PHAS1247 : Classical Mechanics
In-Course Assessment Test #1 : Mon. 3 November 2014

Answer as many of the questions as you can, in any order. None of the questions require a calculator. The approximate distribution of marks is given in square brackets on the right of the page. There are 5 questions: they continue **ON THE OTHER SIDE OF THE PAGE**. *The maximum mark is 33.*

1. A particle of mass 1 kg has a position vector $\underline{\mathbf{r}}$ (in meters) as a function of time, t, (in seconds) given by:

$$\underline{\mathbf{r}} = \cos(2t)\,\hat{\underline{\mathbf{i}}} + 2t^2\,\hat{\underline{\mathbf{j}}} + \mathrm{e}^{t\sqrt{2}}\,\hat{\underline{\mathbf{k}}}$$

   Find (i) its velocity and (ii) its acceleration at a time t.
   Determine the unit vector defining the direction of the force on the particle at time $t = 0$. **[4]**

2. A particle moving in two dimensions with position coordinates (x, y) has a potential energy, V, given by:

$$V = 2x^2y^3 + y^2$$

   Find an expression for the force acting on the particle at position (x, y). **[2]**

   The force acting on a particle is:

$$\underline{\mathbf{F}} = -2xy^3\,\hat{\underline{\mathbf{i}}} - 3x^2y^2\,\hat{\underline{\mathbf{j}}}.$$

   Find an expression for the potential energy of the particle at position (x, y) assuming $V = 0$ at $x = y = 0$. **[3]**

3. A mass $m_1 = 5\,\mathrm{kg}$ has an initial velocity in the laboratory frame (i.e. relative to a stationary observer) $\underline{\mathbf{u}}_1 = 3\,\hat{\underline{\mathbf{i}}}\,\mathrm{ms}^{-1}$ while a second mass $m_2 = 2\,\mathrm{kg}$ has a velocity $\underline{\mathbf{u}}_2 = -3\,\hat{\underline{\mathbf{i}}}\,\mathrm{ms}^{-1}$. Find (i) the total momentum of the system, (ii) the velocity of the centre of mass, and (iii) the relative velocity of particle 1 to particle 2. **[3]**

   The two particles undergo an elastic collision and continue to move along the x-axis. Find the velocity of particle 1 in both the laboratory and the centre of mass frame after the collision. Express your answers as a fraction. **[4]**

4. What is the definition of a conservative force ? **[1]**
   A particle moving in three dimensions moves in a straight line from the origin to a point with a position vector, in metres, of $\hat{\underline{\mathbf{i}}} + 2\,\hat{\underline{\mathbf{j}}} + \hat{\underline{\mathbf{k}}}$. Find the work done during the displacement by (i) a constant force $\underline{\mathbf{F}}_1 = \left(2\,\hat{\underline{\mathbf{i}}} + 3\,\hat{\underline{\mathbf{j}}} + 2\,\hat{\underline{\mathbf{k}}}\right)\mathrm{N}$ and (ii) by the position dependent force $\underline{\mathbf{F}}_2 = \left(2xy^2\,\hat{\underline{\mathbf{i}}} + 2x^2y\,\hat{\underline{\mathbf{j}}}\right)\mathrm{N}$ **[5]**
   Show that the work done by $\underline{\mathbf{F}}_2$ does not depend on the path between the two points. **[1]**

PLEASE TURN OVER FOR QUESTION 5

5. Nigel Farage stands on a stage directly in front of a person who throws an egg directly at the fine upstanding politician. Sadly the egg is launched with the wrong combination of initial speed and launch angle to hit his lustrous tweed jacket and it goes over Mr. Farage's head and the egg lands behind him on the stage.

   The egg is launched with an initial speed, u (in ms^{-1}), from a horizontal distance, d (in m), from the stage and a vertical height, h (in m), below the stage at an angle of α to the horizontal. Neglecting air resistance, the internal dynamics of the egg and egg-spin show that the maximum height the egg attains (relative to the launch point), $y_{\max}$ is:

$$y_{\max} = \frac{u^2 \sin^2 \alpha}{2g}$$

   where g is the acceleration due to gravity. **[5]**

   Hence show that, x: the distance (in m) from the edge of stage where the egg lands is given by:

$$x = \frac{u \cos \alpha}{g} \left(u \sin \alpha + \sqrt{u^2 \sin^2 \alpha - 2gh} \right) - d.$$

[5]

END OF PAPER