

UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. M.Sci.

Physics 1B28: Thermal Physics

COURSE CODE : **PHYS1B28**

UNIT VALUE : **0.50**

DATE : **27-MAY-03**

TIME : **10.00**

TIME ALLOWED : **2 Hours 30 Minutes**

Answer ALL SIX questions from Section A and THREE questions from Section B.

The numbers in square brackets in the right-hand margin indicate the provisional allocations of maximum marks per sub-section of a question.

Boltzmann's constant $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$
The gas constant $R = 8.31 \text{ J K}^{-1} \text{ mol}^{-1}$
Avogadro's number $N_A = 6.02 \times 10^{23} \text{ mole}^{-1}$
Acceleration due to gravity $g = 9.81 \text{ m s}^{-2}$
Elementary charge $e = 1.60 \times 10^{-19} \text{ C}$
Permittivity of a vacuum $\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}$
Freezing point of water $0^\circ\text{C} = 273.15 \text{ K}$

SECTION A

[Part marks]

1. The Bohr model of the hydrogen atom leads to the equation;

$$E_n = -\frac{e^2}{8\pi\epsilon_0 a_0} \left(\frac{1}{n^2} \right).$$

Explain what this equation represents, and discuss the significance of the constants a_0 and n . If $a_0 = 5.29 \times 10^{-11} \text{ m}$, calculate the energy of the ground state of a hydrogen atom. [3]

The electronic configurations of sodium (Na) and fluorine (F) atoms are $1s^2 2s^2 2p^6 3s^1$ and $1s^2 2s^2 2p^5$ respectively. Explain briefly why the bonding in NaF is ionic, while that in F_2 is covalent. [3]

2. The Lennard-Jones interaction potential can be written;

$$u(r) = 4\epsilon \left[\left(\frac{\sigma}{r} \right)^{12} - \left(\frac{\sigma}{r} \right)^6 \right].$$

Discuss briefly what this function describes, and its physical origins. [2]

Sketch the Lennard-Jones interaction potential as a function of r , marking ϵ and σ on your diagram. Mark also the equilibrium spacing, r_0 , between the particles. [3]

Show that for particles interacting through the Lennard-Jones potential, the equilibrium spacing between particles is given by $r_0 = 2^{1/6} \sigma$. [3]

3. Write down a statement of the *First Law of Thermodynamics*, defining any symbols you use. [2]

Show that for a mole of ideal gas expanding isothermally and reversibly from a volume V_1 to a volume V_2 , the work done by the gas is equal to the heat input, and is given by;

$$W = RT \ln \left(\frac{V_2}{V_1} \right). \quad [4]$$

4. Calculate the final equilibrium temperature if 0.2 kg of copper at 90 °C is placed into 1.0 kg of water at 0 °C, and the system is then thermally isolated. [3]

Calculate also the total entropy change for this process. [3]

You can assume there is no volume change during this process. The heat capacities of copper and water are constant over the temperature range in question, and are 387 J kg⁻¹ K⁻¹ and 4186 J kg⁻¹ K⁻¹ respectively.

5. Describe briefly the classical *Principle of the Equipartition of Energy* and the concept of a *degree of freedom*. [3]

Identify the degrees of freedom for a simple diatomic molecule, such as N₂. [2]

Estimate the molar internal energy of ideal N₂ gas at 300 K. [2]

6. Show that the entropy change for an ideal gas undergoing a change can be written

$$\Delta S = nC_V \ln\left(\frac{T_2}{T_1}\right) + nR \ln\left(\frac{V_2}{V_1}\right). \quad [4]$$

Calculate the entropy change of one mole of monatomic ideal gas when the temperature increases from 270K to 320 K, at a constant pressure of 1×10⁵ Pa. [3]

7. Show that for a mole of ideal gas;

$$C_p - C_v = R. \quad [4]$$

Explain the conditions necessary for a *reversible adiabatic* process. [2]

Show that for a reversible adiabatic process involving an ideal gas;

$$pV^\gamma = \text{constant},$$

and;

$$T^\gamma p^{1-\gamma} = \text{constant}. \quad [6]$$

Give the definition of γ in these equations, and state the possible range of values of γ for ideal gases. Explain why $\gamma = 1.4$ for O_2 at 300 K. [4]

The pressure of oxygen at sea-level is 0.22×10^5 Pa. If the atmosphere is isothermal at 280K, use the *Law of Atmospheres* to calculate the height at which the oxygen pressure is 0.11×10^5 Pa. The mass of an oxygen molecule is 5.31×10^{-26} kg. [4]

8. Give a statement of the *Second Law of Thermodynamics*. [2]

Define the entropy change for a *reversible* process between two equilibrium thermodynamics states. Write down the Clausius inequality for an *irreversible* process. [3]

Sketch on p - V and T - S diagrams a reversible Carnot cycle using an ideal gas as the working substance, operating as a *refrigerator* between two temperatures $T_H > T_C$. In each case label the processes involved, and the heat flow and work done. [6]

Explain what is meant by a *state function*. Calculate the entropy change when the ideal gas is taken around the above ideal Carnot cycle, and use your result to verify that entropy is a state function. [6]

An ideal reversible Carnot cycle operates between -5°C and 25°C . Calculate the coefficient of performance of this cycle when it acts as refrigerator. [3]

9. Using the kinetic theory of gases, derive the relationship; [6]

$$pV = \frac{1}{3}Nm\overline{v^2}.$$

Explain the meaning of all symbols used.

Show then that root mean square speed of gas particles is given by [2]

$$\sqrt{\overline{v^2}} = v_{rms} = \sqrt{\frac{3k_B T}{m}}.$$

Calculate the root mean square speed of helium and argon atoms at 300 K. The molar masses of helium and argon are 4 g mol⁻¹ and 36 g mol⁻¹. [3]

When a balloon with a small leak is inflated with helium gas, it deflates in 360 seconds. How long will this balloon take to deflate if it is inflated to the same pressure, and at the same temperature, with argon? Both gases may be taken as ideal. [2]

The Maxwell-Boltzmann speed distribution can be written;

$$P(v)dv = 4\pi v^2 \left(\frac{m}{2\pi kT} \right)^{3/2} \exp\left(\frac{-mv^2}{2kT} \right) dv.$$

Explain briefly what this distribution represents, and its physical origins. Sketch the Maxwell-Boltzmann distribution for helium and argon atoms at 300 K. Mark the root mean square speeds on your diagram. [7]

10. Explain briefly the origins for the thermal expansion of simple solids and liquids. [3]

A cylindrical copper bar has length 1m and radius 0.1m, and is at equilibrium at 280K. When this bar is heated to a temperature of 300 K, what are its new length and volume? The linear expansion coefficient of copper is $1.7 \times 10^{-5} \text{ K}^{-1}$. [5]

If the copper bar is held so that it cannot expand, what pressure is exerted on the ends of the bar when the temperature is raised from 280 K to 300 K? Young's modulus for copper is $1.1 \times 10^{11} \text{ Pa}$. [3]

Thermal conduction is governed by the equation;

$$\frac{Q}{\Delta t} = \kappa A \frac{\Delta T}{\Delta x}.$$

Explain the meaning of the symbols in this equation. [2]

What power is carried along the above copper bar by thermal conduction when one end is maintained at 280 K and the other end at 300 K? The thermal conductivity of copper is $397 \text{ W m}^{-1} \text{ K}^{-1}$, and you may assume no heat is lost from the sides of the bar. [2]

A cylindrical bar, also of radius 0.1m and total length 1m, is comprised of a 0.5 m length of copper joined to a 0.5 m length of iron. What power is carried along this bar through thermal conduction when one end is maintained at 280 K and one end at 300 K? The thermal conductivity of iron is $79.5 \text{ W m}^{-1} \text{ K}^{-1}$, and you may assume no heat is lost from the sides of the bar. [5]

11. The van der Waals equation of state can be written

$$\left(p + \frac{N^2 a}{V^2} \right) (V - Nb) = NkT.$$

Discuss briefly the purpose of this equation, and the conditions under which you would expect it to be valid. [3]

Explain the physical interpretation of the constants a and b , with reference to the ideal gas equation of state and the interparticle interactions. [4]

Use a p - V diagram to illustrate the behaviour of a simple gas, such as neon, in the (i) ideal, and (ii) van der Waals regimes. [5]

Neon forms a cubic close-packed (face-centred cubic) crystal. If the neon atoms are modelled as hard-spheres, calculate the coordination number and packing fraction in this solid. [5]

Using the *Third Law of Thermodynamics*, describe the behaviour of the entropy and heat capacity of a perfect crystal as the temperature approaches absolute zero. [3]