PHAS1247 : Classical Mechanics
Solutions for In-Course Assessment Test #2 : Mon 9 January 2018

1. When the system is only composed of the first mass ont he table, the maximal friction force, parallel to the table surface, will be $F = \mu m_1 g$, so the maximal acceleration the table can have without the mass moving with respect to it is μm_1.

   In the case of the two masses, the force acting on the system in case of maximal static friction will be $F = m_2 g - \mu m_1 g$, leading to an acceleration
   $$a = g\frac{m_2 - \mu m_1}{m_1 + m_2}$$
   So this is the maximal acceleration that the friction can compensate.

2. In the external, inertial system, the ball is thrown at constant speed v and will not feel any external force. So it will reach the edge of the carousel in time $t = R/v$. The speed of the horse is ωR, so in the time it takes for the ball to reach the edge of the carousel, it will have moved by an angle $\alpha = \omega R/v$. This is therefore the minimal length of the horse for it to still be hit by the ball.

3. In the rotating system , the Coriolis force is $F_C = -2\omega \times v$. The anti-clockwise rotation is represented by a vector $\underline{\omega}$ perpendicular to the plane of the carousel, and pointing upwards. The initial speed of the ball is perpendicular to $\underline{\omega}$, so the absolute value of the Coriolis force is $|F_C| = \omega v$, and its direction is anti-clockwise. Since the force is always perpendicular to the ball's speed, the force is always perpendicular to the displacement, and the work done on the ball is zero (while the centrifugal force will instead make work on the ball).

4. The mass of the stick is
   $$M = \int_{-L}^{L} \alpha x^2 dx = \alpha \frac{2}{3} L^3$$
   The x position of the centre of mass
   $$x_{CM} = \int_{-L}^{L} \alpha x^3 dx = 0$$
   The momentum of inertia around $x = 0$
   $$I = \int_{-L}^{L} \alpha x^4 dx = \alpha \frac{2}{5} L^5$$
   When the force is applied, the final momentum will be $P = F \cdot 1s$ and the total angular momentum, obtained integrating the torque, will be $L = FL \cdot 1s$.

5. The gravitational potential will be given by $V = mg\frac{1}{2}\alpha(r - r_0)^2$ so the force acting of the system due to the section of the racetrack is $F = -k(r - r_0)$ with $k = mg\alpha$. The centrifugal force is $F_C = v^2/r_0$, so equating the two
   $$k(r_{eq} - r_0) = \frac{v^2}{r_0}$$
   $$kr_{eq} = \frac{v^2}{r_0} + kr_0$$
   $$r_{eq} = r_0 + \frac{v^2}{kr_0}$$
   When the system oscillates, it will be equivalent to a spring with elastic constant k, so the angular velocity is $\omega = \sqrt{k/m} = \sqrt{g\alpha}$.