Astrophysics PST (2) - Solutions

1. The mass of the star is M = 5 solar masses = 1 x 10^{31} kg
and its luminosity L = 600 solar luminosity = 2.28 x 10^{29} J s^{-1}

Fraction of mass liberated per H-burning reaction =
Mass deficit/mass of 4 protons = 0.0286/4.0312 = 0.0071

The total energy that the star will be able to radiate is

$$E_{total} = 0.0071 \times 0.1 \times M \times c^2$$
$$= 0.0071 \times 0.1 \times (1 \times 10^{31}) \times (9 \times 10^{16}) \text{ Joule}$$
$$= 6.39 \times 10^{44} \text{ Joule}$$

and it will radiate for

$$\frac{E_{total}}{L} = \frac{6.39 \times 10^{44}}{2.28 \times 10^{29}} \text{ s} = 2.8 \times 10^{15} \text{ s} = 8.9 \times 10^{7} \text{ yr}$$

[Btw, check understanding of what is meant by hydrostatic equilibrium!
I.e. a balance between the force of gravity inward and the pressure of hot gases pushing outward.
A balance or 'equilibrium' must be attained in order for a star to have a stable *size*.]

2.
(a) According to Hubble classification scheme:
SBc galaxy : galaxy has a small nucleus, with a bar-like structure through it. The spiral arms emerge from the ends of the bar and are loosely wound.

Sa galaxy: galaxy has a relatively large nucleus, plus tightly wound spiral arms (no central bar).

(See pictures in the lecture's PPT, and e.g. HST images on http://heritage.stsci.edu/gallery/gallery_category.

(b) To shift Lyα from visible (rest wavelength 121.6 nm) to visible (> 370 nm)

Redshift z = Δλ/λ = (370.0 – 121.6)/121.6 = 2.04

For redshifts 2.04 or greater the Lyα line will be shifted to wavelengths longer than 370 nm.

(c) Quasar 3C 273 distance:

Hubble's law, distance d = v / H_o = c z /H_o

= 3 x 10^5 x 0.16 / 70 = 686 Mpc

3.

$$v_{\rm rot}^2 = \frac{GM}{r}$$

$$v_{\rm rot}^2 = \frac{6.7\times10^{-11}\times1.2\times10^{11}\times2\times10^{30}}{8.5\times10^3\times3.1\times10^{16}}$$

$$v_{\rm rot} = 247 \text{ km s}^{-1}.$$

For circular orbit, $\mathrm{P(orb)} = 2\pi \mathrm{r}/v_{\rm rot}$

$$= \frac{2\pi\times8.5\times10^3\times3.1\times10^{16}}{2.47\times10^5} \times \frac{1}{3.16\times10^7} = 2.1\times10^8 \text{ yr}$$

4. (a) The age of the universe, if a(t) evolves like $t^{2/3}$

$H(t) = (da/dt)/a = (2/3)\ (1/t)$
So at the present day $t_0 = (2/3)\ (1/H_0)$

(b) for $H_0 = 68$ km/sec/Mpc, the age of the universe is model (a) is

$t_0 = (2/3)\ (1/H_0) = (2/3)$ 14.4 Gyr = 9.6 Gyr

which is much younger that the age of 13.8 Gyr derived by Planck.