PHAS1247 : Classical Mechanics
In-Course Assessment Test #1 : Mon. 9 November 2015

Answer as many of the questions as you can, in any order. None of the questions require a calculator. The approximate distribution of marks is given in square brackets on the right of the page. There are 5 questions: they continue **ON THE OTHER SIDE OF THE PAGE**. *The maximum mark is 30.*

1. A particle of mass 1 kg has a position vector $\underline{\mathbf{r}}$ (in meters) as a function of time, t, (in seconds) given by:

$$\underline{\mathbf{r}} = \sin(2t)\,\hat{\underline{\mathbf{i}}} + \cos(t\sqrt{3})\,\hat{\underline{\mathbf{j}}} + te^{2t}\,\hat{\underline{\mathbf{k}}}\ .$$

   Find the particle's (i) velocity and (ii) its acceleration at a time t.

   Determine the unit vector defining the direction of the force on the particle at time $t = 0$. **[4]**

2. A person of mass 70 kg stands at one end of a plank of length 4 m and mass 140 kg. Both the person and plank are initially at rest and the plank sits on a frozen lake that can be considered as a flat and frictionless surface. The person walks along the plank at a constant velocity of $1.5\,\hat{\underline{\mathbf{i}}}\,\mathrm{ms}^{-1}$ *relative to the plank (not the ice)* towards the other end of the plank. The direction of the plank is $\hat{\underline{\mathbf{i}}}$.

   (i) How far is the centre of mass of the plank+person system from the centre of the plank ? **[2]**

   (ii) How far and in what direction has the centre of the plank moved in the time interval of the person walking from one end of the plank to the other end ? **[2]**

   (iii) What is the velocity of the person *relative to the ice* while walking along the plank ? **[2]**

3. A mass $m_1 = 6\,\mathrm{kg}$ has an initial velocity in the laboratory frame (i.e. relative to a stationary observer) $\underline{\mathbf{u}}_1 = 2\,\hat{\underline{\mathbf{i}}}\,\mathrm{ms}^{-1}$ while a second mass $m_2 = 3\,\mathrm{kg}$ has a velocity $\underline{\mathbf{u}}_2 = -3\,\hat{\underline{\mathbf{i}}}\,\mathrm{ms}^{-1}$. Find (i) the total momentum of the system, (ii) the velocity of the centre of mass, and (iii) the relative velocity of particle 1 to particle 2. **[3]**

   The two particles undergo an elastic collision and continue to move along the x-axis. Find the velocity of particle 1 in both the laboratory and the centre of mass frame after the collision. Express your answers as a fraction. **[4]**

4. Give one definition of a conservative force ? **[1]**

   A particle moving in three dimensions moves in a straight line from the position vector (measured in meters): $\underline{\mathbf{r_1}} = \left(\hat{\underline{\mathbf{i}}} + \hat{\underline{\mathbf{j}}} + 3\,\hat{\underline{\mathbf{k}}}\right)$ to a point with a position vector, $\underline{\mathbf{r_2}} = \left(2\,\hat{\underline{\mathbf{i}}} + 2\,\hat{\underline{\mathbf{j}}} + 5\,\hat{\underline{\mathbf{k}}}\right)$. Find the work done during the displacement from $\underline{\mathbf{r_1}}$ to $\underline{\mathbf{r_2}}$ by (i) a constant force $\underline{\mathbf{F}}_1 = \left(3\,\hat{\underline{\mathbf{i}}} + 2\,\hat{\underline{\mathbf{j}}} + \hat{\underline{\mathbf{k}}}\right)$ N and (ii) by the position dependent force $\underline{\mathbf{F}}_2 = \left(2xy^3\,\hat{\underline{\mathbf{i}}} + 3x^2y^2\,\hat{\underline{\mathbf{j}}}\right)$ N **[5]**

   Show that the work done by $\underline{\mathbf{F}}_2$ is not dependent on the path taken. **[1]**

PLEASE TURN OVER FOR QUESTION 5

5. A particle moving in three dimensions with position coordinates (x, y, z) has a potential energy, V, given by:

$$V = y^3 + 2x^2y^2 + z^2 \ .$$

Find an expression for the force $\underline{\mathbf{F}}$ acting on the particle at position (x, y, z). **[2]**

What is the change in KE (initial KE–final KE) of the particle as it moves from a position (0,0,0) to (1,1,2) assuming the only force acting on the particle is $\underline{\mathbf{F}}$. **[2]**

If the force acting on a particle is:

$$\underline{\mathbf{F_1}} = -2xy^3\,\hat{\underline{\mathbf{i}}} - 3x^2y^2\,\hat{\underline{\mathbf{j}}}.$$

Find an expression for the potential energy of the particle at position (x, y) assuming $V = 0$ at $x = y = 0$. **[2]**

END OF PAPER