WAVE CHEAT SHEET

General form of SHO

$$\boxed{m\frac{\partial^2\psi}{\partial t^2} = -s\psi - b\frac{\partial\psi}{\partial t} + F_0\cos\omega t}$$

ψ is the displacement

m is the mass of the oscillator

s is the restoring force constant "stiffness"

b is the damping coefficient

F_0 is the magnitude of driving force

ω is the angular frequency of driving force

$K.E. = \frac{1}{2}m\dot{\psi}^2$

$P.E. = \frac{1}{2}s\psi^2$

Damping Force

↳ Work done against this

$= -2m\gamma\dot{\psi}^2 = -b\dot{\psi}^2$

ω is small $(\omega \ll \omega_0)$

$A \approx \frac{F_0}{m}\frac{1}{\sqrt{\omega_0^2}} = \frac{F_0}{m\omega_0}$ ⟹ Stiffness ~~Particle~~ controlled

ω is large $(\omega \gg \omega_0)$

$A = \frac{F_0}{m\omega_0}$ ⟹ mass controlled

$\omega = \omega_0$ resonance

* $\gamma = \frac{b}{2m}$

Critical damping: $\gamma^2 = \omega_0^2$

Overdamped: $\gamma^2 > \omega_0^2$

Underdamped: $\gamma^2 < \omega_0^2$

Average power provided by driver

$$\boxed{= \frac{F_0^2}{m^2}\frac{m\gamma\omega^2}{(\omega_0^2-\omega^2)^2+4\gamma^2\omega^2}}$$

From $\langle P\rangle = b\langle\dot{\psi}\rangle$

For 2 driving forces

$\psi = A_1\cos(\omega_1 t+\phi_1) + A_2\cos(\omega_2 t+\phi_2)$

$A^2 = A_2^2 + A_1^2 + 2A_1A_2\cos(\phi_2-\phi_1)$

$\tan\theta = \frac{A_1\sin\phi_1 + A_2\sin\phi_2}{A_1\cos\phi_1 + A_2\cos\phi_2}$

Combining 2 oscillations

$\psi = A\cos(\omega_1 t) + A\cos(\omega_2 t)$

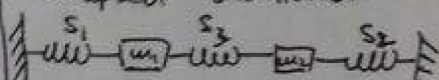
$= 2A\underbrace{\cos\left(\frac{\omega_1+\omega_2}{2}t\right)}_{\text{Fast osc.}}\underbrace{\cos\left(\frac{\omega_1-\omega_2}{2}t\right)}_{\text{Slow osc.}}$

Beat freq $= \omega_1 - \omega_2$

Carrier freq $= \frac{\omega_1+\omega_2}{2}$

Envelope freq $= \frac{\omega_1-\omega_2}{2}$

Coupled oscillator



$m_1\frac{\partial^2\psi_1}{\partial t^2} = -s_1\psi_1 - s_3(\psi_1-\psi_2)$

$m_2\frac{\partial^2\psi_2}{\partial t^2} = -s_2\psi_2 - s_3(\psi_2-\psi_1)$

if $s_1 = s_2 = s_3$

Solve for $\psi_1-\psi_2$ and $\psi_1+\psi_2$

$\psi(x,t) = f\left(\frac{2\pi}{\lambda}x - \frac{2\pi c}{\lambda}t\right)$

$= f(kx+\omega t)$

* k same sign as ω in the $-z$ direction

k diff sign as ω in the $+z$ direction

$k = \frac{2\pi}{\lambda}$ is the wave number

λ is the wavelength

$\omega = kc = \frac{2\pi c}{\lambda}$ is the angular frequency of wave.

$c = \sqrt{\frac{T}{\mu}}$ is the speed of propagation

$Z_0 = \sqrt{T\mu}$

$K.E. = \frac{1}{2}\mu\left(\frac{\partial\psi}{\partial t}\right)^2$

$P.E. = \frac{1}{2}T\left(\frac{\partial\psi}{\partial x}\right)^2$

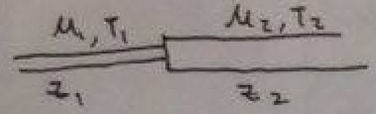
Total $E = \frac{1}{2}\frac{T}{c^2}\left[\left(\frac{\partial\psi}{\partial t}\right)^2 + c^2\left(\frac{\partial\psi}{\partial x}\right)^2\right]$

Powers delivered along wave propagation

↳ $P(x,t) = -T\frac{\partial\psi}{\partial x}\frac{\partial\psi}{\partial t}$

$= -Z_0 c\frac{\partial\psi}{\partial x}\frac{\partial\psi}{\partial t}$

$= -Z_0\left(\frac{\partial\psi}{\partial t}\right)^2$



$\longrightarrow$ transmission direction

Incident wave : $\psi_i(x,t)$

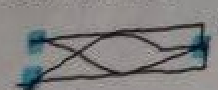
transmitted wave : $\psi_T(x,t) = \frac{2z_1}{z_1+z_2}\psi_i(x,t)$ → T, transmission coefficient

Reflected wave : $\psi_R(x,t) = \frac{z_1-z_2}{z_1+z_2}\psi_i(x,t)$

Reflection coeff R

	T	R
$z_1 > z_2$	>1	>0
$z_1 < z_2$	<1	<0
$z_1 = z_2$	$=1$	$=0$
$z_1 \ll z_2$	0	-1
$z_1 \gg z_2$	2 ?	1 ?
generally	$0 \le T \le 2$	$-1 \le R \le 1$

Standing wave
↳ boundaries



close end is a node.

Open end always anti-node

$\omega \propto \frac{1}{L}$, $\lambda \propto L$, c det. by medium

Longitudinal Waves

Youngs modulus $= Y = \frac{\text{local strain}}{\text{Local stress}} = \frac{\sigma}{\varepsilon}$

↳ units : Pa

$\therefore \frac{\text{Force}}{\text{cross sect. Area}} = Y\frac{\partial\psi}{\partial x}$

$\because \sigma = \frac{F}{A}$, $\varepsilon = \frac{\partial\psi}{\partial x}$

$$\frac{\partial^2\psi}{\partial t^2} = \frac{Y}{\rho}\frac{\partial^2\psi}{\partial x^2}$$

$c = \sqrt{\frac{Y}{\rho}}$ } Propagation velocity.

$z_0 = A\sqrt{\rho Y}$ } impedance

In fluids consider Bulk modulus
↳ $B = \frac{1}{k}$ (k = compressibility of the gas)

$$\frac{\partial^2\psi}{\partial t^2} = \frac{B}{\rho}\frac{\partial^2\psi}{\partial x^2} \Rightarrow c = \sqrt{\frac{B}{\rho}}, \quad z_0 = \sqrt{B\rho}$$

c ⇓ Propagation Velocity

z_0 ⇓ acoustic impedance (impedance / area)

Sound Measurement

Intensity $= I \propto A^2$
(power transmitted per unit area)

Sound level, $\beta = 10\lg_{10}\left(\frac{I}{I_0}\right)$

$I_0 = 10^{-12}\,W/m^2$

unit = dB (deibel)

Doppler effect

$$f' = \left(\frac{V+V_o}{V-V_s}\right)f$$

f': observed frequency

V_o: velocity of observer

V_s: vel. of Source

f: emitted frequency

2 waves propagating together : $\psi_1(x,t) = A\cos(k_1x - \omega_1 t)$

$\psi_2(x,t) = A\cos(k_2x - \omega_2 t)$

Phase velocity $= \frac{\omega_1}{k_1}, \frac{\omega_2}{k_2}$

Carrier velocity $= \frac{\omega}{k} = \frac{\omega_1+\omega_2}{k_1+k_2}$ ← for quick osc.

envelope velocity $= \frac{\Delta\omega}{\Delta k} = \frac{\omega_1-\omega_2}{k_1-k_2}$

group velocity $= \frac{d\omega}{dk} = V_p$ for linear dispersion

LIGHT & OPTICS Cheat Sheet

Snell's Law

$$\boxed{\frac{n_i}{n_t} = \frac{\sin\theta_t}{\sin\theta_i}}$$

n = index of refraction $= \frac{c_{vac}}{c_{medium}}$

Optical path length, $\boxed{\Delta = \frac{nd}{\cancel{\lambda}}}$ $\Delta = nd$

Total internal reflection $(n_i > n_t)$

$$\boxed{\theta_c = \sin^{-1}\left(\frac{n_t}{n_i}\right)}$$

Fraunhofer Limit,

Point of interception of the light rays leaving a diffraction grating is far enough that the light paths are approx. Parallel $\Rightarrow \underbrace{r_2 - r_1}_{\text{optical path diff.}} = d\sin\theta$

Fermat's Theorem
↳ Minimise Optical path length

Huygen's Principle
↳ Every point on a wave front acts as a source of a new wave front

Constructive Interference, $\Rightarrow m\lambda$ phase diff. (in-phase)

$$r_2 - r_1 = \boxed{m\lambda = d\sin\theta}$$

Destructive Interference $\Rightarrow (m+\frac{1}{2})\lambda$ phase diff (out of phase)

$$r_2 - r_1 = \boxed{(m+\tfrac{1}{2})\lambda = d\sin\theta}$$

$$I \propto |A|^2 = E_1^2 + E_2^2 + 2E_1E_2\cos\left(\frac{2\pi}{\lambda}d\sin\theta\right)$$

$r_1 = x - R_2$

↳ for 2 rays

$$\text{Visibility}, V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

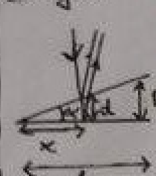
2slit diffraction & 2 rays with same Amp.

$$I \propto |A|^2 = \boxed{4E^2\cos^2\left(\frac{\pi}{\lambda}d\sin\theta\right)}$$

π phase change occur at reflection from less optically to more optically dense

$n_1 < n_2$

Wedges



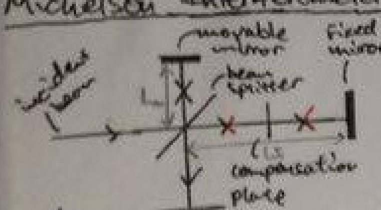
$$\boxed{\tan\alpha = \frac{D}{l} = \frac{d}{nx}}$$

where $2d = n\lambda$
x = spacing between fringes

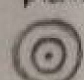
* Spatial Coherence
↳ constant wavelength across all spatial dimensions

* Temporal Coherence
↳ Constant wavelength across the time dimensions

Michelson Interferometer



observation plate



incident beam not perfect

$$\therefore \Delta = 2(L_m - L_f)\cos\theta$$

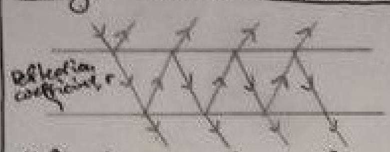
$$\boxed{I(\theta) = 4I_0\cos^2\gamma}$$

where $\gamma = \frac{2\pi}{\lambda}(L_m - L_f)\cos\theta$

$$\boxed{\lambda_2 - \lambda_1 = \Delta\lambda = \frac{\lambda_1\lambda_2}{2\Delta d}} \Big\} \text{resolution}$$

Distance shifted for λ_2 ring to overlap with λ_1 ring

Fabry-Perot interferometer



$$\text{Reflectance} = R = |r|^2$$

$$\text{Finese} = F = \frac{4R}{1-R}$$

$$\boxed{I = I_{max}\frac{1}{1+F\sin^2\gamma}}$$

$$\gamma = \frac{2\pi}{\lambda}d\cos\theta$$

Chromatic resolving power:

$$↳ \boxed{\left|\frac{\lambda_1}{\Delta\lambda}\right| = m\pi\sqrt{F}}$$

Spectral width = $\Delta\lambda$
Coherence length = Δx

$$\boxed{\Delta x = \frac{\lambda^2}{2\Delta\lambda}}$$

Diffraction Grating

$$I(\theta) = I_0\frac{\sin^2(Nx)}{\sin^2(x)}$$

where $x = \frac{\pi d}{\lambda}\sin\theta$

$$I(x \to 0) = I_0 N^2$$

no. of slits = no. of zero + 1

where d is the slit seperation

chromatic resolution

$$\frac{\lambda_1}{\Delta\lambda_{min}} = mN$$

Single Slit Diffraction

$$I = I_0\frac{\sin^2(\frac{\pi}{\lambda}a\sin\theta)}{(\frac{\pi}{\lambda}a\sin\theta)^2}$$

$$I(x \to 0) = I_0$$

a = slit width

Rayleigh Criterion

$$\boxed{\frac{\lambda}{a} = \sin\theta_R} \approx \theta_R$$

Rayleigh Criteria for round lenses: $\theta_R = 1.22\lambda/D$

Maxwell Equations

$$\nabla \cdot \underline{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \underline{B} = 0$$

$$\nabla \times \underline{E} = -\frac{\partial B}{\partial t}$$

$$\nabla \times \underline{B} = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

$$C_{vacc} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

$$C_{medium} = \frac{1}{\sqrt{\varepsilon_r \varepsilon_0 \mu_r \mu_0}} = \frac{C_{vacc}}{n}$$

$$n = \sqrt{\varepsilon_r} \because \mu_r \simeq 1$$

Unpolarised → Polarised

$$I_{polarise} = \frac{1}{2} I_{unpolarise}$$

Polariced → polariser @ ∡θ

$$I' = I\cos^2\theta$$ Malus's Law

$$r_{\parallel} = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$

$$r_{\perp} = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$t_{\parallel} = \frac{2 n_i \cos\theta_i}{n_t \cos\theta_i + n_i \cos\theta_t}$$

$$t_{\perp} = \frac{2 n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$

Brewster's ∡

↳ Occurs when $r_{\parallel} = 0$

↳ $\theta_p = \tan^{-1}\left(\frac{n_t}{n_i}\right)$

↳ when ∡ of incidence $= \theta_p$, ∡ between reflected and transmitted beam $= 90°$, reflected light is polarised

Lens equation

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

object dist   image dist

Lens maker formula

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Position of focal point on a spherical concave or convex mirror

$$f \simeq \frac{1}{2} C$$

Magnification

$$M = \frac{h'}{h} = -\frac{q}{p}$$ *

Compound

↳ $M_c = M_1 \cdot M_2$