PHAS1247 Classical Mechanics
Problem Sheet 1
Submission deadline: Thursday 14 October 2010

1. Give both the *dimensions* (in terms of the fundamental dimensions of length $[L]$, time $[T]$ and mass $[M]$) and the *units* (in the SI system) of the following quantities:

   (a) Acceleration;

   (b) Force;

   (c) Energy (or Work);

   (d) Momentum.

   [4]

2. If the North Atlantic is widening by 3 inches per year because of volcanic activity in the ocean floor, what is the magnitude of the velocity of London relative to New York in SI units? **[3]**

   [You may assume that 1 inch = 2.54 cm and that a year contains exactly 365 days.]

3. The displacement of a particle of mass m moving in one dimension is

   $$x(t) = \alpha t^2 + \beta t + \gamma,$$

   where α, β and γ are non-zero constants.

   What are the dimensions of α, β and γ? **[3]**

   Find expressions for (i) the velocity and (ii) the acceleration of the object at time t. **[4]**

   Explain why there must be an external force acting on the body. What is the power developed by this force at time t? Show that the power is equal to the rate at which the kinetic energy of the particle is changing. **[3]**

4. The driver of a car moving at 90 km/hr sees an obstacle 50 m ahead. Assuming that the driver instantly applies the brakes (i.e. the 'thinking time' is negligible) and that the deceleration produced by the brakes is uniform, find what deceleration is required in order for the car to stop in time to avoid hitting the obstacle. **[2]**

   How would your answer change if the car was moving twice as fast? **[1]**

5. Consider the vectors

   $$\mathbf{a} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}; \qquad \mathbf{b} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}.$$

   Using the definitions of the scalar and vector products given in the lectures, compute (i) $\mathbf{a} \cdot \mathbf{b}$ and (ii) $\mathbf{a} \times \mathbf{b}$. **[5]**

6. What is the value of the scalar product of two orthogonal vectors? **[1]**

   Verify that the following three vectors form an 'orthonormal set' (in other words, that they are of unit length and orthogonal to one another):

   $$\hat{\mathbf{e}}_1 = \frac{1}{2}\begin{pmatrix} \sqrt{3} \\ 1 \\ 0 \end{pmatrix}; \quad \hat{\mathbf{e}}_2 = \frac{1}{2}\begin{pmatrix} -1 \\ \sqrt{3} \\ 0 \end{pmatrix}; \quad \hat{\mathbf{e}}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

   [6]

   An object's position vector is $\mathbf{r} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$. Find the components of $\mathbf{r}$ along the axes corresponding to $\hat{\mathbf{e}}_1$, $\hat{\mathbf{e}}_2$ and $\hat{\mathbf{e}}_3$. (In other words, find numbers c_1, c_2 and c_3 such that $\mathbf{r}$ can be expressed as $c_1\hat{\mathbf{e}}_1 + c_2\hat{\mathbf{e}}_2 + c_3\hat{\mathbf{e}}_3$.) **[3]**