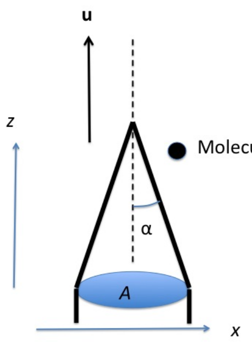
PHAS1247 Classical Mechanics
Problems for Week 5/6 of Lectures (2016)

1. Consider a rocket of mass M moving at a velocity $\mathbf{u} = u\hat{\mathbf{k}}$ through a gas consisting of molecules with mass $m \ll M$. The speed of the rocket, $|\mathbf{u}|$, is very much greater than the speeds of the individual molecules in the gas. The cross-sectional area of the rocket is A, and its nose is a smooth circular cone making an angle α with the axis of the rocket (see diagram).

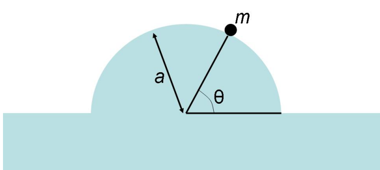


The rocket collides with a molecule displaced in the x-direction from the rocket's axis, as shown. Describe the collision as viewed in their centre-of-mass frame, assuming that the collision is elastic and does not change the component of the molecule's velocity parallel to the nose-cone's surface. Hence show that the rocket experiences an impulse $-2mu \sin\alpha(\cos\alpha\hat{\mathbf{i}} + \sin\alpha\hat{\mathbf{k}})$ from the collision in this frame.

Using this impulse and assuming the density of the gas is ρ, find the total number of such collisions in a time interval δt. Hence show that the rocket experiences an average retarding force due to the net effect of all the collisions equal to

$$\mathbf{F}_{\text{ret}} = -2\rho A u^2 \sin^2\alpha\hat{\mathbf{k}}.$$

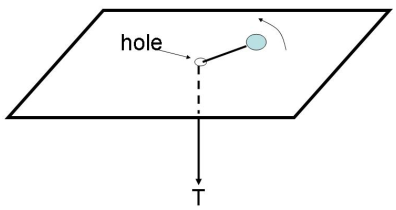
2. A mass m is placed on the top of a smooth hemisphere of radius a such that $\theta = \pi/2$ (see diagram for the definition of θ). It is given a very small impulse and as a result begins to slide down one side of the hemisphere under the influence of the gravitational acceleration g.



State the forces acting on the mass, giving their directions, and write down its radial and angular equations of motion in polar coordinates so long as it remains sliding on the sphere.

Assume the gravitational PE is zero on the ground (base of hemisphere). Find the reaction force between the mass and the surface of the hemisphere as a function of the angle θ, and hence by considering conservation of energy show that the mass flies off the surface of the hemisphere when its vertical height has decreased by $a/3$.

3. A particle of mass m is attached to one end of a light in-extensible string. The other end is passed through a small hole with smooth edges in a smooth horizontal table and is is held fixed while the particle moves on the table in a circular path of radius r_0 with angular velocity ω_0. State the angular momentum L of the particle about the hole.



At time $t = 0$ the string starts to be pulled down through the hole at a constant speed V, so that the distance r of the particle from the hole decreases with time according to

$$r = r_0 - Vt.$$

Which quantity is conserved during the particle's subsequent motion? Use this information to determine at a subsequent time t (a) the angular velocity ω and (b) the velocity vector $\mathbf{v}$ (expressed in polar coordinates) of the particle.

Determine also the tension T in the string and the kinetic energy K of the particle at time t. Show that the rate of change of K with time is equal to the power developed by the person pulling the string.