1228 Thermal Physics - Problem solving class 2

Model Answers

1) 60 g of CO_2 gas at 350 K is confined to a volume of 400 cm^3. For carbon dioxide gas (CO_2), the constants in the van der Waals equations are a = 0.364 J·m^3/mol^2 and $b = 4.27\times10^{-5}$ m^3/mol. The molar mass of carbon is 12 g/mol and the molar mass of oxygen is 16 g/mol.

i) Find the pressure of the gas using both the ideal gas equation and the van der Waals equation. Does gas in these conditions behave as an ideal or a real gas?

ii) If the same amount of CO_2 gas behaves as a real gas, find the volume it should occupy under pressure of 5.0×10^6 Pa at 350 K with accuracy up to 20 cm^3.

V = 4×10^{-4} m^3; T = 350K. The molar mass of CO_2 gas is 12 + 32 = 44 g/mol. Hence the number of moles of the gas is 60/44 = 1.36 mole✓

i) $P = \dfrac{nRT}{V} = \dfrac{1.36\times8.315\times350}{4\times10^{-4}} = \dfrac{39.6\times10^2}{4\times10^{-4}} = 9.89\times10^6\,Pa$ for the ideal gas✓

$$P = \frac{nRT}{V-nb} - \frac{an^2}{V^2} = \frac{1.36\times8.315\times350}{4\times10^{-4} - 1.36\times4.27\times10^{-5}} - \frac{0.364\times1.85}{16\times10^{-8}} = \frac{39.6\times10^2}{3.42\times10^{-4}} - 4.2\times10^6 = 7.37\times10^6\ \text{Pa}$$

for the real gas. ✓✓

The van der Waals equation of state for real gas gives a lower pressure. This means that the gas in these conditions behaves as a real gas.

ii) If the same amount of CO_2 gas behaves as a real gas, find the volume it should occupy under pressure of 5.0×10^6 Pa at 350 K with accuracy up to 20 cm^3.

Using the van der Waals equation of state:

$$V = \frac{nRT}{P + \dfrac{an^2}{V^2}} + nb.$$

This equation is nonlinear and is solved iteratively starting from some approximation for V in the denominator. A natural approximation to start is V given by the ideal gas law:

$nRT = 39.6\times10^2$; $an^2 = 0.673$; nb = 0.58×10^{-4} m^3

$$V_0 = \frac{nRT}{P} = 7.92\times10^{-4}\,m^3 = 792\,cm^3$$

$$V_1 = \frac{nRT}{P + \frac{an^2}{V^2}} + nb = \frac{39.6\times10^2}{5.0\times10^6 + \frac{0.673}{62.7}10^6} + 0.58 = 7.18\times10^{-4}m^3 = 718cm^3$$

Using this value as a better approximation, one obtains V_2 = 686 cm^3

Finally the third iteration gives V_3 = 673 cm^3. This value differs from V_2 by less than 20 cm^3.

One can see that CO_2 at these conditions will occupy the much smaller volume than one predicted by the ideal gas law.

2) A 2 m long brass rod and a 1 m long aluminium rod are aligned linearly. When the temperature is 22 °C, there is a gap of 1.0 x 10^{-3} m separating their ends. No expansion is possible at the other end of either rod. At what temperature will the two rods touch? The coefficient of linear expansion of brass $a_b = 19\times10^{-6}\ ^oC^{-1}$ and the coefficient of linear expansion of aluminium $a_a = 23\times10^{-6}\ ^oC^{-1}$

For brass: $? L_b = a_b \times L_{0b} \times ? T$

For aluminium : $?L_a = a_a \times L_{0a} \times ? T$

The change in temperature is the same for both rods. They will touch when the sum of the two length changes equals the initial width of the gap. Therefore:

$? L_b + ? L_a = a_b \times L_{0b} \times ? T + a_a \times L_{0a} \times ? T = 1.0\times10^{-3}$

So, the temperature change is:

$? T = 16.4\ ^oC$

If the original temperature was 22 °C, the final temperature is 38.4 °C.

3) Consider a flat, two-dimensional doughnut. The doughnut has a hole, with radius r, and an outer radius R. It has a width w, which is simply w = R - r. How does the hole radius change when the donut is heated? Does it get larger or smaller?

If you apply the thermal expansion equation to all three lengths in this problem, the three lengths would change as follows:

$R_f = R\ (1 + a?T)$

$r_f = r\ (1 + a?T)$

$w_f = w\ (1 + a?T)$

The final width should also be equal to the difference between the outer and inner radii. This gives:

$w_f = R_f - r_f = (R – r) + (R – r)\, a?T = w + w\, a\, ?\, T$

This is exactly what we would get by applying the linear therma l expansion equation to the width of the donut above. So, with something like a doughnut, an increase in temperature causes the width to increase, the outer radius to increase, and the inner radius to increase, with all dimensions obeying linear thermal expansion. The hole expands just as if it's made from the same material as the doughnut.

4) An iron circular disk has a diameter of 6 cm and is 0.010 mm too large to pass through a hole in a brass plate when the disk and plate are at a temperature of 30 ^oC. The coefficient of linear expansion of brass is $\alpha_B = 1.9\times10^{-5}\ K^{-1}$ and the coefficient of linear expansion of iron is $\alpha_I = 1.2\times10^{-5}\ K^{-1}$. If the disk and the plate are at the same temperature, at what temperature will the disk just fit into the hole?

The diameter of the iron disk is L_I that of the hole in the brass plate is L_B.

Then $L_I - L_B = 0.001$ cm at T = 30 ^oC.

Since the brass plate and iron disk expands uniformly, the hole must expand in the same proportion and linear expansion law will apply. ✓

Heating both the disk and the plate leads to increases in the diameters of the disk and the hole, with the hole increasing more since $\alpha_B > \alpha_I$.

To solve the problem we require that $?L_B – ?L_I = 0.001$ cm. ✓

$?L = aL?T$. ✓ For simplicity, we can approximate $L_B = L_I = 6$ cm, then

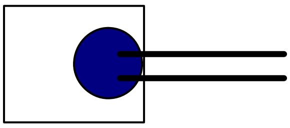
$?L_B – ?L_I = (\alpha_B - \alpha_I) \times L_B \times ?T = 0.001$ cm ✓

$0.07\times10^{-5} \times 6.0 \times ?T = 0.001$. Thus $?T = 23.8\ ^oC$ and $T = 53.8\ ^oC$ ✓

5) (a) A thermometer contains 15 mm^3 of liquid mercury and has a capillary radius of 0.05mm. Calculate how far will the mercury rise in the capillary if the temperature of the mercury increases from 25 ^oC to 27 ^oC, if the expansion of the glass is neglected.

The volume change is $\Delta V = 2 \times 1.82 \times 10^{-4} \times 15mm = 5.46 \times 10^{-3}$ mm. ✓ The rise in the capillary is therefore $\Delta V/\pi r^2 = 0.7$ mm. ✓

(b) Assume that ?l_1 is expansion of mercury in the thermometer in problem (a) and let ? l_2 be the actual expansion which allows for the expansion of the glass. Calculate the numerical value of (? l_1 – ? l_2)/ ? l_1. Assume that only the thermometer bulb is immersed in the object whose temperature is being measured and that the initial volume of mercury is equal to the volume of the bulb. The coefficient of linear expansion of glass is a = 8.3×10^{-6} K^{-1}.



If the expansion of the bulb is neglected, the change ? l_1 in the length of the mercury column is determined as in the previous problem by

$$\Delta l_1 = \frac{b_m V_m \Delta T}{A}$$, where A is the cross-sectional are of the capillary.

The bulb's interior volume will increase by $3a_g V?T$, where V is the volume of the bulb. ✓

Therefore the change in length of the mercury column ? l_2 will be smaller than ? l_1 and given by

$$\Delta l_2 = \frac{(bV_m - 3aV)\Delta T}{A} = (b - 3a)\frac{V\Delta T}{A}$$ ✓ because the V_m = V of the bulb.

Therefore (? l_1 – ? l_2)/ ? l_1 = 3a/ß ✓= 0.14 ✓

6) A 2.2 g lead bullet is moving at 150 m/s when it strikes a bag of sand and is brought to rest. Specific heat capacity of lead is 128 J/kg · °C

i) If all the frictional work is transferred to thermal energy in the bullet, what is the rise in temperature of the bullet as it is brought to rest?
ii) Repeat if the bullet lodges in a 50 g block of wood that is free to move.

i) The kinetic energy of the bullet is fully transferred into heat:

$\frac{1}{2}mv^2 = mc\Delta T$. The kinetic energy is $0.0022(150)^2/2 = 24.8$ J

24.8 = 0.0022×128×?T; ?T = 88 °C

ii) Here some of the kinetic energy is carried away by the bullet-block combination.

From momentum conservation: $mv = (M+m)V$, where M is the mass of the block.

2.2×150 = (50 + 2.2) ×V; V = 6.32 m/s

The loss of the kinetic energy = $0.0522\times(6.32)^2/2 = 1.04$ J

Energy transferred into heat = 24.8 – 1.04 = 23.76 J

Thus ? T = 84.3 °C

7) An iron rocket fragment initially at -100 °C enters the atmosphere almost horizontally and quickly melts completely. Assuming no heat losses by the fragment, calculate the minimum velocity it must have had when it entered. The specific heat of iron c= 448 J/kgK and the specific heat of fusion $L_f = 2.67\times10^5$ J/kg. The melting temperature of iron is 1535 °C.

The kinetic energy of the fragment is all changed to heat. ✓

As a result the fragment is heated from -100 °C to 1535 °C and then melted at that temperature. ✓

Heat due to heating is Q_1 = mc? T and heat required for melting is $m\times L_f$.

$$\frac{1}{2}mv^2 = \left[m\times 2.67\times10^5 + m\times 448(1535+100)\right]$$ ✓;

$v^2 = 1.99\times10^6$ and v= 1.41 km/s ✓

8) You would like to cool 0.25 kg of water, initially at 25°C, by adding ice, initially at -20°C. How much ice should you add so that the final temperature will be 0°C with all ice melted? The heat capacity of the container may be neglected. The specific heat capacity of ice is 2.1×10^3 J·kg^{-1}·K^{-1}, the specific heat of water is 4190 J·kg^{-1}·K^{-1}, and the latent heat of melting ice is $L_f = 334\times10^3$ J·kg^{-1}.

Water loses heat, so the heat needed to cool it down to 0°C is negative. It is equal to:

$$Q_{water} = m_{water}c_{water}\Delta T_{water} = 0.25\times4190\times(-0.25) = -26{,}000\,J$$ ✓

Let the unknown mass of ice be m_{ice}. Then the heat needed to warm it from -20°C to 0°C is positive and equal:

$$Q_1 = m_{ice}c_{ice}\Delta T_{ice} = m_{ice}(2.1\times10^3)(20) = 4.2\times10^4\times m_{ice}$$ ✓

The additional heat needed to melt this mass of ice is:

$$Q_2 = m_{ice}L_f = 3.34\times10^5\times m_{ice}$$ ✓

The sum of these three quantities of heat should be zero:

$$Q_{water} + Q_1 + Q_2 = -26{,}000 + 42{,}000 m_{ice} + 334{,}000 m_{ice} = 0$$ ✓

Thus $m_{ice} = \dfrac{26}{376} = 0.069 kg = 69 g$ ✓

9) 48.0 g of ice at 0 °C is in an aluminium calorimeter can of mass 2.0 g, also at 0 °C. 75.0 g of water at 80 °C are then poured into the can. Specific heat of water is 1.00 cal/gK and that of aluminium is 0.22 cal/gK. Latent heat of fusion of ice is 79.8 cal/g. Assuming that all of the ice has melted, calculate the final temperature of the whole system.

The heat gained by the can and the ice is equal to the heat lost by the water. ✓

Assuming that the hot water is sufficient in quantity to melt all the ice, one can write:

$$m_{ice} L_{ice} + (m_{ice} c_{water} + m_{can} c_{Al})(T_f - T_c) + m_{water} c_{water} (T_f - T_{water}) = 0$$ ✓

where T_c is the initial temperature of the calorimeter = 0°C; T_{water} is the initial temperature of hot water = 80°C, and T_f is the final temperature of the whole system.

$$T_f = \frac{m_{water} c_{water} T_{water} + (m_{ice} c_{water} + m_{can} c_{Al}) T_c - m_{ice} L_{ice}}{m_{water} c_{water} + m_{ice} c_{water} + m_{can} c_{Al}}$$ ✓

$$T_f = \frac{6000 + (48 + 0.44)(0) - 3830}{75.0 + 48.0 + 0.44} = 17.6 \ {}^{o}\mathrm{C}$$ ✓