2016 first attempt

07 April 2019 19:12

1. Describe the process that leads to the formation of absorption lines in the spectrum of a hot object seen through a cooler gas, and give three astrophysical examples of this phenomenon. [7]

A hot object will release a continuous emission spectra of light, as a result of light released due to the black body behaviour of the body.
When this light passes through a cool gas, some light will be absorbed.
Light will only be absorbed if the wavelength/energy of a given photon corresponds with an electron energy level jump in an atom in the gas.
This results in only specific parts of the emission being absorbed.
When we measure this spectra, we see the continuous spectra but with dark gaps in where light has been absorbed.
The dark patches are distinct for various elements, and so the spectra can be used to find the composition of the cooler gas.

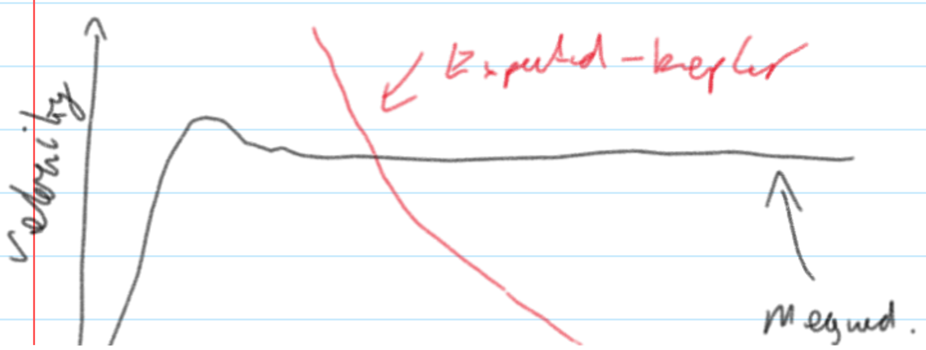
Measure the composition of outer layers of star
Measuring the composition of the atmosphere of exo-planets
Measure the composition of interstella gas clouds / nebula

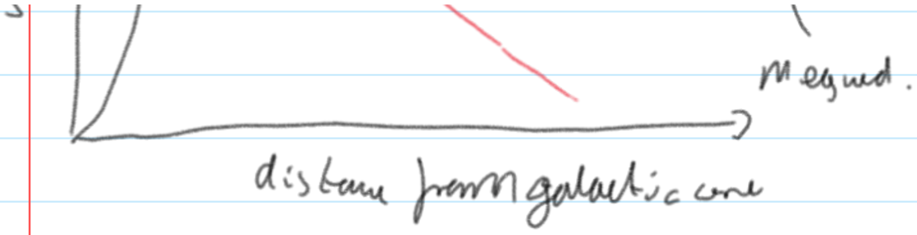
2. Aldebaran is a K5 III star, Sirius is a A1 V star and Rigel is a B8 Ia star. Explain what is meant by these classification terms, including indications of approximate surface temperature. [6]

Letter -
- OBAFGKM
- O is hottest (30,000K +), M coolest (<4000K)
- First number is subclass of hottest
- Roman numeral -
  - Ia is most luminous super giant
  - Ib luminous super giants
  - II luminous giants
  - III giants
  - IV subgitants
  - V main sequence

From this:
- K5 III - quite cool (4000-5000K - closer to 4000K), giant
- A1 V - Quite hot (8000-11000K - top end of this temperature, closer to 11000K), main sequence
- B8 Ia - Very hot (11000-30000K, closer to 11,000), most luminous super giant

3. Sketch a diagram of the 'rotation curve' for our Milky Way Galaxy, carefully labelling the axes. Comment on the shape of the rotation curve, and the main inference to be drawn from it. [7]





By measuring the mass of stars, we can find the mass of the galaxy as a function of distance from its centre.
If we do this, we can use classical mechanics to calculate the velocity of stars at various distances from the galactic centre.
We would expect the relationship to follow Kepler's law.
However, when we measure the velocity of stars by measuring their red/blueshift, we find that they don't follow what we expect
If our understanding of gravity is correct, there must be a very large amount of mass situated in a halo throughout our (and every other) galaxy.
We cannot see this mass, as it doesn't interact with the electromagnetic spectrum
We call this unknown mass dark matter

4. A laser pointer emits green light (wavelength 500nm) with a power output of 1mW. Compute the energy of a single photon at this frequency. How many photons per second are emitted by the laser pointer? [6]

$$E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34} \times 3\times10^{8}}{500\times10^{-9}} = 3.98\times10^{-19}\,J$$

$$\frac{1\times10^{-3}\,J/s}{3.98\times10^{-19}\,J/\text{photon}} = 2.51\times10^{15}\,\frac{\text{Photons}}{s}$$

5. Describe an experiment which implies that light travels as a wave, giving a short but clear argument for why the observations support a wave interpretation. Describe a second experiment which implies that light has particle characteristics, explaining clearly how the observations made in the experiment are incompatible with a wave theory of light. [8]

Light as a wave-
- Young double slit
- Source of coherent light split by 2 slits
- Light emmitted from two slits is allowed to travel and hit a screen
- Screen measures areas of high and low intensity
- This comes from interference of the two light paths, due to phase difference
- This implies light is a wave

Light as a particle-
- Photo electric effect
- Light of high enough energy incident on metal causes electrons to be emmitted
- Electrons are only emitted when light reaches a threshold frequency
- If frequency is lower than this, no electrons emitted, regardless of intensity
- This implies that an electron can only absorb a single photon, and so light must travel in packets of energy - as a particle.

6. Which of the following conditions is fulfilled when a wavefunction $\psi(x)$ is normalised? [6]

i) $\int_{-\infty}^{\infty} \psi(x)dx = 1$

ii) $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$

or iii) $\int_{-\infty}^{\infty} |\psi(x)| dx = 1$.

Name one further condition which $\psi(x)$ must satisfy to be a physical wavefunction. How do we calculate the probability of finding a particle with a normalised wavefunction $\psi(x)$ in a certain interval, e.g. between $x = a$ and $x = b$?

ii) is normalised.

$\psi(x)$ must be contiunous

$\frac{d\psi(x)}{dx}$ must be contiunous

$$P(a \le x \le b) = \int_a^b |\psi(x)|^2 dx$$

7. The time-independent Schrödinger equation (TISE) in one-dimension is:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x).$$

where m is the mass of the particle, E its energy, $\psi(x)$ the wavefunction and $V(x)$ the potential.

(a) Consider a particle moving in a constant potential, i.e. $V(x) = V_0$. Assuming $E > V_0$, show that the following wavefunction

$$\psi(x) = A\sin(\kappa x) + B\cos(\kappa x)$$

is a solution of the TISE and find an expression for E in terms of κ, $\hbar$, m and V_0. [4]

Write down a wavefunction that is a solution to the TISE for the same potential, but when $E < V_0$. [2]

(b) Consider an infinite square well with walls at $x = 0$ and $x = L$. The potential is zero inside the well, i.e. $V(x) = 0$ for $0 \le x \le L$, and is infinite outside the well, i.e. $V(x) = \infty$ for $x < 0$ and $x > L$. By identifying the form of the wavefunction in each of these regions and considering boundary conditions, show that the allowed energies for a particle in the well are [10]

$$E_n = \left(\frac{h^2}{8mL^2}\right)n^2$$

for integer n. Why is the $n = 0$ case not an allowed energy?

(c) Consider an electron trapped in an infinite square well in one dimension. How wide must the well be such that a transition from the $n = 2$ state to the ground state will emit red light at 620nm? [4]

a) $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\left(\psi(x)\right) = \left(E - V(x)\right)\psi(x)$

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$V(x) = V_0$

$E > V_0$

$= (E - V_0)\psi(x)$

$\psi(x) = A\sin(kx) + B\cos(kx)$

$\frac{d^2\psi(x)}{dx^2} = -k^2\psi(x)$

$\cup \rightarrow \quad \frac{\hbar^2 k^2}{2m} = E - V_0$

$E = V_0 + \frac{\hbar^2}{2m}k^2$

$E < V_0 \quad \therefore \quad \frac{\frac{d^2\psi(x)}{dx^2}}{\psi(x)} > 0$

$\psi(x) = e^{-kx}$

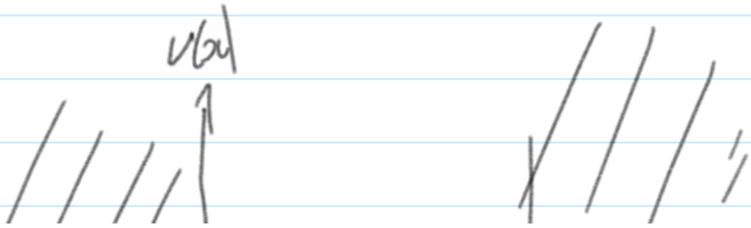
$\rightarrow \frac{d^2\psi(x)}{dx^2} = k^2 e^{-kx}$

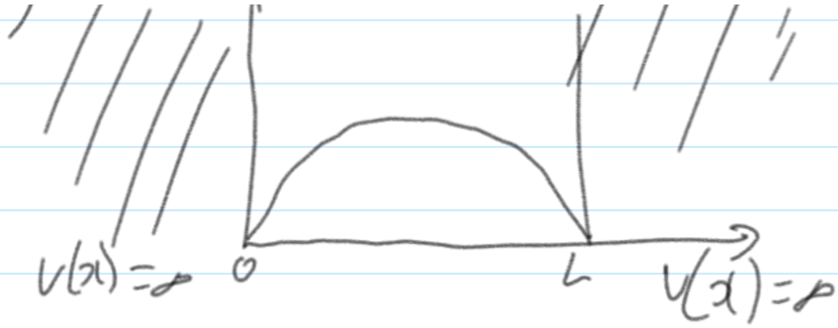
$\rightarrow -\frac{\hbar^2 k^2}{2m} = E - V$

$E < V \therefore E - V < 0$

$\rightarrow \frac{\hbar^2 k^2}{2m} = V_0 - E$

c)





$$\phi(x) = A\sin(kx) + B\cos(kx)$$

$$\phi(0) = 0$$

$$\rightarrow B = 0$$

$$\phi(L) = 0$$

$$\rightarrow A\sin(kL) = 0$$

$$\rightarrow \sin(kL) = 0$$

$$k = \frac{\pi n}{L}$$

$$\rightarrow \phi(x) = A\sin\left(\frac{\pi n x}{L}\right)$$

Normalise:

$$\int_0^L \left|A\sin\left(\frac{\pi n x}{L}\right)\right|^2 dx = 1$$

$$\sin^2(\theta) = \frac{1 - \cos(2\theta)}{2}$$

$$\int_0^L \frac{1}{2} - \cos\left(\frac{2\pi n x}{L}\right) dx = \frac{1}{A^2}$$

$$\left[\frac{1}{2}x - \frac{L}{2\pi n}\sin\left(\frac{2\pi n}{L}x\right)\right]_0^L = \frac{1}{A^2}$$

$$\frac{L}{2} = \frac{1}{A^2} \rightarrow A = \sqrt{\frac{2}{L}}$$

From first part:

$$E = V_0 + \frac{\hbar^2}{2m} k^2$$

$$k = \frac{\pi n}{L}, \quad V_0 = 0$$

$$E = \frac{\hbar^2 \pi^2}{2mL^2} n^2$$

$$\hbar^2 = \frac{h^2}{4\pi^2}$$

$$E = \frac{h^2}{8mL^2} n^2$$

$n = 0?$

c) $$E_{2-1} = \frac{h^2}{8mL^2}(2^2-1)$$

$$= \frac{3}{8}\frac{h^2}{mL^2}$$

$$E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E}$$

$$\lambda = \frac{8mL^2c}{3h}$$

$$\lambda = 620 \times 10^{-9}$$

$$\sqrt{\frac{3\lambda h}{8mc}} = L$$

$$= 7.5 \times 10^{-10} \, m$$

8. (a) Describe the process of quantum tunnelling. Include a sketch of a relevant potential and the form of the wavefunction both inside and outside the barrier. Describe how quantum predictions differ from those of classical physics. [You do not need to derive the exact forms of wavefunctions from the Schrödinger equation – a qualitative sketch of their form is sufficient.] [6]

(b) The Scanning Tunnelling Microscope (STM) is a device which depends crucially on quantum tunnelling. Name a physical process or technology (other than the STM) in which quantum tunnelling plays an important role. [1]

(c) The probability that a quantum particle with mass m and energy E will tunnel through a square barrier of width L and height U is approximately equal to:

$$P = \exp[-2CL]$$

where

$$C = \frac{\sqrt{2m(U-E)}}{\hbar}.$$

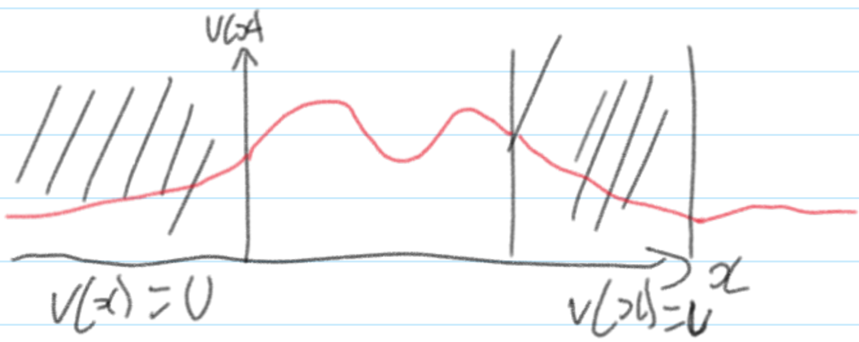
In an STM, electrons tunnel from an electrode across a potential barrier to a surface, completing a circuit whose current I is proportional to the tunnelling probability. Draw a labelled diagram of an STM indicating the path of an electron through such a device. Indicate clearly the position of the tunnelling barrier in your diagram. [4]

(d) If L represents the distance between electrode and surface, the current I passing through the device will be [3]

$$I \propto \exp[-2CL]\,.$$

A microscope is set up so that the tunnelling barrier height is 10eV and incident electrons have energy 2eV. Calculate the constant C for this microscope, justifying its unit.

a)



Wave function must be continuous ∴
at boundary there must be a probability of finding particle in potential $V(x) > E$.

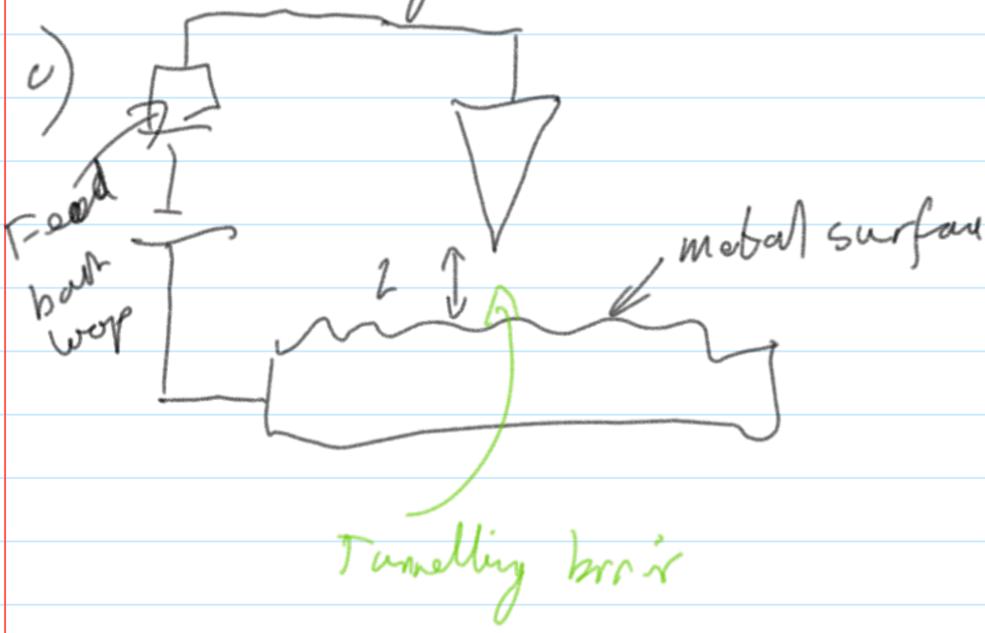
This means there is a probability of finding particle after potential barrier.
The particle can tunnel through the potential
→ quantum tunnelling

pot.

→ quantum tunnelling

b) Nuclear fusion in stars,
Nuclear decay.

c)



d)

$$k = \frac{\sqrt{2\times 9.11\times 10^{-31}\times (10-2)\times 1.6\times 10^{-19}}}{1.05\times 10^{-34}}$$

$$= 1.45\times 10^{10}\ m^{-1}$$

e)

$$0.1A = k e^{-2kL}$$

$$7.7A = k e^{-2k(L-x)}$$

$$= k e^{-2kL} e^{2kx}$$

$$7.7A = 0.1A\, e^{2kx}$$

$$\frac{\ln(77)}{2k} = x$$

$$= 1.498\times 10^{-10}\ m$$

$$= 1.5\times 10^{-10}\ m$$

9. (a) Describe Rutherford's planetary atomic model for Hydrogen and explain two ways in which the model was incompatible with the known properties of Hydrogen at the beginning of the 20th century. [3]

(b) Building on Rutherford's work, Niels Bohr proposed his own atomic model. In Bohr's model, how is the angular momentum of the electron restricted? [1]

Describe one way in which Bohr's model is inferior to a fully quantum mechanical model of the atom. [1]

(c) An ion is called Hydrogen-like when it consists of a single electron orbiting a nucleus of charge $+Ze$, where Z is the ion's atomic number. The magnitude of the Coulomb force $|F|$ and the potential energy V between two objects with charges q_1 and q_2 separated by distance r are as follows,

$$|F| = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2}, \qquad V = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}.$$

Given that a particle in a circular orbit of radius r and speed v undergoes centripetal acceleration v^2/r and has angular momentum $l = mvr$, show that, in the Bohr model for a Hydrogen-like ion, the radii of electron orbits are restricted to the values [6]

$$r_n = \frac{a_0 n^2}{Z}.$$

Derive an expression for the Bohr radius a_0 in terms of $\hbar$, ϵ_0, m_e and e.

(d) Hence, show that, in the Bohr model, the energy levels of the electron in a Hydrogen-like ion have energies that depend on integer n and satisfy [4]

$$E_n = -\frac{Z^2 \times 13.6}{n^2}\ \text{eV}.$$

(e) Consider the nth energy level of the Hydrogen atom and the mth energy level of a Helium+ ion. How must m and n be related for the energies to coincide? Hence identify the lines in the spectrum of Helium+ that coincide with a visible spectral line of Hydrogen. [5]

A) Dense positivly charged nucleus with electrons orbiting like planets

If this were true, acceleration of electron should emit EM radiation - not measured

Electron should spiral down into nucleus as energy is emitted - not seen

B)

Angular momentum in terms of n:

$$L = mvr = \hbar n$$

Quantised energy states forced in with no jsutification - QM model has this fall out as a result of continious wave function.

C)

$q_1 = +Ze$

$q_2 = -e$

$$F = \frac{mv^2}{r} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$$

$mvr = \hbar n$

$$mvr = \hbar n$$
$$v = \frac{\hbar n}{mr}$$

$$\frac{m\left(\frac{\hbar n}{mr}\right)^2}{r} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$$

$$\frac{\hbar^2}{mr^3} n^2 = \frac{Ze^2}{4\pi\epsilon_0 r^2}$$

$$\rightarrow \frac{\hbar^2 4\pi\epsilon_0}{mZe^2} n^2 = r$$

$$\rightarrow a_0 = \frac{\hbar^2 4\pi\epsilon_0}{m_e e^2}$$

d) $E = ke + pE$

$$\frac{1}{2} m v^2 - \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r}$$

$$\frac{mv^2}{r} = \frac{Ze^2}{4\pi\epsilon_0 r^2}$$

$$v^2 = \frac{Ze^2}{4\pi\epsilon_0 mr}$$

$$\frac{1}{2} mv^2 - \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r}$$

$$= \frac{1}{2}\left(\frac{Ze^2}{4\pi\epsilon_0 r}\right) - \frac{Ze^2}{4\pi\epsilon_0 r}$$

$\cdot \frac{\ }{4\pi\varepsilon_0 r} \quad \frac{\ }{4\pi\varepsilon_0 r}$

$$= -\frac{1}{2}\frac{Ze^2}{4\pi\varepsilon_0 r}$$

$$\frac{\hbar^2 4\pi\varepsilon_0}{mZe^2} n^2 = r$$

$$\rightarrow E = -\frac{1}{2}\frac{Ze^2}{4\pi\varepsilon_0} \times \left(\frac{mZe^2}{\hbar^2 4\pi\varepsilon_0 n^2}\right)$$

$$= -\frac{1}{2}\frac{Z^2 m e^4}{16\pi^2\hbar^2\varepsilon_0^2 n^2}$$

$$= -\frac{Z^2}{n^2}\times\left(\frac{m_e e^4}{32\pi^2\varepsilon_0^2\hbar^2}\right)$$

$$= \frac{9.11\times10^{-31}\times(1.6\times10^{-19})^3}{32\pi^2\times(8.85\times10^{-12})^2\times(1.05\times10^{-34})^2}\times(eV)$$

$$= 13.68 \ eV$$

?

$$\Rightarrow -\frac{Z^2}{n^2}\times 13.6 \ eV$$

e)

$$E_m = -\frac{4\times13.6}{m^2} \ eV$$

$$E_n = \frac{-13.6}{n^2} \ eV$$

$$E_n = E_n$$

$$\frac{4 \times 13.6 \text{ eV}}{m^2} = \frac{13.6}{n^2} \text{ eV}$$

$$4n^2 = m^2$$
$$2n = m$$

10. (a) List the stages of the dominant chain reaction that describes the primary energy source in the Sun. [6]

(b) By first outlining the basic physical properties of neutrinos, describe the solar neutrino problem and how it has been solved. [8]

(c) Sirius A has a luminosity 23.5 times that of the Sun and burns H into He. How many kg of H does Sirius A burn into He each second? If Sirius A has 2.3 times the mass of the Sun and 10% of its mass converts from H to He, calculate the lifetime (in years) of Sirius A. [6]

a) $p + p = d$ (duterium) $+ e^+ + \nu_e$
$d + p = {}^3He + \nu$
${}^3He + {}^3He \rightarrow {}^4He + 2p$

b) Nutrino product in first part of above process to balance lepton number.

There are 3 flavours of nutrinos. Electron nutrinos are produced in this reaction.

Nutrinos oscillate between the 3 flavours as they travel.

This was not known at first.

We measured amount of electron-nutrinos hitting earth.

Was $\frac{1}{3}$ of expected value - explained by was $\frac{1}{3}$ of expected value - explained by neutrino oscillation

c) $L = 23.5 \times 3.8 \times 10^{26} = 8.93 \times 10^{27}$

$m = (4 \times 1.0078 - 4.0026) \times 1.66 \times 10^{-27}$

$= 4.7476 \times 10^{-29}$ kg

$M_{4H} = 6.692 \times 10^{-27}$ kg

$E = mc^2$

$= 4.27 \times 10^{-12}$ J

$\frac{E}{kg} = \div$

$\rightarrow 6.4 \times 10^{14}$ J/kg

$\frac{8.93 \times 10^{27} \text{ J/s}}{6.4 \times 10^{14} \text{ J/kg}} = 1.4 \times 10^{13}$ kg/s

$10\% \times 2.3 \times 2.0 \times 10^{30} = 4.6 \times 10^{29}$ kg

$\frac{4.6 \times 10^{29} \text{ kg}}{1.4 \times 10^{13} \text{ kg/s}} = 3.29 \times 10^{16}$ s

$\div (60 \times 60 \times 24 \times 365) = 1 \times 10^{9}$ years

11. (a) Describe the main differences between spiral and elliptical galaxies in terms of content and the properties and motion of stars. [9]

(b) Calculate the critical density of the Universe for a Hubble constant of 75 km s^{-1} Mpc^{-1}. [4]

(c) Calculate the density of visible mass in the Local Group of galaxies, assuming that it contains $4 \times 10^{12}\ M_{\odot}$ of matter within a spherical volume of 3 Mpc. If the Universe has a similar density as the Local Group value, state whether the Universe is open or closed. Explain which fundamental principle is invoked in this estimate. [7]

Elliptical:

- Mostly older redder starts - metal poor (population II)
- Very little star formation - all intersteller gas used up
- Stars have tilted and elongated orbits
- Very little/no disk - bulge like structure
- Very little/no rotation

Spiral:

- Bulge in centre - bulge contains mostly population II stars, metal poor, old and red
- Stars at buldge have tilted and elliptical orbits
- Rest of stars situated in galactic plane
- Stars in galactic plane mostly population I stars - metal rich, young and hot (like yo mumma)
- Star formation mostly in spiral arms
- Spiral arms have larger amount of star formation due to larger density of gas
- Arms in galactic plane orbit centre in regular fashion.

b) $$\rho_{crit} = \frac{3H_0^2}{8\pi G}$$

$$H_0 = 75 km\,s^{-1}\ Mpc^{-1}$$
$$= 7.5\times10^4 m\,s^{-1}\ (Mpc^{-1})$$
$$= \frac{7.5\times10^4\ m\,s^{-1}}{3.1\times10^{16}\times10^6\ m}$$
$$= 2.42\times10^{-18} s^{-1}$$

$$G = 6.67\times10^{-11} m^3\ kg^{-1}\ s^{-1}$$

$$\rho_{crit} = \frac{3\times(2.42\times10^{-18}\ s^{-1})^2}{8\pi\times6.67\times10^{-11}\ m^3\,kg^{-1}\ s^{-2}}$$

$$= 1.05\times10^{-26}\ kg/m^3$$

c) $$\frac{4\times10^{2}\ 2\times10^{30}\ kg}{\frac{4}{3}\pi\ (3\times 3.1\times10^{16}\times10^{6}\,m)^{3}} = 2.37\times10^{-27}\,\frac{kg}{m^{3}}$$

density lower than critical:
open universe

Homogeneity - same mass density & expansion rate everywhere at large enough scale.