PHAS1247 : Classical Mechanics
In-Course Assessment Test #1 : Mon. 7 November 2016

Answer as many of the questions as you can, in any order. The approximate distribution of marks is given in square brackets on the right of the page. The maximum mark is 27.

1. The water in a river moves with velocity $4\,\hat{\underline{\mathbf{i}}}\,\mathrm{ms}^{-1}$. A boat starts at the bank and this bank, bank-A, is along the $\hat{\underline{\mathbf{i}}}$ direction. The boat's captain wants to cross the river to the opposite bank, bank-B, which runs parallel to bank-A. The captain sets the engine such that the boat would have a speed of $5\,\mathrm{ms}^{-1}$ in still water. What is the velocity vector of the boat and the angle of the boat's travel relative to bank-A for the boat to cross the river in the shortest distance? **[4]**

2. A model car has four wheels and a total mass of 100 g. It is initially at rest at one side of a horizontal table. It is given an impulse in the $+\hat{\underline{\mathbf{i}}}$ direction in order to reach the other side of the table in 4 s. The coefficient of kinetic (sliding) friction is 0.1 and g is $9.81\,\mathrm{ms}^{-2}$.

   If the wheels are locked and the car slides across the table, what is the minimum impulse required for the car to cross the table? The car is then placed back on the table and the wheels are unlocked and the wheels, and hence car, roll immediately after an impulse is applied in the $+\hat{\underline{\mathbf{i}}}$ direction. If the radius of the wheels is 5 cm, what is the impulse applied to the car for it to roll 1 m in 4 s and what is the angular speed of rotation of the wheels? **[4]** **[3]**

3. A mass $m_1 = 4\,\mathrm{kg}$ has an initial velocity in the laboratory frame (i.e. relative to a stationary observer) $\underline{\mathbf{u}}_1 = 2\,\hat{\underline{\mathbf{i}}}\,\mathrm{ms}^{-1}$ while a second mass $m_2 = 5\,\mathrm{kg}$ has a velocity $\underline{\mathbf{u}}_2 = -3\,\hat{\underline{\mathbf{i}}}\,\mathrm{ms}^{-1}$. Find (i) the total momentum of the system, (ii) the velocity of the centre of mass, and (iii) the relative velocity of particle 1 to particle 2. **[3]**

   The two particles undergo an elastic collision and continue to move along the x-axis. Find the velocity of particle 2 in both the laboratory and the centre of mass frame after the collision. Express your answers as a fraction. **[4]**

4. Give one definition of a conservative force ? **[1]**

   A particle moving in three dimensions with position coordinates (x, y, z) has a potential energy, V, given by:

$$V = y^2 + 4x^2y^3 + z^2 \ .$$

   Find an expression for the force $\underline{\mathbf{F}}$ acting on the particle at position (x, y, z). **[3]**

   What is the change in KE (initial KE–final KE) of the particle as it moves from a position (0,0,0) to (1,1,2) assuming the only force acting on the particle is $\underline{\mathbf{F}}$. **[3]**

   If the force acting on a particle is:

$$\underline{\mathbf{F_1}} = 2xy^2\,\hat{\underline{\mathbf{i}}} + 2x^2y\,\hat{\underline{\mathbf{j}}}.$$

   Find an expression for the potential energy of the particle at position (x, y) assuming $V = 0$ at $x = y = 0$. **[2]**