Series

- Arithmetic

$$S_n = \sum_{k=0}^{n-1} (a+kd) = a + [a+d] + \ldots + [a+(n-1)d]$$

$$= \frac{n}{2}\left[2a + (n-1)d\right]$$

$\Rightarrow \lim_{n\to\infty} S_n$ diverges (grows indefinitely)

- <u>Geometric</u>

$$S_n = \sum_{k=0}^{n-1} a r^k = a + ar + ar^2 + \ldots + ar^{n-1}$$

$$= a \frac{1-r^n}{1-r}$$

$$\Rightarrow S = \lim_{n \to \infty} S_n \equiv \sum_{k=0}^{\infty} a r^k = \begin{cases} \dfrac{a}{1-r} & \text{for } |r| < 1 \\ \text{diverges/oscillates} & \text{for } |r| \geq 1 \end{cases}$$

- Arithmetic-Geometric

$$S_n = \sum_{k=0}^{n-1} (a+kd)\, r^k = a + (a+d)r + (a+2d)r^2 + \dots + [a+(n-1)d]\, r^{n-1}$$

$$= d\,\frac{1-r^n}{(1-r)^2} + \frac{a-d-[a+(n-1)d]\, r^n}{1-r}$$

$$\Longrightarrow S = \lim_{n\to\infty} S_n = \begin{cases} \dfrac{d}{(1-r)^2} + \dfrac{a-d}{1-r} = \dfrac{a}{1-r} + \dfrac{rd}{(1-r)^2} & \text{for } |r|<1 \\ \text{diverges/oscillates} & \text{for } |r|>1 \end{cases}$$