Sum of integers converges?

$$S = 1+2+3+4+5+6+7+8+\dots$$

$$\Rightarrow 4S = \quad 4 \quad +8 \quad +12 \quad +16+\dots$$

$$\Rightarrow -3S = 1-2+3-4+5-6+7-8+\dots$$

$$-3S = \quad 1-2+3-4+5-6+7+\dots$$

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$$-3S = \quad 1-2+3-4+5-6+\dots$$

$$\Rightarrow -12S = 1+0+0+0+0+0+0+0+\dots = 1$$

$$\Rightarrow 1+2+3+4+\dots = -\frac{1}{12}$$!?!

Sum of integers converges?

$$S = 1+2+3+4+5+6+7+8+\dots$$

$$\Rightarrow 4S \neq 0+4+0+8+0+12+0+16+\dots$$

$$\Rightarrow -3S \neq 1-2+3-4+5-6+7-8+\dots$$

$$-3S \neq 0+1-2+3-4+5-6+7+\dots$$

$$-3S \neq 0+1-2+3-4+5-6+7+\dots$$

$$-3S \neq 0+0+1-2+3-4+5-6+\dots$$

$$\Rightarrow -12S \neq 1+0+0+0+0+0+0+0+\dots = 1$$

NO!!

Inserting zeros and shifting elements not (always) allowed in our "definition of summation" for divergent series

→ We stick to our result: $S = 1+2+3+\dots$ is divergent!

But: Other methods of summation like this can be useful

→ See Ramanujan Summation

http://en.m.wikipedia.org/wiki/Ramanujan_summation

another way of finding the constant is as follows — 41
Let us take the series $1+2+3+4+5+\&c$. Let C be its constant. Then $C = 1+2+3+4+\&c$

$\therefore 4C = 4 + 8 + \&c$

$\therefore -3C = 1-2+3-4+\&c = \frac{1}{(1+1)^2} = \frac{1}{4}$.

$\therefore C = -\frac{1}{12}$.

→ Used in String Theory