Note: Most probably, you will not get through all these questions during the Problem Class time. Nevertheless, you should try them all at some point and check your solutions against those on the course *Moodle* site. If you have any difficulties, please get in touch! *SWZ*

1. (a) Show that $y = y_{\mathrm{m}} \sin(kx - \omega t)$ may be written in the alternative forms:

   i. $y = y_{\mathrm{m}} \sin k(x - vt)$ ii. $y = y_{\mathrm{m}} \sin 2\pi \left(\frac{x}{\lambda} - ft\right)$

   iii. $y = y_{\mathrm{m}} \sin \omega \left(\frac{x}{v} - t\right)$ iv. $y = y_{\mathrm{m}} \sin 2\pi \left(\frac{x}{\lambda} - \frac{t}{T}\right)$

   (b) i. Write an expression describing a sinusoidal transverse wave travelling on a cord in the $-y$ direction with a wavelength of 60 cm, a period of 0.20 s, and an amplitude of 3 mm. Take the transverse direction to be the z direction.

   ii. What is the maximum transverse speed of a point on the cord?

   (c) A wave of frequency 500 Hz has a velocity of 350 m s^{-1}.

   i. How far apart are two points that differ in phase by $\pi/3$?

   ii. What is the phase difference between two displacements at a certain point along the wave at times 1 ms apart?

2. Find the resultant $E(t) = E_1 + E_2 + E_3$ of the following oscillations:

$$E_1 = E_0 \cos(\omega t) \qquad E_2 = 2E_0 \cos\left(\omega t + \frac{\pi}{3}\right) \qquad E_3 = \frac{1}{2} E_0 \cos\left(\omega t - \frac{\pi}{6}\right)$$

3. A flute player hears 4 beats per second when she compares her note to a 523 Hz tuning fork (the note C). She can match the frequency of the tuning fork by pulling out the "tuning joint" to lengthen her flute slightly. What was her initial frequency?

4. An organ pipe open at both ends is 1.5 m long. A second organ pipe that is closed at one end and open at the other is 0.75 m long. The speed of sound in the room is 330 m s^{-1}. Which of the following sets of frequencies consists of frequencies which can be produced by both pipes?

   (a) 110 Hz, 220 Hz, 330 Hz (b) 220 Hz, 440 Hz, 660 Hz (c) 110 Hz, 330 Hz, 550 Hz
   (d) 330 Hz, 440 Hz, 550 Hz (e) 220 Hz, 660 Hz, 1100 Hz

5. By what factor will an intensity change for a 3 dB increase in the sound level?

   (a) 3 (b) 0.5 (c) 2 (d) 4 (e) 0.3

6. How much louder is the intensity of sound at a rock concert in comparison with that of a whisper, if the two intensity levels (in dB) are 120 and 20, respectively?

   (a) 10^{12} (b) 10^{8} (c) 10^{6} (d) 10^{10} (e) 10^{11}

7. The fundamental frequency of an open organ pipe corresponds to middle C, or 261.6 Hz. The third resonance of a closed pipe has the same frequency. What are the lengths of the two pipes? The speed of sound is 343 m s^{-1}.

8. A police siren has a frequency of 550 Hz as the police car approaches you, 450 Hz after it has passed you and is receding. How fast are the police traveling? What is the siren frequency at rest? Assume that the speed of sound is 343 m s^{-1}.

9. While sounding a note on a toy whistle, you notice a friend running toward you. If you want him to hear the same note you hear while he is approaching, you must:

   (a) stay put;

   (b) run towards him at the same speed;

   (c) run away from him at the same speed;

   (d) stay put and play a note of higher frequency; or,

   (e) run towards him and play a note of higher frequency.

10. The phase velocity of a surface wave in a liquid of depth much greater than λ is given by

$$v_{\mathrm{p}} = \sqrt{\frac{g\lambda}{2\pi} + \frac{2\pi Y}{\rho\lambda}}$$

    where $g \equiv$ acceleration due to gravity, $\lambda \equiv$ wavelength, $\rho \equiv$ density, and $Y \equiv$ surface tension. Compute the group velocity of a pulse in the long-wavelength limit (these are called *gravity waves*).

11. The dispersion relation for transverse waves on an elastic beam is $\omega = a/\lambda^2$, where λ is the wavelength and a is a constant depending on the material and dimensions of the beam. Show that for these waves the group velocity is twice the phase velocity.

12. In certain ranges of a piano keyboard, more than one string is tuned to the same note to provide extra loudness. For example, the note at 110 Hz has two identical strings at this frequency. If one string slips from its normal tension of 600 N to 540 N, what beat frequency is heard when the hammer strikes the two strings simultaneously?

13. A whistle used to call a dog has a frequency of 30 kHz. The dog, however, ignores it. The owner of the dog, who cannot hear sounds above 20 kHz, wants to use the Doppler effect to make certain that the whistle is working. She asks a friend to blow the whistle from a moving car while the owner remains stationary and listens.

    How fast must the car move and in what direction for the owner to hear the whistle at 20 kHz? Is the experiment practical? The speed of sound in air is 343 m s^{-1}.

14. The speed of light changes when it goes from ethyl alcohol ($n = 1.361$) to carbon tetrachloride ($n = 1.461$). The ratio of the speeds $v_{\text{carbon tetrachloride}}/v_{\text{ethyl alcohol}}$ is

    (a) 1.99 (b) 1.07 (c) 0.93 (d) 0.51 (e) 0.76

15. If your height is h, what is the shortest mirror on the wall in which you can see your full image? Where must the top of the mirror be hung?

16. A laser beam is aimed at a 1.0-cm-thick sheet of glass, $n_{\text{glass}} = 1.5$, at an angle 30° above the glass.

    (a) What is the laser beam's direction of travel in the glass?

    (b) What is its direction in the air on the other side?

    (c) By what distance is the laser beam displaced?

END OF PAPER