Doppler effect

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If the source or the observer of a wave have a relative velocity between each other, the measured frequency of the wave will differ from the emitted frequency. This is known as the doppler effect or the doppler shift.

If the observer is moving at a velocity v_o relative to a source, then the apparent speed of the wave will be $v' = v + v_o$.

The observed frequency is therefore:

$$f' = \frac{v'}{\lambda} = \frac{v + v_o}{\lambda} = \frac{v}{\lambda} \cdot \frac{v + v_o}{v} = \left(1 + \frac{v_o}{v}f\right)$$

If the source moves at a velocity of v_f then the measured wavelength will decrease, with the change in wavelength per period being the product of velocity and time:

$\Delta\lambda = v_s t$

If the source is moving towards the observer, the wave length will decrease:

$$\lambda' = \lambda - \Delta\lambda = \lambda - v_s t = \lambda - v_s t$$
$$= \frac{1}{f}(v - v_s)$$

And so the observed frequency will be:

$$f' = f\left(\frac{1}{1 - \frac{v_s}{v}}\right)$$

If both the observer and source are moving then both effects are seen, and the new frequency can be found by:

$$f' = \left(\frac{v + v_0}{v - v_s}\right)f$$

For these derivations we assumed the relative velocity was directly towards the observer. If the relative velocity instead occurs at an angle, then the doppler shift takes into account the component of the relative velocities.

If the source has a velocity $\boldsymbol{v_s}$ and position $\boldsymbol{r_s}$ with the observer at a position $\boldsymbol{r_o}$, then we can find v_s by:

$$\frac{\boldsymbol{v_s} \cdot (\boldsymbol{r_s} - \boldsymbol{r_o})}{|\boldsymbol{r_s} - \boldsymbol{r_o}|} = v_s \cos(\theta_{so}(t))$$

The doppler shift also occurs in special relativity, and has a final result of:

$$f_{observerd} = \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}} \cdot f_{source}$$

If a source is moving faster than the velocity of the waves in the medium, a shock wave is produced due to the constructive interference of waves.

This causes a large pressure and results in a sonic boom

The angle between the front of the sock wave and the direction of travel can be found by:

$$\sin(\alpha) = \frac{v_{wave\ max}}{v_{object}}$$