PHAS1247 Classical Mechanics: Problem-Solving Tutorial 1, 2010

Week 8 (18–22 October): vectors, Newton's laws, momentum, and impulse

Objectives: To gain problem-solving experience in

- The applicaton of vectors and calculus to physical problems;
- Application of the concepts of momentum, impulse and energy.

Take the gravitational acceleration $g = 9.81\,\mathrm{ms}^{-2}$ throughout. You may neglect air resistance in all the problems.

Section A—calculus, vectors and motion

1. A particle of mass $m = 2\,\mathrm{kg}$ has an initial velocity $\mathbf{v} = 2\hat{\mathbf{i}}\,\mathrm{m\,s}^{-1}$. It experiences a constant force of $2\hat{\mathbf{j}}\,\mathrm{N}$ for a time of $5\,\mathrm{s}$. What is its final velocity?

2. A particle of mass $m = 2\,\mathrm{kg}$ starts from rest at time $t = 0$. It then experiences, for 2 seconds, a time-dependent force given by

$$\mathbf{F}(t) = [t(2-t)\hat{\mathbf{i}} + 3\hat{\mathbf{j}}]\,\mathrm{N},$$

   where the time t is measured in seconds. After time $t = 2\,\mathrm{s}$ the force is zero (you can neglect the influence of gravity at all times).

   (a) Calculate the total impulse imparted to the body by the force. Hence find the body's velocity at time $t = 2\,\mathrm{s}$.

   (b) Show that by this time ($t = 2\,\mathrm{s}$) the object has moved approximately $3.07\,\mathrm{m}$ from its original position. In what direction has it moved?

   (c) Describe the particle's motion after the force has ceased to act. Hence find its displacement after a total time $t = 5\,\mathrm{s}$ has elapsed (i.e., $3\,\mathrm{s}$ after the force ceased to act).

3. A gardener holds a hose so the water emerges horizontally at a height of $1\,\mathrm{m}$ above the ground. It lands on a flowerbed $2\,\mathrm{m}$ in front of her; what is the speed of the water (a) when it emerges from the hose and (b) just before it hits the ground?

Problem for general discussion

A firework of mass M is launched into the sky at an angle, so that it reaches its highest point when it is $d = 10\,\mathrm{m}$ horizontally away from its launch point. When it is precisely at this highest point the firework explodes, splitting into two fragments of equal mass $M/2$. The two fragments are observed to fall back to earth at exactly the same time. One lands at the launch point; where does the other fragment land?

[You may neglect air resistance and assume that forces from the firework's propulsion systems operate only at the instant of launch and the instant of the explosion.]

Section B—Newton's laws and momentum

1. A boy kicks a football into the air at a speed of $20\,\mathrm{ms}^{-1}$ and at an angle of $30°$ to the horizontal. What are (a) the minimum distance and (b) the maximum distance he can be from the foot of a thin wall, $3\,\mathrm{m}$ high, in order for the ball to clear the top of the wall without bouncing first?

2. A rifle bullet of mass $m = 0.03\,\mathrm{kg}$ is fired horizontally at a speed of $v = 400\,\mathrm{ms}^{-1}$ into a sand-bag of mass $M = 2\,\mathrm{kg}$ (considered as a point particle), suspended by a light inextensible string of length $l = 1.8\,\mathrm{m}$ from a beam. The bullet remains within the sand-bag and no sand falls out.

   What can you say about the horizontal momentum of the system (bullet + sand-bag) during the collision? Hence determine the (i) the velocity of the sand-bag with the embedded bullet after the collision, (ii) the initial kinetic energy, and (iii) the final kinetic energy. Explain any difference between your answers to (ii) and (iii).

   Also determine the maximum height h_{max} attained by the sand-bag after the collision.

3. A bucket of mass m is being drawn up a well by a rope which exerts a steady force F. Initially (at time $t = 0$) the bucket is at rest and contains a mass m_0 of water, but this leaks out at a constant rate so that after a time $t = T$ (before it reaches the top) the bucket is empty. Show that the velocity v_{final} of the bucket at the instant it becomes empty is
$$v_{\mathrm{final}} = \frac{FT}{m_0} \ln\left[\frac{m + m_0}{m}\right] - gT.$$

Problem for general discussion

The magnetic force on a particle of charge q moving with velocity vector $\mathbf{v}$ through a magnetic field $\mathbf{B}$ is
$$\mathbf{F}_{\mathrm{mag}} = q\mathbf{v} \times \mathbf{B}.$$
Show that this force never does any work on the particle.

In fact it is observed that the potential energy of a magnetic material, such as iron, is lowered when its magnetization is aligned with an external magnetic field, and rises when the moment is anti-aligned with the field. What does this fact tell us about the magnetic forces that are operating?