otherwise} \end{cases}$



$\psi_A(x) = Ae^{\kappa x} \quad \psi_B(x) = B_1\cos(kx) \quad k = \sqrt{2m(E-U)}/\hbar$
$\psi_C(x) = Ce^{-\kappa x} \quad \text{or} = B_2\sin(kx) \quad k = \sqrt{2mE}/\hbar$

If Classical:	If Quantum:
- $E < V \Rightarrow$ Bound	-
- $E > V \Rightarrow$ Can Escape	

3D TISE: $-\frac{\hbar^2}{2m}\nabla^2\psi(x,y,z) + V(x,y,z)\psi(x,y,z) = E\psi(x,y,z)$

Stellar Classification:

O	30–50000K	Blue/Violet	He II
B	11–30000K	Blue/White	He I
A	7500–11000K	White	H, Fe II, Si II, Mg II
F	6000–7500K	Yellow/White	Ca II
G	5000–6000K	Yellow	Ca II, Fe I, CH
K	3000–5000K	Orange	Ca II, Fe I
M	≤3000K	Orange/Red	Fe I, TiO

A0 → A9
Hot → Cold

Core: $0 \le r \le 0.25R_\odot$
Rad: $0.4R_\odot \le r \le 0.8R_\odot$
Conv: $0.8R_\odot \le r \le R_\odot$

Our Sun is a G2V
$R_\odot \approx 7 \times 10^8\,m$

Birth Mass Matters

EM Spectrum:

Radio ~ 10 m
Micro ~ 10^{-3}m
IR ~ 10^{-6}m
Visible ~ $(4\to7)\times10^{-7}$m
UV ~ 10^{-8}m
X-Rays ~ 10^{-10}m
γ ~ 10^{-12}m

T_{EFF} = Temp of BB which emits same Total Energy as the star.

Continuous Spectra for BB (Ideal)
Emission Spectra for Hot gases
Absorption Spectra for Cold gases

Morgan-Keenan Classification:

Ia: Lum. Supergiants
Ib: Supergiants
II: Lum. Giants
III: Giants
IV: Subgiants
V: Main Sequence

H-R Diagrams:

1) White Dwarves
2) Main Sequence
3) Giants
4) Supergiants

Lifespan ≅ E_{tot}/L

Fusion of H→He requires:
- $P \sim 10^{14}$Pa, $T \sim 10^{7}$K

otherwise, protons repelled by EM.

PP1 Chain:

① $p + p \to d + e^+ + \nu_e$
② $d + p \to {}^3He + \gamma$
③ ${}^3He + {}^3He \to {}^4He + 2p$

Most Common Fusion Chain

HS Equilibrium:

Thermal Pressure Balances Gravity:

$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}$

Protostar – Clouds of gas accrete
↓
Pre MS – Core heats up, Hydrostatic Equilibrium commences, $\rho\uparrow$
↓
MS – Central T↑

White Dwarf If Below C. Limit: 1.4$M_\odot$
Neutron Star (Pulsar if spinning) If between: 1.4$M_\odot$ – 3$M_\odot$
Black Hole if between 3$M_\odot$ – 8$M_\odot$
Supernova if >8$M_\odot$

Big Bang Models

- Supported by CMBR
- Expansion + Evolution of the Universe
- Abundance of Light Elements.

Expansion:

- Expansion speed depends on matter density, more dense ⇒ more gravity so expansion slows down.
- ρ_c is critical density required to stop expansion at ∞. $\Omega = \rho/\rho_c$

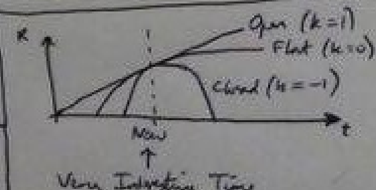
Friedman Equation:

$\dot{R}^2(t) = \frac{8\pi G\rho}{3}R^2(t) - kc^2 + \frac{\Lambda}{3}R^2(t)$

k measures curvature of the universe:

$k = 1 \Rightarrow$ Hyperbolic, Open Universe ← $\Omega < 1$ (Light Diverge)
$k = 0 \Rightarrow$ Flat, Euclidean Universe ← $\Omega = 1$ (Light Parallel)
$k = -1 \Rightarrow$ Spherical, Closed Universe ← $\Omega > 1$ (Light Converge)

$\rho_c = 3H_0^2/8\pi G$
$\Omega_m = \Omega_B + \Omega_{DM}$
$\Omega = \Omega_m + \Omega_\Lambda$ ← CC



Current Estimates:

Dark Energy ~ 73%
Dark Matter ~ 23%
Baryonic Matter ~ 4%

of all matter in the Universe.