# Testing the convergence of infinite series

1) **Preliminary Test**: If $\lim\limits_{k\to\infty} u_k \neq 0 \Rightarrow \sum^{\infty} u_k$ diverges

2) **Comparison Test**: a) If $\sum^{\infty} v_k$ converges and $u_k \leq v_k$

$\Rightarrow \sum^{\infty} u_k$ converges

b) If $\sum^{\infty} v_k$ diverges and $v_k \geq u_k$

$\Rightarrow \sum^{\infty} u_k$ diverges

3) **D'Alembert Ratio Test**:

$$\boxed{\rho = \lim_{k\to\infty} \left| \frac{u_{k+1}}{u_k} \right|} \qquad S = \sum_{k}^{\infty} u_k$$

$\Rightarrow$ If $\rho < 1$, $S$ converges

If $\rho > 1$, $S$ diverges

If $\rho = 1$, test is inconclusive

# Power Series

Series of the form

$$\boxed{P(x) = \sum_{k=0}^{\infty} a_k x^k = a_0 + a_1 x + a_2 x^2 + \dots}$$

defining a function $P(x)$

$P(x)$ converges if (d'Alembert)

$$\boxed{|x| < \left[ \lim_{k \to \infty} \left| \frac{a_{k+1}}{a_k} \right| \right]^{-1}} \Rightarrow \text{interval of convergence in } x$$

(radius of convergence for complex numbers $z$)