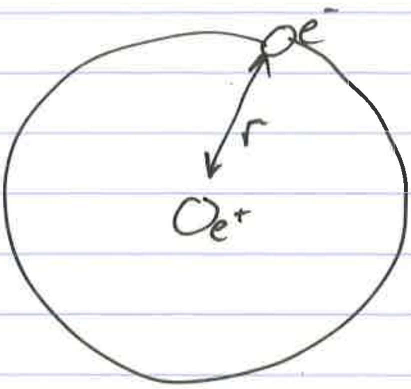
Bohr model



r: radius of orbit

Angular momentum:

$l = mvr$

Balance forces:

Coulomb force = centripetal force.

Coulomb force $F = -k\dfrac{q_1 q_2}{r^2} = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r^2}$

$q_1 = e \quad q_2 = -e$

$$\frac{mv^2}{\cancel{r}} = \frac{1}{4\pi\varepsilon_0}\frac{ee}{r\cancel{^2}} \xrightarrow[\text{rewrite}]{} \boxed{v^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{m}\frac{1}{r}} \quad *$$

Bohr's quantisation rule

$l = \hbar n \qquad n = 1, 2, 3, \ldots$

$l = mvr = \hbar n$

$$v = \frac{\hbar n}{mr} \rightarrow \boxed{v^2 = \frac{\hbar^2 n^2}{m^2 r^2}} \quad **$$

Equate $*$ and $**$

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\cancel{m}}\frac{1}{\cancel{r}} = \frac{\hbar^2 n^2}{m^{\cancel{2}} r^{\cancel{2}}}$$

$$r = \overbrace{\left(\frac{\hbar^2 (4\pi\varepsilon_0)}{me^2}\right)}^{a_0} n^2$$

$= a_0 n^2$

$a_0 = 5.3 \times 10^{-11}$ m

$\approx \frac{1}{2}$ Å $\qquad$ Å $= 10^{-10}$ m.

a_0: radius of $n=1$ orbit

"Bohr radius"

Energy of electron

Energy = Kinetic energy + Potential Energy.

$$= \frac{1}{2}mv^2 + \left(-\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} \right)$$

Substitute * for v^2:

$$E = \frac{1}{2}\cancel{m}\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{e^2}{r}\left(\frac{1}{\cancel{m}}\right) - \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

$$= \frac{-1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{e^2}{r}$$ Use: $r = a_0 n^2$

$$= \underbrace{\frac{-1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{e^2}{a_0}}_{\text{Fundamental constants}} \frac{1}{n^2}$$

$$= \frac{-2.2 \times 10^{-18}\ \text{J}}{n^2} = \frac{-13.6\ \text{eV}}{n^2}$$