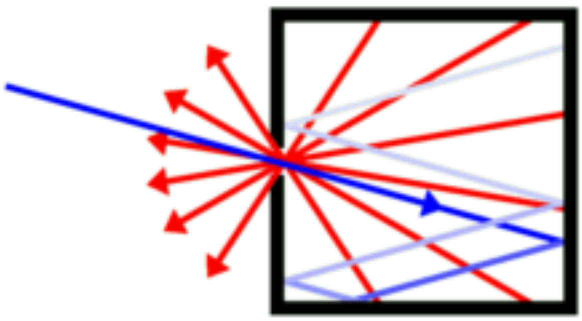
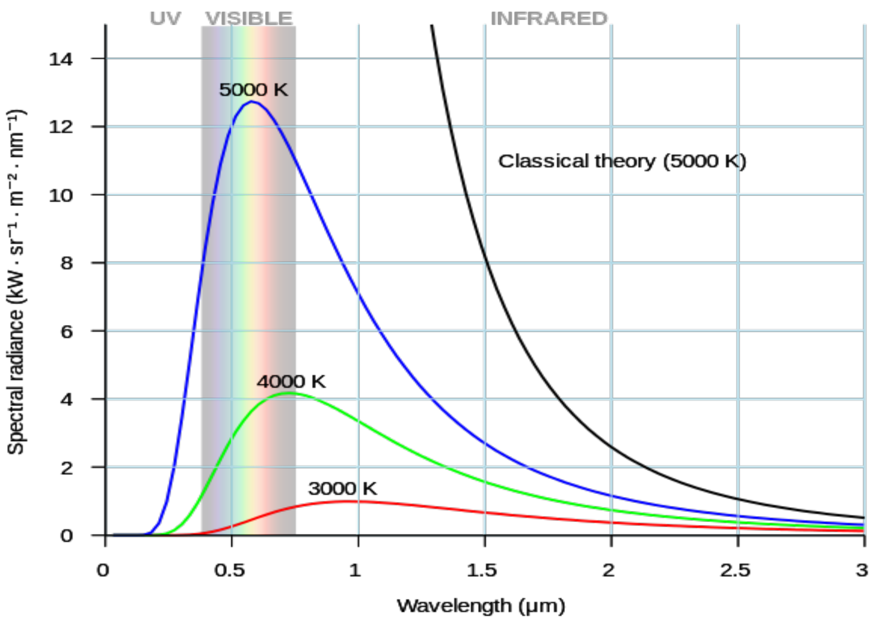
Blackbody Radiation

Objects that are hot give off light. Even "cool" objects behave this way, but at room temperature most of the light is in the infrared part of the spectrum so we can't see it with our eyes.

To understand this behaviour, we define the concept of a "black body", which is an ideal system that absorbs all radiation that is incident on it. Such objects give off what we call "blackbody radiation", and while most objects are not perfect black bodies many can be well approximated by this simplification.



If we measure the energy radiated as a function of wavelength, we find



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We can immediately see two things:

1) The total power radiated per unit area (which is the integral of the intensity over all wavelengths) increases dramatically with T. Empirically, this is called Stefan's law:

$$\frac{P}{A} = \sigma\, \epsilon\, T^4$$

where ϵ is the emissivity (which for a perfect black body is 1) and $\sigma = 5.670 \times 10^{-8} W/(m^2 K^4)$ is the Stefan-Boltzmann constant, whose value was determined from experiments.

2) The peak of the wavelength distribution is found at lower wavelengths as T increases. Empirically, this is called Wien's displacement law:

$$\lambda_{max} T = b$$

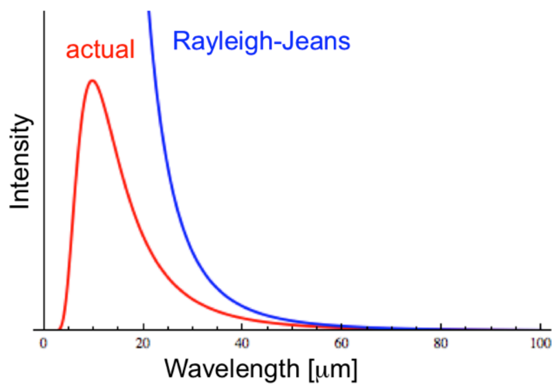
where $b = 2.898 \times 10^{-3}\ m\ K$ is Wien's displacement constant.

Qualitatively, you know that the latter one is true from your own experience since you have probably seen a metal being heated (e.g. the filament in a light bulb) and observed that it's colour starts as a dull red and then goes to orange until finally becoming white hot.

One useful model of a black body is a box with a small hole leading inside. Any radiation that enters the hole is unlikely to come out since the hole is small, so this is a good approximation of the behaviour of a black body. Using this model, we can then try to calculate the intensity of the emitted radiation as a function of temperature. Rayleigh and Jeans did this in the 1900s, using the assumption that each standing wave mode inside the box has an average energy proportional to $k_B\, T$, where $k_B = 1.381 \times 10^{-23} J/K$ is the Boltzmann constant. By counting the number of modes they derived

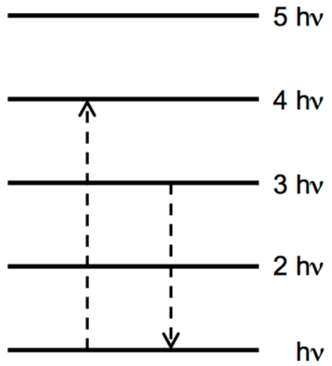
$$I(\lambda, T) = \frac{2\pi\, c\, k_B\, T}{\lambda^4}$$

Note that the units of this are $\frac{\frac{m}{s} J}{m^4} = \frac{W}{m^2\, m}$, which is power per area per wavelength as expected. Let's compare this with what is actually observed:



At very long wavelengths this is reasonable, but at shorter wavelengths (or higher frequencies) this does not work. This is obvious if you look at the formula because it diverges (goes to infinity) as $\lambda \to 0$. This represents a complete breakdown of the classical description, and was called the "ultraviolet catastrophe".

Around the same time, Planck developed a different theory of blackbody radiation that was consistent will all prior observations, reproducing the blackbody intensity curve as well as Stefan's law and Wien's law. To do this, he had to make three dramatic and unprecedented assumptions:



1) The radiation in the box came from a set of tiny oscillators that had a discrete (or quantised) energy spectrum with spacing

   $E = h\,\nu$

   where h is an arbitrary constant that would be determined from comparison to experiments.
2) Energy was emitted or absorbed when the light interacted with the oscillators and caused them to make transitions between their states. This energy was not emitted or absorbed continuously but rather in discrete (or quantised) chunks.
3) Instead of having the energy of each mode be proportional to $k_B\,T$, Planck made it proportional to the energy difference E between the oscillator levels. Furthermore, rather than having equal populations of all modes, Planck weighted them according to the Boltzmann distribution law, making the probability of being in a mode proportional to $e^{-\frac{E}{k_B T}}$. This means that modes with higher energy (i.e. higher frequency or smaller wavelengths) are less likely to be occupied.

Based on this, Planck derived the following formula for the intensity of blackbody radiation:

$$I(\lambda, T) = \frac{2\pi\,h\,c^2}{\lambda^5}\frac{1}{e^{\frac{h\,c}{\lambda\,k_B\,T}} - 1}$$

Note that this formula is sometimes written without the π because it then refers to the power per area per wavelength per angle, while for this version we've already integrated over all of the relevant angles (appropriately weighted) to get just power per area per wavelength.

This function produces exactly the right curve when we set $h = 6.626\,\times\,10^{-34}\,J\,s$; this value is now known as Planck's constant. Furthermore, if we integrate over all wavelengths, we find

$$\int_0^\infty d\lambda\,I(\lambda, T) = 2\pi\,h\,c^2\int_0^\infty d\lambda\frac{1}{\lambda^5}\frac{1}{e^{\frac{h\,c}{\lambda\,k_B\,T}} - 1} = \frac{2\,\pi^5\,k_B^4}{15\,c^2\,h^3}\,T^4 = \sigma\,T^4$$

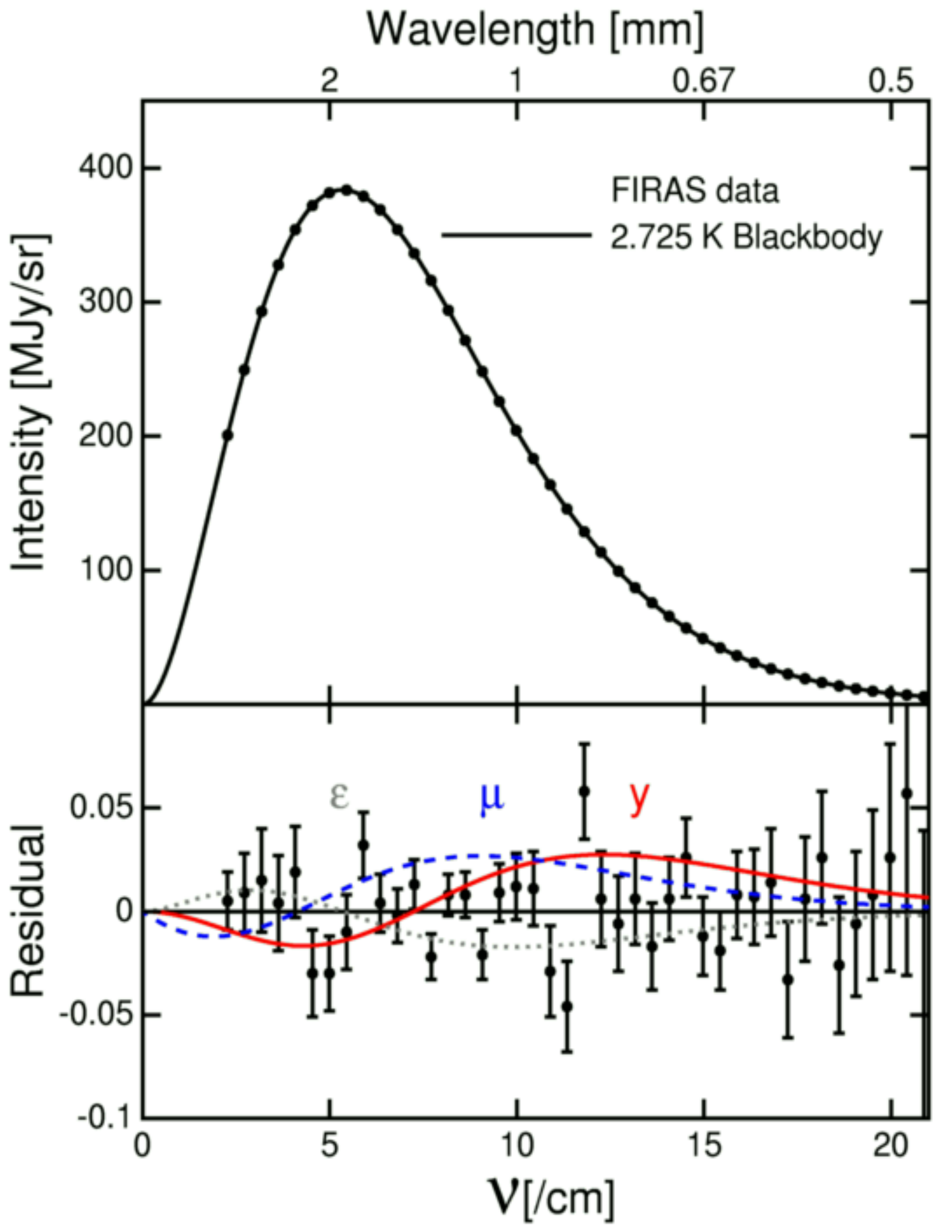
where we have reproduced Stefan's law and the Stefan-Boltzmann constant σ. As a final proof of its correctness, we can find the peak wavelength in the Planck intensity formula as follows:

$$\frac{d}{d\lambda}I(\lambda,T)=0\ \Rightarrow\ \frac{hc}{\lambda k_B T}\frac{e^{\frac{h\,c}{\lambda\,k_B\,T}}}{e^{\frac{h\,c}{\lambda\,k_B\,T}}-1}-5=0\ \Rightarrow\ \lambda_{max}=\frac{hc}{x}\frac{1}{k_B T}=\frac{b}{T}$$

where the value of x is determined numerically to be 4.9651… and yields the correct value of Wien's displacement constant.

Although this was a stunning success, Planck and his colleagues initially thought that quantising the energy was just a mathematical trick that produced the correct answer, and that some more basic explanation would provide an answer that was consistent with classical physics. This was not to be the case, however, and in fact these results laid the groundwork for an even larger departure from classical behaviour.

The Cosmic Microwave Background radiation has a nearly perfect Blackbody spetrcum, with T=2.725 K.



The Spectrum of the Cosmic Microwave Background from COBE