# In-course Assessment (1) - Solutions

**1.**

Assuming the two stars emit as blackbodies and have the same radius, using
the Stefan-Boltzmann law, we know that the total energy flux from the stars is proportional to their temperature to the 4th power. Thus:

$$\frac{F_{20000}}{F_{5000}} = \left(\frac{2\times10^4}{5\times10^3}\right)^4 = 256$$

[2]

so the hotter star is emitting over two orders of magnitude more energy.

Wien's displacement law gives the wavelength of the blackbody peak:

$$\lambda_{\max} \approx \frac{3000}{T}\,\mu m$$

[1]

So the hotter star's spectrum peaks at 0.15 μm, or 1500 angstroms, i.e. in the UV, and the cooler star's spectrum at 0.6 μm, or 6000 angstroms, i.e. in the visible band. [2]

*[5 marks total]*

**2.**

(i)

The mass of the star is $M$ = 10 solar masses = 2 x $10^{31}$ kg
and its luminosity $L$ = $10^4$ solar luminosity = 3.8 x $10^{30}$ J $s^{-1}$

Fraction of mass liberated per H-butning reaction = [1]
Mass deficit/mass of 4 protons = 0.0286/4.0312 = 0.0071

[some students might start from next line; i.e. quote efficiency = 0.0071]

The total energy that the star will be able to radiate is

$$E_{total} = 0.0071\times0.1\times M\times c^2$$

[1]

$$= 0.0071\times0.1\times(2\times10^{31})\times(9\times10^{16})\text{ Joule}$$

$$= 1.3\times10^{45}\text{ Joule}$$

[1]

and it will radiate for

$$\frac{E_{total}}{L} = \frac{1.3\times10^{45}}{3.8\times10^{30}}\text{ s} = 3.4\times10^{14}\text{ s} \sim 10^7\text{ yr}$$

**[1]**

The end-state of the star will be a neutron star. **[1]**

**(ii)**

*Luminosity* = $4\Pi R^2 \sigma T^4$ [1]

(Constant $\sigma$ not needed, students should work in ratios!)

$R^2_{rg}/R^2_{ms} = [L_{rg}/L_{ms}] \times [T_{ms}/T_{rg}]^4 = 2 \times 5^4 = 1.25 \times 10^3$ [1]

*Rrg/Rms = 35.4 i.e. factor of 35 increase in radius* [1]

*[8 marks total]*

**3.**

(a) Spica [1]
(b) Antares [1]
(c) Alpha Centauri A [1]
(d) Antares [1]
(e) Alpha Centauri A [1]

*[5 marks total]*

**4.**

According to Hubble classification scheme:

*SBa galaxy* -- galaxy has a large nucleus, with a bar-like structure through it. The spiral arms emerge from the ends of the bar and are tightly wound. [2]

*Sc galaxy* -- galaxy has a relatively small nucleus, plus loosely wound spiral arms (no central bar). [2]

*E6 galaxy* -- elliptical galaxy, whose apparent shape is very elongated or flattened. [1]

*[5 marks total]*

**5.**

Velocity = distance/time = circumference/period:

$$T(orbit) = \frac{2\pi 8000 \text{ pc } \times 3.1 \times 10^{13} \text{ km pc}^{-1}}{220 \text{ km s}^{-1} \times 3.16 \times 10^{7} \text{ s yr}^{-1}}$$
$$= 2.24 \times 10^{8} yr$$

[3]

Approximate age of the solar system is $4.5 \times 10^9$ yr

Thus $4.50 \times 10^9 / 2.24 \times 10^8 \sim 20$ orbits completed [1]

The spiral arms cannot be fixed, permanent feature because they would quickly tighten (wind-up) After 20 revolutions. They must instead represent a density wave phemomenon which helps to compress interstellar clouds and trigger star formation along the arms. [3]

*[7 marks total]*

**6.**

Distance to galaxy:
radial velocity

$$v = \left(\frac{(\lambda - \lambda_0)}{\lambda_0}\right) c$$

[1]

$V = [(402.8 - 393.30)/393.30] \times 3 \times 10^5 = 7246 \text{ km s}^{-1}$ [2]

Hubble law, distance $d = v / H_o = 7246 / 75 = 96.6$ Mpc [2]

*[5 marks total]*

**7.**

Using Hubble's expansion relation:
$v = Ho.\ d$   and redshift $z = v/c = Ho.\ d / c$

Time to travel distance d is therefore
$T = d / c = z / Ho$ [2]

T (yrs) = ( 0.15 / 75 ) Mpc s km$^{-1}$ x km Mpc$^{-1}$ x yr s$^{-1}$

= (0.15 / 75) x (3.1 x 10 $^{19}$ km/3.16 x 10$^{7}$ s) [1]
= 2.0 x 10$^{9}$ yr. [1]

This is just an estimate as we are assuming Ho is constant in time, which is not true over the age of the Universe. [1]

*[5 marks total]*