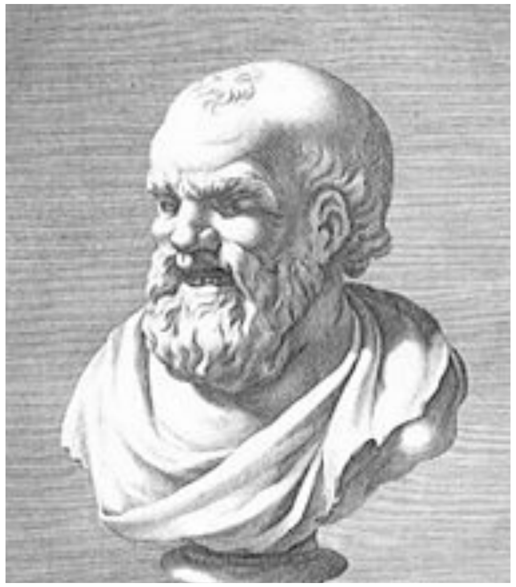
Recap of Last Lecture

Part 2: Atomic theory from 400 BC to 1913

History of the Atom



Democritus

John Dalton

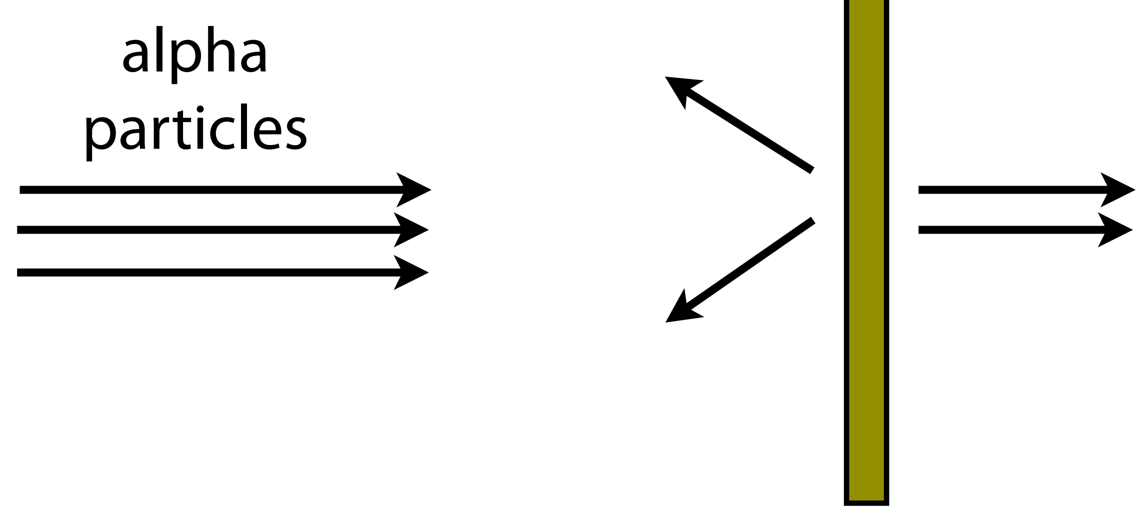
- ***Democritus: c. 400 BC***
    - All matter composed of indivisible “atoms”. (No experimental evidence to support this)
- ***John Dalton: c. 1800***
    - All matter is composed of atoms.
    - There are a limited number of “types” of atoms - called elements - which have differing mass.
    - Atoms corresponding to the same element are identical.
    - Evidence: mass ratios in chemical reactions.

Images from Wikipedia

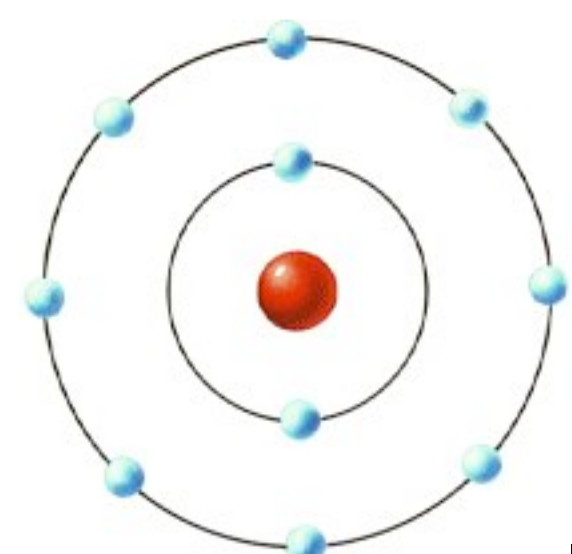
History of the Atom

Ernest Rutherford

- ***Rutherford: 1907***
  - Geiger–Marsden experiment
  - Alpha particles (He^{2+} ions) scattered from Gold foil.



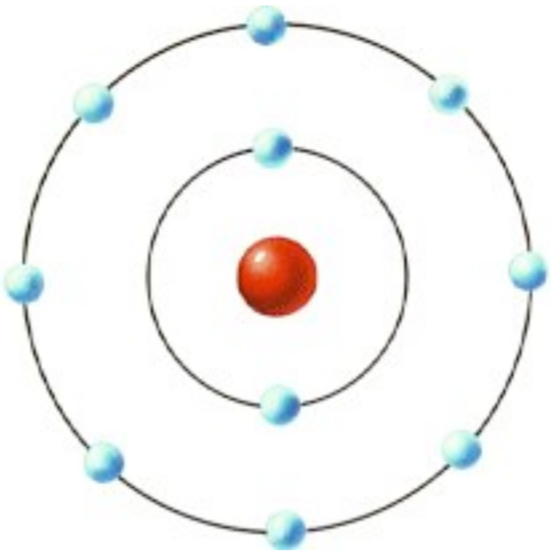
Evidence of a
+ve charged nucleus



- Atomic model: "**Planetary Model**"
  - Atom: a tiny **positively charged nucleus** orbited by **electrons**.

Images from Wikipedia

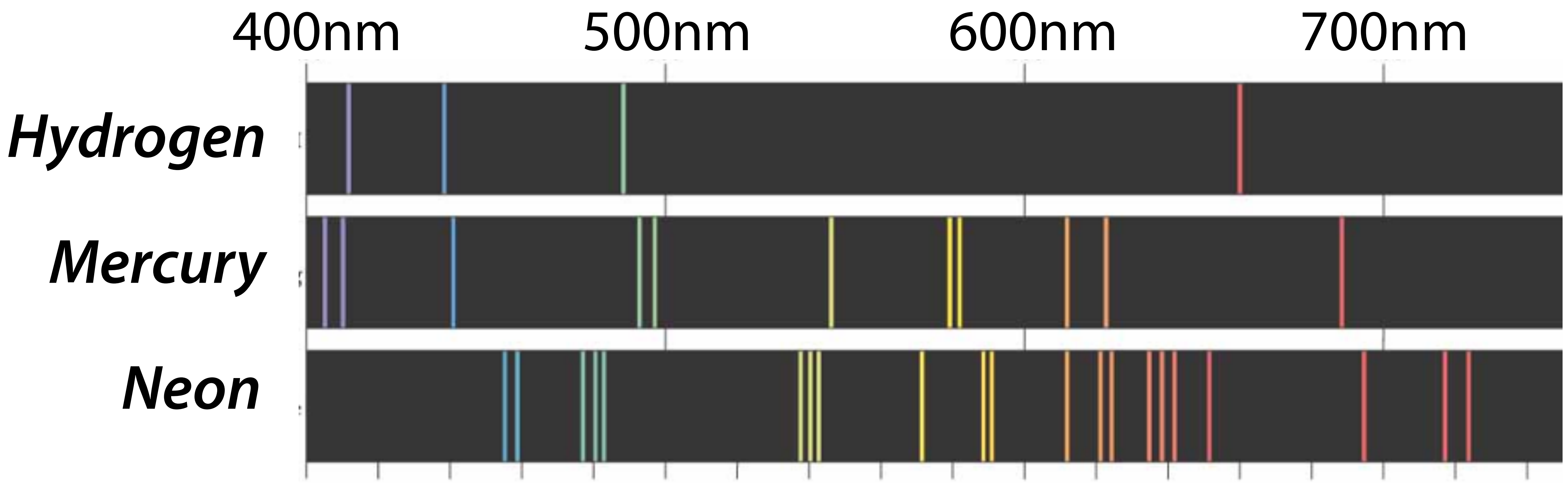
Problems with the planetary model



- **Stability of Matter**
  - **Orbiting electrons** must be **constantly accelerating** (centripetal acceleration).
  - But electromagnetic theory (Maxwell's equations) predict that **accelerating charges** always create **electromagnetic radiation**.
  - Electrons should **lose energy** as they radiate and **spiral** into the nucleus.
- **Atomic spectrometry**
  - Elements known to have **unique signatures** of emitted light.
  - These spectra **not explained at all** by this model..

Atomic spectra

- We call the frequencies at which a gas absorbs and emits light its **spectrum**.
- The spectrum differs from element to element, from molecule to molecule.
- It is a **unique signature** which helps us, for example, study the chemical composition of distant stars.



The emission spectra (in visible range) for Hydrogen, Mercury and Neon (from Jewett / Serway, p. 1253).

Spectrum of Hydrogen

Balmer series

- In 1885, Jacob Balmer realised a **remarkably simple formula** can predict the spectrum of Hydrogen.

Jacob Balmer

$$\frac{1}{\lambda} = R_H \left(\frac{1}{2^2} - \frac{1}{n^2} \right)$$ for ***n*** = 3,4,5,…,

where $R_H = 1.1 \times 10^7 \mathrm{m}^{-1}$ $(2\ \mathrm{s.f.})$ is now called the **Rydberg constant**.

n	λ (nm)
3	656
4	486
5	434
6	410
7	396

- Balmer's formula predicts **further** (Ultra-violet) **lines** outside the visible region.
- These were later **confirmed** in experiment.
- These spectral lines are now called the **Balmer series**.

Spectrum of Hydrogen

Johannes Rydberg

Balmer Formula

$$\frac{1}{\lambda} = R_H \left(\frac{1}{2^2} - \frac{1}{n^2} \right)$$

for ***n*** = 3,4,5,…,

Rydberg Formula

- In 1888, **Rydberg** proposed a generalisation of Balmer's formula:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$$

for ***m*** = 1,2,3,4,5,…,
and ***n*** = m+1,m+2,…,

- The **Balmer series** corresponded to the **m=2** case.
- Rydberg's formula predicts many (infinitely!) more spectral lines but none in the visible spectrum, all are **ultraviolet** or **infrared**.
- **None** of the non-visible spectral lines predicted had been observed prior to **1888**.
- But **all** were subsequently confirmed.

Spectrum of Hydrogen

Understanding the Rydberg formula

$$\frac{1}{\lambda} = R_H \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$$ for ***m*** = 1,2,3,4,5,… and ***n*** = m+1,m+2,…

$$E = hf = \frac{hc}{\lambda}$$

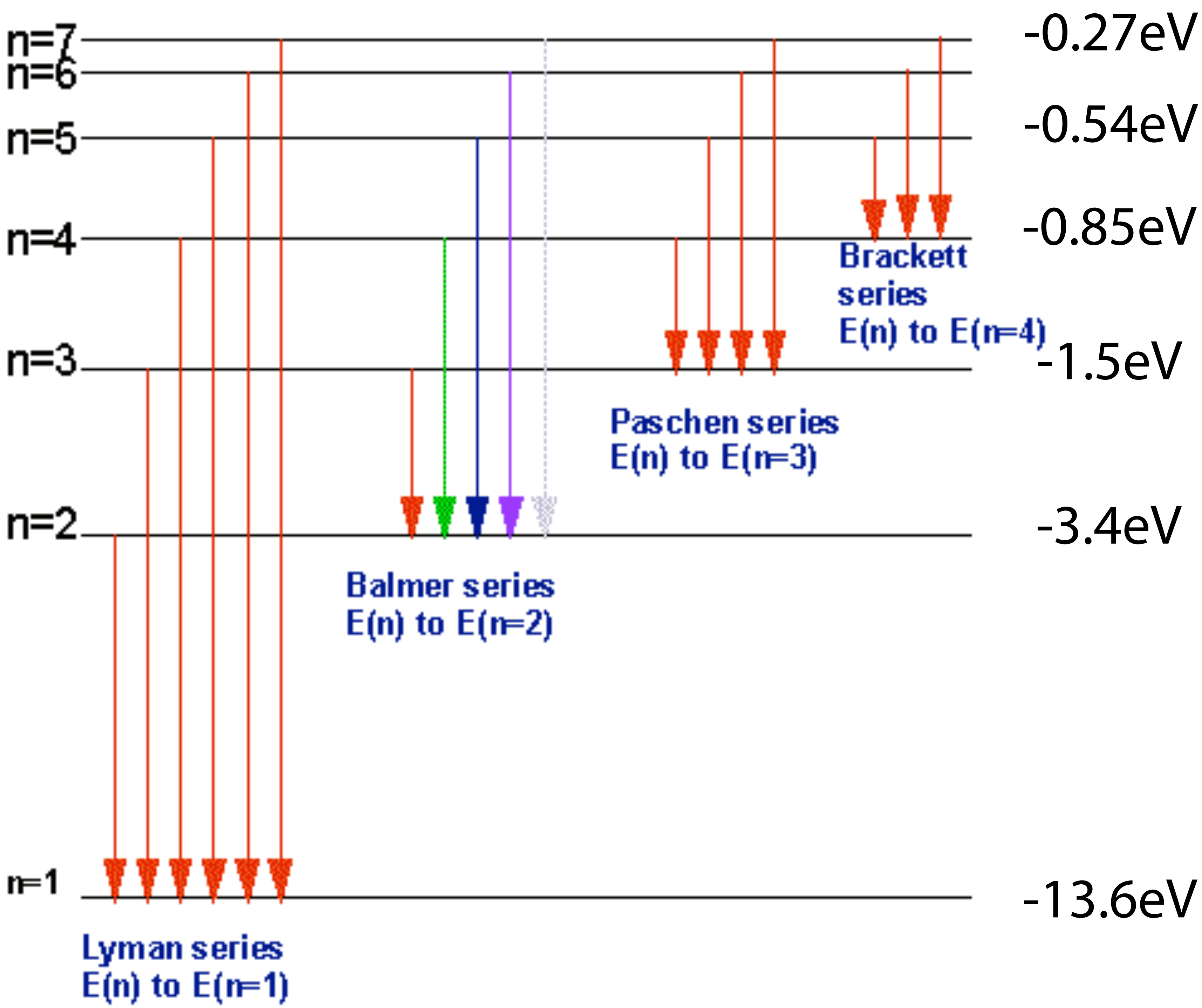
- The Rydberg formula is therefore consistent with the Hydrogen atom possessing "energy states" with energies

$$E_k = -hc\frac{R_H}{k^2} = \frac{-13.6\mathrm{eV}}{k^2}$$

where ***k*** is an **integer** from 1 to infinity and an **electron Volt (eV)** is a convenient unit of **energy.**

$$1\mathrm{eV} = 1.6 \times 10^{-19}\mathrm{Joules}$$

Spectrum of Hydrogen



$$\frac{1}{\lambda} = R_H \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$$

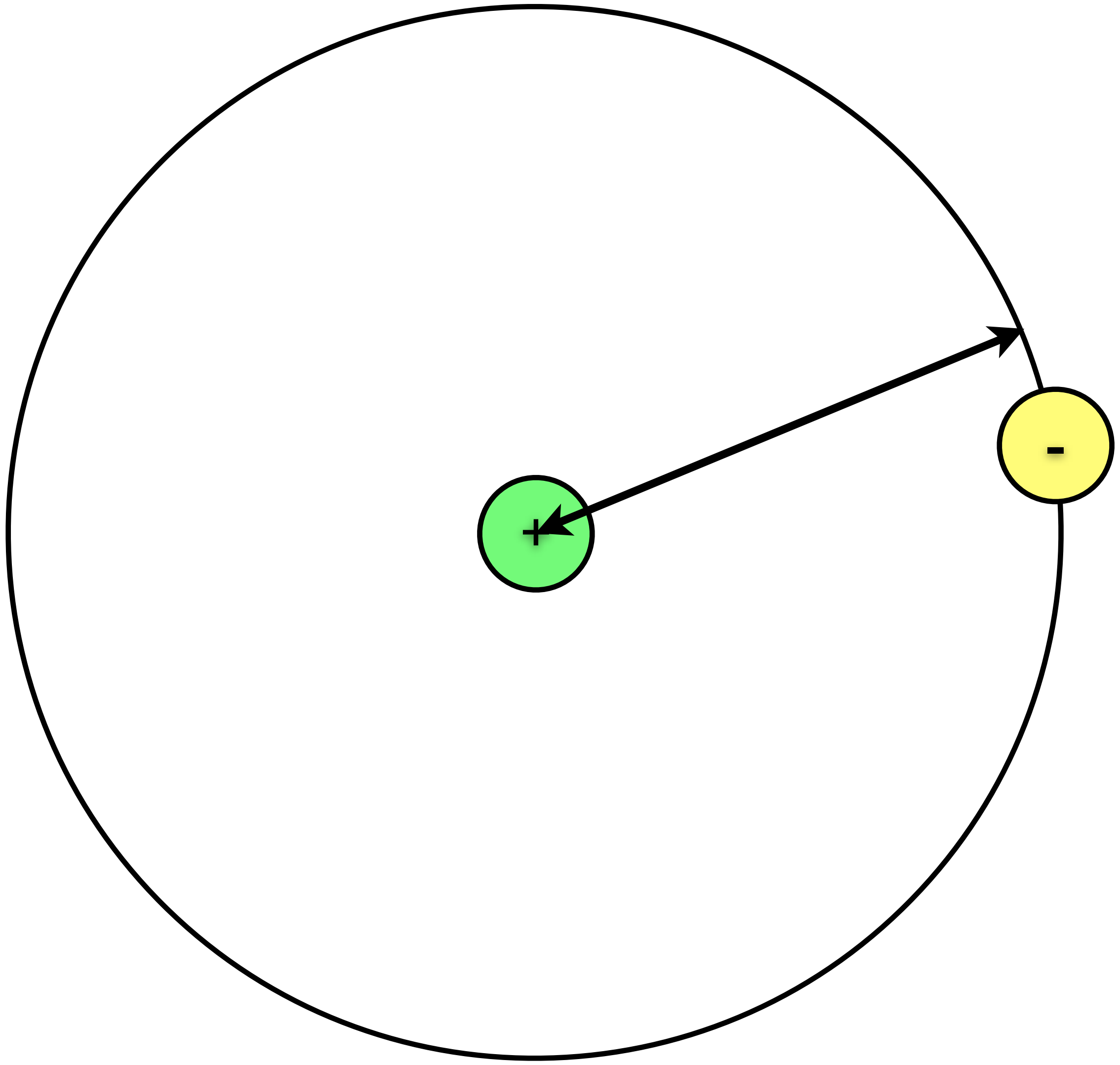
for ***m*** = 1,2,3,4,5,…

and ***n*** = m+1,m+2,…

$$E_k = -hc\frac{R_H}{k^2}$$

$$= \frac{-13.6\text{eV}}{k^2}$$

The Bohr Model



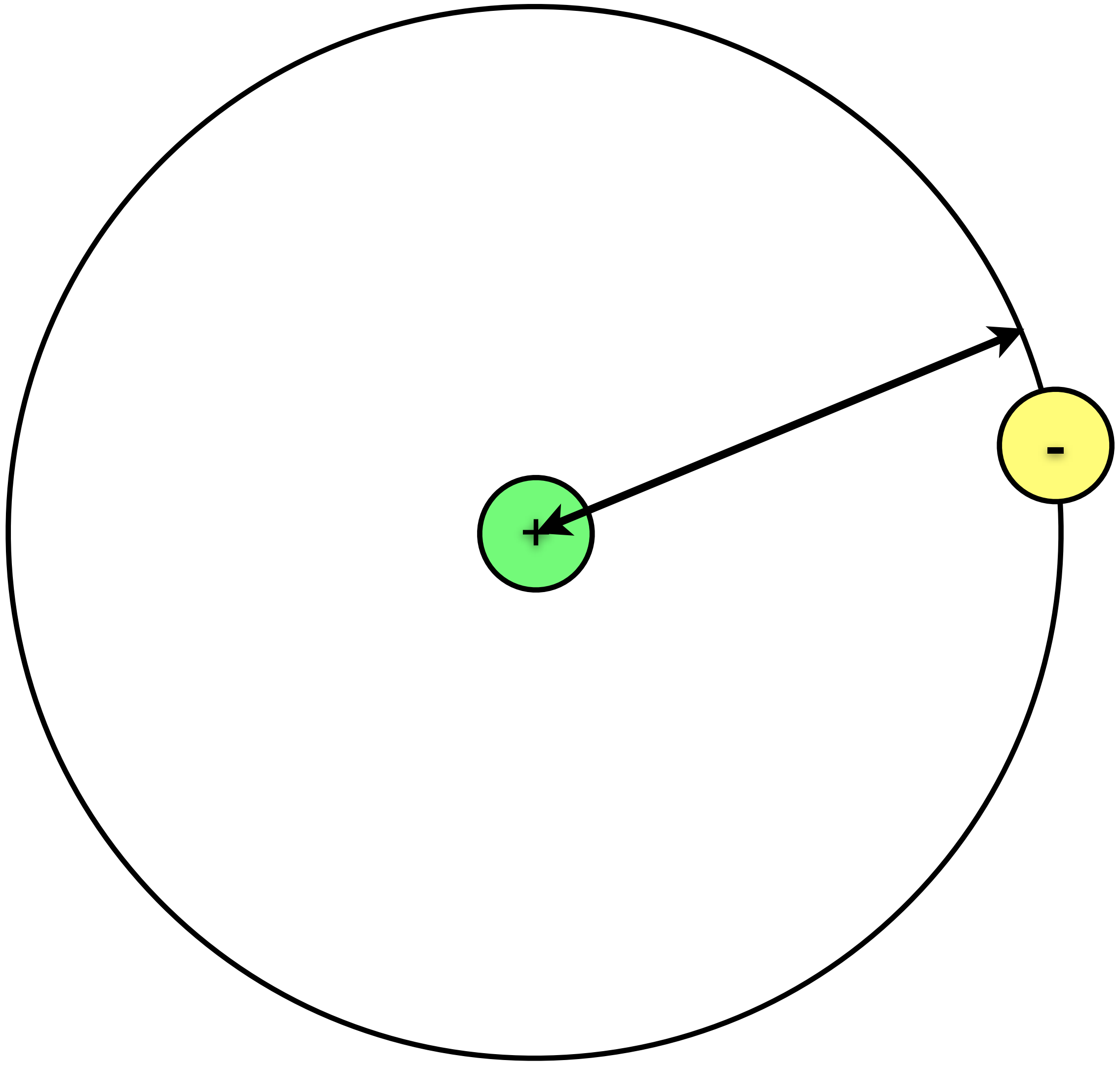
- 1. Electrons travel in **stable** circular orbits.
- 2. Orbits are **quantised.** The only allowed orbits satisfy an **angular momentum rule:**

$$l = mvr = \hbar n = \frac{h}{2\pi} n$$

where ***n*** = 1,2,3,…

- 3. Electrons in an orbit **do not** emit light due to their **acceleration**, and their orbit does not decay.
- 4. Electrons may change orbits by **absorbing** or **emitting** a photon of **equal energy** to the **energy difference** between the orbits.

The Bohr Model



$$l = mvr = \hbar n = \frac{h}{2\pi} n$$

where $\boldsymbol{n}$ = 1,2,3,…

- Using this **angular momentum quantisation rule** together with the classical mechanics of a charged particle in a circular orbit, we derived:
- Orbital radius:

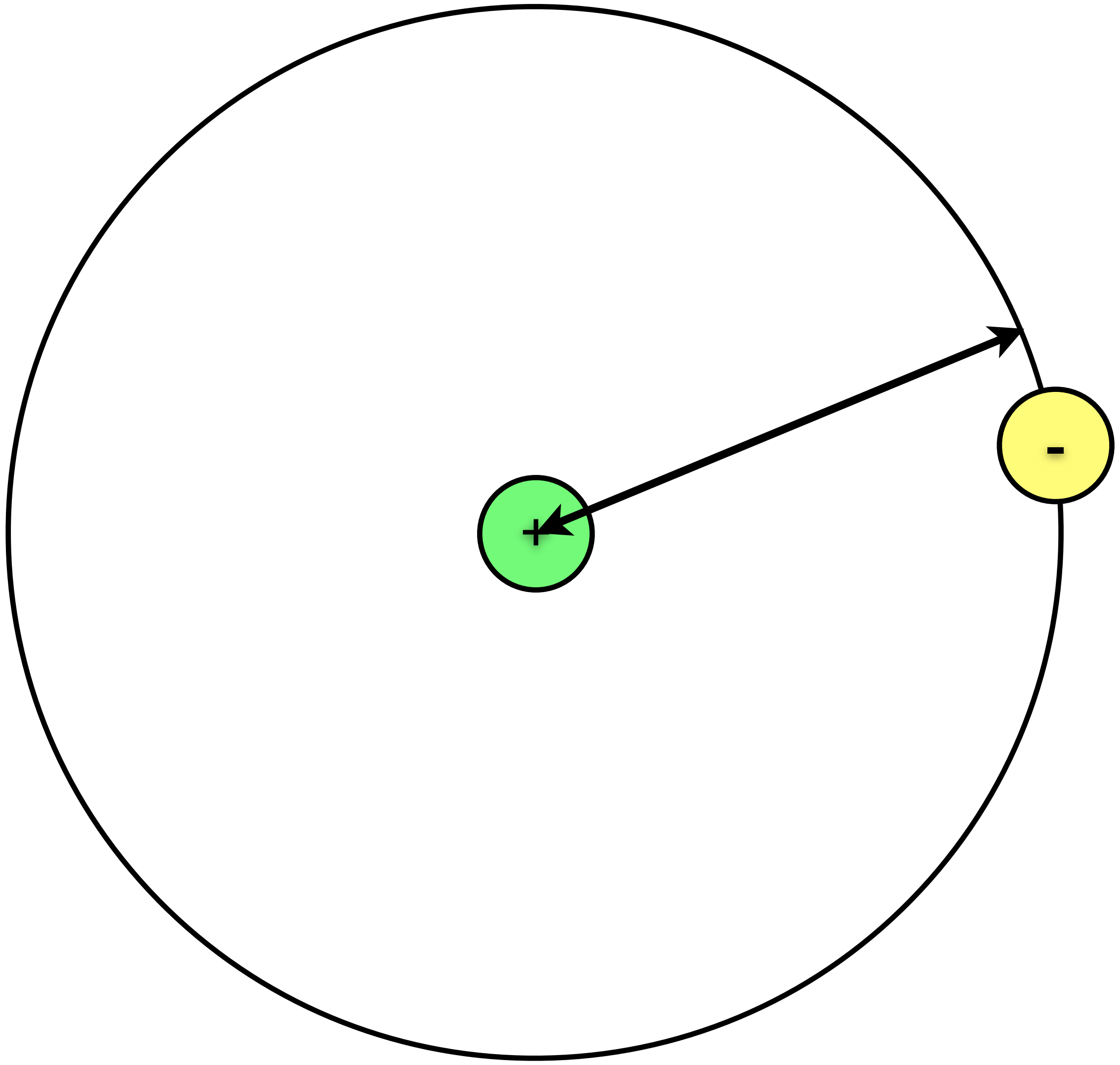
$$r_n = a_0 n^2$$

- where a_0 is the **Bohr radius**

$$a_0 = \frac{\hbar^2 (4\pi\epsilon_0)}{me^2}$$

- which depends only on constants of nature:
  - **hbar** (Planck's constant), **epsilon$_0$** (permittivity of free-space), **m** (electron mass) and **e** (electron charge).

The Bohr Model



$$l = mvr = \hbar n = \frac{h}{2\pi} n$$

where $\boldsymbol{n}$ = 1,2,3,…

- We also used it to derive an **energy** for each allowed orbit (i.e. each value of n).

$$E_n = -\left(\frac{1}{2}\frac{1}{4\pi\epsilon_0}\frac{e^2}{a_0}\right)\frac{1}{n^2}$$

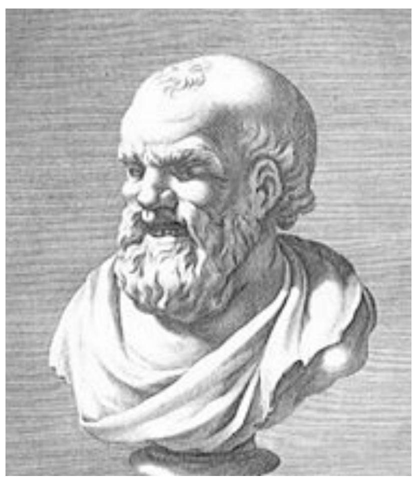
$$= -\frac{2.2 \times 10^{-18}}{n^2} \text{ Joules}$$

$$= -\frac{13.6}{n^2} \text{ eV}$$

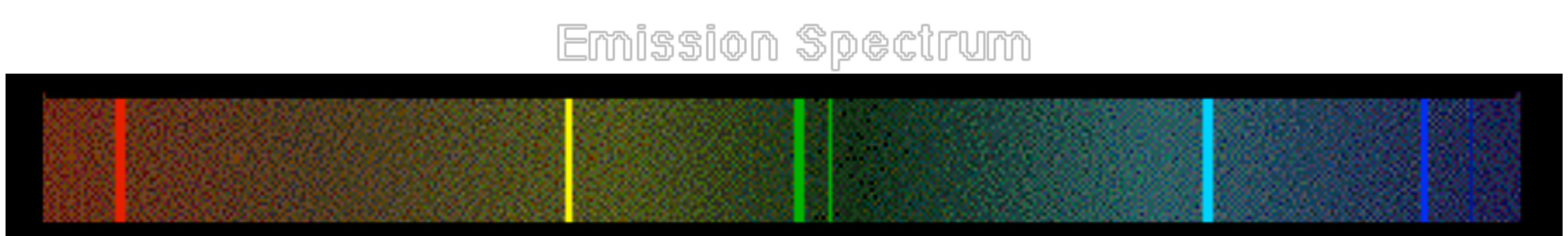
- which precisely coincides with the predictions of **Rydberg's formula** plus **Planck's** photon energy:

$$E_n = -\frac{hcR_H}{n^2} \approx -\frac{2.2 \times 10^{-18}}{n^2} \text{ Joules}$$

Summary of Part 2



- We saw how the development of the pre-quantum models of the atom, from **Democritus** to **Bohr**.
- **Atomic spectroscopy** provided the key test. No models prior to Bohr could derive **Rydberg's formula**.
- **Bohr model** could do so - but it had **many failings**.
- To do **better** than Bohr,
  - to develop a **modern theory** of the atom compatible with **all** spectroscopic predictions and other experiments
  - we need a new theory - **quantum mechanics**.



Part 3: Particles as Waves

de Broglie waves

Louis de Broglie

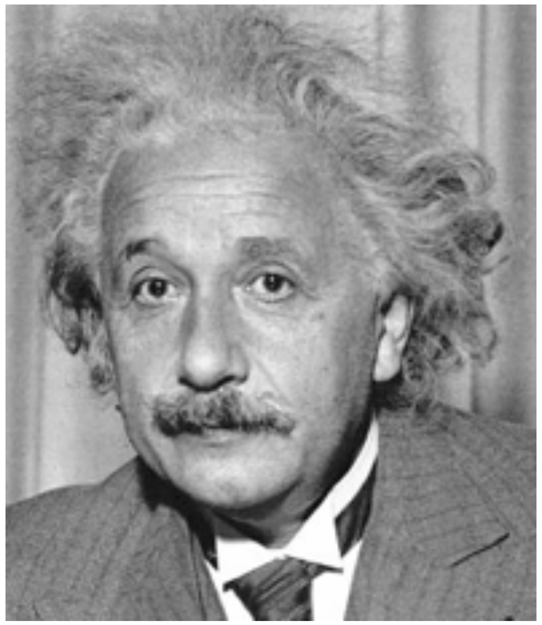
- **1920s: Bohr model** the **best attempt** at an atomic model, but its **flaws** were acknowledged.
- A **better model** was needed.
- By 1920s, **wave-particle duality** of **photons** was growing in acceptance (though before Compton Effect).
- de Broglie (1926: Nobel-prize-winning PhD thesis). Matter (atoms, electrons, etc.) should satisfy **wave-particle** duality **too**.

Images from Wikipedia

Straw poll

- Have you studied **de Broglie waves** before?
- 1. Yes
- 2. No

Photon Momentum



- In part 1, we saw that special relativity predicted that photons, as **massless particles,** should carry **momentum**

$$p = \frac{E}{c} \qquad \text{where} \qquad E = hf$$

- And thus:

$$p = h\frac{f}{c} = \frac{h}{\lambda}$$

de Broglie waves

- de Broglie's idea: (1924 - PhD Thesis)
  - Matter should exhibit "**wave-particle duality"** just like photons.
  - Particles (e.g. electrons, atoms etc.) with momentum **p** should have wave-like behaviour with a **de Broglie wavelength**

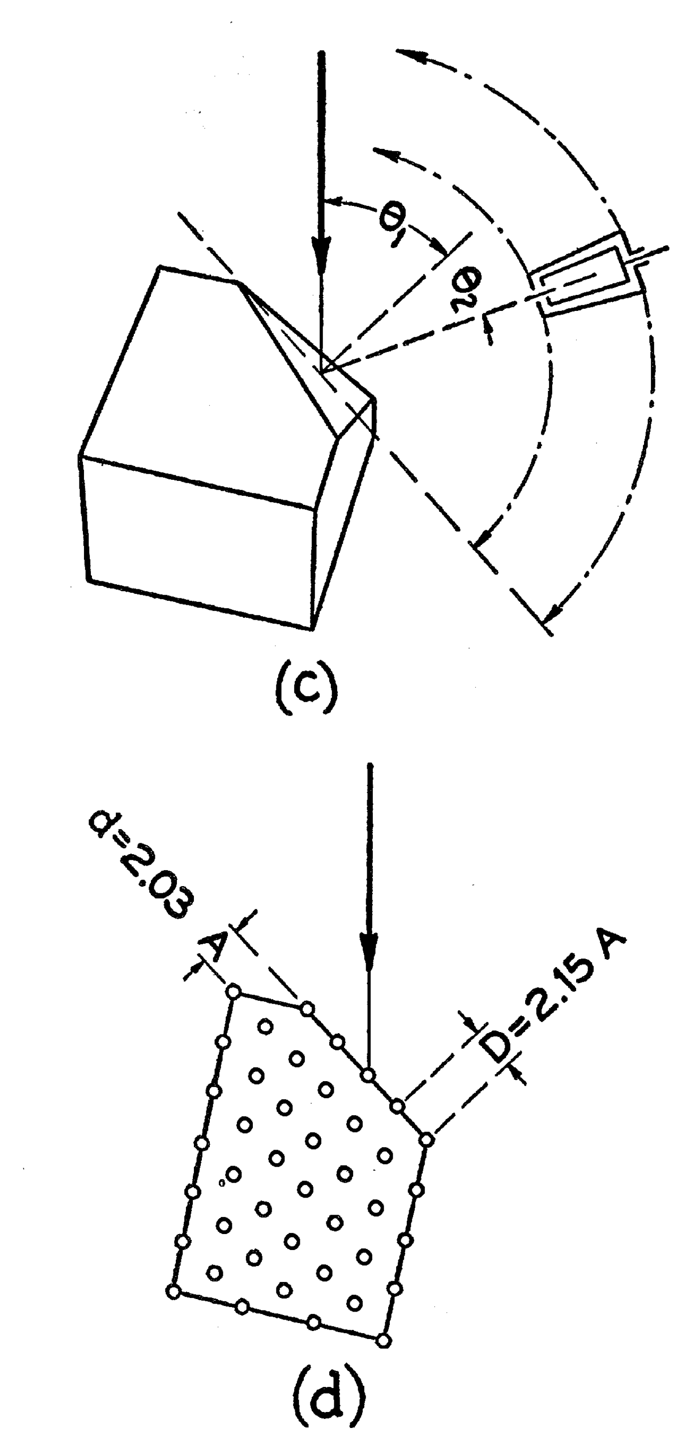
$$\lambda = \frac{h}{p}$$

  - NB This is precisely the **same relationship** as **photon** momentum:

$$p = \frac{h}{\lambda}$$

de Broglie waves

- **de Broglie wavelength** $\lambda = \dfrac{h}{p}$
- This startling suggestion, was confirmed experimentally surprisingly quickly.
- In 1927 **Davisson** and **Germer** scattered an electron-beam from Nickel.
- Their aim was not to investigate de Broglie's formula.
- They thought the electrons would scatter as particles, allowing them to image the surface of the crystal.



Figures from Davisson and Germer, PNAS, vol 14, page 317 (1928)

de Broglie waves

- This is what Davisson and Germer saw:

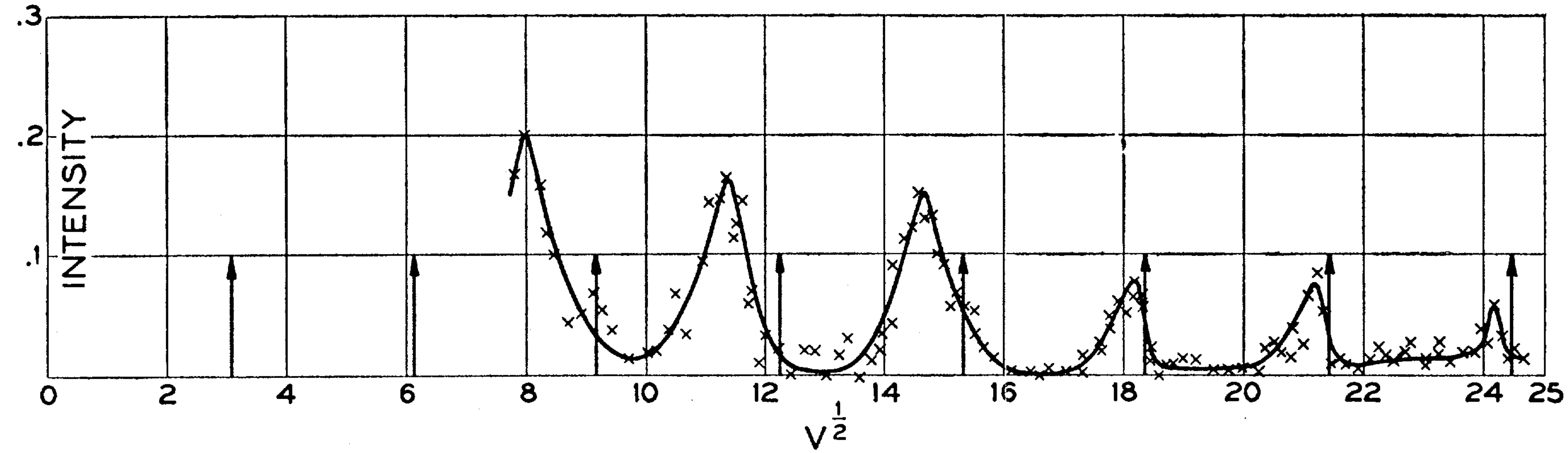


FIGURE 3

Variation of the intensity of the regularly reflected electron beam with bombarding potential, for 10° incidence—Intensity vs. $V^{1/2}$.

Figure from Davisson and Germer, PNAS, vol 14, page 317 (1928)

- It has the same structure as the **diffraction pattern** from a **grating**.

- The distance between peaks was consistent with de Broglie!

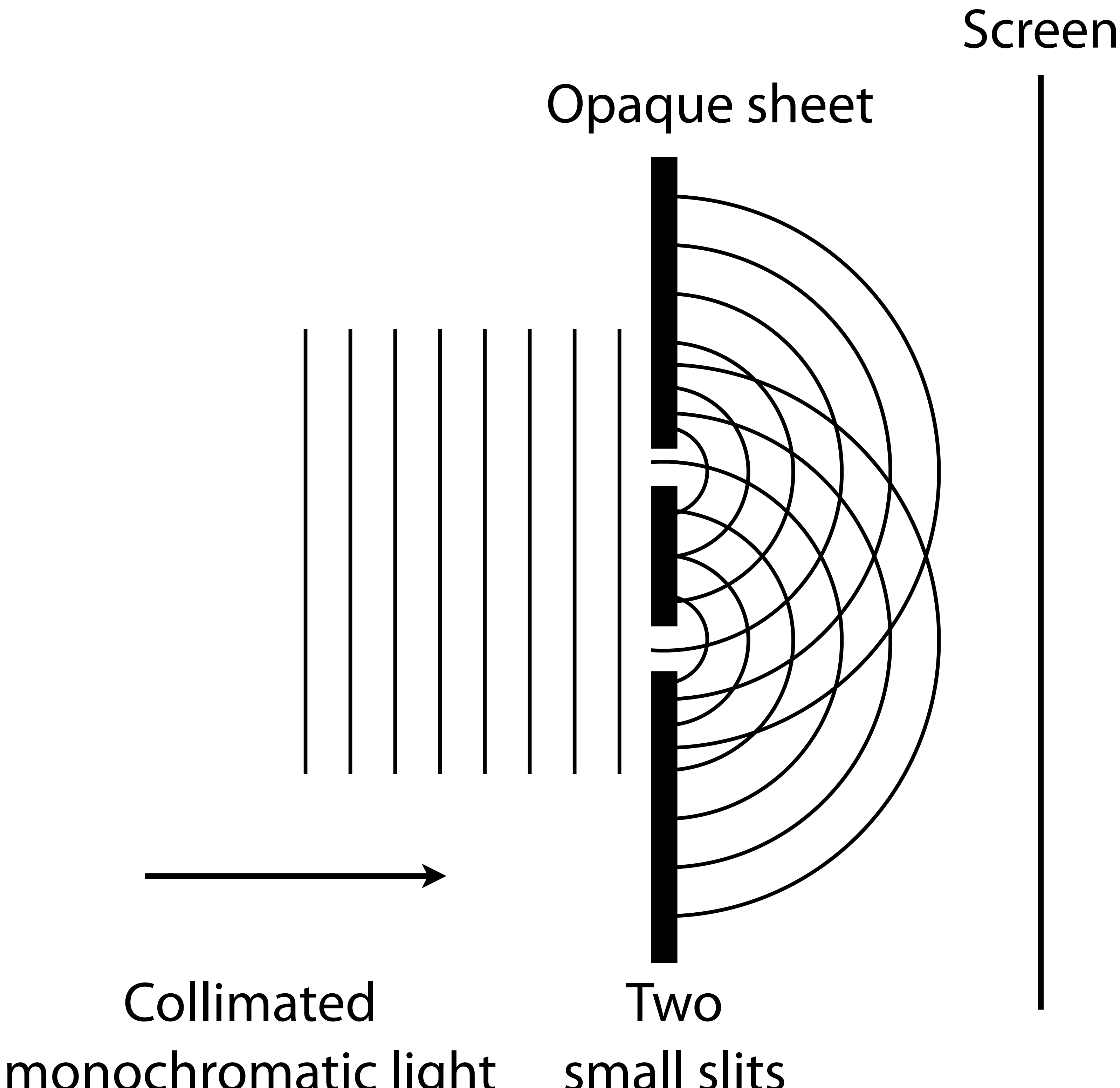
λ(OBS.)	λ(CAL.)	$\frac{\lambda(\text{OBS.})}{\lambda(\text{CAL.})} - 1$
0.956 Å	0.953 Å	+0.003
1.064	1.074	−0.01

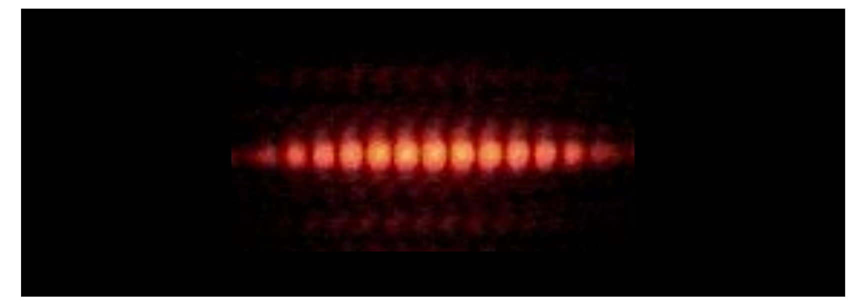
Davisson and Germer, PNAS, vol 14, page 317 (1928)

The Young Two-slit experiment

- The Young experiment is the classic demonstration of the wave nature of light.

T. Young

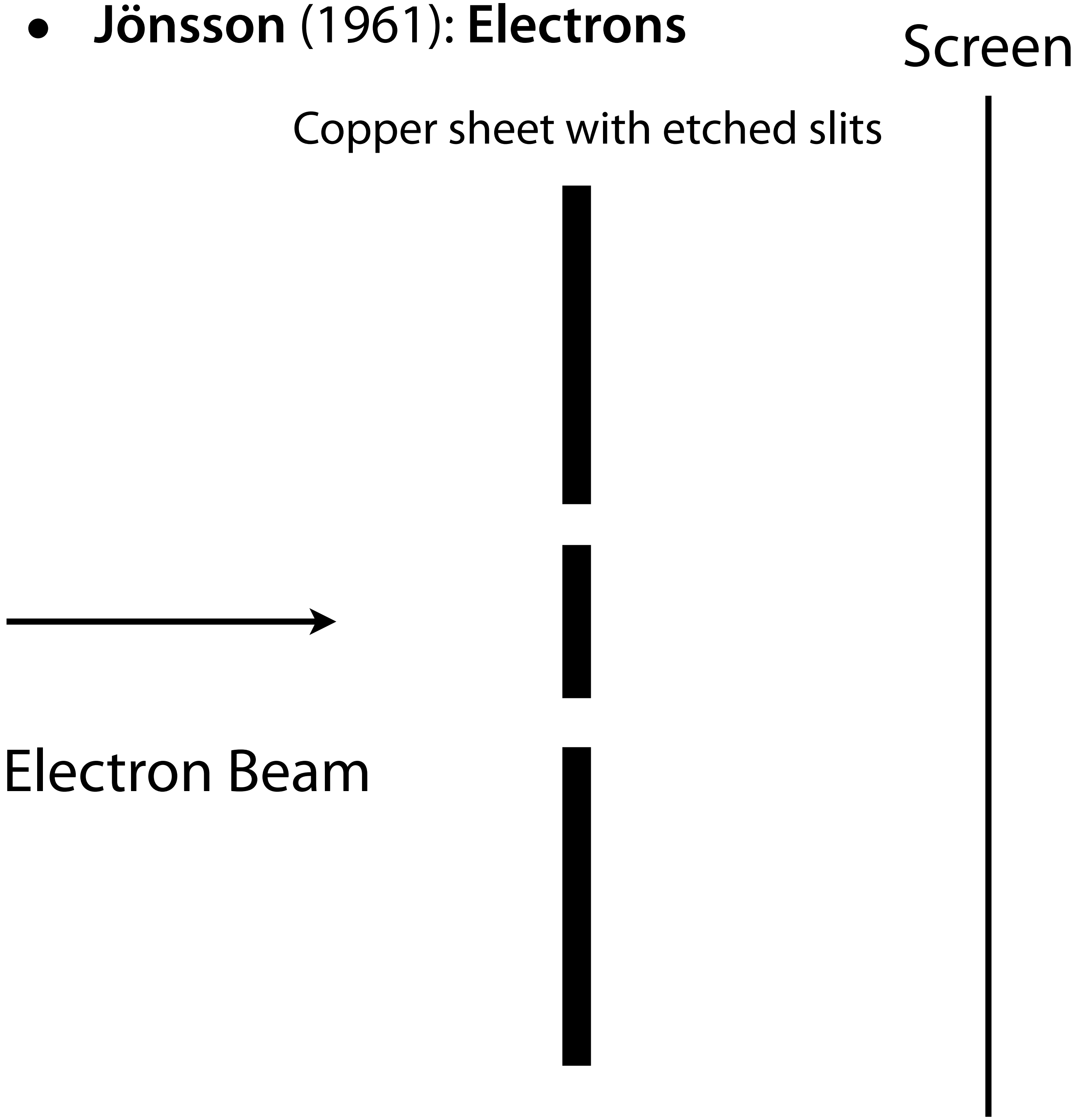




Young two slit diffraction pattern observed when **slit separation** is on the order of **wavelength**.

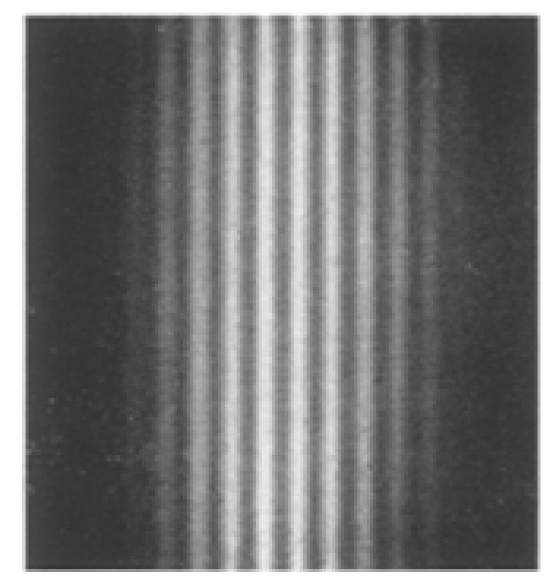
The Young Two-slit experiment

- Can it be performed with matter ?
- **Jönsson** (1961): **Electrons**



Claus Jönsson

Results



C. Jönsson, Zeitschift für Physik, vol. 161, page 454 (1961)

de Broglie wavelength

$$\lambda = \frac{h}{p}$$

$$h = 6.626 \times 10^{-34}\,\mathrm{Js}$$

- **Examples**
    - An **electron** in an old TV screen (cathode ray):
        - Mass: 9.1×10^{-31} kg
        - Velocity: 100 ms^{-1}
        - Momentum: 9.1×10^{-29} kg ms^{-1}
        - de Broglie wavelength: 7.3 microns (= 7.3×10^{-6}m)
    - A **tennis ball** during serve
        - Mass: 60g = 6×10^{-2} kg
        - Velocity: 233 kms^{-1} = **64.7** ms^{-1}
        - Momentum: 3.9 kg ms^{-1}
        - de Broglie wavelength: 1.7×10^{-34} m

de Broglie wavelength

$$\lambda = \frac{h}{p}$$

$$h = 6.626 \times 10^{-34}\ \mathrm{Js}$$

- **Examples**
  - Usain Bolt
    - Mass: 94 kg
    - Velocity: **12.4** ms^{-1}
    - Momentum: 1184.4 kg ms^{-1}
    - de Broglie wavelength: 5.6×10^{-37} m

Discussion

- Why do we **not** see wave-like behaviour in matter ineveryday life?

de Broglie wavelength

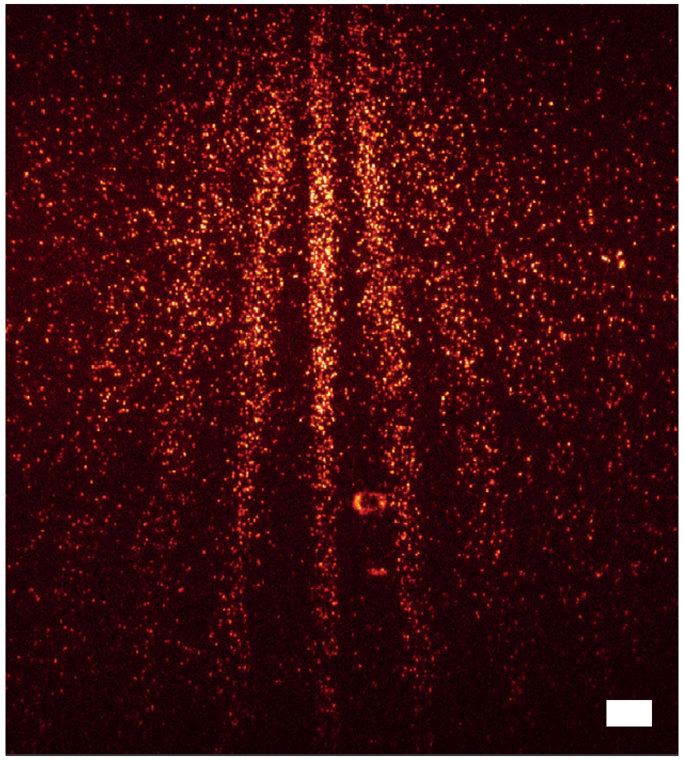
- To observe two-slit interference effects, slit separation must be approximately the **same size** as the wavelength.
- Human-scale objects have such tiny de Broglie wavelength that interference effects are never observed.
- Still, scientists are managing to achieve 2-slit interference with ever larger objects.

T. Juffman et al, Nature Nanotechnology, (2012)

- **Examples**
  - **Phthalocyanine**
    - Mass: **8.5×10^{-25} kg = approx 1 million electron masses**
    - Velocity: **150** ms^{-1}
    - Momentum: 1.2×10^{-22} kg ms^{-1}
    - de Broglie wavelength: 5.2 pm = 5.2×10^{-12} m

Single molecules
in a quantum movie

Wave-particle Duality



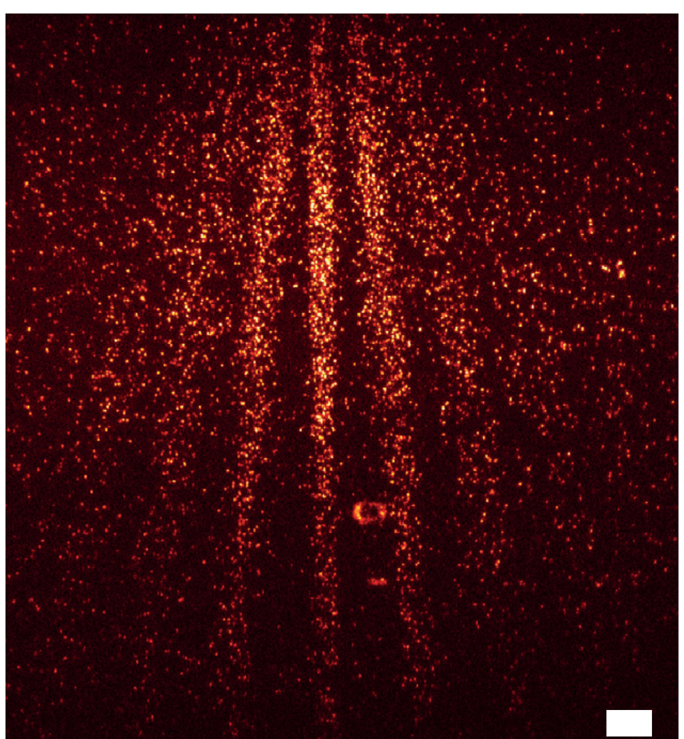
- In a Young's double slit experiment with matter:
    - Behaviour is analogous to photons.
        - ***Wave-like*** behaviour (while **unobserved**)
        - ***Particle-like*** behaviour (when detected).

Summary of Part 3

- Matter is observed to undergo wave-like interference with **de Broglie wavelength**:

$$\lambda = \frac{h}{p}$$

  - There is nothing in classical physics which can behave like this.



- **Fundamentally new physics** is needed:
  - **Quantum Mechanics!**
    - the dramatic **new theory** which, in Parts **4** and **5**, we start to introduce.