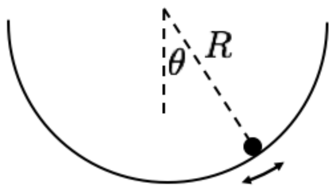
PHAS1247 : Classical Mechanics
In-Course Assessment Test #2 : Mon. 9 January 2017

Answer as many of the questions as you can, in any order. The distribution of marks is given in square brackets on the right of the page. There are 4 questions: they continue **ON THE OTHER SIDE OF THE PAGE**. *The maximum mark is 31.*

1. A mass on the end of a spring is released from rest at a position x_0 from its equilibrium position and oscillates with an (angular) frequency ω_o. This is repeated but the system is now immersed in a fluid that provides a retarding force $-2m\gamma\dot{x}$ and causes the motion to be *over-damped*. Show that the maximum velocity in the over-damped case occurs when: **[8]**

$$t = \frac{1}{2\omega}\ln\left(\frac{\gamma+\omega}{\gamma-\omega}\right) \quad \text{where} \quad \omega^2 = \gamma^2 - \omega_0^2$$

2. A small ball with radius r and uniform density, *rolls*, under the influence of gravity, without slipping near the bottom of a fixed cylinder of radius R (see below).



By considering the tangential acceleration of the centre of mass and the torques, show that, for $R \gg r$ and small θ, that the ball executes simple harmonic motion with angular speed: **[6]**

$$\omega = \sqrt{\frac{5g}{7R}}.$$

The moment of inertia of the ball is: $\frac{2}{5}mr^2$.

3. Making America great again, The Donald starts to build a wall on the edge of a cliff at a lattitude of 31.6^0N. The prototype wall can be approximated as a square lamina with a height of 100 m and a mass per unit area of 16 gcm^{-2}.

   (a) Determine the horizontal centrifugal force on the base of the wall due to the earth's rotation.

   (b) There is also a centrifugal force at the top of the wall. Consider the difference between the two centrifugal forces and the resultant torque about the CM to determine the angular acceleration of The Donald's wall due to the earth's rotation. **[5]**

   With the costs of the wall escalating, The Donald decides to use cheap cement but his wall crumbles due to the torque and buries Kid Rock who is rehearsing "Another Brick In The Wall" in Bigot Valley below.

   (c) The bricks fall vertically down the cliff with an initial speed of 20 ms^{-1}. Using this speed, determine the magnitude and direction of the Coriolis force on a single brick of mass 4 kg. **[2]**

   The moment of inertia of a square lamina with a side of length l is: $\frac{1}{12}ml^2$ and the earth's radius is 6400 km.

CONTINUED ON OTHER SIDE OF PAGE

4. Using the definition of $\underline{\mathbf{v}}$ in polar coordinates show that the energy E for a particle of mass m and angular momentum L in a potential, $V(r)$, is: **[2]**

$$E = \frac{1}{2}m\dot{r}^2 + \frac{L^2}{2mr^2} + V(r) = \frac{1}{2}m\dot{r}^2 + V_{\text{eff}}(r)$$

(a) For the case $E = 0$ and a spiral trajectory: $r = r_0 e^{a\theta}$, write an expression for V(r) in terms of L, a, m and r. **[2]**

A particle moves in a potential:

$$V(r) = -\frac{C}{3r^3}$$

where C is a constant.

(b) Draw a sketch of V_{eff} and for a given L, find the maximum value of V_{eff} ($V_{\text{eff}}^{\text{max}}$).

A particle comes from $r = \infty$ with speed v_0 and is acted on by this potential.

(c) In the two cases: $E < V_{\text{eff}}^{\text{max}}$ and $E > V_{\text{eff}}^{\text{max}}$ what happens to the particle. **[6]**

(d) If the closest the particle gets to $r = 0$ is b, determine the maximum value of b that results in the particle being captured by the potential.

END OF PAPER