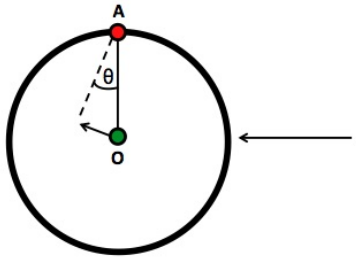
PHAS1247 : Classical Mechanics
In-Course Assessment Test #2 : Mon. 12 January 2015

Answer as many of the questions as you can, in any order. The approximate distribution of marks is given in square brackets on the right of the page. There are 4 questions: they continue **ON THE OTHER SIDE OF THE PAGE**. *The maximum mark is 41.*
The % awarded will be calculated as: $\mathrm{MIN}(100, 100 \times \frac{\mathrm{mark}}{41} \times \frac{60}{\text{test duration (mins)}})$.

1. A ring of mass m and radius r is suspended in the vertical plane from an axis through the point A, shown in the figure below.



   (a) Show by integration that the moment of inertia of the ring about the centre of mass is mr^2. **[2]**

   (b) Determine the moment of inertia about the axis A. **[2]**

   (c) The ring is displaced by an impulse acting from the right as shown such that the centre of mass is rotated through an angle θ. Show, for small values of θ, that the subsequent motion of the ring is simple harmonic and determine the period (T) of oscillation. **[4]**

2. Nigel Farage is back and is stood stationary at the mid-point of a horizontal rod of length A drinking a pint of illegally imported beer and secretly enjoying it. Just as the press arrive to take a photo, the rod starts to rotate around a vertical axis at one end, and fearing a bad photo opportunity, Farage, starts to walk along the rod at a constant speed v away from the axis of rotation. The angular speed of rotation of the rod, ω, is given by: $\omega = \omega_o t$ where ω_o is a constant. The rod starts to rotate and Farage stars to walk along the rod at $t = 0$.

   In polar coordinates, the velocity, $\underline{\mathbf{v}}$, and acceleration, $\underline{\mathbf{a}}$, are:

$$\underline{\mathbf{v}} = \dot{r}\,\hat{\underline{\mathbf{r}}} + r\dot{\theta}\,\hat{\underline{\boldsymbol{\theta}}} \quad \text{and} \quad \underline{\mathbf{a}} = (\ddot{r} - r\dot{\theta}^2)\,\hat{\underline{\mathbf{r}}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{\underline{\boldsymbol{\theta}}}.$$

   The mass of Farage is m and the mass of the rod can be neglected relative to Farage, although it's intellect is higher.

   (a) Determine the angular momentum vector, $\underline{\mathbf{L}}$, of Farage at time t. **[6]**

   (b) Determine the acceleration vector, $\underline{\mathbf{a}}$, at time t. **[2]**

   (c) The rod is at an angle of $\theta = \alpha$ when Farage gets to the end of the rod and steps off the rod. If $\frac{A^2\omega_0}{v^2} = \sqrt{4/3}$: show that Farage will continue walking at an angle $\theta = \alpha + 30^\circ$ when he steps off the rod. **[4]**

   Sadly this sudden change in direction causes the ex-Banker with the German wife to break his ankle and he collapses in a heap with beer ruining his tweed suit. He is taken to hospital but there are no English nurses left to treat him and he starts to cry.

PLEASE TURN OVER FOR QUESTIONS 3 & 4

3. Unperturbed by his broken ankle Farage is at it again meddling with fictitious forces that he doesn't understand.

   (a) He is handed a bucket filled with liquid and he attaches a rope and starts to maniacally swing the bucket in a vertical plane. By considering the motion of the liquid in the rotating frame show that the water stays in the bucket provided the velocity at the highest point, v_h, satisfies:

   $$v_h > \sqrt{gL}$$

   where L is the length of the rope and g is the acceleration due to gravity. **[3]**

   Sadly Farage cannot make the bucket attain this velocity and he is drenched with the liquid that is a mixture of bile and vomit.

   (b) Farage decides to recuperate by spending his not-so hard earned EU salary on a trip to planet Nigel. Planet Nigel came into existence in 1957 and all its inhabitans are the same. On planet Nigel he gets into a taxi of mass 1000 kg which is initially stationary at a latitude of 45° S. Planet Nigel's radius is 1000 km, the acceleration due to gravity on the planet is $5\,\mathrm{ms}^{-2}$, and its period of revolution is $2\pi \times 10^3$ s.

   - What is the magnitude of the horizontal centrifugal force due to Planet Nigel's rotation on the stationary taxi ? **[4]**
   - What is the magnitude of the Coriolis force in the horizontal plane if the taxi's speed is $10\sqrt{2}\,\mathrm{m}^{-1}$ and it travels exactly due South. **[3]**
   - Unfortunately Farage's taxi skids and collides with an EU juggernaut: if the coefficient of sliding friction between the taxi and the road is 0.5 what is the approximate size of the frictional force when the taxi is skidding. **[2]**

   Sadly on Planet Nigel nobody is qualified to treat him after his EU juggernaut accident but fortunately for Farage, and despite his travel insurance being inadequate, he gets flown back to Earth and, after waiting for three days for an English surgeon, he undergoes emergency surgery.

4. As a result of the belated surgery Farage's head and brain have to be replaced by a hollow spherical ball of mass, m and his body by a small spring of negligible mass with a spring constant k. As a result his mental ability is greatly enhanced and he can easily jump over the bar at his local pub. But without his tweed waistcoat the press are no longer interested.

   Farage is turned upside down and the bottom of his spring body attached to a vertical hook. At equilibrium his shiny steel head is a distance x_0 below the hook. His head is then pulled down vertically a further distance x and released.

   (a) Write down the differential equation satisfied by the displacement x from the equilibrium position as long as Farage's head remains moving and show that his head executes simple harmonic motion with angular frequency $\omega = \sqrt{\frac{k}{m}}$ about the equilibrium position. **[3]**

   (b) Verify that the differential equation is satisfied by a solution of the form:

   $$x(t) = A\cos(\omega t) + B\sin(\omega t)$$

   and find the values of the constants A and B for the data given. **[3]**

   (c) For the range of displacements, x, Farage's shiny little head is now also immersed in a viscous liquid of emulsified UKIP tripe that provides a retarding force: $-\beta\dot{x}$. Write down the new equation of motion of Farage's shiny, vacuous head. Illustrate with a sketch of x as a function of time, how the solution for x found in (b) changes if $\frac{\beta}{2m} < \sqrt{\frac{k}{m}}$. **[3]**

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