Answer ALL SIX questions from section A
and THREE questions from section B.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

You may need these:

Law of cosines:

$$a^2 = b^2 + c^2 - 2bc\cos\alpha,$$

where α is the angle across from the side of length a.

Law of sines:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma},$$

where the sides of length a, b and c are across from the angles α, β and γ, respectively.

The *binomial series* for $|x| < 1$:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \ldots.$$

Expansion of sin:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots.$$

Addition of sines:

$$\sin A \pm \sin B = 2\sin\left[\frac{1}{2}(A \pm B)\right]\cos\left[\frac{1}{2}(A \mp B)\right].$$

SECTION A

[Part marks]

1. (a) Draw a ray diagram showing the formation of the image of a slide on a screen by a simple converging lens. **[3]**

   (b) A thin converging lens of focal length 50 mm is to be used to form an image of a slide on a screen 5 m from the lens.

   i. How far from the lens should the slide be placed? **[2]**

   ii. If the slide is 24 mm high, how high will the image be? **[2]**

2. The fundamental frequency of an open organ pipe corresponds to middle C, or 261.6 Hz. The third resonance of a closed pipe has the same frequency. What are the lengths of the two pipes? The speed of sound is 343 m s^{-1}. **[6]**

3. (a) State three techniques for producing polarized light from unpolarized light. **[3]**

   (b) Unpolarized light passes through three polaroid sheets. The axis of the first is vertical. The axis of the second is at 30° to the vertical. The axis of the third is vertical. What fraction of the incident light is transmitted? **[3]**

4. (a) In a Young's slit experiment, two narrow slits have a separation d and are illuminated by a source with wavelength λ. Fringes are observed on a screen a distance D from the slits. Positions of maxima on the screen are given by

$$d \sin\theta = m\lambda \qquad m = 0, 1, 2, \ldots.$$

Show that the positions of adjacent maxima are given by **[3]**

$$\begin{aligned} y_m &= m\frac{\lambda D}{d} \\ y_{m+1} &= (m+1)\frac{\lambda D}{d}. \end{aligned}$$

(b) Use a phasor diagram to find the resultant (magnitude E and phase angle ϕ) of two fields represented by **[4]**

$$\begin{aligned} E_1 &= 12\sin(\omega t) \\ E_2 &= 18\sin(\omega t + 60^\circ). \end{aligned}$$

5. (a) Write down an expression for sound level in terms of sound intensity, explaining each of the terms in your equation. **[3]**

(b) If you are given two sound intensities, I_1 and I_2, how is the difference in the sound levels β_1 and β_2 related to the intensities? **[2]**

(c) By what factor has the intensity increased if the sound level has risen by 10 dB? **[2]**

6. (a) State the change of phase which occurs when a light wave is reflected from

i. an optically more dense material, and

ii. an optically less dense material. **[2]**

(b) A soap-bubble film with refractive index $n = 1.33$ is floating in air. Assume that we are looking at a sufficiently small area of the bubble so that the curvature of the bubble is negligible.

i. Draw a diagram to show the interference in light reflected from the film. **[3]**

ii. Calculate the minimum thickness of the film that results in constructive interference in the reflected light, when the film is illuminated with light whose wavelength in air is 550 nm. **[2]**

SECTION B

7. (a) Explain what is meant by *the principle of superposition* and by *dispersion.* **[2]**

(b) Consider the superposition of the following two waves:

$$\begin{aligned} y_1(x,t) &= A\sin(k_1 x - \omega_1 t) \\ y_2(x,t) &= A\sin(k_2 x - \omega_2 t)\,. \end{aligned}$$

i. Explain the terms *carrier wave* and *envelope*; and **[6]**

ii. Derive expressions for the phase and group velocities of a dispersive wave. **[4]**

(c) The angular frequency ω of longitudinal waves with wavevector k on a chain of beads, each of mass m, joined together by springs with unstretched length a and spring constant κ is

$$\omega = \sqrt{\frac{\kappa}{m}}\sin\left(\frac{ka}{2}\right).$$

i. In the limit of long wavelength, show that the phase and group velocities are both equal to **[4]**

$$v = \frac{a}{2}\sqrt{\frac{\kappa}{m}}.$$

ii. Calculate this limiting velocity if $m = 10$ g, $a = 1$ cm, and $\kappa = 1$ N m^{-1}. **[2]**

iii. Find the group velocity of a wave with wavelength $2a$? **[2]**

8. Light of wavelength λ incident on a diffraction grating produces interference maxima at angles given by

$$\sin\theta_{\mathrm{m}} = m\lambda/D.$$

where D is the spacing of the diffraction grating and m is an integer.

(a) A diffraction grating has 49,000 lines spread over a width of 42 mm. If the wavelength of the visible spectrum extends from $\lambda = 400$ nm to $\lambda = 700$ nm, find the angular width of the first-order visible spectrum. **[5]**

(b) i. Show that the dispersion of a diffraction grating may be written as **[4]**

$$\frac{\mathrm{d}\theta}{\mathrm{d}\lambda} \approx \frac{m}{D\cos\theta_m}.$$

What does this expression become for small θ_m?

ii. Also prove that the resolving power R of a diffraction grating is given by

$$R = \frac{\lambda}{\Delta\lambda} = mN,$$

where m is the order of the spectrum and N is the number of slits. **[8]**

(c) How many lines must a diffraction grating have in order to resolve the sodium D lines, with wavelengths $\lambda_1 = 588.995$ nm and $\lambda_2 = 589.592$ nm, in second order? **[3]**

9. (a) A compound microscope is formed from two converging lenses, an objective with focal length 4 mm and an eyepiece with focal length 20 mm. The microscope is adjusted by altering the separation of the lenses.

   i. Draw a ray diagram (not necessarily to scale) showing the formation of an image 250 mm from the eye (at the near point), of an object placed 4.1 mm from the objective. **[6]**

   ii. What must the separation of the lenses be in this setting? **[4]**

   iii. What is the magnification of the microscope when adjusted in this way? **[2]**

   iv. Why is it more usual to adjust the microscope so that the image is at infinity? **[1]**

(b) A simple terrestrial telescope that produces an upright image consists of a converging objective lens and a diverging eyepiece at opposite ends of the telescope tube. For distant objects, the tube length is the objective focal length less the absolute value of the eyepiece focal length.

   i. Does the user of the telescope see a real or virtual image? **[1]**

   ii. Where is the final image? **[2]**

   iii. If a telescope is to be constructed with a tube 10.0 cm long and a magnification of 3.00, what are the focal lengths of the objective and the eyepiece? **[4]**

10. (a) A source emits a sound wave with frequency $f_{\rm S}$. The frequency with which a listener hears the sound wave is $f_{\rm L}$. The speed of sound in the medium is v.

If the listener is moving toward a stationary source with velocity $v_{\rm L}$, the observed frequency is

$$f_{\rm L} = \left(1 + \frac{v_{\rm L}}{v}\right) f_{\rm S}.$$

If a source moves toward a stationary listener with velocity $v_{\rm S}$, the observed frequency is

$$f_{\rm L} = \left(\frac{1}{1 - \frac{v_{\rm S}}{v}}\right) f_{\rm S}.$$

Derive **one** of the above equations. **[6]**

(b) A police car with its 300-Hz siren is moving toward a warehouse at 30 m s^{-1}, intending to crash through the door. What frequency does the driver of the police car hear reflected from the warehouse? The speed of sound in air is 343 m s^{-1}. **[5]**

(c) i. Write down the expression for the observed frequency when both the listener and source are moving towards each other, with speeds $v_{\rm L}$ and $v_{\rm S}$, respectively. **[4]**

ii. If a light source and an observer approach each other with a relative speed v, the frequency $f_{\rm obs}$ measured by the observer is given by the relativistic equation:

$$f_{\rm obs} = \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}}\, f_{\rm source} \qquad \text{[do not derive this equation]},$$

where c is the speed of light and $f_{\rm source}$ is the frequency of light emitted by the source.

Show that the non-relativistic equation agrees with this to first order in v/c. **[5]**

11. (a) With respect to interferometry, what is an etalon? With which type of interferometer is the term mostly linked? **[3]**

(b) Draw a diagram of a Michelson spectral interferometer. Include in your diagram a compensating plate, and explain why it is important for observing fringes with white light. **[6]**

(c) Monochromatic light is beamed into a Michelson interferometer. The movable mirror is displaced 0.382 mm, causing 1700 fringes to move across a line in the field of view. Determine the wavelength of the light. What colour is it? **[4]**

(d) One leg of a Michelson interferometer contains an evacuated cylinder of length $L = 5.00$ cm, having glass plates on each end. A gas is slowly leaked into the cylinder until a standard pressure is reached. If 48 bright fringes pass on the screen when light from a mercury-vapor lamp is used ($\lambda = 546$ nm), what is the index of refraction n of the gas? **[7]**

END OF PAPER