PHAS1228/2011

Problem Set 1

ANSWERS

1. Sketch the $p - T$ phase diagram of water, clearly labelling axes, areas and special points.

Answer: See the first figure in the document `Phase_diagram_of_water.pdf` on Moodle. Make sure you can draw this clearly.

2. Sketch the main components of a constant volume gas thermometer, and a graph to show how the measured temperature depends on the mass and type of gas used.

Answer: See Fig. 1.6 in `Finn_Thermal_Physics_Extract.pdf` on Moodle. Note that in the lower figure, the pressure of the gas at the triple point of water P_{TP} is proportional to the mass or amount of gas, so the graph requested looks the same. Don't memorise the slopes of the individual gases - just remember the idea that different gases have different slopes, but the same intercept.

3. Calculate the average volume occupied by a gas molecule in (nm)^3 at 25 ^{o}C and 1 atm (101325 Pa) pressure. Estimate how many argon atoms (radius 0.71Å) could fit in this volume if they were closely packed.

Answer: $V_{\text{molar}} = RT/P, V_{per\ molecule} = k_{\text{B}}T/P = 1.38 \times 10^{-23} \times 298/101325 = 4.06 \times 10^{-26}$ m$^3 \approx 41$ nm^3.
$V_{\text{argon}} = 1.333\pi 0.071^3 = 0.0015$ nm^3.$N_{\text{argon}} = 41/0.0015 \approx 27,000$.

4. Bulb A has volume 0.1 m^3 and bulb B has volume 0.3 m^3. They are connected by a narrow tube, with a tap, and maintained at 27 ^{o}C. Initially bulb A contains 1 mole of gas and bulb B is empty. The tap is then opened and the system allowed to come to equilibrium. Calculate the amount of gas and the pressure of gas in bulb B.

Answer: $PV = nRT$. Initially in bulb A, $P = nRT/V = 3000R$. When the tap is opened, the amount of gas and temperature remain constant, but the volume effectively changes to 0.4 m^3. Therefore $P' = nRT/V' = 750R$. Pressure is intensive and uniform at equilibrium, so $P_B = 750R = 6233$ Pa. Amount is extensive and uniformly distributed at equilibrium so 3/4 of it is in bulb B. $n_B = 0.75$ mol.

5. An ideal gas at equilibrium is enclosed in a container with two sections of equal volume (labeled 1 and 2), separated by a diathermal wall. Section 1 contains 0.1 mol gas at 1 atm. pressure. If Section 2 contains 0.3 mol of gas, what is its pressure?

Answer: The wall is diathermal so there is thermal contact and $T_1 = T_2$ at equilibrium. Volumes are equal so $V_1 = V_2$. Therefore $P \propto n$ and $P_2 = 3$ atm.

6. (a) Write an equation to define the ideal gas temperature T explaining any terms you introduce.

Answer:

$$T = 273.16 \lim_{P_{TP} \to 0} \left(\frac{P}{P_{TP}} \right)$$

.

T_{TP} is the pressure of the gas when it is in thermal contact with water at its triple point. P is the pressure of the gas when it is at temperature T .

(b) Write an equation to define the Celsius temperature t.

Answer: t = T - 273.15

(c) Explain, in not more that 20 words and no equations the key conceptual difference between 100 degrees Celsius and 100 degrees centigrade.

Answer: 100 degrees Celsius is measured as 373.15 K while 100 degrees centigrade is defined as 373.15 K.

Comment: This is the main point. Make sure you understand the difference and try to explain it in your own words.

7. The bulb of a constant volume gas thermometer is immersed in an ice-water-vapour mixture and the recorded pressure is 0.500 atm. It is then immersed in a boilng liquid and the pressure is 0.720 atm. Some gas is removed from the bulb and the experiment repeated, registering 0.250 and 0.350 atm respectively. Estimate, as accurately as possible, the boiling temperature of the liquid.

Answer:
$(P/P_{TP})_1 = 0.72/0.5 = 1.44, (P/P_{TP})_2 = 0.7/0.5 = 1.4$
By simple proportions, $(P/P_{TP})_{P_{TP} \to 0} = 0.68/0.5 = 1.36$.
$T = 273.16 \times 1.36 = 371.5$ K.

Comment: see the answer to 6a above and the graph in Q2 above.

8. 100 g of a liquid at 90 ^{o}C are mixed with 30 g of the same liquid at 10 ^{o}C. Calculate the final temperature.

Answer: The final temperature is a weighted mean. Note that we can work work in degrees Celsius (t) rather than kelvin (T).
$m_1c(t_1 - t_f) = m_2c(t_f - t_2) \rightarrow t_f(m_2 + m_1) = t_1m_1 + t_2m_2 \rightarrow t_f = (t_1m_1 + t_2m_2)/(m_1 + m_2)$.
$\rightarrow t_f = (9000 + 300)/(130) = 71.5^o$ C.

9. The coefficient of linear thermal expansion of steel is 11 $\times 10^{-6}$ ^{o}C^{-1}. A sheet of steel has dimensions 1 m $\times$ 1 m $\times$ 0.01m. Calculate the change in area and change in volume if the sheet is heated through 100 ^{o}C.

Answer: $\Delta L = L\alpha\Delta T = 1 \times 11 \times 10^{-6} \times 100 = 11 \times 10^{-4}$ m.
$A + \Delta A = (L + \Delta L)^2 \rightarrow \Delta A \approx 2L\Delta L \rightarrow \Delta A \approx 22 \times 10^{-4}$ m^2.
$\Delta W = W\alpha\Delta T = 2 = 0.01 \times 11 \times 10^{-6} \times 100 = 11 \times 10^{-6}$ m.
$V = AW \rightarrow dV = WdA + AdW = 0.01 \times 22 \times 10^{-4} + 1 \times 11 \times 10^{-6} = 33 \times 10^{-6}$ m^3.

10. An iron rod of cross sectional area 4 cm^2 has one end maintained at 0 ^{o}C and the other at 90 ^{o}C. If the rate of heat flow down the rod is 7.2 J s^{-2}, calculate the length of the rod. ($k_{\rm iron} = 80$ W m^{-1}K^{-1}).

Answer: $\Delta x = kA\Delta T/P \rightarrow \Delta x = 80 \times 4 \times 10^{-4} \times 90/7.2 = 0.40$ m $= 40$ cm.