PHAS1202 - Atoms, Stars and The Universe
Problem Solving Tutorial Sheet 1 - 2017

All questions (or variations of them) may appear in the In-Course-Assessment test. Questions are made available approximately one week before the PST. Please attempt the problem sheet in advance of the PST class. A solution sheet will be made available after all PSTs have taken place. **Please print this question sheet and bring it to the PST.**

Objectives:

1. Gain further experience in the energy scales of single photons in comparison with the energy scales of classical Physics.
2. Practise physicist's methods of order-of-magnitude calculation. Please attempt these questions **without** the use of a calculator, make suitable approximations and aim for your numerical answers to be correct to the first significant figure.
3. Gain insights into the wavelength scales in the Compton Effect.
4. Explore the mathematical properties of Rydberg's formula.
5. Explore how adding optic components affects the Mach-Zehnder interferometer.
6. See a modern application of Bohr's atomic model and gain intuition for the atomic size-scales the model predicts.

Useful constants

Planck's constant h is 6.6×10^{-34} Js (2 s.f.).
The speed of light is 3×10^{8} ms^{-1}.
The mass of a Hydrogen atom is 1.7×10^{-27} kg.
The mass of an electron is 9.1×10^{-31}kg.
1 electronVolt is 1.6×10^{-19} J.

1: Quantum scale

1.a) Without using a calculator, give an order-of-magnitude estimate of how many photons are emitted per second by a 5mW green laser pointer?

You may assume that 100% of the power supplied to the laser is converted to green light at a wavelength of 500 nm.

1.b) Many modern lighthouses utilise 1kW bulbs coupled with rotating mirror and lens assemblies. Assuming the wavelength of the light is 570nm calculate the energy of a single photon and the number of photons emitted per second.

2: Compton Effect

Compton's derivation predicted that, after scattering from an electron in a material, if the incoming photon wavelength is λ, the outgoing wavelength λ' of a photon scattered through angle θ would satisfy

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta).$$

2.a) The quantity $\frac{h}{m_e c}$ is called the Compton wavelength and represents frequency shift of light scattered at $90°$. Without using a calculator provide an estimate (in metres) for this wavelength.

2 b.i) Consider xray light with a wavelength of 0.1nm. What is the ratio between the Compton wavelength and this wavelength?

2 b.ii) Consider visible light with a wavelength of 500nm. What is the ratio between the Compton wavelength and this wavelength?

2 c) Consider light scattering from a proton. Assuming a similar physical process to Compton scattering takes place calculate the equivalent "Compton wavelength" for scattering from a proton. If you wanted to build an experiment to measure the proton Compton wavelength what region of the electromagnetic spectrum would you use?

3: Macroscopic photons?

In the lectures, we have always assumed that photons have an extremely small energy, and this is why we do not observe the "granularity" of photons in classical (i.e. human-scale) light. But why is this the case? Planck's photon energy formula does not have an upper bound. In principle at least, photons could have any energy. In this question, you will explore the reasons why we do not see photons with the energies of macroscopic objects in nature.

3a) Estimate (to the nearest order of magnitude) the kinetic energy of a cricket ball, bowled by James Anderson (England's leading wicket taker).

> What order of magnitude is the mass of a cricket ball? 1g? 10g? A kilogram? How fast is it likely to go? Try to use your own intuition to estimate these (i.e. not wikipedia!). Without using a calculator, give an estimate for the kinetic energy in Joules.

3b) What frequency would a photon need to have to possess the same energy?

3c) Wien's law gives the peak wavelength of the emission spectrum of a black-body at temperature T.

$$\lambda_{\text{max}} = \frac{2.9 \times 10^{-3}}{T} \text{m} \cdot \text{K}$$

where m · K (meter Kelvins) are the SI unit for this constant. How hot must a black body be such that its peak emission will be at the frequency of the photon you just calculated?

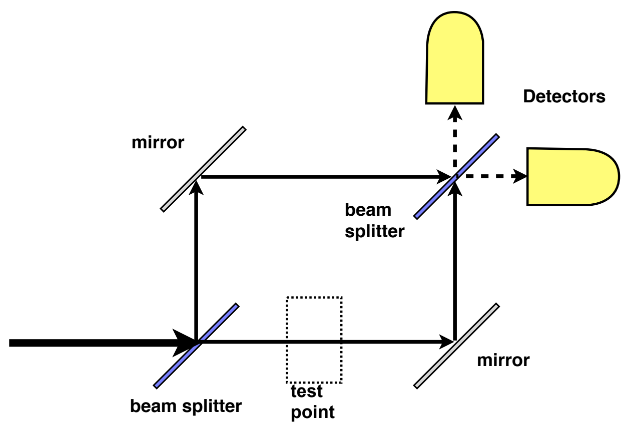
3d) Consider Rydberg's formula for the Hydrogen atom emission spectrum.

$$\frac{1}{\lambda} = R_H \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$$

where $R_H = 1.1 \times 10^7 \mathrm{m}^{-1}$, where n and m are positive integers and where $n > m$. What is the highest frequency of of all Hydrogen emission lines?

4: Mach-Zehnder Interferometer

In the lectures we discussed the Mach-Zehnder interferometer, as pictured in the image below. As a reminder the beam splitters transmit 50% of the incident light with no phase shift, whilst the remaining 50% is reflected with a phase shift of $\lambda/4$. The mirrors reflect 100% of the incident light with no phase shift.



4a) What is the probability that a photon will reach the right-most detector if there is nothing placed in the test point?

4b) What is the probability that a photon will reach the right-most detector if an absorbing block is placed in the test point (the absorbing block absorbs 100% of the light incident upon it).

A quarter-wave plate is a device that transmits 100% of the light but does so whilst adding a phase shift of $\lambda/4$.

4c) What is the probability that a photon will reach the right-most detector if a single quarter-wave plate is placed in the test point?

4d) What is the probability that a photon will reach the right-most detector if two quarter-wave plates are placed in the test point?

5: Nobel-prize winning Bohr-model atoms

In part 2 of the course, you learnt about Niels Bohr's model of the Hydrogen atom, where electrons lie circular orbits, which satisfy an angular momentum quantisation rule – $l = n\hbar = nh/(2\pi)$ where n is an integer from 1 to infinity. Although the model has been superceded by a fully quantum mechanical treatment (which you'll study in detail in PHAS2222), it remains a good approximation for certain cases.

Prof Serge Haroche was one of the winners of the 2012 Nobel Prize in Physics. Prof Haroche performs experiments with *circular Rydberg states* of Rubidium atoms. We say an atom is in a Rydberg state when one of the electrons has been promoted into a very high energy state. Circular Rydberg states are states where the outer electron's properties are well approximated by the Bohr model for Hydrogen.

The Hydrogen Bohr model works very well for these states due to certain special properties that they possess. Rubidium has atomic number 37 - i.e. it has 37 electrons orbiting a nucleus containing 37 protons. In a circular Rydberg state, the outer electron is very far away from all other electrons and the nucleus. The outer electron, therefore, experiences the other electrons and protons together as a single object of charge $+e$ (this effect is known as *electron shielding* of the nucleus). The energy of these states and average radius of the outer electron are very well approximated by Bohr's Hydrogen model, and we shall use Bohr's model to study them in this question.

5.a) In Prof Haroche's experiments, photons were generated via the transition from the $n = 51$ circular Rydberg state to the $n = 50$ state (n here can be taken to mean the same n that appears in Bohr's angular momentum rule). What is the radius of these orbits in the Bohr model?

5.b) What is the frequency of the photons emitted in this experiment? In what region of the electromagnetic spectrum do they lie (e.g. are they infrared? ultraviolet? etc.)?

5.c) A human hair has a thickness of approx 10^{-5}m. Approximately which orbit (i.e. which value of n) would you need to excite the electron so its orbit had a diameter similar to that of a human hair?

Ryan Nichol, October 2017