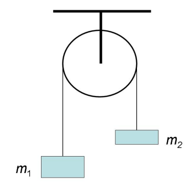
PHAS1247 Classical Mechanics
Problem-Solving Tutorial 2, 29 Oct – 2 Nov 2017
Rotations and oscillations

1. Two blocks, having masses m_1 and m_2 (with $m_1 > m_2$), are connected by a light, inextensible string of length L which hangs over a light, frictionless pulley. The two blocks hang vertically under gravity.

   Find the accelerations of the blocks indicating the forces acting on each block and writing down the equation of motion for each. Remembering the constraint that the length L of the string should remain constant, combine your two equations into one and solve it to find the acceleration of each block and the tension in the string.

   If the blocks are released from rest when m_2 is a distance l below the pulley, how long is it before the pulley jams?

   For general discussion.

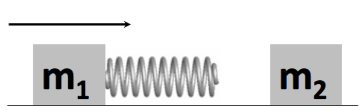
   Find the accelerations of the masses in the previous quesion by the following alternative method. Write down the total energy of the system (kinetic energy plus gravitational potential energy, choosing any convenient zero of potential energy). Differentiate your expression with respect to time to find the acceleration of each block; confirm that the result is the same as found previously.

2. A body of mass m moves in space under the influence of a force

   $$\mathbf{F} = xy\hat{\mathbf{i}} + 4y^2\hat{\mathbf{j}} - 7z\hat{\mathbf{k}}$$

   Is this force conservative? Calculate the work done if the body moves from the origin to point (1,2,3) along a straight line.

3. A mass $m_1 = 0.3\,\mathrm{kg}$ slides on a frictionless horizontal surface with speed $2\,\mathrm{ms}^{-1}$. It collides with a stationary mass, $m_2 = 0.7\,\mathrm{kg}$. A spring of negligible mass with a spring constant $k = 20\,\mathrm{Nm}^{-1}$ attached to m_1 compresses and then relaxes during the collision during which energy is conserved. See the figure below.

   By considering energy conservation, determine the maximum compression of the spring during the collision, that happens when the two masses move with the same speed before being separated again by the effect of the spring.

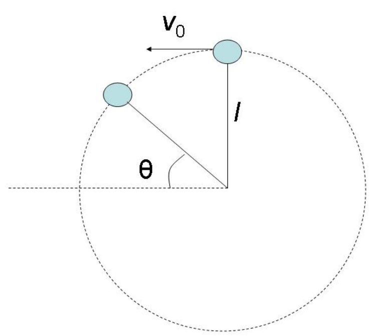
   The PE of a spring extended a distance x from equilibrium is: $\frac{1}{2}kx^2$.

4. An object of mass m moves around a circle of radius R. Its angular velocity increases linearly with time according to the relation: $\omega = \alpha t$, where α is a constant. Find the object's acceleration vector in terms of the unit vectors $\hat{\mathbf{r}}$ and $\hat{\theta}$.

   Find also the object's angular momentum as a function of time and the torque acting on it (in both cases, about the centre of the circle).

5. A conker of mass m attached to a string of length l is launched horizontally with speed v_0 at the top of a circular arc, with the string vertical and held rigidly at the bottom with his wizard hand (see diagram).

   Assuming the string remains taut and the conker moves in a circle, find (i) the speed of the conker, (ii) the centripetal force required to keep it moving in a circle, and (iii) the tension in the string, all as a function of the angle θ. [**HINT** for part (i)—think about which quantity is conserved during the motion.]



   If the breaking tension of Steve Prescott's string is 10 N, $l = 0.3$ m and $m = 0.02$ kg, what is the maximum speed at which he can safely launch his conker? What is the minimum speed that will keep the string taut at all times?

   [The acceleration due to gravity is $g = 9.81\,\mathrm{ms}^{-2}$.]

6. A series of parallel hills can be described by a sinusoidal expression for their altitude as a function of position:

$$z = A\sin(x)$$

   A ball is placed on the top of the first hill at $x = \pi/2$ and falls without friction along the direction of negative x. Calculate the velocity as a function of its position x, neglecting frictions. If the ball is instead placed at a distance $d = 0.1$ to the right of the bottom of the valley, calculate the its equation of motion assuming small oscillations around the equilibrium position.