MM1 CHEATSHEET:

Vectors: $\underline{a}\cdot\underline{b} = |\underline{a}||\underline{b}|\cos\theta$ $\quad \underline{a}\times(\underline{b}\times\underline{c}) = (\underline{a}\cdot\underline{c})\underline{b} - (\underline{a}\cdot\underline{b})\underline{c}$

$\underline{a}\times\underline{b} = |\underline{a}||\underline{b}|\sin\theta$ $\quad (\underline{a}\times\underline{b})\times\underline{c} = (\underline{a}\cdot\underline{c})\underline{b} - (\underline{b}\cdot\underline{c})\underline{a}$

$[\underline{a}, \underline{b}, \underline{c}] = \underline{a}\cdot(\underline{b}\times\underline{c})$ = Cyclic Perm = − Other Perm = Volume of Parallelipiped.

Point to line : $d = |\underline{p}-\underline{a}|\sin\theta = |(\underline{p}-\underline{a})\times\hat{\underline{b}}|$

Point to plane : $d = |(\underline{a}-\underline{p})\cdot\hat{\underline{n}}|$

Complex Numbers:

$z_1 z_2 = r_1 r_2 e^{i(\theta_1+\theta_2)}$

$z_1/z_2 = \frac{r_1}{r_2} e^{i(\theta_1-\theta_2)}$

$\cosh(ix) = \frac{1}{2}(e^{ix}+e^{-ix}) = \cos(x)$

$\sinh(ix) = \frac{1}{2}(e^{ix}-e^{-ix}) = i\sin(x)$

Differentiation: See Table for Common Derivatives.

$$\frac{df}{dx} = \lim_{\delta x\to 0}\frac{f(x+\delta x)-f(x)}{\delta x}$$

For constraint ϕ: $df + \lambda d\phi = 0$

$$\frac{\partial^2 f}{\partial x\partial y} = \frac{\partial^2 f}{\partial y\partial x} \qquad \underline{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$$\int u\,dv = uv - \int v\,du$$

$$I_{ab} = \lim_{\delta x\to 0}\sum_{x=a}^{x=b} f(x_i)\,\delta x$$

Integration: See Table for Common Integrals.

$$\int\frac{1}{ax^2+bx+c}\,dx \quad \begin{cases} b^2-4ac<0 \Rightarrow \text{Trig Substitution} \\ b^2-4ac>0 \Rightarrow \text{Partial Fractions}\end{cases}$$

$$\bar{f} = \frac{1}{b-a}\int_a^b f(x)\,dx \qquad E[X] = \int_a^b x f(x)\,dx \qquad V = \pi\int_a^b f(x)^2\,dx$$

$$I_{LINE} = \int_C \underline{F}\cdot d\underline{r} = \int_{t_A}^{t_B}\left[\underline{F}(\underline{r}(t))\cdot\frac{d\underline{r}}{dt}\right]dt$$ (Parametric force and path).

Series: Arithmetic: $S_n = \frac{n}{2}(2a+(n-1)d)$

Geometric: $S_n = \frac{a(1-r^n)}{1-r}$

Arithmetic-Geometric: $S_\infty = \frac{a}{1-r} + \frac{rd}{(1-r)^2}$

Difference Method: $u_k = f(k) - f(k-n)$ Then cancel terms.

Convergence Tests: $\lim_{k\to\infty} u_k = 0$ Necessary for convergence.

Comparison Tests (u_k compared to v_k).

$$\rho = \lim_{k\to\infty}\left|\frac{u_{k+1}}{u_k}\right| \quad \begin{cases}\rho<1 \Rightarrow \text{Converge} \\ \rho>1 \Rightarrow \text{Diverge}\end{cases}$$ Else different method.

Power Series: $|x| < \frac{1}{\rho}$, Taylor: $f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \dots$

L'Hôpital's: $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$ for limit converge to $\frac{0}{0}$ or $\frac{\infty}{\infty}$ etc.