# PHYS1B24 Waves, Optics and Acoustics
# Solutions to Final Exam 2004

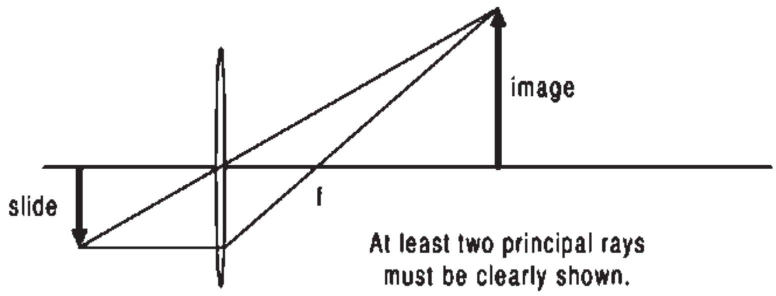
**Answer ALL SIX questions from section A and THREE questions from section B.**

The numbers in square brackets in the right-hand margin indicate the provisional allocation of part marks per sub-section of a question.

A "BW" beside the mark indicates that the answer is mainly "bookwork". A "US" indicates that the answer is to an "unseen" question.

## SECTION A **[Part marks]**

1. (a) Ray diagram showing the formation of the image of a slide on a screen by a simple converging lens: **[3 BW]**




(b) i. Applying the lens formula **[1 BW]**

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

gives

$$p = \frac{qf}{q-f} = \frac{5000 \times 50}{5000 - 50} = 50.5 \text{ mm}, \qquad \left( \frac{1}{p} = 0.0198 \text{ mm}^{-1} \right)$$

that is, the slide is 50.5 mm from the lens. **[1 US]**

ii. From the figure it is then obvious that the height of the image is **[1 US]**

$$h' = -\frac{q}{p} h = -\frac{5000}{50.5} 24 = -2376 \text{ mm},$$

or $-2.4$ m. The minus sign indicates that the image is inverted. **[1 US]**

2. The wavelength is $\lambda = v/f = 343/261.6 = 1.31$ m, so the length of the open pipe vibrating in its simplest mode (A–N–A) is **[1 BW]**

$$d_{\text{A to A}} = \frac{1}{2}\lambda = 0.656 \text{ m}.$$ **[1 US]**

A closed pipe pipe has (N–A) for its simplest resonance, (N–A–N–A) for the second, and (N–A–N–A–N–A) for the third. Here, the pipe length is **[1 US]** **[1 US]**

$$5d_{\text{N to A}} = \frac{5}{4}\lambda = \frac{5}{4}(1.31) = 1.64 \text{ m}.$$

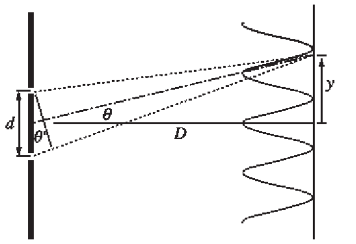
**[2 US]**

3. (a) Three of:
   - selective absorption,
   - double refraction,
   - scattering, or,
   - reflection.

   . **[3 BW]**

   (b) After the first sheet: $I_1 = \frac{1}{2}I_0 = 0.5I_0$. **[1 BW]**
   After the second sheet: $I_2 = I_1 \cos^2(30°) = \frac{1}{2}I_0 \times \frac{3}{4} = \frac{3}{8}I_0 = 0.375I_0$. **[1 BW]**
   After the third sheet: $I_3 = I_2 \cos^2(30°) = \frac{3}{8}I_0 \times \frac{3}{4} = \frac{9}{32}I_0 = 0.281I_0$. **[1 US]**

4. (a)




For a distant screen assumption: **[2 BW]**

$$\tan\theta \approx \sin\theta \approx \theta = \frac{y}{D}.$$

From the condition for the $m^{\mathrm{th}}$ maximum: $d\sin\theta_{\mathrm{m}} = m\lambda$ gives us, using the above:

$$\sin\theta_{\mathrm{m}} \approx \frac{y_{\mathrm{m}}}{D} = m\frac{\lambda}{d},$$

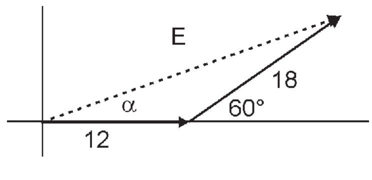
or

$$y_{\mathrm{m}} = m\frac{\lambda D}{d}.$$

For the adjacent, or $(m+1)^{\mathrm{th}}$ fringe, we have **[1 BW]**

$$y_{\mathrm{m+1}} = (m+1)\frac{\lambda D}{d}.$$

(b) **[2 US]**




By Law of Cosines, the final magnitude is given by: **[1 US]**

$$E = \sqrt{12^2 + 18^2 - 2(12)(18)\cos(120°)} = 26.2$$

The angle is given by the Law of sines: **[1 US]**

$$\frac{\sin\alpha}{18} = \frac{\sin(120°)}{26.2}$$

so that $\alpha = 36.6°$.

5. (a) Sound level $\beta$, in decibels (dB), is defined by

$$\beta = 10 \log \left( \frac{I}{I_0} \right)$$ **[1 BW]**

where $I_0$ is the reference intensity, taken to be at the threshold of hearing, $I_0 = 1.0 \times 10^{-12}$ W m$^{-2}$ (quoted value not necessary for mark) and $I$ is the intensity in W m$^{-2}$ [must have some idea of units]. **[1 BW]** **[1 BW]**

(b) We can write $\beta_1 = 10 \log(I_1/I_0)$ and $\beta_2 = 10 \log(I_2/I_0)$. Thus **[1 US]**

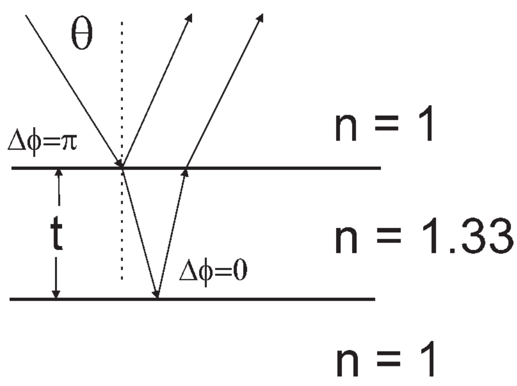
$$\begin{aligned} \beta_2 - \beta_1 &= 10 \log(I_2/I_0) - 10 \log(I_1/I_0) \\ &= 10 \log \left( \frac{(I_2/I_0)}{(I_1/I_0)} \right) \\ &= 10 \log \left( \frac{I_2}{I_1} \right). \end{aligned}$$

**[1 US]**

(c) If $\beta_2 - \beta_1 = 10$ then, from above, $10 = 10 \log(I_2/I_1)$, or $\log(I_2/I_1) = 1$ so that $I_2/I_1 = 10$. **[1 US]**
Thus, $I_2 = 10 I_1$. **[1 US]**

6. (a) i. $\pi$ phase change
ii. 0 phase change **[2 BW]**

(b) i.




**[3 BW]**

ii. Because of the extra $\pi$ phase change we must use **[1 BW]**

$$2nt = \left( m + \frac{1}{2} \right) \lambda.$$

For minimum thickness, $m = 0$, and the thickness is given by

$$t_{\min} = \frac{\lambda}{4n} = \frac{550}{4 \times 1.33} = 103.4 \text{ nm}.$$

**[1 BW]**

7. (a) The *principle of superposition* states that if two disturbances $y_1$ and $y_2$ are solutions of the wave equation, then their sum $y_1 + y_2$ is also a solution. *Dispersion* is the dependence of wave speed on frequency. **[2 BW]**

(b) i. From the sum of the two waves, **[2 BW]**

$$\begin{aligned} y(x,t) &= y_1(x,t) + y_2(x,t) \\ &= A\sin(k_1x - \omega_1 t) + A\sin(k_2x - \omega_2 t) \\ &= 2A\sin\left(\frac{k_1+k_2}{2}x - \frac{\omega_1+\omega_2}{2}t\right)\cos\left(\frac{k_1-k_2}{2}x - \frac{\omega_1-\omega_2}{2}t\right) \end{aligned}$$

which consists of a wave, the *carrier wave*, having the average of the component wavevectors and frequencies, modulated by an *envelope* function characterised by half the differences between the component wavevectors and frequencies. **[2 BW]** **[2 BW]**

ii. The speed of the carrier wave gives the phase velocity, $v_\mathrm{p}$, and is **[2 BW]**

$$v_\mathrm{p} = \frac{\omega_1 + \omega_2}{k_1 + k_2} \approx \frac{\omega}{k}.$$

The speed of the envelope gives the group velocity, $v_\mathrm{g}$, **[2 BW]**

$$v_\mathrm{g} = \frac{\omega_1 - \omega_2}{k_1 - k_2} = \frac{\Delta\omega}{\Delta k} \approx \frac{\partial\omega}{\partial k}.$$

(c) i. If $\omega = \sqrt{\kappa/m}\sin(ka/2)$, then **[1 US]**

$$v_\mathrm{p} = \sqrt{\kappa/m}\frac{\sin(ka/2)}{k}$$

and, either from the expansion of sin or from l'Hôpital's rule, for small $k$ this gives **[1 US]**

$$v_\mathrm{p} = \frac{a}{2}\sqrt{\kappa/m}.$$

Also, **[1 US]**

$$v_\mathrm{g} = \sqrt{\kappa/m}\frac{a}{2}\cos(ka/2)$$

giving at $k = 0$ **[1 US]**

$$v_\mathrm{g} = \frac{a}{2}\sqrt{\kappa/m}.$$

ii. For the values given, the velocity is **[2 US]**

$$v = \frac{10^{-2}}{2} \times \sqrt{\frac{1}{10^{-2}}} = 0.05 \text{ m/s}.$$

iii. If $\lambda = 2a$, $\cos(ka/2) = \cos(2\pi a/(2 \times 2a)) = 0$. The group velocity is thus zero. **[2 US]**

8. (a) $m = 1$ **[1 US]**

$$D = \frac{42 \times 10^{-3}}{49000} = 8.57 \times 10^{-7} \text{ m}.$$

Using $\sin\theta = m\lambda/D$: **[1 BW]**
For $\lambda = 400$ nm, $\sin\theta_1 = 400 \times 10^{-9}/8.57 \times 10^{-7} = 0.467$ so that $\theta_1 = 27.82°$. **[1 US]**
For $\lambda = 700$ nm, $\sin\theta_1 = 700 \times 10^{-9}/8.57 \times 10^{-7} = 0.817$ so that $\theta_1 = 54.77°$. **[1 US]**
The angular width is then $\Delta\theta = 54.77 - 27.82 = 26.95° = 0.47$ rad. **[1 US]**

(b) i. Taking the differential of the condition $D\sin\theta = m\lambda$ we get **[2 BW]**

$$D\cos\theta \mathrm{d}\theta = m\mathrm{d}\lambda \quad \text{or} \quad \frac{\mathrm{d}\theta}{\mathrm{d}\lambda} = \frac{m}{D\cos\theta}$$

For small $\theta_m$, $\cos\theta_m \approx 1$ and we have that **[1 BW]** **[1 BW]**

$$\frac{\mathrm{d}\theta}{\mathrm{d}\lambda} \approx \frac{m}{D}.$$

ii. The phase difference $\phi$ between adjacent slits is given by **[3 BW]**

$$\phi = \frac{2\pi D\sin\theta}{\lambda} \quad \text{so} \quad \mathrm{d}\phi = \frac{2\pi D\cos\theta \mathrm{d}\theta}{\lambda}$$

but this must equal $2\pi/N$ for this to correspond to the angular interval **[1 BW]**
$\mathrm{d}\theta$ between a maximum and the first adjacent minimum. Thus **[1 BW]**

$$\frac{2\pi}{N} = \frac{2\pi D\cos\theta \mathrm{d}\theta}{\lambda} \quad \text{or} \quad D\cos\theta \mathrm{d}\theta = \frac{\lambda}{N}.$$

But from the differential from the maximum condition $D\sin\theta = m\lambda$ we have that **[2 BW]**

$$D\cos\theta \mathrm{d}\theta = m\mathrm{d}\lambda.$$

Dividing the last equation into the previous one, and letting $\mathrm{d}\lambda \approx \Delta\lambda$ for small $\Delta\lambda$, we get

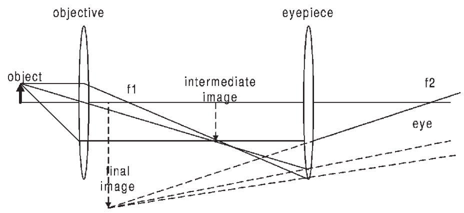
$$1 = \frac{\lambda}{Nm\Delta\lambda}$$

or, as required, **[1 BW]**

$$R = \frac{\lambda}{\Delta\lambda} = Nm.$$

(c) $\Delta\lambda = 589.592 - 588.995 = 0.597$ and $\lambda = \overline{\lambda} = (\lambda_1 + \lambda_2)/2 = 589.293$ **[1 US]**
Then, with $m = 2$, $N = \lambda/(\Delta\lambda m) = 589.293/(2\times 0.597) = 494$ lines (*integer*). **[2 US]**

9. (a) i. The diagram shows a compound microscope with an objective with focal length 4 mm and an eyepiece with focal length 20 mm, forming an image, 250 mm from the eye, of an object placed 4.1 mm from the objective. *(The diagram must show the intermediate and final images, which must be located using two or more principal rays for each image.)* **[6 US]**




ii. The image at 250 mm gives us, from the lens formula for the eyepiece

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

the position of the intermediate image as **[2 US]**

$$\frac{1}{p} = \frac{1}{230} + \frac{1}{20} \quad \text{or} \quad p = 4600/250 \text{ mm} = 18.4 \text{ mm}.$$

This point, 18.4 mm to the left of the eyepiece, is $(L - 18.4)$ mm to the right of the objective. But for the objective **[1 US]**

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{4} - \frac{1}{4.1} = \frac{0.1}{16.4},$$

or $q = 164$ mm. Thus the separation of the lenses in this setting is $18.4 + 164 = 182.4$ mm. **[1 US]**

iii. The magnification of the microscope when adjusted in this way is the product of the objective and eyepiece magnifications

$$M = M_o m_e = -\frac{L}{f_o}\frac{250}{f_e} = -\frac{182.4}{4}\frac{250}{20} = -570.$$

(The sign denotes that the image is inverted.) **[2 US]**

iv. It is more usual to adjust the microscope so that the image is at infinity because the eye is then relaxed, and so use of the microscope is less tiring. **[1 US]**

(b) i. Final image is virtual. **[1 US]**

ii. Final image is at infinity since $p_2 = f_e$, or the object for the eyepiece is at the eyepiece's focal point. **[2 US]**

iii. Since $m = -f_o/f_e$, we have $f_o = 3|f_e|$.
Also, $L = f_o - |f_e|$, so $f_o = 10 + |f_e|$. **[1 US]**
Combining with the above equation gives $3|f_e| = 10 + |f_e|$, or $|f_e| = 5$ cm, so $f_e = -5$ cm . **[2 US]**
Then $f_o = 3|f_e|$ gives $f_o = 15$ cm. **[1 US]**

10. (a) A source emits a sound wave with frequency $f_\mathrm{S}$. The frequency with which a listener hears the sound wave is $f_\mathrm{L}$. The speed of sound in the medium is $v$. If the listener is moving toward a stationary source with velocity $v_\mathrm{L}$, the speed of the sound waves relative to the listener is $v' = v + v_\mathrm{L}$. The wavelength, $\lambda$, remains unchanged. So the frequency observed by the listener is **[2 BW]** **[4 BW]**

$$\begin{aligned} f_\mathrm{L} &= \frac{v'}{\lambda} \\ &= \frac{v + v_\mathrm{L}}{\lambda} \\ &= \frac{v}{\frac{v}{f_\mathrm{S}}} + \frac{v_\mathrm{L}}{\frac{v}{f_\mathrm{S}}} \\ &= \left(1 + \frac{v_\mathrm{L}}{v}\right) f_\mathrm{S} \end{aligned}$$

This then gives the required

$$f_\mathrm{L} = \left(1 + \frac{v_\mathrm{L}}{v}\right) f_\mathrm{S}.$$

.............................. OR ..........................

If a source moves toward a stationary listener with velocity $v_\mathrm{S}$, the wavefronts observed by the listener are closer together than if the source were still. Thus $\lambda_\mathrm{L} < \lambda$.

For each vibration with period $T$, the source moves **[1 BW]**

$$v_\mathrm{S} T = \frac{v_\mathrm{S}}{f_\mathrm{S}} = \Delta\lambda.$$

Then **[2 BW]**

$$\begin{aligned} \lambda_\mathrm{L} &= \lambda - \Delta\lambda \\ &= \lambda - \frac{v_\mathrm{S}}{f_\mathrm{S}} \\ &= \frac{v}{f_\mathrm{S}} - \frac{v_\mathrm{S}}{f_\mathrm{S}} \end{aligned}$$

Then, substituting **[3 BW]**

$$\begin{aligned} f_\mathrm{L} &= \frac{v}{\lambda_\mathrm{L}} \\ &= \frac{v}{\frac{v}{f_\mathrm{S}} - \frac{v_\mathrm{S}}{f_\mathrm{S}}} \\ &= \left(\frac{1}{1 - \frac{v_\mathrm{S}}{v}}\right) f_\mathrm{S} \end{aligned}$$

as required.

Either way, it should be obvious as to which one they are doing!

(b) The stationary wall "observes" the following frequency:

$$f_{\text{wall}} = \left(\frac{1}{1-\frac{v_{\text{car}}}{v}}\right) f_{\text{car}}$$ **[1 US]**

The wall reflects this frequency so that the moving driver observes **[1 US]**

$$f_{\text{driver}} = \left(1+\frac{v_{\text{car}}}{v}\right) f_{\text{wall}}$$

or **[2 US]**

$$f_{\text{driver}} = \left(1+\frac{v_{\text{car}}}{v}\right)\left(\frac{1}{1-\frac{v_{\text{car}}}{v}}\right) f_{\text{car}}$$

Inserting the values leads to

$$f_{\text{driver}} = \left(1+\frac{30}{343}\right)\left(\frac{1}{1-\frac{30}{343}}\right) \times 300 = 357.5 \text{ Hz}$$

**[1 US]**

(c) i. From the expressions above,

$$f_{\text{obs}} = \frac{1+\frac{v_{\text{L}}}{v}}{1-\frac{v_{\text{S}}}{v}}.$$

**[4 US]**

ii. In free space the above expression for the observed frequency is (writing $v$ as $c$)

$$f_{obs} = f\frac{c+v_{\text{L}}}{c-v_{\text{S}}} = f\left(1+\frac{v_{\text{L}}+v_{\text{S}}}{c}+\frac{v_{\text{L}}v_{\text{S}}}{c^2}+\frac{{v_{\text{S}}}^2}{c^2}+\ldots\right)$$

**[2 US]**

whereas the relativistic form expands to

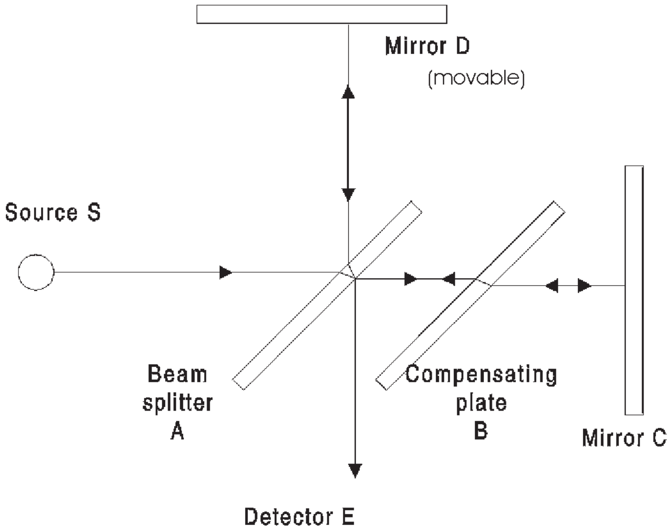
$$f_{obs} = f\left(1+\frac{v}{c}+\frac{1}{2}\frac{v^2}{c^2}+\ldots\right)$$

**[2 US]**

Clearly the two forms agree to first order in $v/c \equiv (v_{\text{S}}+v_{\text{L}})/c$. **[1 US]**

11. (a) Etalon: two plane, parallel, highly reflecting surfaces separated by some distance $d$.
It is mostly associated with the Fabry-Perot interferometer. **[3 BW]**

(b) The Michelson interferometer **[3 BW]**




As drawn, the mirrors C and D are silvered on their front faces, and the beam splitter consists of a glass plate partsilvered on its rear surface.
The glass in the beam splitter will be dispersive. This plate, however, is traversed thrice by the beam which traverses the path *source - beam splitter - mirror D - beam splitter - detector* but only once by the beam *source - beam splitter - mirror C - beam splitter - detector*. This results in a wavelength dependent optical path difference. If the compensator plate is identical (apart from any silvering) to the beam splitter, and is aligned parallel to it, it will introduce an extra path length into the second path which is exactly equivalent to the **two extra traverses** [must have some sort of count] of the beam splitter in the first path. **[3 BW]**

(c) Distance $= 2(3.82 \times 10^{-4}$ m$) = 1700\lambda$ **[2 US]**
$\lambda = 4.49 \times 10^{-7}$ m $= 449$ nm.
The light is blue. **[2 US]**

(d) Counting light going in both directions, the number of wavelengths originally in the cylinder is $m_1 = 2L/\lambda$. It changes to $m_2 = 2L/(\lambda/n) = 2nL/\lambda$ as the cylinder is filled with gas. **[2 US]** **[2 US]**
If $N$ is the number of bright fringes passing,

$$N = m_2 - m_1 = \frac{2L}{\lambda}(n-1),$$

or the index of refraction of the gas $n$ is **[2 US]**

$$n = 1 + \frac{N\lambda}{2L} = 1 + \frac{48 \times 546 \times 10^{-9}}{2 \times 50 \times 10^{-2}} = 1.000026.$$

**[1 US]**

 **END OF PAPER**