PHAS1247 : Classical Mechanics
In-Course Assessment Test #1 : Mon. 6 November 2017

Answer as many of the questions as you can, in any order. There are 5 questions, and each complete question is worth 6 marks, for a maxmimum mark of 30. Try to write the solution on the same page as the question.

1. A particle of mass 1 kg has a position vector $\underline{\mathbf{r}}$ (in meters) as a function of time, t, (in seconds) given by:

$$\underline{\mathbf{r}} = \cos(t)\,\hat{\underline{\mathbf{i}}} + \sin(\sqrt{3}t)\,\hat{\underline{\mathbf{j}}} + t\mathrm{e}^{10t}\,\hat{\underline{\mathbf{k}}}\ .$$

   Find the particle's (i) velocity and (ii) its acceleration at a time t.

   Determine the vector of the force acting on the particle at time $t = 0$. **[6]**

2. A mass of 1 kg is on a plane, inclined with an angle α with respect to the horizontal direction. The plane has a static friction coefficient of $\mu = 0.2$. Calculate the components of the gravitational force parallel and perpendicular to the plane. Calculate the minimal value of the angle α for which the parallel component of the gravitational force is stronger than friction, and the body starts moving.

   Assuming that once the body is in motion the friction coefficient reduces to 0.1, calculate the acceleration of the mass along its direction of motion. **[6]**

3. The shape of the bottom of a valley can be described by the function $y = -\cos(x)$ where x is the horizontal distance from lowest point. Write the gravitational potential energy acting on a pointlike and frictionless ball of mass m = 1 kg, as a function of the position x, both in the exact case and in the parabolic approximation. Calculate the force acting on the ball as a function of x. Describe the motion of the ball around its equilibrium position, as a function of time, if the ball starts it motion at the bottom of the valley with a horizontal speed of 1 m/s. **[6]**

4. A particle with mass $m = 1$ kg moves in a space with potential energy given by

$$V(x, y, z) = x^2 + 4y^3 - y^2 z^4$$

Calculate the vector expressing the force acting on the particle in a position (x, y, z). Calculate the change in kinetic energy if the particle moves from point $(1, 1, 1)$ to point $(0, 0, 0)$, with no other force applied.

How are the potential and the resulting force modified if the system also feels a constant gravity, pointing downwards along the z direction, and assuming a gravitational constant g $= 10\ m/s^2$? **[6]**

5. A system is made of two masses $m_1 = 10$ kg with a vertical position $y = 0$ and $m_2 = 5$ kg at a vertical position $y = 5$ m.

The second mass m_2 is left free falling at time $t = 0$. Calculate the time it takes to reach $y = 0$, and its speed. At that moment, the first mass (m_1)is also left free falling. Calculate the vertical positions of both masses and of their centre of gravity as a function of time.

Assume that instead of falling independently, once the second mass reaches the position $y = 0$ it sticks to the first mass, such that the two continue their motion together. Calculate the vertical position of the two masses as a function of time for this case. Is it different from the motion of the centre-of-mass in the non-collision case described above? **[6]**

END OF PAPER