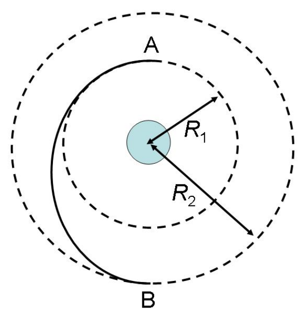
PHAS1247 Classical Mechanics Problem Sheet 4
Submission deadline: 1700 Thursday 16 December 2010

Please fasten your answer sheets together and remember to give **your name** and **your problem-solving tutorial group** at the top.

1. A Hohmann transfer orbit is a way of transferring a spacecraft between two planetary orbits (which we shall assume are circular) by using one half of an elliptical orbit about the Sun, shown as the solid line in the diagram.



Suppose the spacecraft is initially moving around the Sun with the orbital speed V_1 of the first planet, at radius R_1, and we wish to move it to a larger orbital radius R_2. Let the orbital speeds of the spacecraft at perihelion (point A) and aphelion (point B) in the elliptical orbit be v_A and v_B respectively. Write down the conditions on v_A and v_B coming from (i) the conservation of energy and (ii) the conservation of angular momentum, on the assumption that the gravitational fields of the planets have a negligible effect on the spacecraft compared to the gravitational field of the Sun. **[2]**

Hence show that the velocity boost required to accelerate the spacecraft into the transfer orbit is

$$\Delta v = v_A - V_1 = \sqrt{\frac{GM_\odot}{R_1}}\left[\sqrt{\frac{2R_2}{R_1+R_2}} - 1\right],$$

where $M_\odot$ is the mass of the Sun. **[3]**

Show also, using the expression derived in the lectures for the period of an elliptical orbit in an inverse-square-law force, that the time taken for the transfer is

$$T_{\text{transfer}} = \pi\sqrt{\frac{(R_1+R_2)^3}{8GM_\odot}}.$$

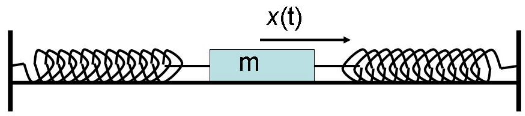
[2]

Evaluate T_{transfer} and Δv for a Hohmann transfer between the (approximately circular) orbits of Earth (radius $R_1 = 1.50 \times 10^{11}$ m) and Jupiter (radius $R_2 = 7.79 \times 10^{11}$ m). **[2]**

What is the eccentricity of the transfer orbit in this case? **[1]**

[Mass of Sun: $M_\odot = 1.99 \times 10^{30}$ kg; gravitational constant $G = 6.67 \times 10^{-11}\,\text{m}^3\text{kg}^{-1}\text{s}^{-2}$.]

2. A mass m moves along a line on a rough table and is attached on either side to a stretched spring; the springs have equal spring constants k. Suppose the coefficients of static and sliding friction between the mass and the table are equal and have the value μ.



(a) Show that in the absence of friction (i.e. if $\mu = 0$), the particle executes simple harmonic motion with angular frequency $\omega = \sqrt{\frac{2k}{m}}$. **[2]**

(b) Now we include the effect of friction. Suppose the particle is released from rest at time $t = 0$ with a positive displacement x_0 from equilibrium. Describe its initial motion, distinguishing between the cases (i) $2kx_0 > \mu mg$ and (ii) $2kx_0 \leq \mu mg$. **[2]**

(c) For case (i), write down the differential equation satisfied by the displacement x of the mass as long as it remains moving. Verify that it is satisfied by a solution of the form

$$x(t) = A\cos(\omega t) + B\sin(\omega t) + C,$$

and find the values of the constants A, B and C for the data given. **[5]**

(d) Find the time t_1 and position x_1 at which the particle next comes to rest. **[2]**

(e) Find the condition for the particle to move again after it has stopped at position x_1. What happens subsequently? Assuming $2kx_0 \gg \mu mg$, outline how the amplitude of the oscillation varies with time, and how the frequency of the mass's oscillation is affected by the friction. **[4]**

3. Show that the moment of inertia of a rectangular plane of mass M, having length l and width w, about an axis passing through its centre and at right angles to the plane is

$$I = \frac{M(l^2 + w^2)}{12}.$$

[5]

4. A bicycle wheel of mass m and radius a is suspended from just inside its rim so that it is free to rotate about an axis perpendicular to the wheel. Assuming all the mass of the wheel is concentrated uniformly around the rim and that the rim is very thin, find the moment of inertia of the wheel about the axis of rotation. [HINT—use the Parallel Axes Theorem.] **[2]**

With the wheel pivoted at the point of suspension, its centre is displaced to one side (see diagram below). Find the torque about the axis of suspension when the wheel is displaced through an angle ϕ. Hence show that for small displacements the wheel executes simple harmonic motion, and find its period in terms of a and the gravitational acceleration g. **[3]**