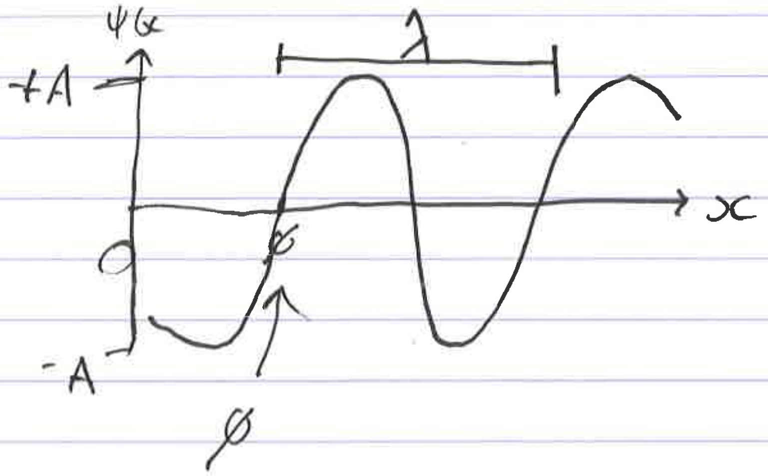
TISE for a free particle.

$$\frac{-\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x)$$

$$\frac{d^2\psi(x)}{dx^2} = -\left(\frac{2mE}{\hbar^2}\right)\psi(x)$$

<u>Trial solution</u> (general sinusoidal function).

$$\psi(x) = A\sin\left(\frac{2\pi(x-\phi)}{\lambda}\right)$$



Calculate $d^2\psi/dx^2$

$$\frac{d\psi}{dx} = A\cos\left(\frac{2\pi(x-\phi)}{\lambda}\right)\frac{2\pi}{\lambda}$$

$$\frac{d^2\psi}{dx^2} = A\left(-\sin\left(\frac{2\pi(x-\phi)}{\lambda}\right)\right)\left(\frac{2\pi}{\lambda}\right)^2$$

$$\frac{d^2\psi}{dx^2} = -\left(\frac{2\pi}{\lambda}\right)^2 \underbrace{A\sin\left(\frac{2\pi(x-\phi)}{\lambda}\right)}_{\psi(x)} = -\left(\frac{2\pi}{\lambda}\right)^2\psi(x)$$

$$\boxed{\frac{1}{\lambda} = \frac{p}{h}}$$

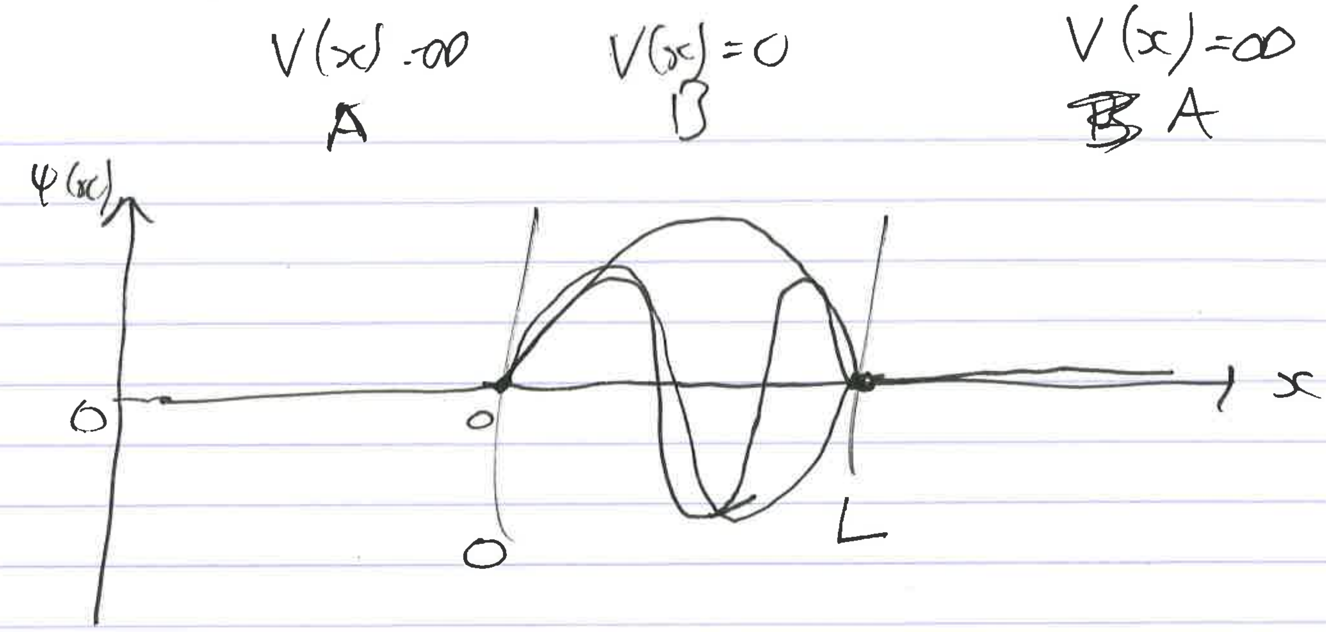
subst into TISE:

$$\cancel{-}\left(\frac{2\pi}{\lambda}\right)^2\cancel{\psi(x)} = \cancel{-}\left(\frac{2mE}{\hbar^2}\right)\cancel{\psi(x)}$$

$$\frac{1}{\lambda} = \frac{\sqrt{2mE}}{h}$$

$$\left(\frac{2\pi}{\lambda}\right)^2 = \left(\frac{2mE}{\hbar^2}\right)$$

$$\frac{2\pi}{\lambda} = \frac{\sqrt{2mE}}{\hbar}$$

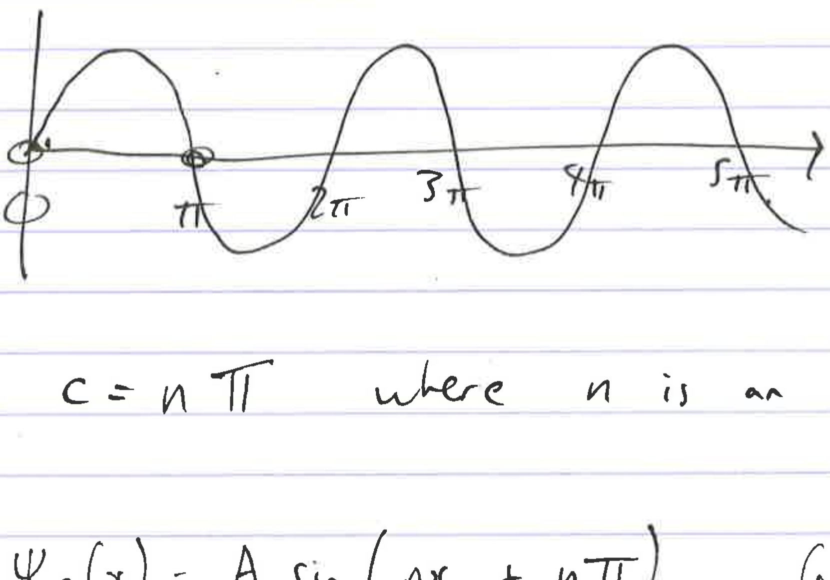


$\psi_A(x) = 0$ $\qquad\qquad$ $\psi_B(x) = A \sin\left(\frac{px}{\hbar} + c\right)$

$\psi_A(0) = 0 = \psi_B(0) = A \sin\left(\cancel{\frac{p0}{\hbar}} + c\right) = A \sin(c)$

Can't have $A = 0$ (no particle) hence to solve this:

$\sin(c) = 0$

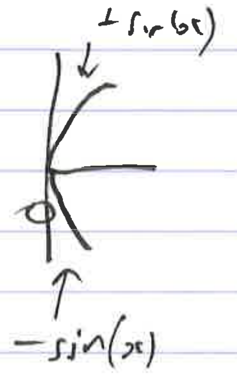


$c = n\pi$ where n is an integer.

$\psi_B(x) = A \sin\left(\frac{px}{\hbar} + n\pi\right)$ for any integer n.

$\sin(f(x) + 2\pi) = \sin(f(x))$ $\qquad$ $\sin(f(x) + \pi) = -\cancel{A}\sin(f(x))$

$$A \sin\left(\frac{px}{\hbar} + n\pi\right) = \pm A \sin\left(\frac{px}{\hbar}\right)$$



Will see later $\psi(x) = f(x)$ and $\psi(x) = -f(x)$ have same physical properties.

Hence, ignore -1 solution for now.

Let: $\psi_B(x) = A \sin\left(\frac{px}{\hbar}\right)$

<u>Second B.C.:</u> is $\psi_A(L) = \psi_B(L)$.

$$A \sin\left(\frac{pL}{\hbar}\right) = 0$$

By same reasoning $A \neq 0$ hence.

$$\sin\left(\frac{pL}{\hbar}\right) = 0 \quad \text{hence} \quad \frac{pL}{\hbar} = n\pi$$

for any integer n.

$n = 0$ not allowed since $\sin(0) = 0$ is not a physical wavefunction "No particle"

$$\psi_B(x) = \sin\left(\frac{p\,x}{\hbar}\right) \quad \text{where} \quad \frac{pL}{\hbar} = n\pi$$

$$\text{Set} \quad \frac{pL}{\hbar} = n\pi \qquad p = \frac{n\pi\hbar}{L}$$

$$\psi_B(x) = \sin\left(\cancel{\frac{n\pi\hbar}{L}\,\frac{x}{\hbar}}\right)$$

$$= \sin\left(\frac{x}{\hbar}\,\frac{n\pi\hbar}{L}\right) = \sin\left(\frac{n\pi x}{L}\right)$$

$n = \pm 1, \pm 2, \pm 3, \ldots\ldots$

Since $\sin(-x) = -\sin(x)$

Since "global" minus signs on a wavefunction have no physical effect, ignore -ve values of n.

→ Wavefunctions for a particle in an ∞ square well

$$\psi(x) = A\sin\left(\frac{n\pi x}{L}\right) \qquad n = 1, 2, 3, \ldots\ldots$$