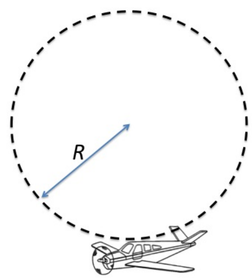
PHAS1247 Classical Mechanics
Problem-Solving Tutorial 4, 3–7 December 2018
Orbits and Fictitious Forces

Section A: centrifugal forces

1. A stunt pilot pulls out of a dive in such a way that her plane moves at a speed of $V = 100\,\mathrm{ms}^{-1}$ around part of a vertical circle of radius $R = 500\,\mathrm{m}$ (see diagram). If her mass is $m = 50\,\mathrm{kg}$, what is her apparent weight when the plane is at its lowest point on the circle?



Acceleration due to gravity: $g = 9.81\,\mathrm{m\,s^{-2}}$

2. Consider an object of mass 1 kg at a latitude of 50° (i.e. roughly that of London). Find the magnitude and direction of the centrifugal force on the object arising from the Earth's rotation.

   Find also the resultant force arising from the combination of the object's weight and the centrifugal force. What angle to the vertical does it make? How would this affect the determination of a 'horizontal' plane given by a spirit level?

   Acceleration due to gravity: $g = 9.81\,\mathrm{m\,s^{-2}}$; rotation period of Earth is 24 hours; radius of Earth $R_E = 6.37 \times 10^6\,\mathrm{m}$.

3. A truck of mass M is moving horizontally at constant speed v around a curve of radius R (as measured from the centre of curvature to the centre of mass of the truck) The centre of mass of the truck is a height h above the ground and the distance between the inner and outer wheels is D. Sketch a diagram of the truck, viewed from behind, and indicate on it the real forces acting on the truck. State the magnitude and point of application of the ficitious force which, when introduced, allows you to consider the truck to be in equilibrium in its own frame of reference.

   By taking moments about any convenient point, determine the normal reactions from the ground at the inner and outer wheels of the truck. Hence determine the maximum speed the truck can travel without overturning, assuming the tyres are sufficiently good that the truck does not skid sideways first.

For general discussion.

In 2011 the OPERA experiment reported it had found neutrinos travelling faster than the speed of light. Despite several desperate Nobel attemps to predict this effect theoretically from new physics sources this turned out to be a cable not connected correctly. One effect on the motion of the neutrinos is due to the Coriolis force. The neutrinos were travelling in the South East direction from CERN to Gran Sasso over a distance of approximately 730 km. What is the Coriolis acceleration (magnitude and direction) as the neutrinos leave CERN ? What effect does this have on the path length and hence time of flight of the neutrinos : how large is this compared to the accuracy of the GPS systems in the experiment (approx. 2 ns) ? Reminder: in a rotating reference frame, a body experiences radial and tangential fictious forces, with the second tangential term called Coriolis force:

$$F_r = m(\ddot{r} - r\dot{\theta}^2)$$

$$F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

Section B: orbits

1. A satellite of mass m is moving in a circular orbit of radius r_0 about the Earth, whose mass is M_E. Determine the kinetic energy KE, potential energy V, the total energy E and the angular momentum L of the satellite in terms of m, M_E, r_0 and the gravitational constant G. (You may assume that $m \ll M_E$.)

   A booster rocket on the satellite is fired for a short time, increasing the speed of the satellite to 4/3 of its original value without significatly altering its mass. Determine the new total energy E_1 and angular momentum L_1 of the satellite.

   Determine the eccentricity of the satellite's new orbit and sketch this orbit superimposed on the original orbit. Justify qualitatively why the new orbit has the form it does.

   What shape would the orbit have been if the velocity boost had been by a factor of $\sqrt{2}$, rather than 4/3?

   The eccentricity of the orbit of a particle of mass m in an inverse-square-law force $F(r) = -|K|/r^2; K = GM_E m$ is given by

   $$e = \sqrt{1 + \frac{2EL^2}{mK^2}}$$

   where the other symbols have the meanings above.

For general discussion.

Suppose I set a simple pendulum bob oscillating in a horizontal plane. In view of the fictitious forces it experiences from the Earth's rotation, will its vertical plane of oscillation remain the same relative to an observer on the Earth? If not, how quickly and in what direction will the plane of oscillation change? How does your answer depend on the latitude of the pendulum?