Probability Calculation

$$P(a \leqslant x \leqslant b) = \int_a^b |\psi(x)|^2 dx$$

Normalisation

$$\int_{-\infty}^{+\infty} |\psi(x)|^2 dx = 1$$

Our $\psi(x)$,

$$\psi(x) = \sin(\pi x) \qquad \text{for } 0 \leq x \leq 2$$
$$= 0 \qquad \text{for all other } x$$

$$I = \int_{-\infty}^{\infty} |\psi|^2 dx = \cancelto{0}{\int_{-\infty}^{0} |\psi(x)|^2 dx} + \int_0^2 |\psi(x)|^2 dx + \cancelto{0}{\int_2^{\infty} |\psi(x)|^2 dx}$$

$$I = \int_0^2 \sin^2(\pi x)\, dx$$

But $$\sin^2\theta = \frac{1 - \cos(2\theta)}{2}$$

$$I = \int_0^2 \frac{1}{2}\,dx - \int_0^2 \frac{\cos(2\pi x)}{2}\,dx$$

$$= \left[\frac{x}{2}\right]_0^2 - \cancelto{0}{\left[\frac{\sin 2\pi x}{4\pi}\right]_0^2}$$

$$I = \frac{2}{2} - \frac{0}{2} = 1$$

$\therefore \psi(x)$ is normalised.

Probability Calculation

$$P(0.5 \le x \le 1) = \int_{0.5}^{1.0} |\psi|^2 \, dx$$

$$\psi(x) = \sin(\pi x) \quad \text{for } 0 \le x \le 2$$

$$0 \quad \text{for all other } x$$

$$P(0.5 \le x \le 1) = \int_{0.5}^{1} \sin^2(\pi x) \, dx$$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$P(0.5 \le x \le 1) = \int_{0.5}^{1} \frac{dx}{2} - \int_{0.5}^{1.0} \frac{\cos 2\pi x}{2} \, dx$$

$$= \left[\frac{x}{2}\right]_{0.5}^{1} - \left[\frac{\sin 2\pi x}{4\pi}\right]_{0.5}^{1}$$

$$= \frac{1}{4}$$