PHAS 1102/1423
Physics of the Universe

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Partial Notes, Part 2, Section 3

Cosmology

These partial notes should be used in conjunction with your own notes, and the powerpoint slides which will be made available on-line.

1 The Big Bang Model

1.1 The Hubble Flow

We've already mentioned that all distant galaxies are moving away from us: the 'Hubble Flow'. This observation alone demonstrates that the universe is expanding.

1.1.1 Redshift

The velocities of distant gaaxies are measured through their *redshifts*. The redshift, z, is *defined* as

$$1 + z \equiv \frac{\lambda}{\lambda_0} = \frac{\lambda_0 + \Delta\lambda}{\lambda_0} = 1 + \frac{\Delta\lambda}{\lambda_0}$$

where λ_0 is the emitted wavelength and λ is the observed wavelength.

If $v \ll c$ then

$$1 + z = \frac{\lambda}{\lambda_0} \simeq 1 + \frac{v}{c}$$

but if v is not $\ll c$ the relativistic form must be used:

$$1 + z = \frac{\lambda}{\lambda_0} = \frac{1 + v/c}{\sqrt{1 - v^2/c^2}}.$$

For redshifts $z \lesssim 0.1$ or so, the value is usually expressed as a velocity (in which case it is always just cz); Larger values are normally given just as z (or sometimes $1 + z$).

1.1.2 Hubble's Law

In 1929 Hubble combined his own and other's observations of galaxy redshifts with new determinations of galaxy distances (besed on Cepheids and other methods), and established observationally that the velocity of recession is proportional to distance. We now call this proportionality Hubble's Law,

$$v = H_0 d$$

where v is the velocity of recession, d is the distance, and H_0 is 'Hubble's constant', ca. 70 km s^{-1} Mpc^{-1}). Hubble's constant H_0 has dimensions of 1/time, but it is usually given in units of km s^{-1} Mpc^{-1} (i.e., v in km s^{-1}, d in Mpc).

All galaxies outside the Local Group[1] ($\sim$ 30+ galaxies within $\sim$ 1 Mpc) are moving away from us (and we from them), and hence they show redshifted spectral lines. We believe that this 'Hubble Flow' shows that the Universe is expanding from a 'Big Bang' event early in its history.

> [Because the value of H_0 was, until relatively recently, quite poorly known (of order 100 km s^{-1}, but factor $\sim$2 uncertainty!), it has often been parameterized as
>
> $$H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$$
>
> Thus, for example, if a cosmological redshift of 500 km s^{-1} is measured for a galaxy, its distance may be expressed as
>
> $$d = v/H_0 = v/(100h) = 5h^{-1}\text{Mpc}.$$
>
> The last few years have seen a growing consensus that $H_0 \simeq 72 \pm 5$ km s^{-1} (i.e., $h \simeq 0.72 \pm 0.05$), through projects such as the HST Key Programme, Supernova Cosmology, WMAP etc.]

The expansion of the universe implies that at some time in the past everything was in the same vicinity. Suppose some galaxy, at a distance d, has been receding from us since the Big Bang with its *current* observed velocity (a very rough approximation). Then it has taken the age of the universe, designated t_0, to get to that distance, so

$$d \simeq v t_0;$$

but

$$v = d H_0$$

[1]Local Group galaxies have individual motions determined by the gravitational pull of other Local-Group galaxies.

whence

$$t_0 \simeq H_0^{-1} \equiv \tau_0$$

where the 'Hubble time', τ_0, is roughly the age of the Universe (more precisely, it is the time required for the Universe to double its size at the current expansion rate).

It's of interest to note that Hubble estimated $H_0 = 530$ km s^{-1} Mpc^{-1}. This gives

$$\tau_0 = \frac{9.77 \times 10^{11}}{530}\text{yrs} \approx 2 \times 10^9\text{yr}$$

– implying that the age of the Universe is less than the age of old rocks!

1.2 Evolution of source counts

Hypothetically, if a set of sources of the same luminosity L is distributed with number density n (per unit volume) then the number of sources per unit solid angle to distance r is

$$N(r) = \frac{4\pi r^3}{3}.\frac{n}{4\pi} = \frac{nr^3}{3}$$

and their brightness is greater than $\ell \propto L/r^2$ (i.e., $r^2 \propto L/\ell$). The number brighter than ℓ is thus

$$N(\ell) \propto \frac{nL^{3/2}}{\ell^{3/2}}$$

We can test the implicit assumption that the number of sources per unit volume is constant with distance (and hence with time; when we view distant sources we see them as they were, not as they are, because of the finite light-travel time), by plotting $N(\ell)$ as a function of ℓ.[2]

As first shown by Ryle in the 1950s, observed source counts do *not* follow the expected $N \propto \ell^{-3/2}$ trend. Typically, for many classes of extragalactic sources, there were more in the past than now; for example, there are *no* quasars in the nearby universe, but many at redshift 1. This constitutes powerful evidence for an evolving universe.

1.3 The microwave background

In 1965 Penzias and Wilson, engineers at Bell Laboratories, accidentally discovered the cosmic microwave background (CMB) – microwave 'noise' characterized by a black-body temperature of

[2]Historically, this was an important test of the now-discredited 'steady-state model', advocated in particular by Fred Hoyle.

3K and uniformly distributed across the sky. Penzias and Wilson got the Nobel Prize for this discovery (while the astrophysicist Robert Dicke, who told them what it was they'd discovered, who'd already conducted experimental searches for cosmic background radiation that had just failed to detect the 3K radiation, and who was in the process of building a new experiement which would've made the discovery, got nothing).

The natural interpretation of the evolution of the Universe is that it had some phase when it was extremely compact and hot; that it has expanded from that state in (very loosely speaking) an 'explosion'; and that the '3K' background is the remnant of the explosion.

Up to 3×10^5 yr after the Big Bang, the Universe was filled with high-energy photons which kept the matter (mostly hydrogen) ionized, and the Universe opaque (because free electrons have a large photon scattering cross-section).

Eventually, when the temperature fell to ∼3000K, electrons and nucleons combined to form atoms. The coupling between radiation and matter ceased (H atoms interact with photons at a much lower rate than free electrons). This took place at redshift $z \sim 1000$.

The Universe then became transparent; the CMB represents the 'last scattering surface' that photons saw, and carries an image of the Universe when it was one thousandth of its present size[3] (and about one hundred thousandth of its present age).

Observations, notably with the COBE satellite, have shown that, after corrections for local motions, the background is highly isotropic (to 1 part in 10^5!) on large scales; it is indistinguishable from black-body radiation characterized by $T = 2.725 \pm 0.001$K; and has it has low-level small-scale structure (on $\sim 1^\circ$ scales), studied in detail by the BOOMERanG and WMAP experiments. The precision of present-day observations, particularly of the small-scale structure, allows strong constraints to be placed on cosmological models.

1.3.1 A more quantitative analysis

As the early universe cooled, the initially ionized hydrogen would 're'combine with free electrons. We might expect this to occur roughly when

$$kT \sim 13.6\text{eV} \rightarrow T \sim 1.5 \times 10^5\text{K}$$

[3]What is 'the size of the Universe'? The answer is that it is very probably infinite. Usually, however, when we talk about 'the size of the Universe', we mean the size of the *observable* universe – that is, *roughly,* a sphere whose radius is given by the age of the universe times the speed of light.

(where 13.6eV is the ionization potential of hydrogen). However, photons outnumber electrons by a factor $\sim 10^9$ (then, and now).[4] Also, there is a distribution of photon energies at a given temperature (described by the Planck function). In consequence there were still plenty of photons capable of ionizing hydrogen, even at temperatures well below 10^5K, and a better (if still rough) estimate of the temperature at which the radiation field just failed to ionize hydrogen is

$$\sim (13.6\mathrm{eV}/k)/\ln(10^9) \simeq 7600\mathrm{K}.$$

More detailed calculations show that the hydrogen first recombined at $T \sim 3000$K.

Because free electrons have much larger scattering cross-sections than electrons bound in hydrogen, the radiation and matter interacted strongly when the universe was hotter than this 'decoupling temperature'. The properties of radiation and matter were tightly coupled at this time, enforcing thermal equilibrium and giving the radiation a black-body spectrum.

1.4 Cosmic abundances: primordial nucleosynthesis

All stars in our galaxy, and throughout the nearby universe, appear to be formed from an initial composition which is ~73% H, 25% He by mass.[5] This 'primordial' composition appears to be universal (and there is much more helium than can be explained by nuclear processing in stars). It is therefore natural to suppose that this composition is a consequence of initial conditions in the Universe; and indeed, early on in the Big Bang conditions were right for proton-proton chains to occur – exactly the same hydrogen fusion process that we have seen occurs in the cores of solar-type stars. Observed H/D/He/(Li) abundances provide strong constraints on the evolution of the Universe (while all heavier elements have formed subsequently, in stars).

At an age of 100 s, the Universe had cooled to 10^9 K and D, He and Li were synthesised from the plasma. At higher temperatures, high energy gamma rays broke up any deuterium, inhibiting nucleosynthesis. When the Universe reached an age of 1000 s, all nucleosynthesis stopped as the temperature was then too low (6×10^4 K).

The standard model of the Big Bang can account for the observed primordial abundances of He, D and Li for a specific value of the density of ordinary (baryonic) matter in today's universe. This is

[4]The universe today is said to be 'matter dominated'. This is because although the number of photons greatly exceeds the number of baryons + electrons, the *energy* of the photons compared to that of the mater is tiny. (The universe was 'radiation dominated' for the first $\sim 10^4$ yr after the Big Bang.)

[5]By convention, astronomers refer to the mass fractions of hydrogen, helium, and 'metals' (all elements except hydrogen and helium) as X, Y, Z; these abundances vary rather little throughout most of the nearby universe, with $X \simeq 0.73$, $Y \simeq 0.25$, $Z \simeq 0.02$.

because the baryonic density now is related to the baryonic density at the time of nucleosynthesis, and the baryonic density at that time determined the amount of deuterium (and, less sensitively, helium) produced – loosely speaking, the higher the density, the more deuterium. So by observing the ratio of deuterium to hydrogen atoms today, we infer the baryonic density at the time of nucleosyntheses, and hence the baryonic density today.

1.4.1 Another more quantitative analysis

As the temperature of the expanding Universe falls, a point is reached where neutrons and protons can combine into nuclei. The important points for our discussion of associated processes are:

- Protons are lighter than neutrons (938.3MeV vs. 939.6MeV; $\Delta m = 1.3$MeV)
- Free neutrons decay ($n \rightarrow p + e^-$), with a half-life of $t_{1/2}$ ~940s
- Bound neutrons (i.e., those in nuclei) are stable, as are protons.

At high temperatures the numbers of neutrons and protons are practically identical (since the mass-energy difference between them is negligible compared to kT):

$$n \leftrightarrow p + e^- + \overline{\nu}_{\mathrm{e}}(+0.8\mathrm{MeV})$$

(plus other processes). However, when $kT \sim 0.8$MeV neutrons can convert to protons, but not vice versa. Thus protons begin to outnumber neutrons, by a factor

$$\frac{n_{\mathrm{p}}}{n_{\mathrm{n}}} \sim \exp\left(\frac{\Delta m}{kT}\right) \sim \exp\left(\frac{1.3}{0.8}\right) \sim 5$$

The protons and neutrons combine to build up complex nuclei. Densities are too low at this stage for many-body collisions to be important, so nuclei build up through chains of two-body collisions, starting with $p + n \rightarrow^2 D + \gamma$ and then proceeding through a variety of routes, schematically as follows:

^{2}D gives...	^{3}He gives...	^{3}T gives...
$^2\mathrm{D} + p \rightarrow^3 \mathrm{He} + \gamma$	$^3\mathrm{He} + n \rightarrow^4 \mathbf{He} + \gamma$	
or $^2\mathrm{D} +^2 \mathrm{D} \rightarrow^3 \mathrm{He} + n$	or $^3\mathrm{He} + n \rightarrow^3 \mathrm{T} + p$	$^3\mathrm{T} + p \rightarrow^4 \mathbf{He} + \gamma$
or $^2\mathrm{D} + n \rightarrow^3 \mathrm{T} + \gamma$		
or $^2\mathrm{D} +^2 \mathrm{D} \rightarrow^4 \mathbf{He} + \gamma$		

The key stage is the production (and destruction) of deuterium, which has a low binding energy (~2.2MeV), and so is destroyed at $T \geq 10^9$K. Most nucleosynthesis actually occurs at around

$kT \sim 0.1$MeV ($T \sim 3 \times 10^8$K), for a period around $t_\mathrm{n} \sim 400$s. This timescale is long enough for the decay of free neutrons to be significant, but not complete; these decays reduce the neutron:proton number ratio by a further factor of $\sim \exp(400/940)$ to give a final neutron:proton ratio of

$$n_\mathrm{n}/n_\mathrm{p} \simeq \exp\left(\frac{-\Delta m}{kT}\right) \exp\left(\frac{t_\mathrm{n}}{t_{1/2}}\right) \simeq 1/7.$$

The only elements produced in bulk are ^{1}H and ^{4}He (the only stable low-mass nuclei), with virtually all the neutrons in helium. Thus the number density of helium atoms is $n(^4\mathrm{He}) = n_\mathrm{n}/2$; each weighs 4 hydrogen masses (to a very good approximation – for this rough calculation we can equate the masses of the neutron and proton) so the mass density is $2n_\mathrm{n}m_\mathrm{H}$. The total mass density is $m_\mathrm{H}(n_\mathrm{n} + n_\mathrm{p})$, so the mass fraction of helium is

$$Y_4 = \frac{2n_\mathrm{n}}{n_\mathrm{n} + n_\mathrm{p}} = \frac{2}{1 + n_\mathrm{p}/n_\mathrm{n}} \simeq \frac{2}{1+7} = 0.25$$

A more detailed, numerical treatment keeps track of the whole temperature-dependent reaction network, and yields mass fractions of

^{2}H $\sim 10^{-4}$

^{3}He $\sim 2 \times 10^{-5}$

^{7}Li $\sim 10^{-10}$

with almost nothing else produced cosmologically.

1.4.2 Comparison with observations

The precise abundance of (especially) ^{4}He constrains the temperature (and its rate of change) at the epoch of nucleosynthesis, and thereby constrains the details of Big-Bang models. There are two important input parameters which affect the abundance:

- The number of neutrino species (which affects the expansion t–T relation and hence how the reactions go out of thermal equilibrium)
- The density of baryonic matter, from which the nuclei form. This is often expressed in terms of a quantity η, the ratio of baryon to photon numbers:

  $$\eta = 2.76 \times 10^{-8} \Omega_\mathrm{B} h^2$$

  where Ω_B is the baryon density expressed as a fraction of the critical density, discussed further below (Section **??**).

The observed present-day abundance of helium, combined with standard Big-Bang models, led to a firm prediction that *only three neutrino species exist.* This prediction was subsequently confirmed experimentally by particle-physics experiments.

Overall agreement between light-element abundances and Big-Bang models is achieved for

$$0.010 \leq \Omega_{\rm B} h^2 \leq 0.022$$

Note the significance of this result: $\Omega_{\rm B} \ll 1$ **for any plausible value of h!** If $h \simeq 0.7$ then $\Omega_{\rm B} \lesssim 0.05$, which compares with the total matter content of the universe suggested by other observations (such as 'weighing' clusters of galaxies): $\Omega_{\rm M} \gtrsim 0.3$. The implication is that large quantities of non-baryonic matter must exist in the Universe.

(Neutrinos have mass and are not baryons, hence might be considered a possible source of this matter; however, galaxy-formation models currently require 'cold' (slowly-moving) material to explain large-scale structure. Moreover, direct measurements of neutrino mass show them to contribute very little to the overall mass budget of the universe. Hence, at present, some sort of ill-understood, exotic non-baryonic mass is implied, generically labelled as "weakly interacting massive particles", or WIMPs.)

2 The Dynamical Evolution of the Universe

We know that the Universe is expanding (the 'Hubble Flow'); what are the details?

In 1915 Einstein published his General Theory of Relativity. In seeking solutions to his General Relativity equations that described the entire Universe, Einstein (wrongly) assumed that the Universe was static. Because the natural tendency for any group of bodies (such as galaxies) is to fall together under gravity, Einstein was therefore compelled to introduce a 'cosmological constant' term – essentially, a repulsive force to counteract the tendency of gravity to collapse the Universe. On hearing of Hubble's demonstration that the Universe is expanding, Einstein is reported (perhaps apocryphally) to have called the cosmological constant his 'greatest blunder'.

In 1922 Alexandr Friedmann produced the first GR solutions for a dynamic Universe. The 'Freidmann equation' embodies the dynamical evolution of the Universe; we give it for reference (and not because you're expected to remember it!):

$$\dot{R}^2(t) = \frac{8\pi G\rho R^2(t)}{3} - kc^2 + \frac{\Lambda}{3}R^2(t).$$

where R is the 'scale factor' (characterizing the distance separating two galaxies compared to the separation now), ρ is the density, k is the so-called 'curvature term', discussed below, and Λ is the cosmological constant term. (G, π, and c have their usual meanings.) The units employed by cosmologists are such that k is normally allowed only to take values 0 or ± 1.

Since R has units of distance, $\dot{R}$ (the rate of change of R) has units of velocity. Thus $\dot{R}/R$ is velocity per unit distance. Sound familiar? The Hubble constant is also 'velocity per unit distance' (normally expressed in km s^{-1} Mpc^{-1}); and in fact, $H_0 = \dot{R}/R$.

2.1 The Density Parameter, Ω

We can consider the Big Bang in terms of a traditional explosion (a simplifying, but dangerous, picture – don't forget that the Bang didn't occur at one place, but *everywhere*, in a probably infinite universe...which is still infinite, but bigger...). This doesn't give us the full picture, but it does allow us to imagine three straightforward possibilities:

(i) If the explosion is powerful enough, it will hurtle galaxies apart with such force that gravity will never pull them together again. The universe will expand forever – an *open universe* (curvature term $k = -1$).

(ii) If the explosion is weak, gravity will eventually bring galaxies back together. The universe will collapse in a 'Big Crunch' – a *closed universe* (curvature term $k = +1$).

(iii) For the special intermediate case, the rate of expansion will forever slow under gravity, but will only come to a halt after infinite time – a *critical universe* (curvature term $k = 0$).

These models can, in principle, be distinguished by the rate at which the expansion is changing with time, which can in turn be determined by the rate of recession of very distant sources – naively, we expect gravity to be slowing down the rate of expansion, so the most distant sources (seen not as they are now, but as they were at an early stage of the Universe) should be receding 'too fast'.

The deceleration of the universal expansion due to the gravitational influence of the matter content of the Universe is characterized by a *density parameter*, Ω. This parameter was originally developed to characterize the density of matter in the Universe, $\Omega_{\rm M}$, compared to the density needed to give a critical universe (by definition, $\Omega \equiv 1$). The critical density of matter (i.e., neglecting the effect of the cosmological constant) is easily obtained by simply rearranging the Friedmann equation, whence

$$\rho_{\rm crit} = 3H_0^2/8\pi G = 8 \times 10^{-27} \text{kg m}^{-3}$$

(for $H_0 = \dot{R}/R = 72$ km/s Mpc^{-1}).
Again neglecting the cosmological-constant term, then if the Universe is of sufficiently high density ($\Omega_{\rm M} > 1$), the force of gravitational attraction would be enough halt the expansion and the Universe would collapse in a 'Big Crunch'.

$\Omega_{\rm M} < 1$ would give an 'open' universe that would continue to expand forever ($\Omega_{\rm M} = 0$ is a universe containing no matter at all, so the rate of expansion would not slow down).

$\Omega_{\rm M} = 1$ corresponds to a critical-density universe that would come to a halt after infinite time (and is geometrically flat; see below).

If t_0 corresponds to the present age of the Universe, then for $H_0 = 70$ km/s Mpc^{-1}:

$\Omega_{\rm M} = 0$ gives $t_0 = 1/H_0 \simeq 1.4 \times 10^{10}$ yr
$\Omega_{\rm M} = 1$ gives $t_0 = (2/3)/H_0 \simeq 0.9 \times 10^{10}$ yr
$\Omega_{\rm M} = 2$ gives $t_0 = 0.57/H_0 \simeq 0.8 \times 10^{10}$ yr
Primordial nucleosynthesis and observational limits on the present-day deuterium abundance gives a limit on the baryon content, $\rho_{\rm B}$, of about 4% of $\rho_{\rm crit}$; i.e, $\Omega_{\rm B} = 0.04$.

We know that there is also *dark,* non-baryonic, matter (recall: rotation curves of spiral galaxies, masses of clusters of galaxies). We believe that there is about 10 times as much dark as visible matter; i.e., in total, $\Omega_{\mathrm{M}} \simeq 0.3$ from observations.

For $\Omega_{\mathrm{M}} = 0.3, \rho_{\mathrm{M}} = 2.4 \times 10^{-27}$ kg m^{-3}.

The energy density of 3 K cosmic background microwave radiation (CMB), expressed as a mass ($E = mc^2$), is $\rho_{\mathrm{R}} = 4.5 \times 10^{-31}$ kg m^{-3}. (Stars contribute about the same radiation density – fewer photons, but each more energetic on average.)

Because $\rho_{\mathrm{M}} >> \rho_{\mathrm{R}}$, the present-day universe is said to be **matter dominated** (although the number of photons exceeds the number of nucleons by a factor of about 10^9).

In the early Universe, the reverse was true (radiation dilutes faster than matter, because radiation appears to lose energy by being red-shifted); the early Universe was **radiation dominated**.

However, we have a potential problem – for $\Omega_{\mathrm{M}} > 0$ (i.e., for a Universe that isn't completely devoid of matter!), the implied age of the universe is less than the age of the oldest observed stars! (The solution to this problem is revealed in Section **??**.)

2.2 Geometry of the Universe

According to General Relativity, the *geometry* of space-time is also determined by the mass density (more properly, the mass–energy density) in the Universe. There are 3 possible geometries, determined by Ω, and related to the curvature parameter:

- $\Omega > 1$: the geometry is closed and spherical (the sum of the angles of a triangle $> 180°$); $k = +1$.
- $\Omega = 0$: the geometry is open and hyperbolic (the sum of the angles of a triangle $< 180°$); $k = -1$.
- $\Omega = 1$: the geometry is flat (Euclidean space, with the sum of the angles of a triangle $= 180°$); $k = 0$.

2.3 Recent Results

So what *is* the density parameter? What is the future fate of the Universe? Recent results give the surprising answers

- **Small-scale structure in the CMB**

  The large-scale structure in the present-day universe (i.e., the distribution of galaxies in space; see later notes) can be mapped back to a prediction of the length scales of structure in the cosmic microwave background. Translating that *linear* scale into an observed *angular* scale depends sensitively on the geometry of space. Results from BOOMERanG and WMAP show that Ω (total) is within a few per cent of 1 – the Universe has a flat, Euclidean geometry.

  But we know, from primordial nucleosynthesis, that ordinary (baryonic) matter accounts for only $\Omega_{\rm B} \simeq 0.04$, and all matter (baryonic plus 'dark matter') only $\Omega_{\rm M} \simeq 0.3$. What makes up the other 70% (since we know that radiation has a negligible role)?

- **Supernova Cosmology**

  When we look at very distant galaxies, we see them as they were in the past. If gravity is slowing the rate of expansion of the universe, then in the past those galaxies were moving faster. If we can measure the velocities of, and distances to, remote galaxies reliably, then we can measure the rate of change of velocity with time (i.e., we can determine the deceleration parameter).

  Determining distances accurately is difficult, but is possible by using supernovae as 'standard candles' (specifically, type Ia supernovae). The results from 'supernova cosmology' are stunning – the rate of expansion of the universe is not slowing down, but *speeding up!*

  What can possibly cause this? We believe that the answer lies with Einstein's Cosmological Constant (or something closely related to it), which he introduced to counter the force of gravity. This cosmological constant represents the (non-zero) energy density of the vacuum – popularly known as the 'dark energy' (by analogy with 'dark matter'; we know almost nothing about either!).

We believe that the dark energy, designated Λ, contributes about 70% of the total mass-energy content of the universe. Our final inventory is, therefore:

$\Omega_{\rm Tot} = 1$ – the universe has a flat geometry, and is made up of:

$\Omega_{\rm M} \simeq 0.27$, of which

$\Omega_{\mathrm{B}} \simeq 0.04$ – familiar matter accounts for 4% of the stuff of the universe

$\Omega_{\Lambda} \simeq 0.73$

When the cosmological constant is taken into acount, the simple, gravity-dominated, ideas about the dynamical evolution of the universe no longer apply; matter exerts a gravitational pull, and slows down the rate of expansion, but the cosmological constant, Λ, exerts an 'anti-gravity', and speeds it up. The relative importance of matter and Λ changes with time, because Λ is determined by the energy constant of the vacuum; as the universe gets bigger, there's more vacuum, and the cosmological constant becomes more important.

We believe that in the early, compact (but still infinite!) universe, matter dominated over Λ, and the universe expanded at a slowing rate. As it expanded, the cosmological constant became more important, until Ω_{Λ} exceeded Ω_{M}. The dominance of the dark energy over matter in the resent-day universe implies that the rate of expansion of the universe will increase exponentially in the future.

2.4 Problems with the Big Bang model

The Big Bang model is successful in accounting for the evolution of the Universe from 0.01 sec to the present time. The observational evidence that support this model includes:

(1) The uniform expansion of the Universe.

(2) The Cosmic Microwave Background (CMB).

(3) The primordial abundances of D, ^{3}He, ^{4}He and ^{7}Li.

But there are, nonetheless, problems that indicate that the Big Bang model is incomplete. These include:

- Horizon Problem

  Two regions separated by less than ct are within their mutual *horizon;* they can communicate with each other in time t. Regions more than ct apart are outside the horizon and so cannot communicate.

  For the CMB, the horizon corresponds to 1°; i.e., regions separated by $> 1°$ should have different temperatures since they cannot communicate.

  The CMB is extremely uniform on large angular scales; how could well-separated regions 'know' that they had to end up looking the same in all directions?

- Flatness Problem

  Why does the Universe have $\Omega = 1$, when it could have taken any value at all? $\rightarrow$ This implies the only stable value for Ω is 1.

- Structure Problem

  What was the origin of the fluctuations from which galaxies and the large scale structure of the Universe formed?

  Density perturbations grow so slowly in an expanding Universe that there have to be some 'seed' fluctuations in the very early Universe to produce the large scale structures we see today. Where did these 'seeds' come from?

- Monopole Problem

  As the universe cooled, successively less massive particles should 'freeze out' of the matter-energy soup. This should lead to a significant population of very massive particles predicted by advanced (Grand Unified) theories, and exemplified by magnetic monopoles. These are not observed – where are they?

The answer to all these problems lies in a new ingredient: *inflation*.

3 THE INFLATIONARY UNIVERSE

The idea of an inflationary Universe was first proposed in the Western world by Alan Guth, in 1982 (and by Linde and Starobinsky in the Soviet world at around the same time, or slightly earlier).

The central premise is that the Universe expanded at an exponential rate in its *very* early history, increasing in size (scale factor) by an enormous factor in its very early stages. There are a number of variations on the basic inflationary theory, but they share many basic characteristics.

The inflationary era occured between $t = 10^{-35}$ and 10^{-32} sec, during which the size of the Universe (i.e., the size of that part of the Universe that is observable today) increased from 10^{-23} cm to 10 cm, give or take a few orders of magnitude.

To produce a theory explaining why inflation occurs, it's therefore necessary for all known physics to be extrapolated back from to 10^{-35} s – a realm of very high energies ($> 10^{14}$ GeV) and temperatures (10^{27} K).

Near this 'beginning', Quantum Theory meets the General Theory of Relativity; we have no knowledge of what happens before this time, which is called the Planck time, $t = (Gh/c^5)^{1/2} = 10^{-43}$ sec. This is called the Quantum Gravity era; there is currently no theory of Quantum Gravity (GR is a field theory).

Theories of particle physics aim at unifying the forces of nature. Maxwell unified electricity and magnetism into electromagnetism; Weinberg and Salaam unified electromagnetism and the weak force into the electroweak force. 'Grand Unified Theories' (GUTs; they exist, but there are many, and none is accepted as 'the' theory) unify the electroweak and strong forces. The greatest challenge is to incorporate gravity – this would be a Theory of Everything (TOE).

In GUT theories, at 10^{-35} sec, the strong nuclear force separated from the electroweak force. This phase transition or symmetry-breaking led to a period when the gravitational field became negative and acted as a repulsive force – inflation – as the false vacuum decayed to the true vacuum.

Eventually the energy locked in during this process is released $\rightarrow$ hot unified soup of particles/energy $\rightarrow$ Universe expands as a power-law instead of exponentially (i.e., inflation then joins onto traditional Big Bang theory)

PROBLEM (1): regions that were originally very close together are separated exponentially rapidly by the inflationary expansion. Thus causally connected regions (i.e. those $< ct$) are swept beyond their local horizons by the inflationary expansion.

PROBLEM (2): The very fast expansion has the effect of straightening out the geometry of the Universe so that when the inflationary period ends, the geometry is flat, i.e., the curvature radius grows exponentially so that it becomes enormous compared to the Hubble radius of the Universe today.

PROBLEM (3): The seeds of structure formation are supposed to have been formed by quantum energy oscillations during the quantum gravity era. One prediction of inflation is that the size of fluctuations in the CMB is the same on all angular scales because of the very rapid expansion.

COBE and WMAP searched for temperature fluctuations in the CMB on angular scales of 10° and above and found a single level of fluctuations at 1×10^{-5}. This finding therefore strongly supports the theory of inflation (but does not prove it).

PROBLEM (4): If monopoles freeze out before inflation, they are diluted to a density of fewer than one particle per observable universe.

4 FORMATION OF STRUCTURE IN THE UNIVERSE

Today we see galaxies, clusters of galaxies and superclusters. How did these structures form?

COBE and WMAP have found density fluctuations in the CMB ($\delta\rho/\rho = 1 \times 10^{-5}$). When the Universe becomes matter-dominated, these density inhomogeneities are amplified by gravity into the structures we see today. This amplification cannot happen until radiation-matter decoupling because radiation interacts with the matter.

There is a major problem with this basic picture: the observed fluctuations are too small. If only ordinary baryonic matter exists in the Universe, the level has to be $\sim 10^{-3}$ for there to be sufficient time to form the observed structures.

The solution is non-baryonic dark matter (DM) which only interacts gravitationally. Thus large unseen fluctuations can be present prior to matter-radiation decoupling. When decoupling occurs, the baryonic matter falls into the deep gravitational wells of DM and structures are created.

The way in which the structures are created depends on whether the DM is 'cold' (slow-moving) or 'hot' (relativistic).

Cold Dark Matter (CDM) – hypothetical massive, slow moving exotic particles with names like axions, gravitinos, photinos, or, generically, WIMPs (Weakly Interacting Massive Particles).

Hot Dark Matter (HDM) – fast moving particles (e.g., neutrinos) with a small rest mass (< 10 eV).

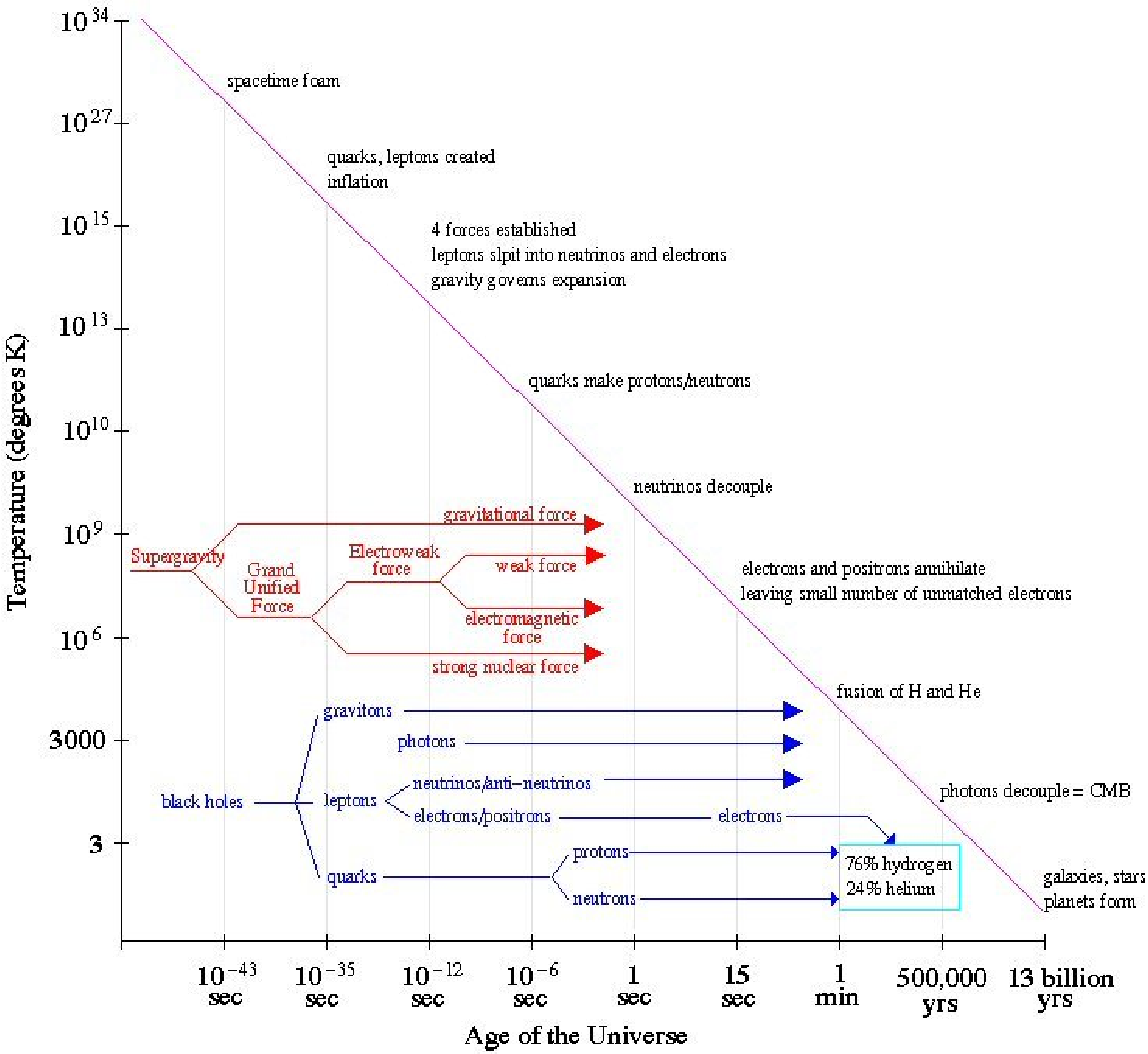
CDM evolution – fluctuations collapse and form galaxies first and then clusters and superclusters i.e. from bottom up.

HDM evolution – fine scale structure (galaxies) is obliterated by free-streaming of fast-moving neutrinos and large scale structure forms first, and the galaxies form by fragmentation i.e. from top down.

The nature of dark matter remains largely unknown, but numerical simulations of structure formation strongly favour <u>cold</u> dark matter.

Appendix: A Brief History of the Universe

Epoch	Time after Big Bang	Main events
Radiation Era ($\rho_{\rm R} > \rho_{\rm M}$)		
Planck	0–10^{-43}s	unknown physics; quantum gravity
GUT	10^{-43}–10^{-35}s	Strong, weak and EM forces unified; gravity freezes out
Hadron	10^{-35}–10^{-4}s	strong and weak forces freeze out
Lepton	10^{-4}–10^{2}s	Only light particles still in thermal equilibrium
Nuclear	10^{2}s–10^{3}yr ($1{\rm yr} = 3.16\times10^{7}$s)	Deuterium and helium formed, in the first few minutes
Matter Era ($\rho_{\rm R} < \rho_{\rm M}$)		
Atomic	10^{3}–10^{6}yr	Matter dominates. Atoms form, leading to the CMB forming at $\sim 3\times10^{5}$ yr
Galactic	10^{6}–10^{9}yr	Galaxies and superclusters form.
Stellar	10^{9}–1.4×10^{10}yr	All galaxies have formed. Stars continue to form. Jeremy Bentham born, died, stuffed



Temperature (degrees K)
10 34
10 27
10 15
10 13
10 10
10 9
10 6
3000
3
spacetime foam
quarks, leptons created
inflation
4 forces established
leptons slpit into neutrinos and electrons
gravity governs expansion
quarks make protons/neutrons
neutrinos decouple
electrons and positrons annihilate
leaving small number of unmatched electrons
fusion of H and He
photons decouple = CMB
galaxies, stars
planets form
Supergravity
Grand
Unified
Force
Electroweak
force
gravitational force
weak force
electromagnetic
force
strong nuclear force
black holes
gravitons
photons
leptons
neutrinos/anti-neutrinos
electrons/positrons
electrons
quarks
protons
neutrons
76% hydrogen
24% helium
10−43 sec
10−35 sec
10−12 sec
10−6 sec
1 sec
15 sec
1 min
500,000 yrs
13 billion yrs
Age of the Universe