PHAS 1102
Physics of the Universe

1 - Radiation

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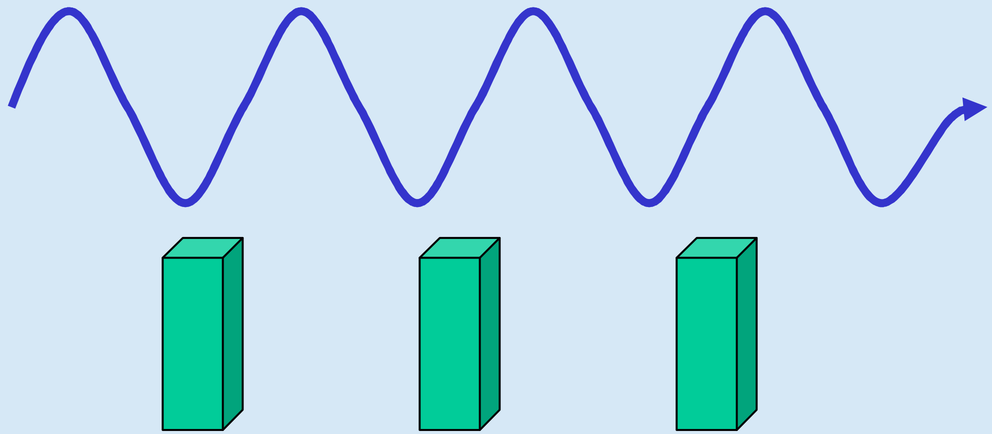
Radiation *(ZG Ch. 8; FK Ch. 5)*

When an electric charge is accelerated, electromagnetic energy is produced. This energy can be thought of as propagating as a wave – or, equally, as a particle:

wave-particle duality

The waves are usually referred to as **light waves** or **radiation**.

The particles are known as **photons** or **quanta of light**.

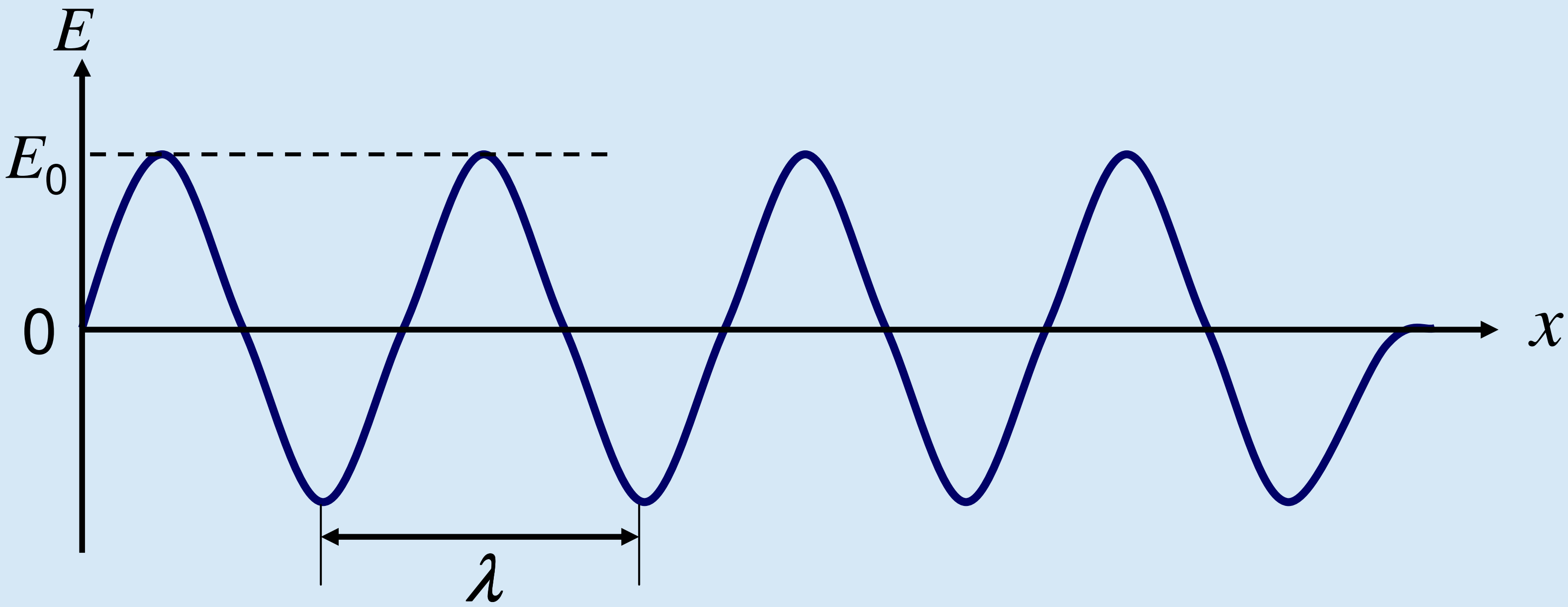


Electromagnetic waves (1)

Electromagnetic waves are transverse sine waves. For an EM wave travelling in the x direction, the electric field $\underline{E}$ at time t is given by:

$$\boxed{\underline{E} = \underline{E}_0 \sin\left[\frac{2\pi}{\lambda}(x - ct)\right]}$$

λ = wavelength
c = speed of light
t = time



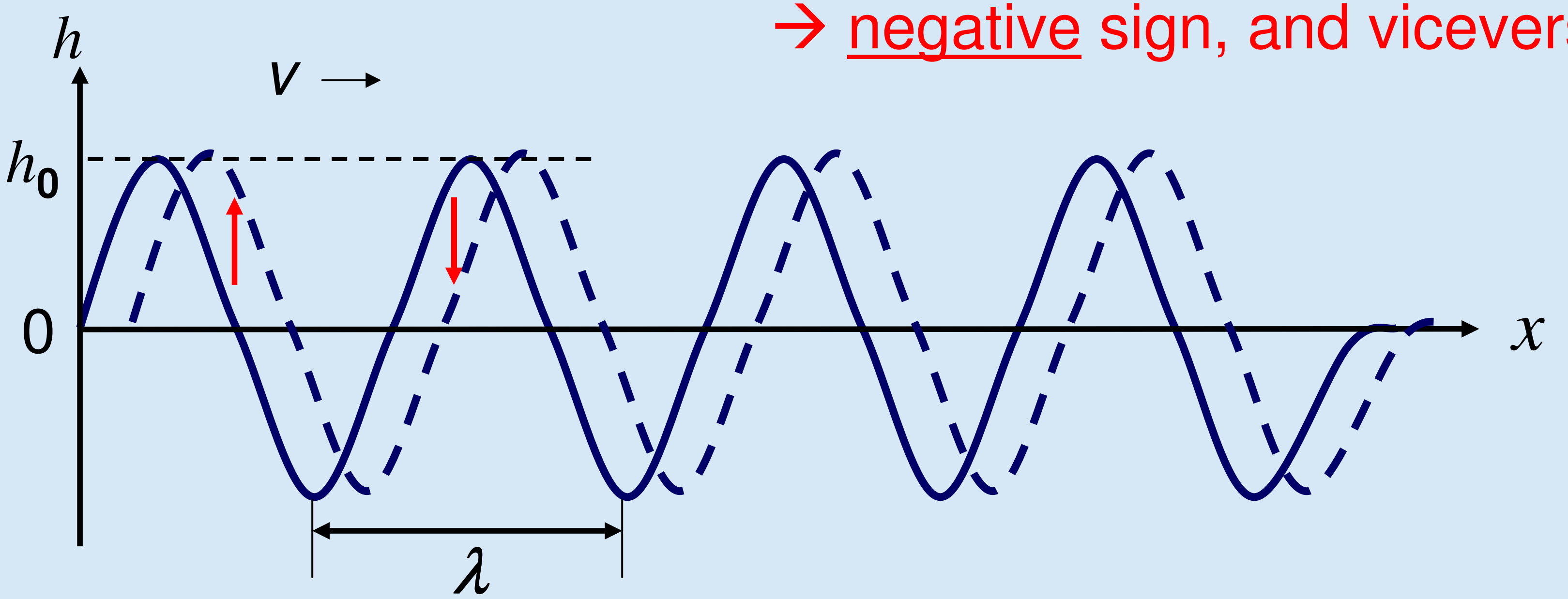
Electromagnetic waves (2) *(ZG Ch. 8)*

General equation, for e.m., acoustic, seismic, etc. waves:

$$\boxed{h = h_0 \sin\left[\frac{2\pi}{\lambda}(x - vt)\right]}$$

λ = wavelength
v = propagation speed
t = time

If motion along positive x axis (progressive wave)
→ negative sign, and viceversa



Two things happening: local oscillations and wave propagation

Electromagnetic waves (3)

General equation, for e.m., acoustic, seismic, etc. waves:

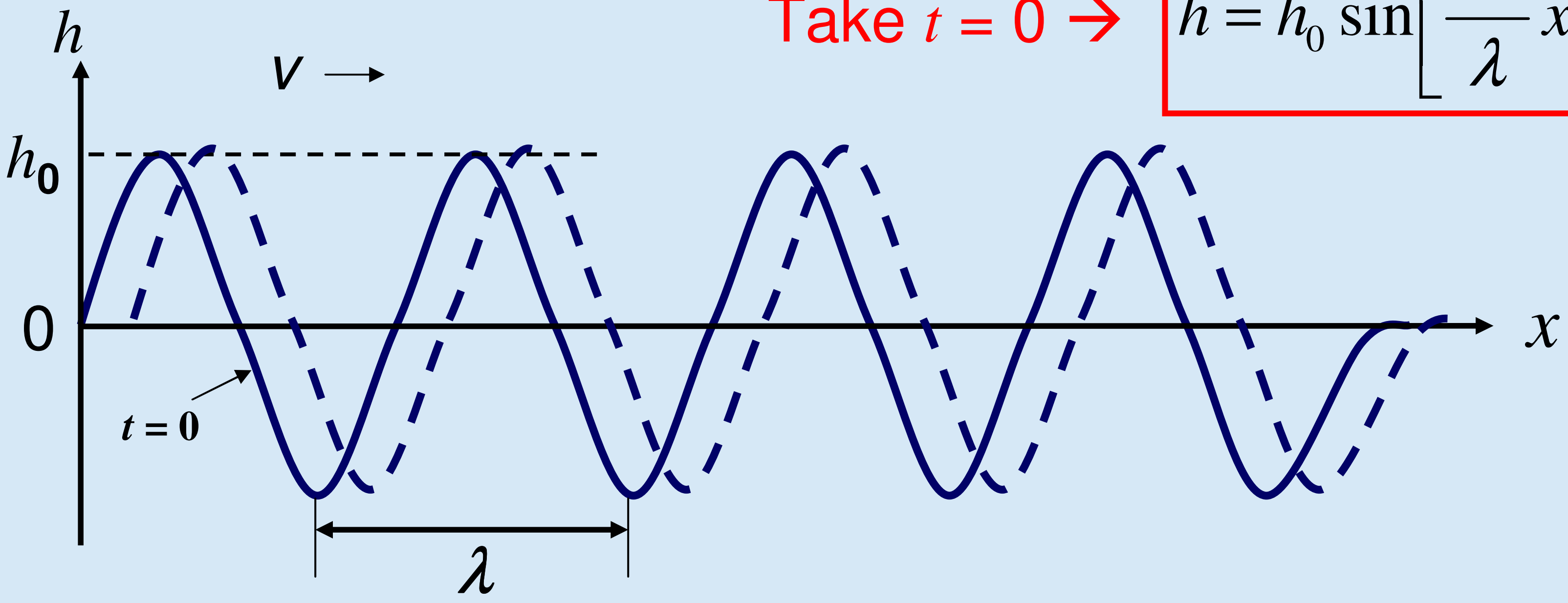
$$h = h_0 \sin\left[\frac{2\pi}{\lambda}(x - vt)\right]$$

λ = wavelength
v = propagation speed
t = time

Freeze t → snapshot of wave at an instant

Take t = 0 →

$$h = h_0 \sin\left[\frac{2\pi}{\lambda}x\right]$$



Electromagnetic waves (4)

General equation, for e.m., acoustic, seismic, etc. waves:

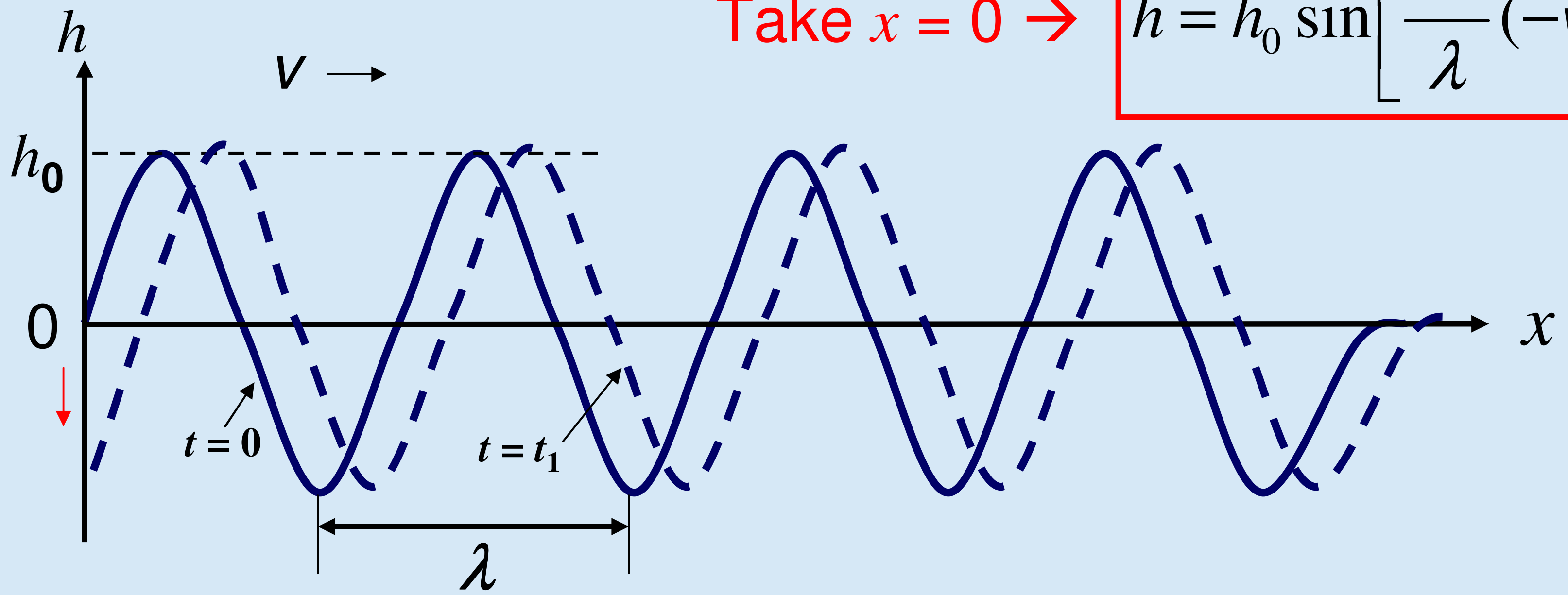
$$h = h_0 \sin\left[\frac{2\pi}{\lambda}(x - vt)\right]$$

λ = wavelength
v = propagation speed
t = time

Freeze x → oscillation with time at that point

Take $x = 0$ →

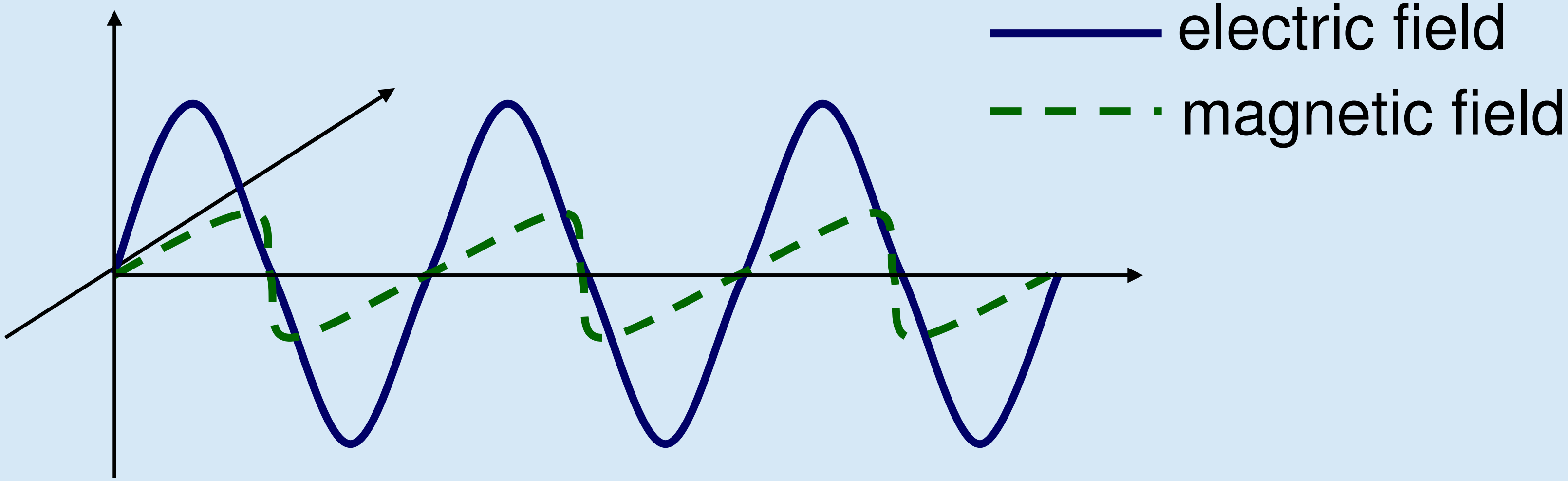
$$h = h_0 \sin\left[\frac{2\pi}{\lambda}(-vt)\right]$$



As t starts increasing, with $v > 0$, h goes < 0 → convention OK!

Electromagnetic waves (5)

Time varying electric field produces perpendicular time-varying magnetic field (Maxwell's equations).



EM waves are self-propagating, i.e. they need no medium.

c = speed of light in m s^{-1}

λ = wavelength in m

ν = frequency in Hz

$$\boxed{c = \lambda \nu}$$

c = 2.998 x 10^8 m s^{-1}

Note difference: c (speed of light) ⟺ v (source velocity)!

Doppler effect

The Doppler effect is of fundamental importance in astrophysics.

The observed wavelength, λ, is different from the emitted wavelength, λ_0, due to the radial velocity of the emitter with respect to the observer:

$$\boxed{\frac{(\lambda - \lambda_0)}{\lambda_0} \equiv \frac{\Delta\lambda}{\lambda_0} = \frac{v}{c}}$$

λ = observed wavelength
λ_0= 'rest' wavelength
v = source's radial velocity

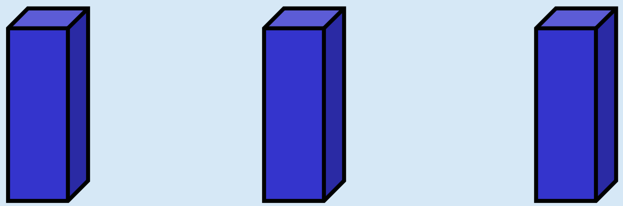
$\lambda > \lambda_0$ implies a 'redshift' of the light, $v > 0$, the emitter is moving away from the observer

$\lambda < \lambda_0$ implies a 'blueshift' of the light, $v < 0$, the emitter is moving towards the observer

'Astronomical redshift' $\boxed{z \sim \frac{v}{c}}$ when $z << 1$

Quantum nature of light

Alternatively, light can be thought of as packets (or 'quanta') of energy called ***photons***.



Photon energy, E:

$$\boxed{E = h\nu = \frac{hc}{\lambda}}$$

ν = frequency (Hz) $\boldsymbol{h}$ = Planck's constant (= 6.63 x 10^{-34} J s)

high frequency

$\Rightarrow$ short wavelength

$\Rightarrow$ high energy

Examples of particle nature of light can be seen in:

- Photo-electric effect
- Atomic spectra

Units

Wavelength: SI units – metre, m

Optical/UV: **Angstrom, A** 1A = 10^{-10} m = 10^{-8} cm = 0.1nm

nanometre, nm 1nm = 10^{-9} m

Infrared: **micron, μm** 1μm = 10^{-6} m

Frequency: SI units – Hertz, Hz

Radio: **Gigahertz, GHz** 1GHz = 10^{9} Hz

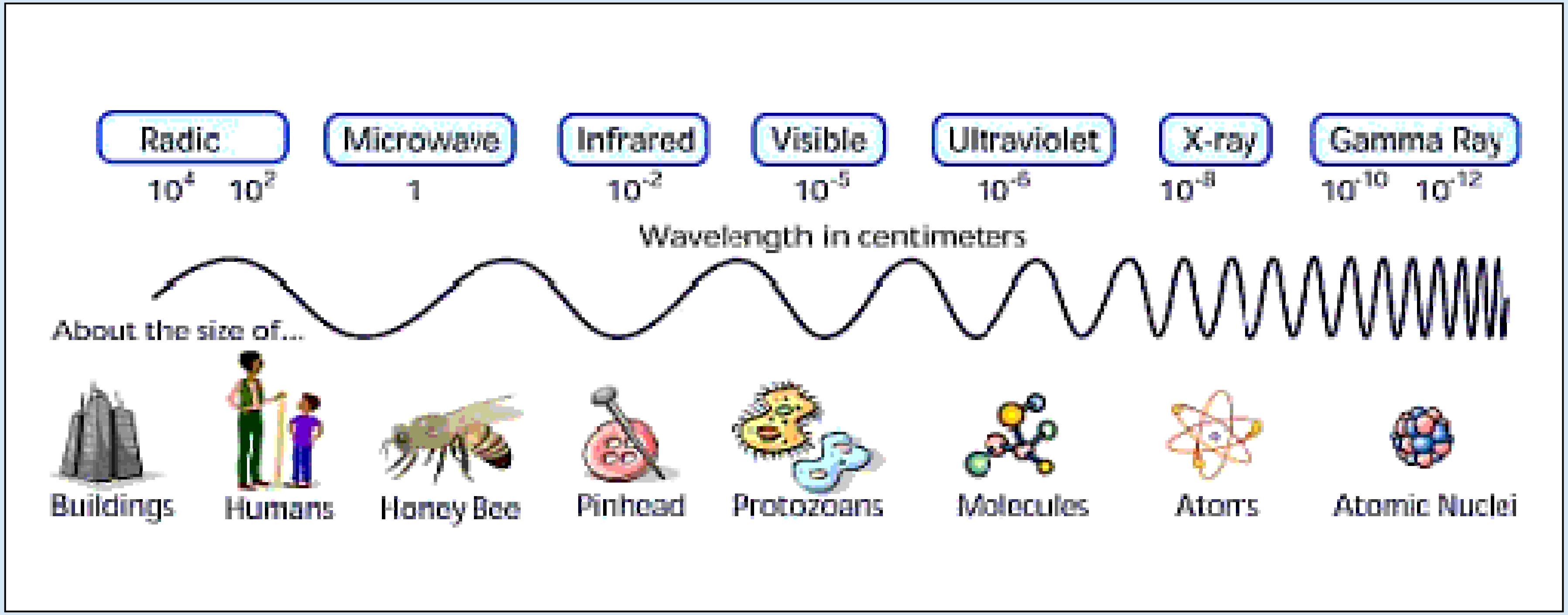
Energy: SI units – Joules, J

X-ray: **electron volts, eV** 1eV = $1.6\text{x}10^{-19}$ J

1keV = $1.6\text{x}10^{-16}$ J

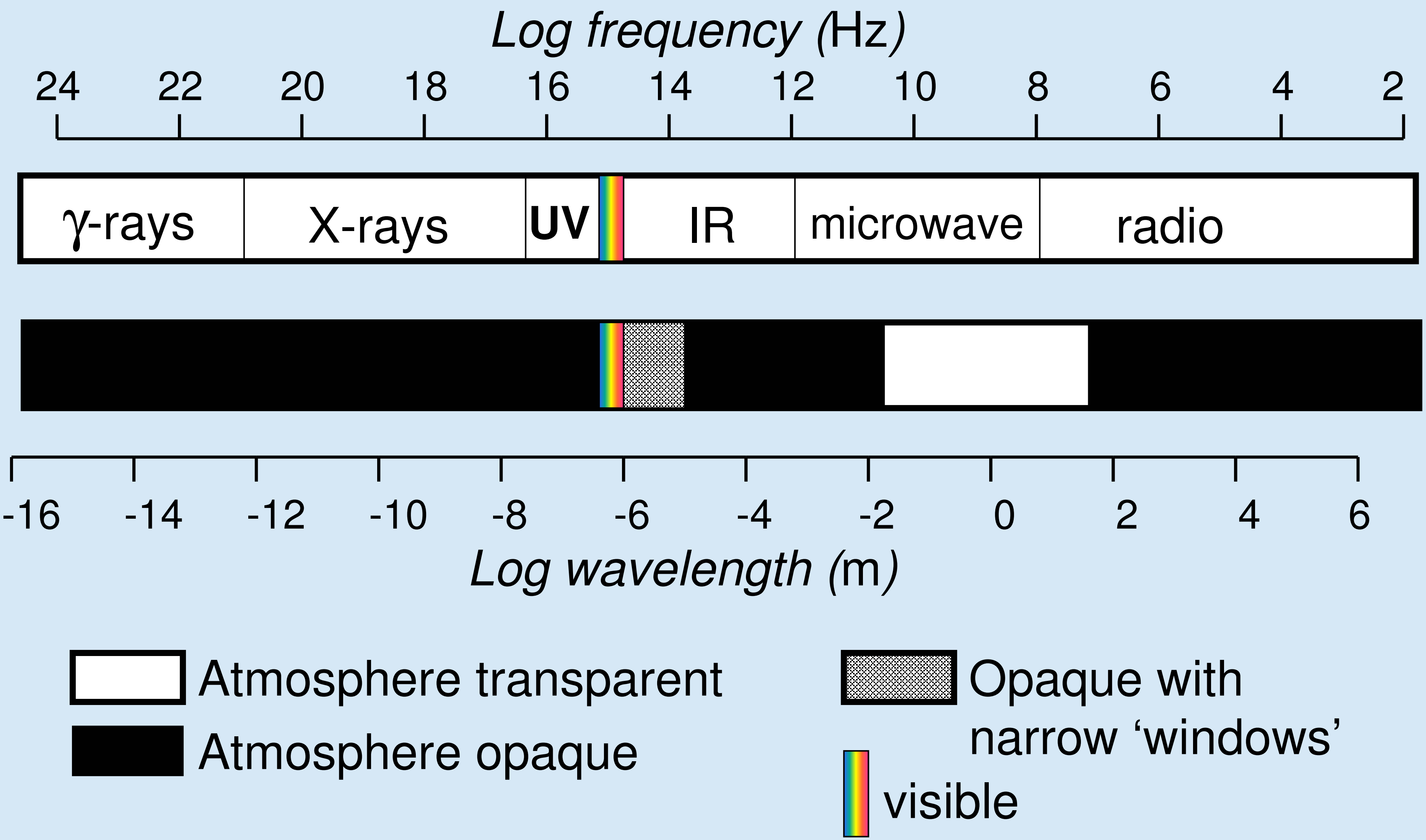
The electromagnetic spectrum

The electromagnetic spectrum

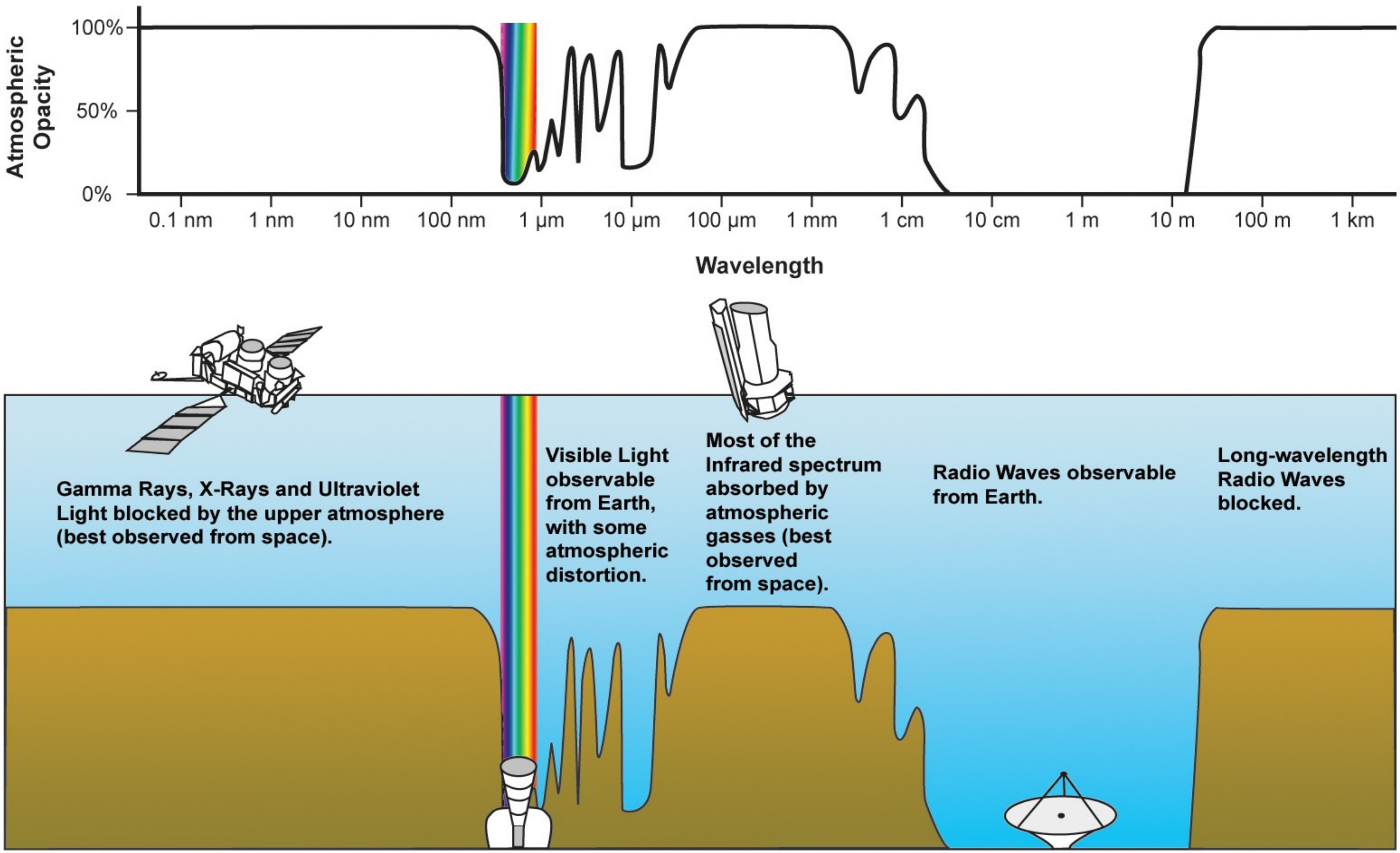


(Image created by NASA)

'Map' of the electromagnetic spectrum

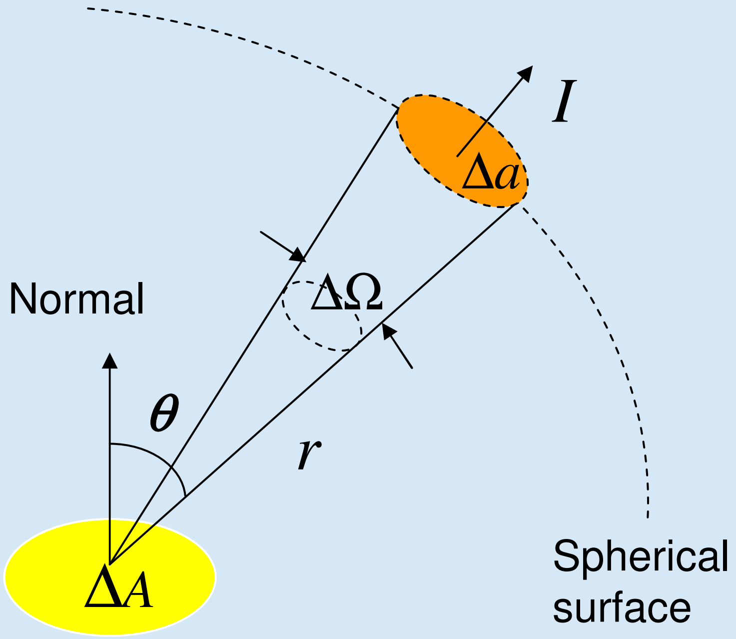


Atmospheric absorption



Intensity, Luminosity and Flux (1)

Intensity I : Amount of energy emitted per unit time Δt,
per source unit area ΔA,
per unit frequency interval $\Delta \nu$,
per unit solid angle $\Delta\Omega$ in a given direction
(depends on direction!)



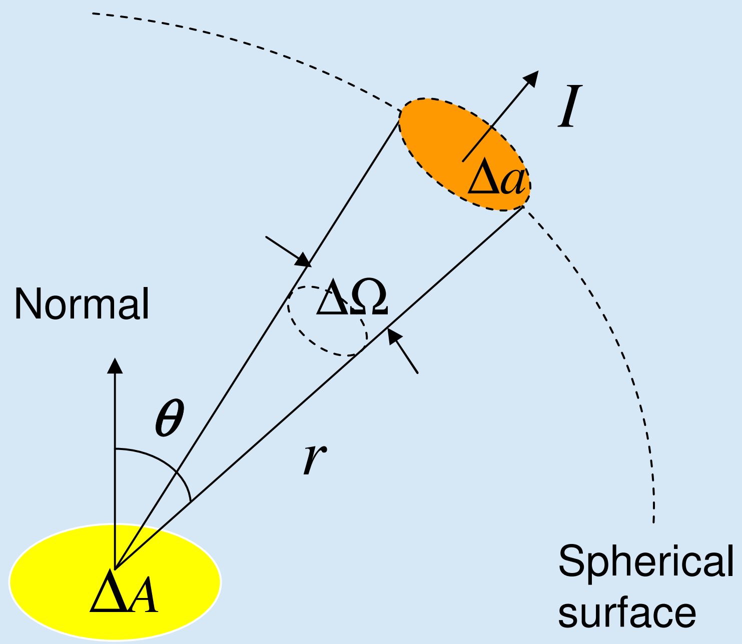
$$\boxed{\Delta\Omega = \frac{\Delta a}{r^2}}$$

Steradian (sr):
Unit of solid angle
(entire spherical surface:
$\Delta a = 4\pi r^2$ ➔ 4π sr)

Intensity units (e.g.): Joule s^{-1} m^{-2} Hz^{-1} sr^{-1}

Intensity, Luminosity and Flux (2)

Luminosity L : Total energy emitted per unit time (second) from a spherical star of total surface A, over all frequencies and in all directions



$$\boxed{L = 4\pi \int_0^\infty A\, I(\nu)\, d\nu}$$

$I(\nu)$: Monochromatic intensity (i.e. intensity emitted at specific frequency ν)

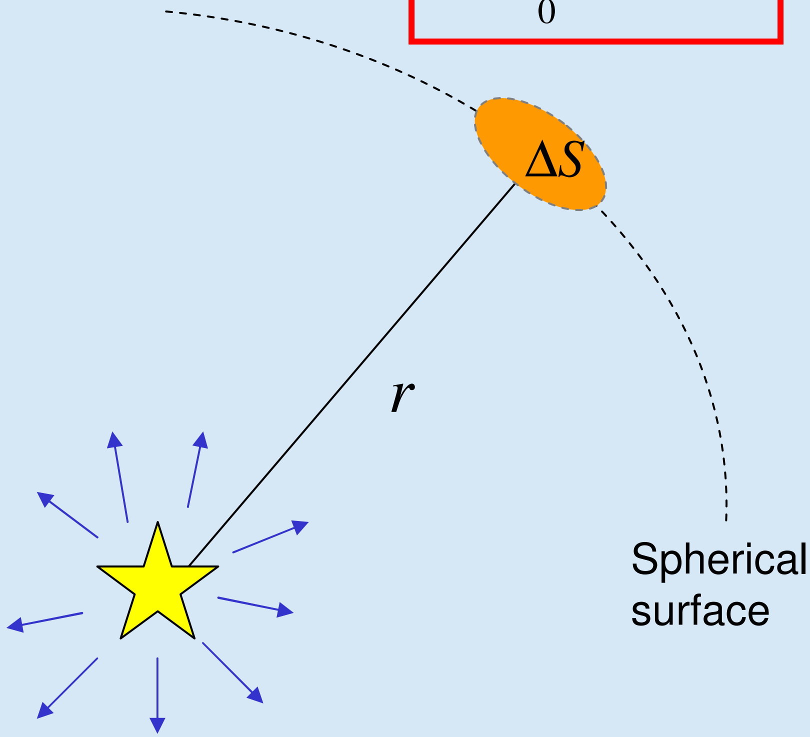
Unit of luminosity: Watt = Joule s^{-1}

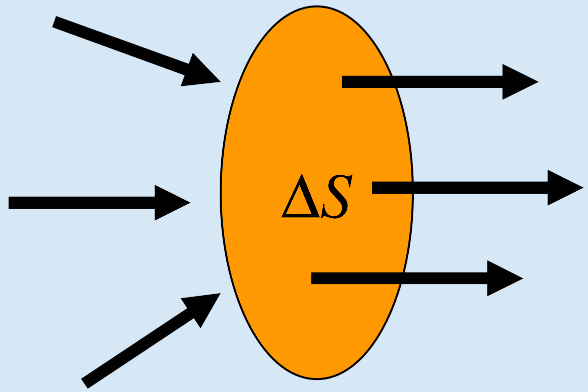
Intensity, Luminosity and Flux (3)

Monochromatic flux $F(\nu)$: Amount of energy at frequency ν received per unit time Δt, passing through a unit surface ΔS of the detector, per unit frequency interval $\Delta \nu$

Total flux: $\boxed{F = \int_0^\infty F(\nu)\,d\nu}$

Unit of flux $F(\nu)$: Joule s^{-1} m^{-2} Hz^{-1}





Energy flux from a star through concentric sphere at distance r:

$$\boxed{F = \frac{L}{4\pi r^2}}$$ in Joule s^{-1} m^{-2}

‘Thermal’ spectrum of radiation: Blackbody (1)

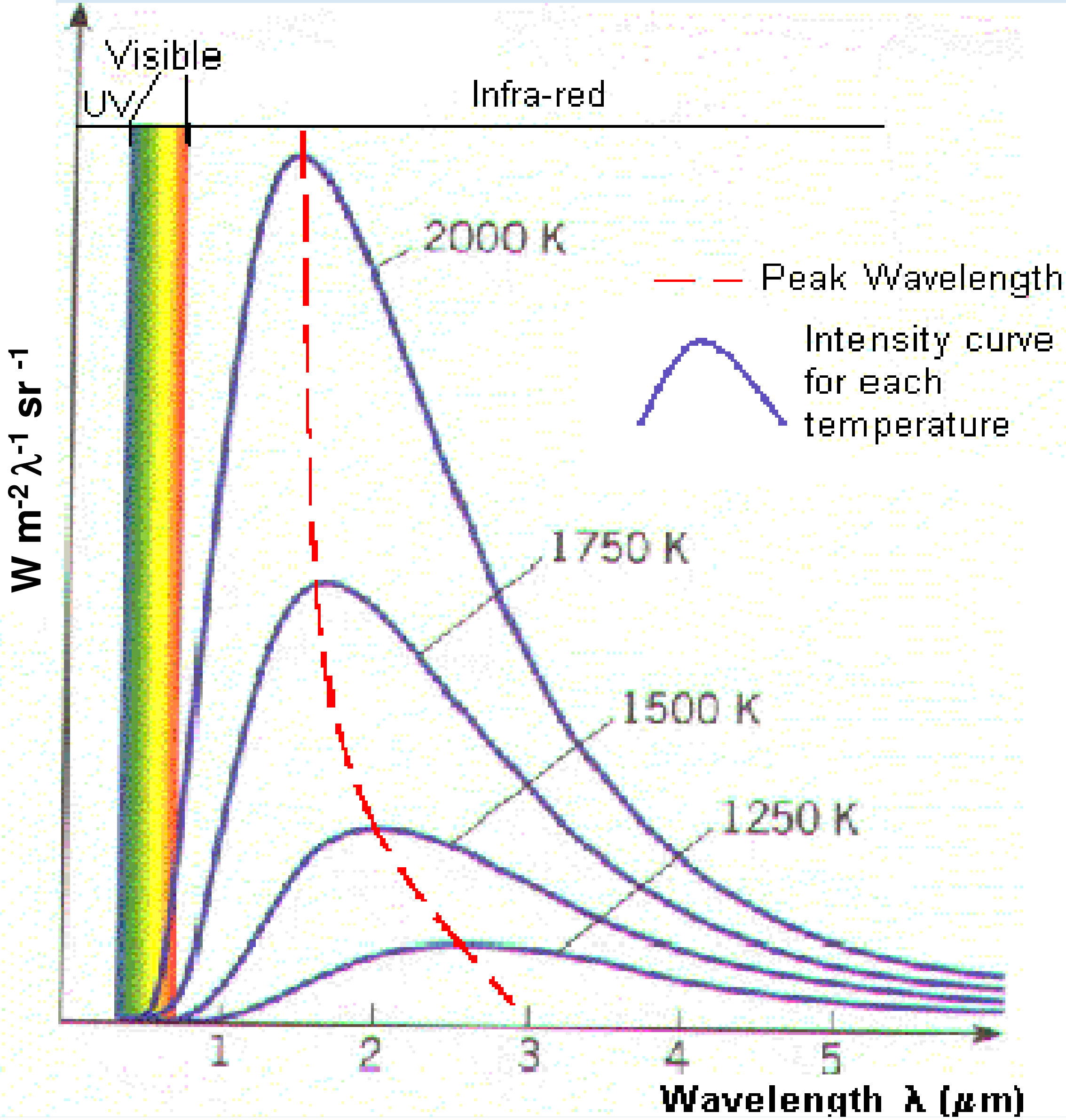
Blackbody: A body which absorbs all radiation incident upon it.
To be in perfect thermal equilibrium, it must also emit radiation at the same rate it absorbs it
➔ its temperature is maintained

Example: Perfectly insulated enclosure within which radiation is in equilibrium with the enclosure walls
➔ observe blackbody radiation through a pinhole

Gases in the interior of stars are opaque to all radiation

➔ Surfaces of stars emit very closely to a *blackbody spectrum*

‘Thermal’ spectrum of radiation: Blackbody (2)



In 1900 Planck postulated e.m. energy propagates in quanta, and derived the blackbody radiation law:

$$I(\lambda)\,\Delta\lambda = \frac{2hc^2}{\lambda^5}\left[\frac{1}{e^{hc/\lambda kT}-1}\right]\Delta\lambda$$

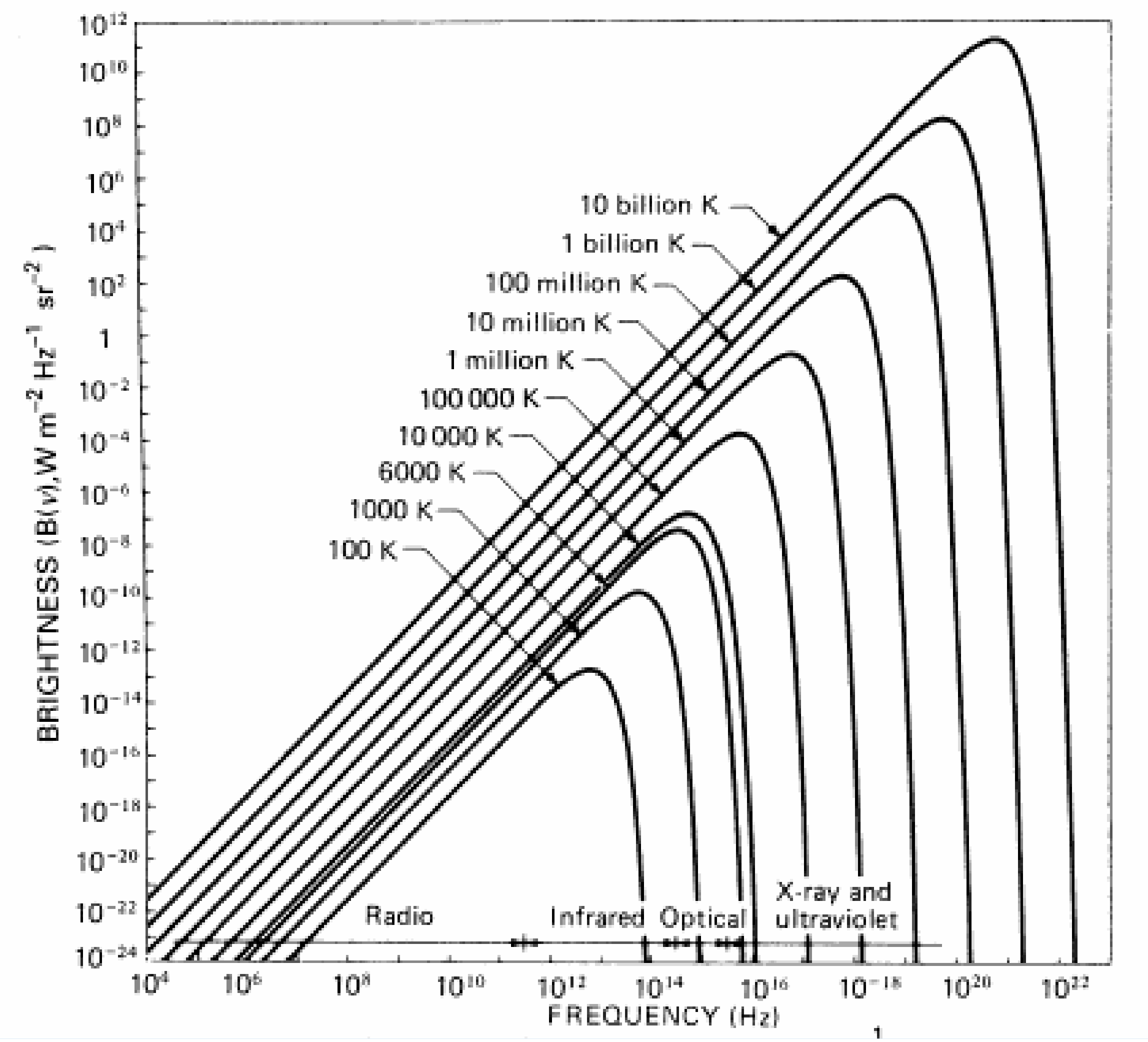
where $I(\lambda)$ is the intensity emitted by a blackbody at temperature T in the range of wavelength λ and $\lambda+\Delta\lambda$

h: Planck’s constant
c: speed of light
k: Boltzmann’s constant
T: temperature in Kelvin

‘Thermal’ spectrum of radiation: Blackbody (3)

Alternatively, we can express the blackbody radiation law (also called Planck’s function) in terms of frequency:

$$I(\nu)\Delta\nu = \frac{2h\nu^3}{c^2}\left[\frac{1}{e^{h\nu/kT}-1}\right]\Delta\nu$$

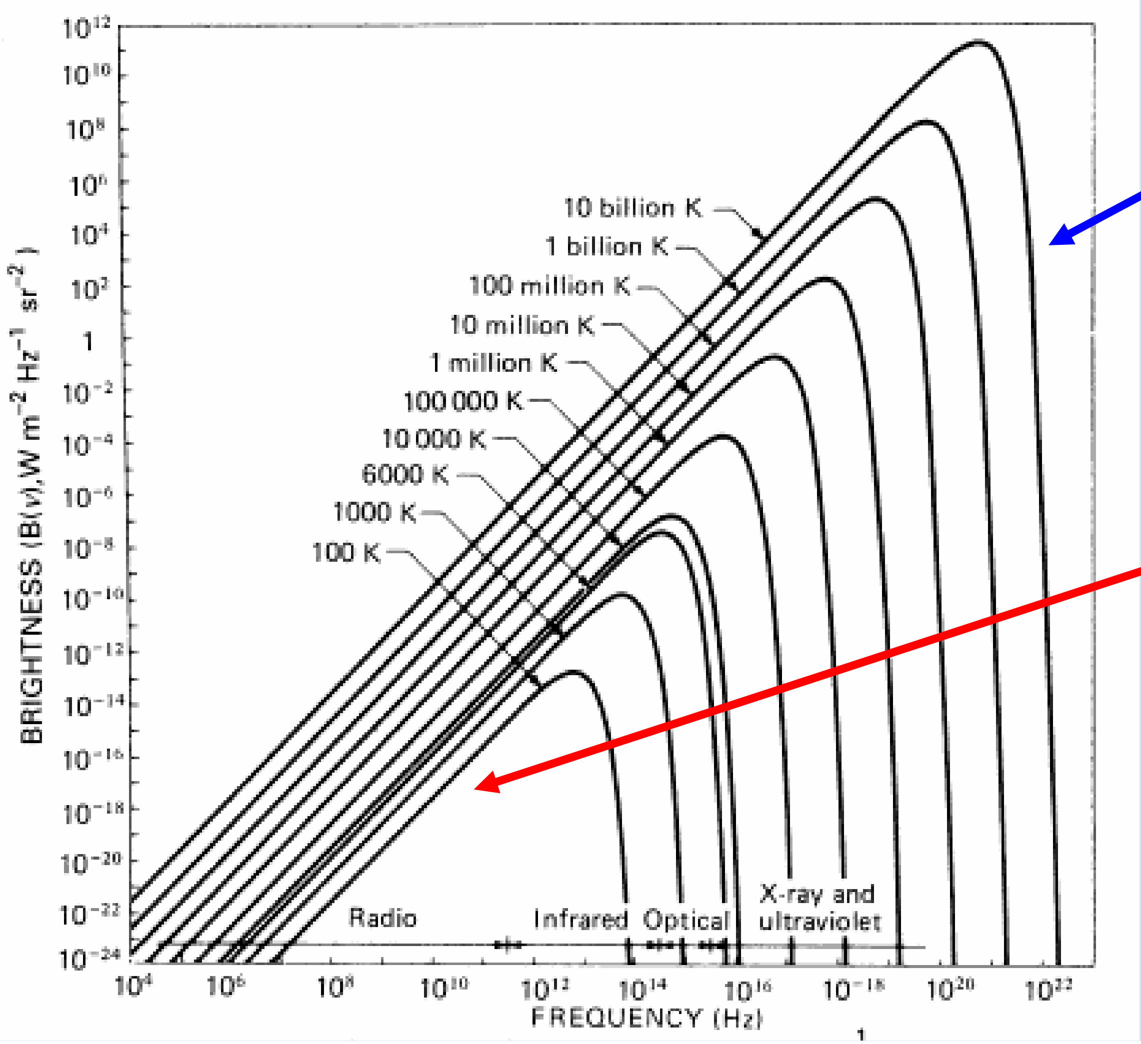


where $I(\nu)$ is the intensity emitted by a blackbody at temperature T in the range of frequency ν and $\nu+\Delta\nu$

h: Planck’s constant
c: speed of light
k: Boltzmann’s constant
T: temperature in Kelvin

'Thermal' spectrum of radiation: Blackbody (4)

Useful approximations - At high frequencies, Wien distribution:



$$I(\nu) = \frac{2h\nu^3}{c^2} e^{-h\nu/kT}$$

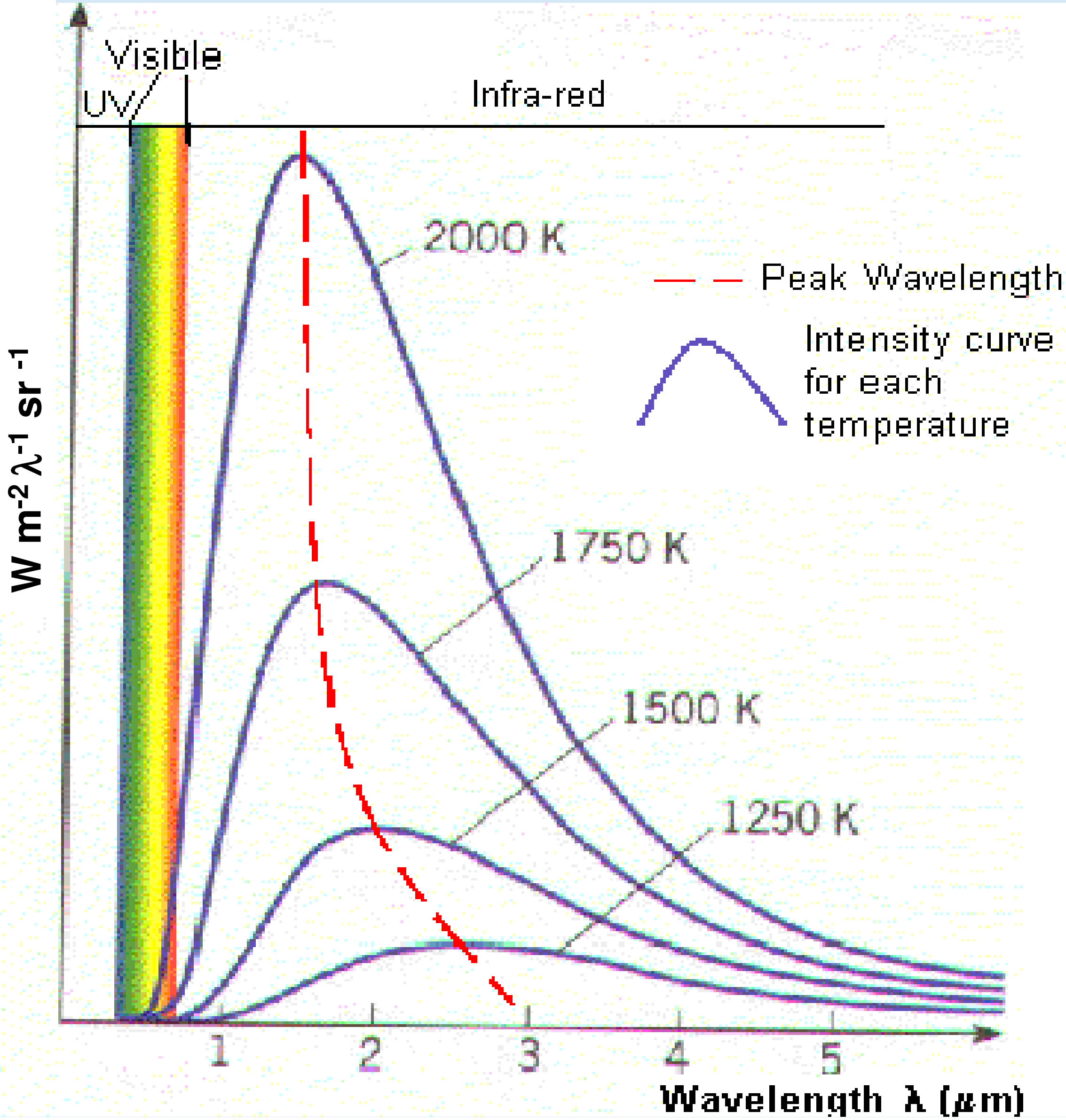
$$I(\lambda) = \frac{2hc^2}{\lambda^5} e^{-hc/\lambda kT}$$

At low frequencies, the Rayleigh-Jeans distribution applies:

$$I(\nu) = \frac{2\nu^2 kT}{c^2}$$

$$I(\lambda) = \frac{2ckT}{\lambda^4}$$

‘Thermal’ spectrum of radiation: Blackbody (5)



Wien displacement law expresses wavelength at which maximum intensity of blackbody radiation is emitted as a function of temperature; it is obtained by setting

$$\frac{dI(\lambda)}{d\lambda} = 0$$

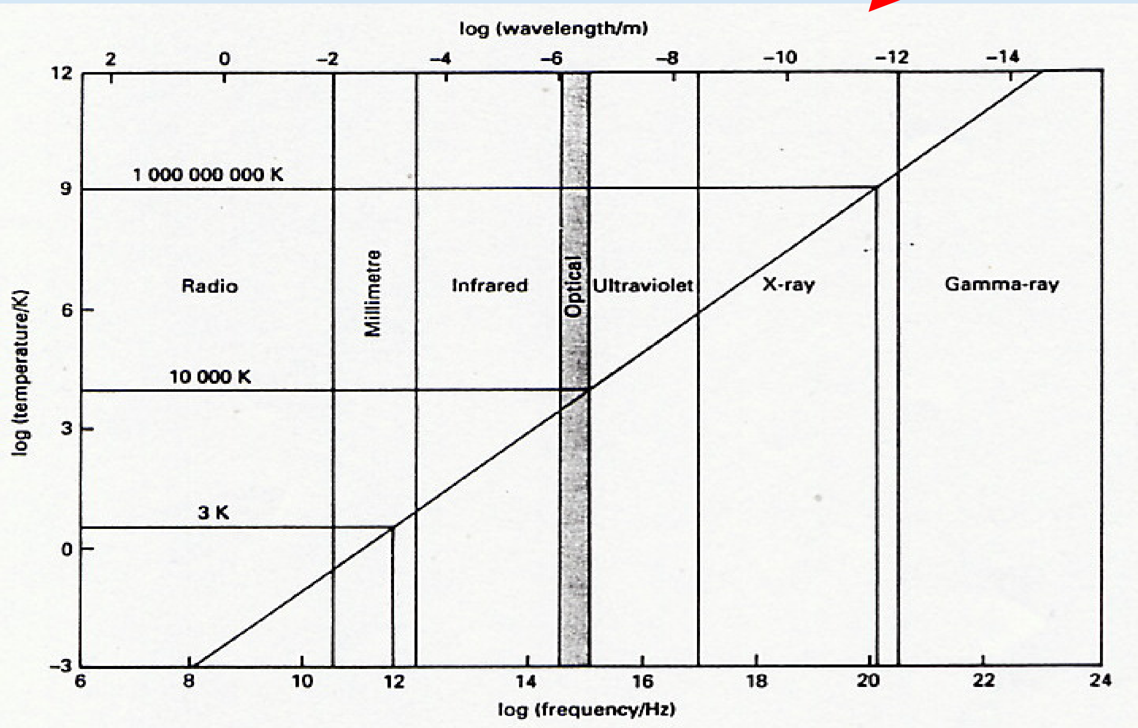
$$\rightarrow \boxed{\lambda_{\max} \approx \frac{3\times 10^{-3}}{T}}$$

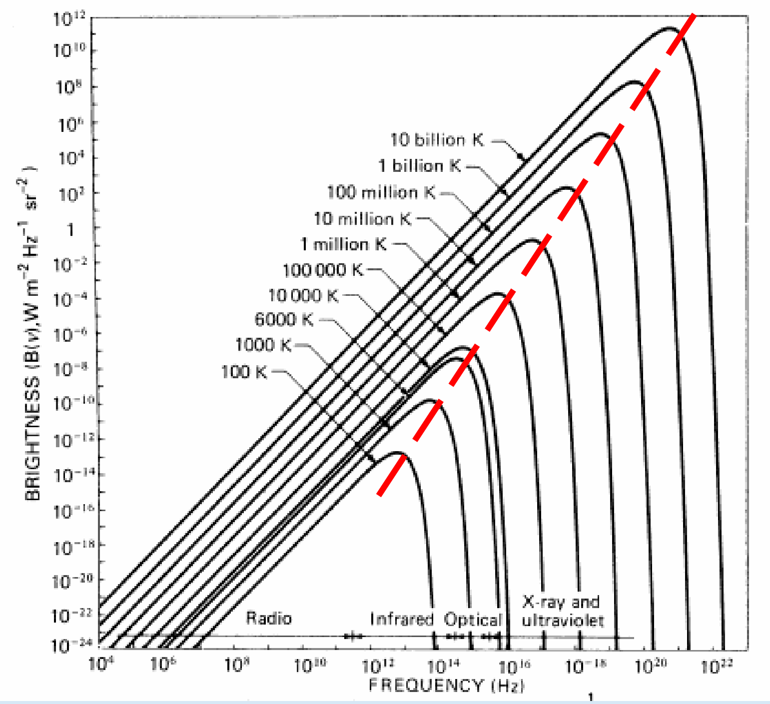
where $\lambda_{\max}$ in m
T in Kelvin

‘Thermal’ spectrum of radiation: Blackbody (6)

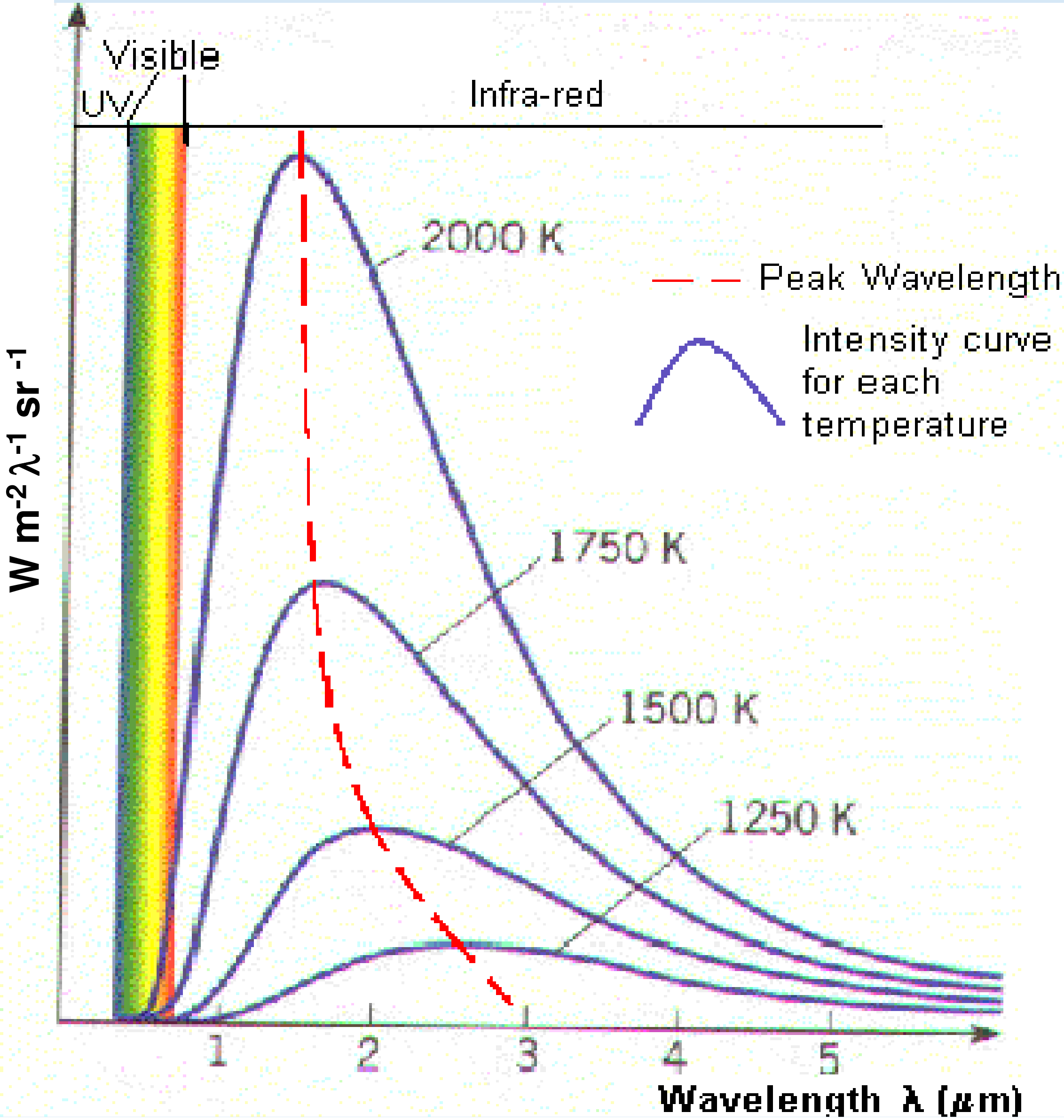
Wien displacement law: $\boxed{\lambda_{\max} \approx \dfrac{3\times10^{-3}}{T}}$ or $\boxed{\nu_{\max} = 10^{11}T}$ (ν in Hz)

(remember $\lambda\nu = c = 3$ x 10^8 m s^{-1})





'Thermal' spectrum of radiation: Blackbody (7)



Stefan-Boltzmann law

Area under Planck's curve (by integrating Planck's function over wavelength and all solid angles) is the <u>total power emitted per unit area</u>:

$$\boxed{B(T) = \sigma T^4} \text{ W m}^{-2}$$

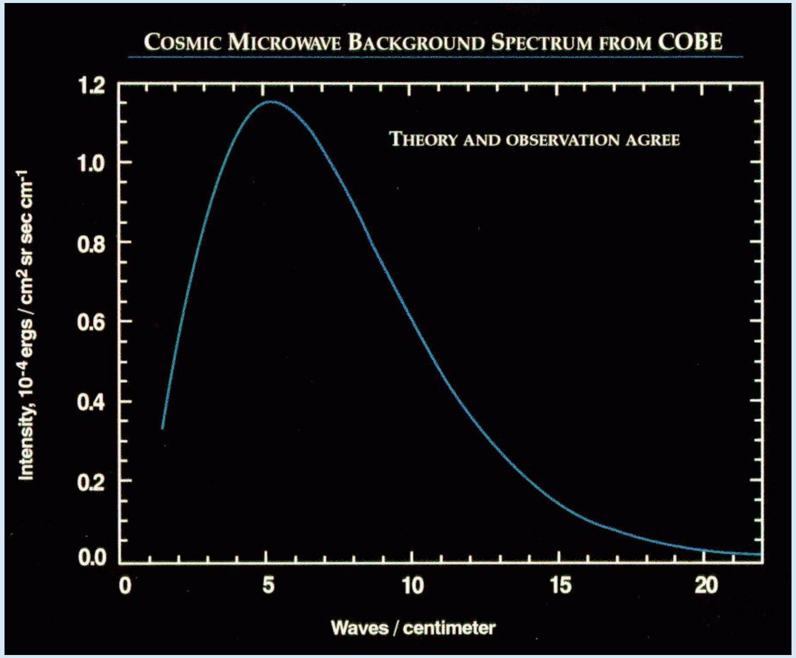
$\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}
Stefan-Boltzmann's constant

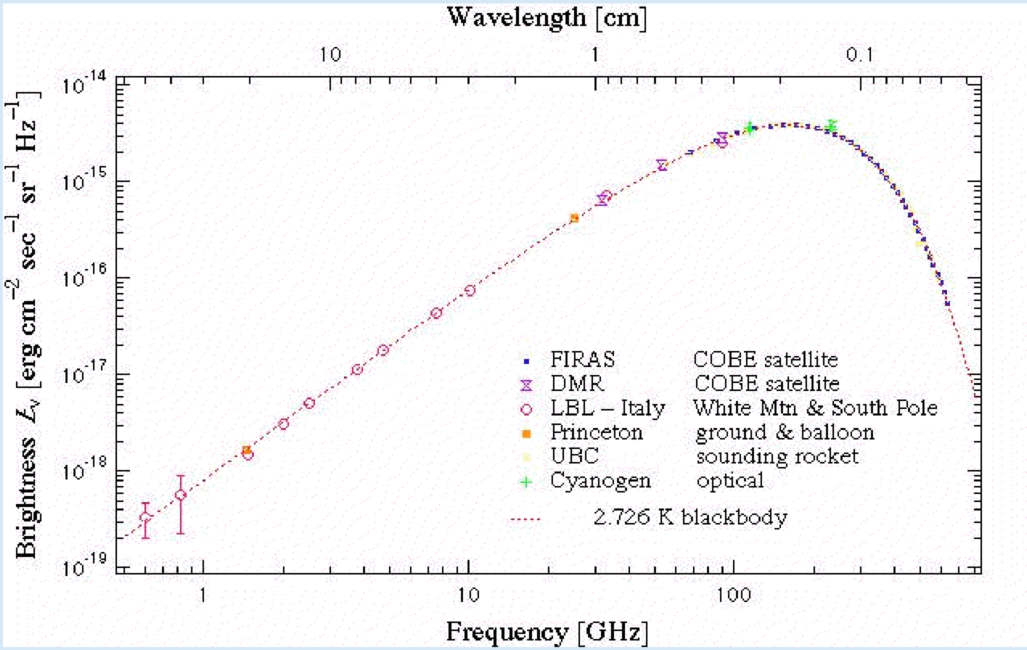
Luminosity of star of radius R (emitting as a blackbody)

$$\boxed{L = 4\pi R^2 \sigma T^4} \text{ W}$$

‘Thermal’ spectrum of radiation: Blackbody (8)

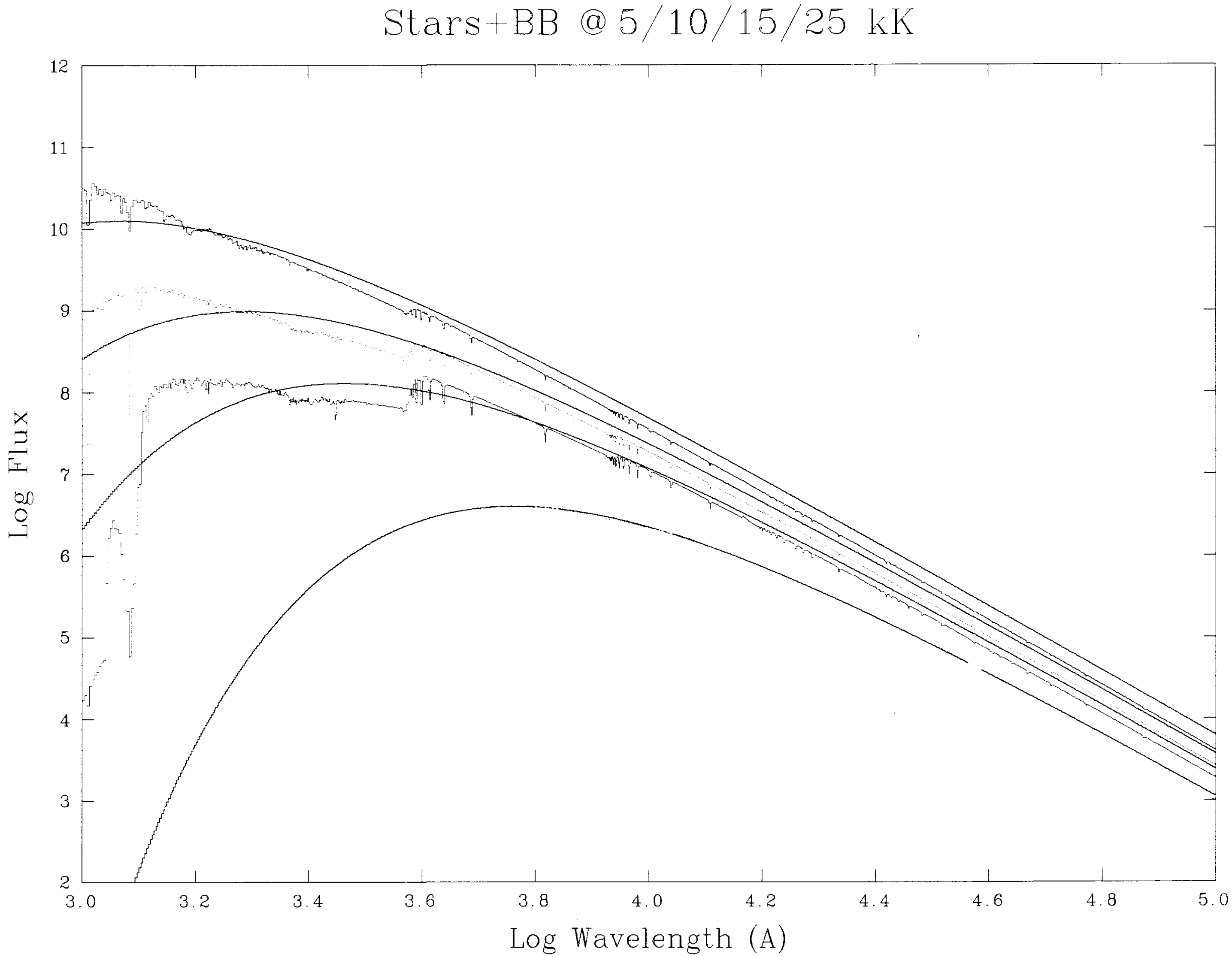
The best example of a blackbody is provided by the spectrum of the microwave background, the relic of the Big Bang, known to be now at 2.7 K.





‘Thermal’ spectrum of radiation: Blackbody (9)

Effective temperature T_{eff} of a star: Temperature of the blackbody that would emit the same total energy as the star



For the Sun:
$T_{\text{eff}} \approx 6000$ K