PHAS1247 Classical Mechanics
In-Course Assessment Test 1
(Monday 4 November 2013)

Answer as many of the questions as you can, in any order. Calculators may be used. Marks are given in square brackets on the right of the page; the maximum mark is 30. The unit vectors along the Cartesian axes are $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$; take $g = 9.81\,\mathrm{m\,s^{-2}}$.

1. A child is travelling in a car with a velocity (as measured by an external observer) of $\mathbf{v}_1 = 15\hat{\mathbf{i}}\,\mathrm{m\,s^{-1}}$. The child throws a ball with a velocity (as measured by the child) of $\mathbf{v}_2 = (-2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})\,\mathrm{m\,s^{-1}}$. What are (a) the velocity and (b) the speed of the ball, as measured by the external observer? **[3]**

2. A particle moving in two dimensions with position coordinates (x, y) has a potential energy given by
$$V(x, y) = Ax^2y^2 - By^4,$$
where A and B are constants. Find an expression for the force acting on the particle at position (x, y). **[3]**

3. A particle moving in three dimensions moves in a straight line from the origin to the point with position vector $2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$. Find the work done during the displacement by (a) a constant force $\mathbf{F} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and (b) by the position-dependent force $\mathbf{F} = 3x^2y^2\hat{\mathbf{i}} + 2x^3y\hat{\mathbf{j}}$. **[5]**

   Would the work done for each force depend on the path taken between the end points? **[2]**

4. A mass $m_1 = 4\,\mathrm{kg}$ has an initial velocity in the laboratory frame (i.e. relative to a stationary observer) $\mathbf{u}_1 = 6\hat{\mathbf{i}}\,\mathrm{ms^{-1}}$, while a second mass $m_2 = 2\,\mathrm{kg}$ has velocity $\mathbf{u}_2 = -3\hat{\mathbf{i}}\,\mathrm{ms^{-1}}$. Find (i) the total momentum of the system, (ii) the velocity of the centre of mass, and (iii) the relative velocity of particle 1 relative to particle 2. **[3]**

   The two particles undergo an elastic collision. Assuming they continue to move along the x-axis, find the velocities of each after the collision in the lab frame. **[4]**

5. A child at a fair throws a coconut at a plate on a stall; hitting the plate wins a prize. The child throws the coconut from a starting height h above the ground, with initial speed u and at an angle of elevation α to the horizontal; neglecting air resistance, find the height above the ground and the horizontal distance moved by the coconut as a function of time t. **[3]**

   Hence show also that the vertical height z above the ground is related to the horizontal displacement x by the quadratic equation
$$\frac{gx^2}{2u^2}\tan^2(\alpha) - x\tan(\alpha) + z - h + \frac{gx^2}{2u^2} = 0.$$
[3]

   If $u = 4\,\mathrm{ms^{-1}}$ and $h = 1\,\mathrm{m}$, while the plate is a distance $d = 2\,\mathrm{m}$ away and at a height of $0.3\,\mathrm{m}$ above the ground, find two possible values of the angle α which would enable the child to hit the plate and win the prize. **[4]**

END OF PAPER