Limits

Find $\lim\limits_{x \to a} f(x)$

- $= f(a)$ if $f(x)$ is defined and continuous at $x=a$
- otherwise use Taylor expansion (around $x=a$) or L'Hôpital's rule
- Limit may not exist, i.e. $f(x) \to \pm\infty$ as $x \to a$

L'Hôpital's Rule

$$\boxed{\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}}$$

- applies if $f(a) = g(a) = 0$
  or $f(x), g(x) \to \infty$ as $x \to a$
- also applies for $x \to \pm\infty$
- may be necessary to use repeatedly

Basic Examples

- $$\lim_{x\to\pi} \frac{\sin x}{x} = \frac{0}{\pi} = 0$$

- $$\lim_{x\to 0} \frac{\sin x}{\ln x} = \frac{"0"}{-\infty} = 0$$

- $$\lim_{x\to 0} \frac{\ln x}{\sin x} = \frac{"-\infty"}{0} \to -\infty$$

- $$\lim_{x\to 0} \frac{\sin x}{x} \overset{L'H}{=} \lim_{x\to 0} \frac{\cos x}{1} = 1$$

- $$\lim_{x\to 0} \frac{\sin x}{\sqrt{x}} \overset{L'H}{=} \lim_{x\to 0} \frac{\cos x}{\frac{1}{2}x^{-1/2}} = \lim_{x\to 0} \left[2\sqrt{x}\cos x\right] = 0$$

- $$\lim_{x\to 0} \frac{\sin x}{x^2} \overset{T}{=} \lim_{x\to 0} \frac{x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \dots}{x^2} = \lim_{x\to 0} \left[\frac{1}{x} - \frac{1}{3!}x + \frac{1}{5!}x^2 - \dots\right]$$

$$\to \infty$$