PHAS0012 (1449) Computing for Mathematical Physics 2018/19

Homework 1

Mark for homework 1: /42

(to be competed by your marker)

Feedback from marker:

(to be competed by your marker)

Which **feedback from your last homework** are you **employing in this homework**?

Marks will be deducted if you do not complete this section. - Note that this section should not be used for your first homework as you will not have received any feedback. You must complete this section for all subsequent homework.

Give your answers in the code cells labelled "(*your solution here*)"

1. Integrate $\sin((\pi t^2/3) + a\, t)$ with respect to t from 0 to ∞.

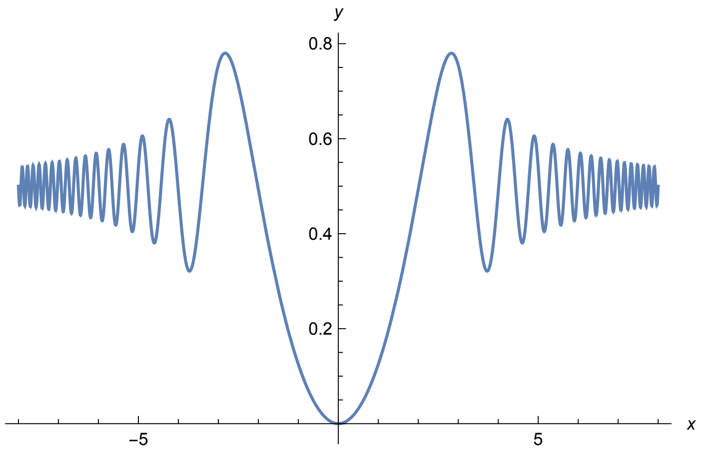
In[34]:= `q1 = Integrate[Sin[(π t^2/3) + a t], {t, 0, Infinity}]`

Out[34]= $$\text{ConditionalExpression}\left[\frac{1}{2}\sqrt{\frac{3}{2}}\left(\text{Cos}\left[\frac{3\,\text{False}^2}{4\pi}\right]\left(1-2\,\text{FresnelS}\left[\frac{\sqrt{\frac{3}{2}}\ \text{False}}{\pi}\right]\right)+\left(-1+2\,\text{FresnelC}\left[\frac{\sqrt{\frac{3}{2}}\ \text{False}}{\pi}\right]\right)\text{Sin}\left[\frac{3\,\text{False}^2}{4\pi}\right]\right),\ 3x+19z==1+3y\in\text{Reals}\right]$$

2. The answer to Question 1 for real *a* contains two functions you may not have met before. They are called the Fresnel integrals S and C. Plot `FresnelC[(a^2)/8]` for -8 ≤ a ≤ 8. Label the axes with the labels “x” and “y” - you will need to use Wolfram Documentation to learn how to label the axes - search for “AxesLabel”.

In[35]:= `Plot[FresnelC[(a^2)/8], {a, -8, 8}, AxesLabel → {x, y}]`

Out[35]=



3. Differentiate `FresnelC[(x^2)/8]` once with respect to `x`, and plot the result and FresnelC`[(x^2)/8]` over the range -8≤x≤8 on the same graph. As in the question above label the axes with “x” and “y”. Since you now have two lines on the same graph you will also need to add a plot legend to distinguish the lines. Look at the information for “Plot” and “PlotLegends” in the Wolfram Documentation and add a legend to your graph which labels each line according to its expression.

In[36]:= `q3eq1 = D[FresnelC[(x^2)/8], x]`

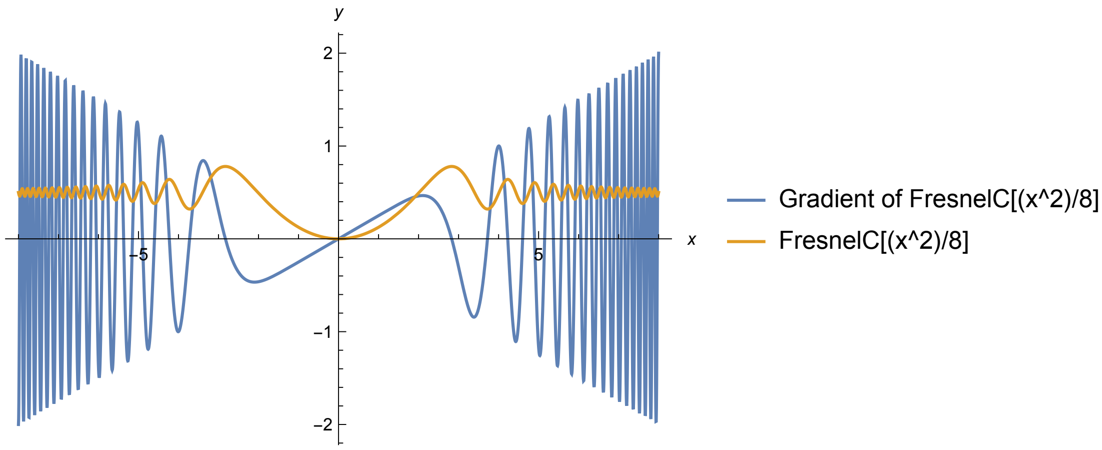
Out[36]= $\frac{1}{4}\,x\,\text{Cos}\left[\frac{\pi x^4}{128}\right]$

In[37]:= `q3eq2 = FresnelC[(x^2)/8]`

Out[37]= $\text{FresnelC}\left[\frac{x^2}{8}\right]$

In[38]:=
```
Plot[{q3eq1, q3eq2}, {x, -8, 8}, AxesLabel → {x, y},
 PlotLegends → {"Gradient of FresnelC[(x^2)/8]", "FresnelC[(x^2)/8]"}]
```

Out[38]=



4. Solve the equation $(x+4)^4+(x-2)^4 = 60$.

In[39]:=
```
Solve[ (x + 4)^4 + (x - 2)^4 == 60, x]
```

Out[39]= $\left\{\left\{x \to -1 - i\sqrt{27-\sqrt{678}}\right\}, \left\{x \to -1 + i\sqrt{27-\sqrt{678}}\right\}, \left\{x \to -1 - i\sqrt{27+\sqrt{678}}\right\}, \left\{x \to -1 + i\sqrt{27+\sqrt{678}}\right\}\right\}$

5. Solve the simultaneous equations

$3x - 3y + 19z = 1$

$y - 5x + 9z = -102$

$x + y - 3z^2 = 22$

In[40]:=
```
q5eq1 = -3 y + 19 z + 3 x == 1
```

Out[40]= $3x - 3y + 19z == 1$

In[41]:=
```
q5eq2 = y - 5 x + 9 z == -102
```

Out[41]= $-5x + y + 9z == -102$

In[42]:=
```
q5eq3 = x + y - 3 z^2 == 22
Solve[{q5eq1, q5eq2, q5eq3}, {x, y, z}]
```

Out[42]= $x + y - 3z^2 == 22$

Out[43]= $\left\{\left\{x \to \frac{1}{36}\left(1237 - 23\sqrt{538}\right), y \to \frac{1}{36}\left(1757 - 61\sqrt{538}\right), z \to \frac{1}{6}\left(14 - \sqrt{538}\right)\right\}, \left\{x \to \frac{1}{36}\left(1237 + 23\sqrt{538}\right), y \to \frac{1}{36}\left(1757 + 61\sqrt{538}\right), z \to \frac{1}{6}\left(14 + \sqrt{538}\right)\right\}\right\}$

6. Find the limit of $\sinh\left(\frac{1}{x}\right)\log(x)$ as x tends to zero from above and from below.

In[44]:=
```
Limit[Sinh[1 / x] Log[x], x → 0, Direction → "FromAbove"]
Limit[Sinh[1 / x] Log[x], x → 0, Direction → "FromBelow"]
```

... Limit: Value of Direction –> FromAbove should be a number or Automatic.

Out[44]= Limit$\left[\text{Log}[x] \,\text{Sinh}\left[\frac{1}{x}\right], x \to 0, \text{Direction} \to \text{FromAbove}\right]$

... Limit: Value of Direction –> FromBelow should be a number or Automatic.

Out[45]= Limit$\left[\text{Log}[x] \,\text{Sinh}\left[\frac{1}{x}\right], x \to 0, \text{Direction} \to \text{FromBelow}\right]$

7. Expand tan(x) sin($\sqrt{x}$) in powers of x up to x^5 around the value x = 0.

In[46]:=
q7eq1 = Sin[$\sqrt{x}$] Tan[x]

Out[46]= Sin[$\sqrt{x}$] Tan[x]

In[47]:= Sin[$\sqrt{x}$] Tan[x]

Out[47]= Sin[$\sqrt{x}$] Tan[x]

In[48]:= Series[q7eq1, {x, 0, 5}]

Out[48]= $x^{3/2} - \frac{x^{5/2}}{6} + \frac{41\,x^{7/2}}{120} - \frac{281\,x^{9/2}}{5040} + O[x]^{11/2}$

8. Integrate $1/(15\,x^2 - 7\,x - 2)$ with respect to x analytically. Then express in terms of partial fractions, in which the denominators are linear in x. Integrate the partial fractions with respect to x. Verify that the two integrals are the same by writing an expression which *Mathematica* will evaluate to True.

In[49]:= q8int1 = Integrate[$1/(15\,x^2 - 7\,x - 2)$, x]

Out[49]= $\frac{\text{Log}\left[7 + \sqrt{57} - 2\,x\right] - \text{Log}\left[-7 + \sqrt{57} + 2\,x\right]}{15\,\sqrt{57}}$

In[50]:= q8eq1 = Apart[$1/(15\,x^2 - 7\,x - 2)$]

Out[50]= $\frac{3}{13\,(-2 + 3\,x)} - \frac{5}{13\,(1 + 5\,x)}$

In[51]:= q8int2 = Integrate[q8eq1, x]

Out[51]= $\frac{1}{13}\,\text{Log}[2 - 3\,x] - \frac{1}{13}\,\text{Log}[1 + 5\,x]$

In[52]:= q8int1 == q8int2

Out[52]= $\frac{\text{Log}\left[7 + \sqrt{57} - 2\,x\right] - \text{Log}\left[-7 + \sqrt{57} + 2\,x\right]}{15\,\sqrt{57}} == \frac{1}{13}\,\text{Log}[2 - 3\,x] - \frac{1}{13}\,\text{Log}[1 + 5\,x]$

9. Solve the differential equation $x^2 \frac{d^2}{d\,x^2}\,y + (x^2 - 3)\,y = 0$.
You will find that the solution contains two terms, involving special functions of mathematical physics which you may not have met before, and two coefficients C[1] and C[2].

In[53]:= q9eq1 = (x^2 y''[x]) + ((x^2 - 3) y[x]) == 0

Out[53]= $\left(-3+x^2\right) y[x]+x^2 y''[x]==0$

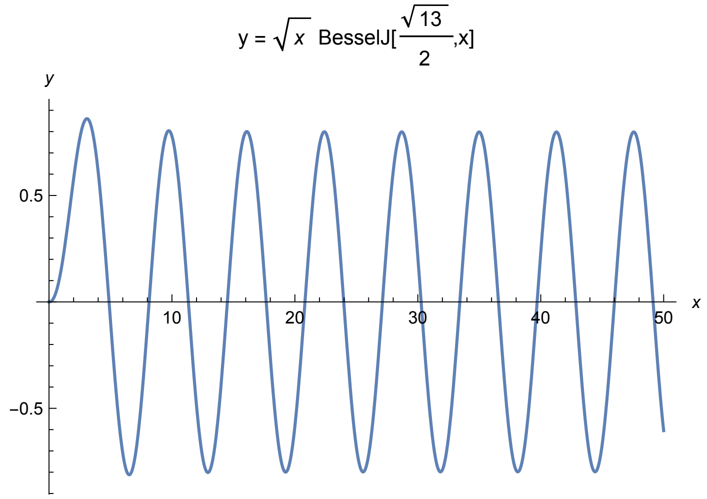
In[54]:= q9eq2 = DSolve[q9eq1, y[x], x]

Out[54]= $\left\{\left\{y[x]\to\sqrt{x}\,\text{BesselJ}\left[\frac{\sqrt{13}}{2},x\right]C[1]+\sqrt{x}\,\text{BesselY}\left[\frac{\sqrt{13}}{2},x\right]C[2]\right\}\right\}$

Plot the functions which appear multiplied by C[1] and C[2] (without the coefficients C[1] and C[2]) over the range 0 ≤x≤50.
Consider using the option 'PlotRange -> Full' and explain why it is or is not useful in this situation.
Confirm what the graphs seem to show by finding the limits of the special functions as $x \to 0$. You may need to use the option, 'Direction.'
Be sure to label the axes of your plots, and to label your plots (hint: the PlotLabel argument can be used to title graphs).

In[55]:= Plot$\left[\sqrt{a}\,\text{BesselJ}\left[\frac{\sqrt{13}}{2},a\right]\right.$, {a, 0, 50}, AxesLabel → {x, y},
PlotLabel → Style$\left[\text{"y = }\sqrt{x}\text{ BesselJ[}\frac{\sqrt{13}}{2}\text{,x]"}\right]\Big]$

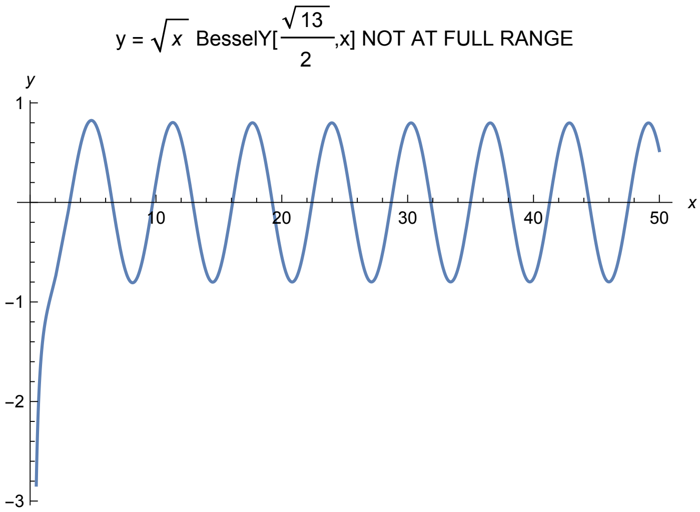
Out[55]=



In[56]:=

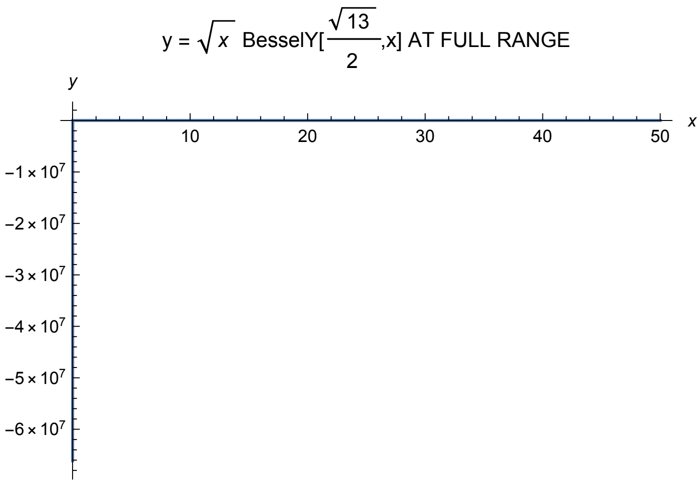
In[57]:= Plot[$\sqrt{x}$ BesselY[$\frac{\sqrt{13}}{2}$, x] , {x, 0, 50}, AxesLabel → {x, y},
PlotLabel → Style["y = $\sqrt{x}$ BesselY[$\frac{\sqrt{13}}{2}$,x] NOT AT FULL RANGE"]]

Out[57]=



In[58]:= Plot[$\sqrt{x}$ BesselY[$\frac{\sqrt{13}}{2}$, x] , {x, 0, 50}, AxesLabel → {x, y}, PlotRange → Full,
PlotLabel → Style["y = $\sqrt{x}$ BesselY[$\frac{\sqrt{13}}{2}$,x] AT FULL RANGE"]]

Out[58]=



In[59]:= The Graph above, "y = $\sqrt{x}$ BesselY[$\frac{\sqrt{13}}{2}$,x] AT FULL RANGE" shows that as x approacl

Syntax::sntxf: "The Graph above" cannot be followed by
", "y = \!\(*SqrtBox[\(x\)]\) BesselY[\!\(*FractionBox[SqrtBox[\(13\)], \(2\)]\".

Syntax::sntxi: Incomplete expression; more input is needed .

In[59]:= Limit[$\sqrt{x}$ BesselY[$\frac{\sqrt{13}}{2}$, x], x → 0, Direction → "FromAbove"]

Limit: Value of Direction -> FromAbove should be a number or Automatic.

Out[59]= Limit[$\sqrt{x}$ BesselY[$\frac{\sqrt{13}}{2}$, x], x → 0, Direction → FromAbove]

In[60]:= Limit[$\sqrt{x}$ BesselJ[$\frac{\sqrt{13}}{2}$, x] , x → 0, Direction → "FromAbove"]

... Limit: Value of Direction -> FromAbove should be a number or Automatic.

Out[60]= Limit[$\sqrt{x}$ BesselJ[$\frac{\sqrt{13}}{2}$, x], x → 0, Direction → FromAbove]

Deduce what the value of C[2] must be if the solution is to equal 0 at *x*=0 and is equal to 1 at *x*=1. Find an expression for C[1] satisfying the same conditions.

In[61]:= DSolve[{q9eq1, y[0] == 0, y[1] == 1}, y[x], x]

... DSolve: Unable to resolve some of the arbitrary constants in the general solution using the given boundary conditions. It is possible that some of the conditions have been specified at a singular point for the equation.

Out[61]= {{y[x] → $\frac{1}{\text{BesselY}\left[\frac{\sqrt{13}}{2}, 1\right]}$
$\left(\sqrt{x}\ \text{BesselY}\left[\frac{\sqrt{13}}{2}, x\right] + \sqrt{x}\ \text{BesselJ}\left[\frac{\sqrt{13}}{2}, x\right] \text{BesselY}\left[\frac{\sqrt{13}}{2}, 1\right] C[1] - \sqrt{x}\ \text{BesselJ}\left[\frac{\sqrt{13}}{2}, 1\right] \text{BesselY}\left[\frac{\sqrt{13}}{2}, x\right] C[1]\right)$}}

In[23]:= q9eq3 = y[x] == $\frac{1}{\text{BesselY}\left[\frac{\sqrt{13}}{2}, 1\right]}$
$\left(\sqrt{x}\ \text{BesselY}\left[\frac{\sqrt{13}}{2}, x\right] + \sqrt{x}\ \text{BesselJ}\left[\frac{\sqrt{13}}{2}, x\right] \text{BesselY}\left[\frac{\sqrt{13}}{2}, 1\right] C[1] - \sqrt{x}\ \text{BesselJ}\left[\frac{\sqrt{13}}{2}, 1\right] \text{BesselY}\left[\frac{\sqrt{13}}{2}, x\right] C[1]\right)$

Out[23]= y[x] == $\frac{1}{\text{BesselY}\left[\frac{\sqrt{13}}{2}, 1\right]}$
$\left(\sqrt{x}\ \text{BesselY}\left[\frac{\sqrt{13}}{2}, x\right] + \sqrt{x}\ \text{BesselJ}\left[\frac{\sqrt{13}}{2}, x\right] \text{BesselY}\left[\frac{\sqrt{13}}{2}, 1\right] C[1] - \sqrt{x}\ \text{BesselJ}\left[\frac{\sqrt{13}}{2}, 1\right] \text{BesselY}\left[\frac{\sqrt{13}}{2}, x\right] C[1]\right)$

In[62]:= q9eq4 = y[x] == $\sqrt{x}$ BesselJ[$\frac{\sqrt{13}}{2}$, x] C[1] + $\sqrt{x}$ BesselY[$\frac{\sqrt{13}}{2}$, x] C[2]

Out[62]= y[x] == $\sqrt{x}$ BesselJ[$\frac{\sqrt{13}}{2}$, x] C[1] + $\sqrt{x}$ BesselY[$\frac{\sqrt{13}}{2}$, x] C[2]

In[25]:=

```
Solve[{q9eq3, q9eq4}, {C[1], C[2]}]
```

Out[25]= $\left\{\left\{C[1] \to -\frac{\sqrt{x}\ \text{BesselY}\left[\frac{\sqrt{13}}{2}, x\right] - \text{BesselY}\left[\frac{\sqrt{13}}{2}, 1\right] y[x]}{\sqrt{x}\left(\text{BesselJ}\left[\frac{\sqrt{13}}{2}, x\right] \text{BesselY}\left[\frac{\sqrt{13}}{2}, 1\right] - \text{BesselJ}\left[\frac{\sqrt{13}}{2}, 1\right] \text{BesselY}\left[\frac{\sqrt{13}}{2}, x\right]\right)},\right.\right.$

$\left.\left.C[2] \to \frac{\sqrt{x}\ \text{BesselJ}\left[\frac{\sqrt{13}}{2}, x\right] - \text{BesselJ}\left[\frac{\sqrt{13}}{2}, 1\right] y[x]}{\sqrt{x}\left(\text{BesselJ}\left[\frac{\sqrt{13}}{2}, x\right] \text{BesselY}\left[\frac{\sqrt{13}}{2}, 1\right] - \text{BesselJ}\left[\frac{\sqrt{13}}{2}, 1\right] \text{BesselY}\left[\frac{\sqrt{13}}{2}, x\right]\right)}\right\}\right\}$

Total marks available: 42

Solutions are due by **4pm on Monday January 14th**.
Make a copy of your solutions with the output deleted (`Cell|Delete All Output`) and upload that file to Moodle.
Please name the file to include your family name and first name, for example I would use `hw1_-Jasvir_Bhamrah`.

The first thing I shall do when I get the file is to click `Evaluation|Evaluate Notebook`, so make sure the file you send me will survive that.

J Bhamrah, J Underwood, L McKemmish
UCL
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