PHAS1247 : Classical Mechanics
In-Course Assessment Test #2 : Mon. 11 January 2016

Answer as many of the questions as you can, in any order. The approximate distribution of marks is given in square brackets on the right of the page. There are 4 questions: they continue **ON THE OTHER SIDE OF THE PAGE**. *The maximum mark is 46.*

1. (a) A car travels around a horizontal circular track of radius r and the coefficient of static friction is μ_s.
Derive an expression for the maximum speed the car can attain if it is to stay on the circular track. You can assume a single normal force between the car and the track through the car's centre of mass. **[3]**

   (b) If the track is now inclined at an angle θ to the horizontal but is now **frictionless**, determine a new expression for the maximum speed of the car that maintains circular motion around the track. **[3]**

   (c) The track remains inclined but **friction is now magically added** with the coefficient of static friction being, again, μ_s. Find expressions for the minimum ($V_{\min}$) and maximum velocities ($V_{\max}$) where circular motion is possible and show that:

$$V_{\max}^2 - V_{\min}^2 = \frac{2rg\mu_s}{\cos^2\theta - \mu_s^2 \sin^2\theta}$$

[8]

2. (a) Show, by integration, that the moment of inertia of a thin disc of radius R and mass M about an axis passing through its centre and perpendicular to the plane of the disc is:

$$I = \frac{1}{2}MR^2$$

[3]

   (b) A vertical disc, initially at rest, is released and rolls without slipping down a plane inclined at an angle θ to the horizontal. Using conservation of energy or by considering the torque about the point at which the disc touches the plane, show that the acceleration of the centre of mass, a_{CM}, is given by:

$$a_{\mathrm{CM}} = \frac{2}{3}g\sin\theta$$

[5]

   (c) A giant, spherical star of radius 15,000 km rotates with a period of 30 days about an axis through its centre. The star undergoes a supernova explosion and collapses into a neutron star of radius 11.0 km. The only forces during the transition from the giant star to the neutron star act radially.
Determine the period of rotation of the neutron star assuming that all the mass of the original star remains in the neutron star. **[3]**

PLEASE TURN OVER FOR QUESTIONS 3 & 4

3. (a) A cylindrical bucket of radius 10 cm containing water is rotated about its vertical symmetry axis until the water is 3 cm lower at the centre of the bucket than at the edge. Determine the frequency of rotation. **[5]**

   (b) An aircraft flies at a speed of 700 km/h in a direction due East at a latitude of 60° North. You may assume that the only vertical force is due to the weight of the aircraft and the only horizontal force is due to the Coriolis force. At what angle should the pilot tilt the wings to correct for the effect of the Coriolis force. *Hint: The direction of the wings should be perpendicular to the resultant: weight + Coriolis force.* **[7]**

4. (a) A planet of mass m is subject to a force from the sun of $\frac{k}{r^2}$. The planet moves in an elliptical orbit given by the conic section equation:

$$\frac{1}{r} = \frac{1}{r_o}\left(e\cos\theta + 1\right) \quad \text{and} \quad r_o = \frac{L^2}{mk}$$

   If the angular momentum per unit mass of the planet is λ show that:

$$\frac{2GM}{\lambda^2} = \frac{1}{r_C} + \frac{1}{r_F}$$

   where r_C and r_F are the closest and furthest distances the planet gets to the sun, M is the mass of the sun and G is Newton's gravitational constant. **[3]**

   (b) A space shuttle is initially in a circular orbit of radius 6500 km about the centre of the Earth. It then accelerates to meet up with a satellite in a circular orbit (about the Earth's centre) of radius 4.2×10^4 km. Use the result from part (a) to calculate the increase in speed required to put the shuttle into the appropriate elliptical-transfer orbit. In the elliptical-transfer orbit the closest the shuttle gets to the Earth is the radius of the initial shuttle orbit and the furthest it gets from the Earth is the radius of the satellite orbit. You should assume that the increase in speed occurs essentially instantaneously, that the only gravitational interactions are between the Earth and the shuttle and the Earth and the satallite, and that the mass of the Earth is = 5.97×10^{24} kg and $G = 6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$. **[6]**

END OF PAPER