PHAS1247 Classical Mechanics
Problems for Week 7/8 of Lectures (2016)

1. A bucket of water rotates with constant angular velocity, ω, around a vertical axis through the centre of the bucket. The water is subject to the earth's gravitational field. Consider a small mass of water at a distance r from the axis of rotation and determine the force, in the rotational frame of reference, on the mass due to the centrifugal force and its weight. You should assume that all elements rotate at ω i.e. we are neglecting frictional effects with the bucket's wall and also the earth's rotation. This force is balanced by the normal reaction of the rest of the water. Show that the angle the water makes to the horizontal, α, is given by"

$$\tan\alpha = \frac{r\omega^2}{g}$$

and hence that the height of the water depends on r as:

$$h(r=0) + \frac{1}{2g}\omega^2 r^2$$

Show that the same result can be achieved by considering the conservation of energy where is assumed that the rotating liquid is in equilibrium i.e. there is no relative motion of any part of the liquid with respect to the rest.

2. A small mass $m = 0.1\,\text{kg}$ is at the bottom of a $20\,\text{m}$ string that is attached to the top of UCL's portico. If the mass is stationary, what fictitious force does it experience as a result of the Earth's rotation? Given that the latitude of London is approximately 50° N and the radius of the Earth is approximately $6400\,\text{km}$, what is the force's magnitude and direction? Find the horizontal component of the fictitious force and hence find how far, and in what direction, the mass is displaced from the 'true' vertical joining the string's point of suspension on the portico to the centre of the Earth. (You may neglect any corrections arising from the non-spherical mass distribution of the Earth.)

3. Suppose a second identical mass is dropped from the same height (and the same location) that the string is attached to in Q2 and that it falls under gravity. What additional fictitious force does it experience while it is moving? Assuming the velocity of the falling mass is approximately vertical, as viewed by an observer in the quad, find the magnitude and direction of the additional force as a function of its speed v. Show that, as a result of this additional force, the dropped mass passes approximately $1.26\,\text{mm}$ to the East of the suspended one. (Assume you can neglect air resistance, and any small corrections to the mass's vertical velocity arising from the fictitious forces.)