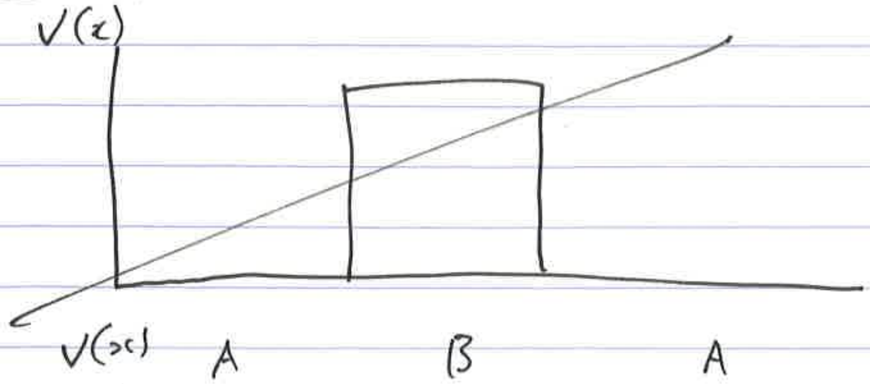
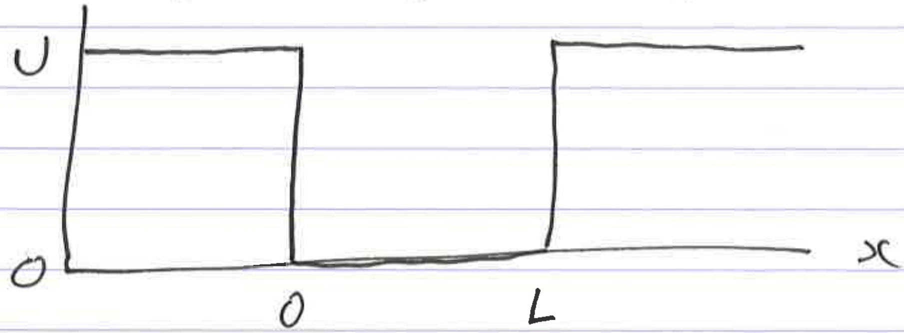
Finite well





$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U\psi(x) = E\psi(x)$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = (E-U)\psi(x)$$

$$\frac{d^2\psi(x)}{dx^2} = -\frac{2m}{\hbar^2}(E-U)\psi(x) \qquad *$$

Two cases:

i) $E > U$	ii) $E < U$
$(E-U) > 0$	$(E-U) < 0$
RHS of $*$ is $-$ve	RHS of $*$ is $+$ve.

Use trial solution:

$$\psi(x) = Ae^{(Bx+C)}$$

$$\frac{d\,\psi(x)}{dx} = ABe^{(Bx+C)} \qquad \frac{d^2\psi}{dx^2} = AB^2 e^{(Bx+C)}$$

$$= B^2\psi(x)$$

$$\frac{d^2\psi(x)}{dx^2} = \frac{-2m(E-U)}{\hbar^2}\psi(x) = B^2\psi(x)$$

$$\rightarrow \quad B^2 = \frac{-2m(E-U)}{\hbar^2} = \frac{2m}{\hbar^2}(U-E)$$

$$B = \pm\sqrt{\frac{2m}{\hbar^2}(U-E)}$$

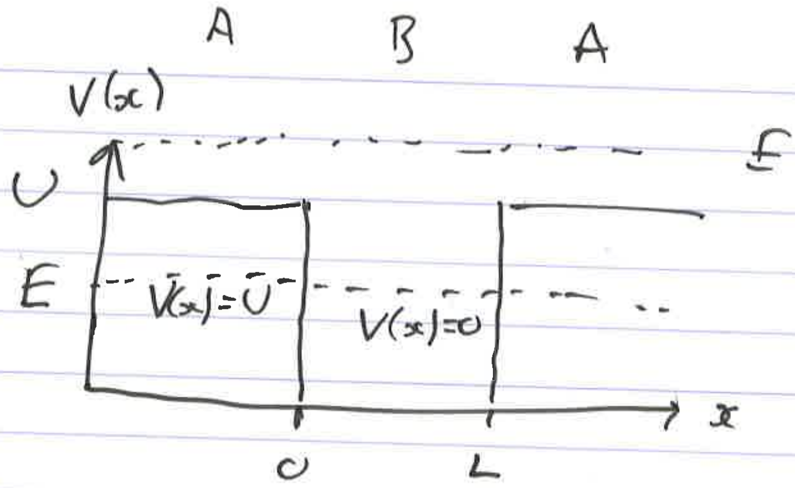
$$\text{Let } K = \sqrt{\frac{2m}{\hbar^2}(U-E)}$$

$$B = \pm K$$

Hence solns of TISE for $E < U$ are:

$$\psi(x) = Ae^{\pm kx + c}$$

Finite Square Well



In region A: $\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U\psi(x) = E\psi(x)$

$$\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} = (E-U)\,\psi(x)$$

$$\frac{d^2\psi}{dx^2} = \frac{-2m}{\hbar^2}(E-U)\,\psi(x)$$

Two cases:

i) $E > U$
$(E-U) > 0$
$\downarrow$
RHS: $-ve$

or ii) $E < U$
$(E-U) < 0$
$\downarrow$
RHS: $+ve$

Case ii) Recall: $\frac{d^2}{dx^2}e^x = e^x$

Use trial solution: $\psi(x) = Ae^{Bx+C}$

$$\frac{d\psi}{dx} = ABe^{Bx+C} \qquad \frac{d^2\psi}{dx^2} = AB^2e^{Bx+C}$$

$$= B^2\psi(x)$$

$$\psi(x) = A e^{Bx+C} \qquad \frac{d^2\psi}{dx^2} = B^2 \psi(x)$$

$$B^2 \cancel{\psi(x)} = \frac{d^2}{dx^2}\psi(x) = \frac{-2m}{\hbar^2}(E-U)\,\cancel{\psi(x)}$$

$$B^2 = \frac{-2m(E-U)}{\hbar^2}$$

Recall $\underline{\underline{E < U}}$

$$B^2 = \frac{2m(U-E)}{\hbar^2} \qquad B = \pm\sqrt{\frac{2m(U-E)}{\hbar^2}}$$

Let $K = \sqrt{\frac{2m(U-E}{\hbar^2}}$

Hence $B = \pm k$

Subst into trial solution: $\boxed{\psi(x) = Ae^{\pm kx + C}}$