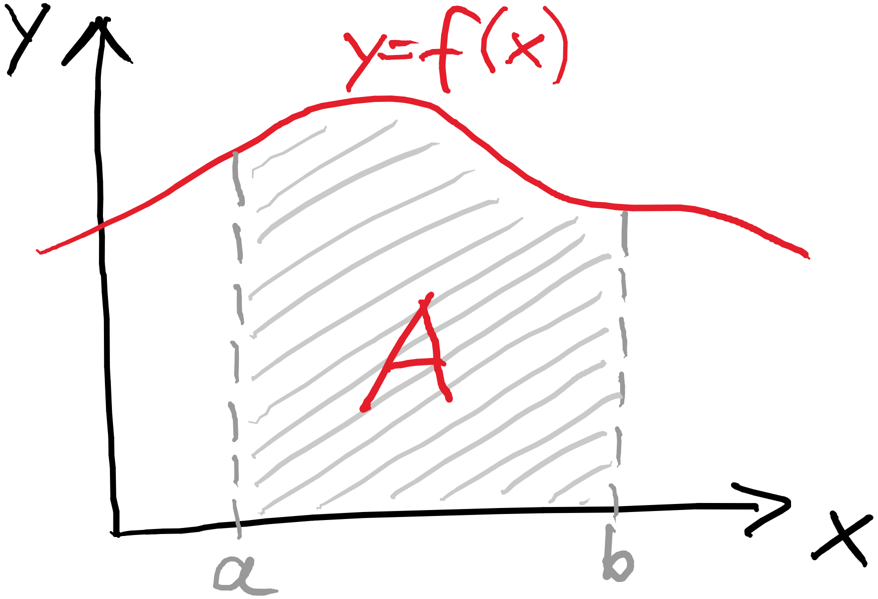
4) Integration

Motivation

- Area under a curve

$$\boxed{A = \int_a^b f(x)\,dx}$$



- „Integration" of quantity over time, position, ...

e.g. work: $W = F \cdot \Delta x \longrightarrow W = \int_{x_1}^{x_2} F(x)\,dx$

constant force

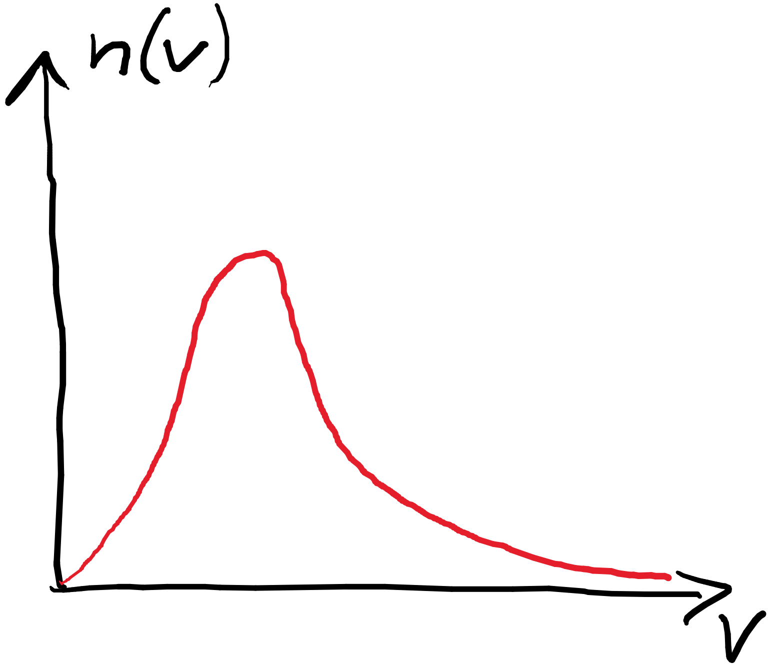
force depends on x

- Averages, statistics, distributions

e.g. velocity distribution $n(v)$ of particles in a gas

$\Longrightarrow$ total number of particles

$$N = \int_0^\infty n(v)\,dv$$



$\Longrightarrow$ average kinetic energy of a particle

$$\langle E_{kin} \rangle = \int_0^\infty \frac{1}{2} m v^2 n(v)\,dv$$

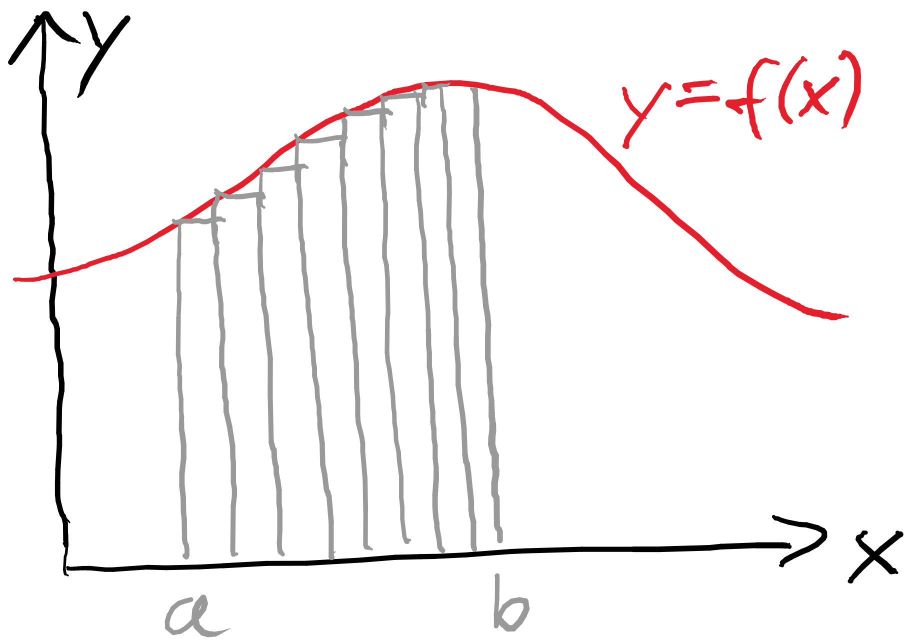
Definite Integral

- Infinite Limit of sum

$$\boxed{I_{ab} = \int_a^b f(x)\,dx = \lim_{\Delta x \to 0} \sum_{x=a}^{x=b} f(x_i)\,\Delta x}$$

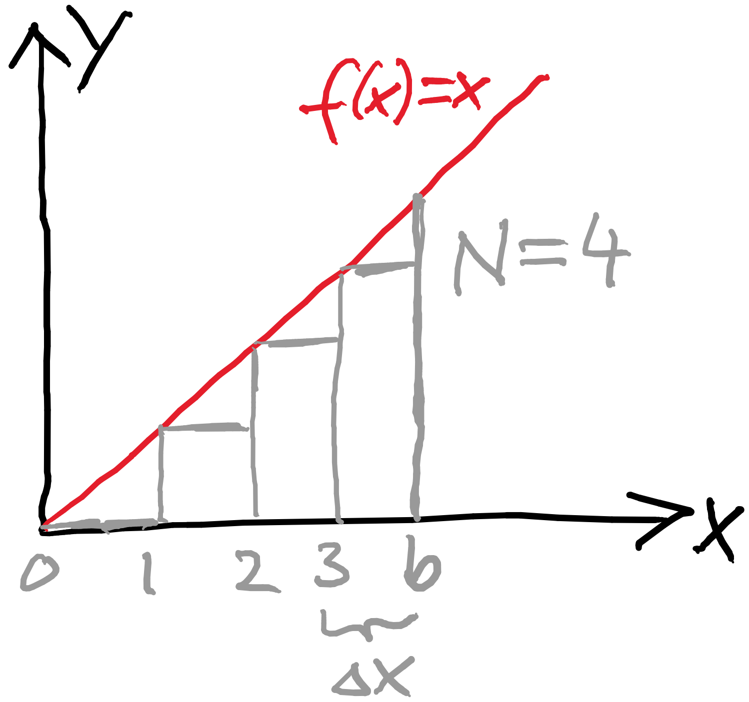
Definite integral is a number (= area under curve)

Subdivide area into rectangles
→ Limit of infinitely many infinitely narrow strips



Explicit example

$$f(x) = x$$



$$I = \int_0^b x\,dx$$

$$= \lim_{N\to\infty} \sum_{i=0}^{N-1} x_i \Delta x$$

$$= \lim_{N\to\infty} \sum_{i=0}^{N-1} i\frac{b}{N}\cdot\frac{b}{N}$$

$$\Delta x = \frac{b}{N}$$

$$x_i = i\cdot\Delta x = i\frac{b}{N}$$

$$= \lim_{N\to\infty} \frac{b^2}{N^2} \sum_{i=0}^{N-1} i$$

$$= \lim_{N\to\infty} \frac{b^2}{N^2} \frac{(N-1)N}{2} = \lim_{N\to\infty} \left[\frac{1}{2}b^2 - \frac{b^2}{2N}\right] = \frac{1}{2}b^2 \checkmark$$