PHAS1224 Waves, Optics and Acoustics

Problem Class IV

1. The bright fringes are seen when $d \sin\theta = m\lambda$. At a given point on the screen $\sin\theta$ is a constant so if we see the second fringe for light of wavelength λ_1 at the same place as the third fringe for light of wavelength λ_2 then

$$2\lambda_1/d = 3\lambda_2/d \rightarrow \lambda_2 = 2/3\lambda_1. \tag{1}$$

So if $\lambda_1 = 600$nm then $\lambda_2 = 400$nm.

2. Primary maximum at $d \sin\theta = m\lambda$ since both $\sin^2((\pi/\lambda)Nd\sin\theta)$ and $\sin^2((\pi/\lambda)d\sin\theta) \rightarrow 0$ but ratio

$$\frac{\sin^2((\pi/\lambda)Nd\sin\theta)}{\sin^2((\pi/\lambda)d\sin\theta)} \rightarrow \left(\frac{((\pi/\lambda)Nd\sin\theta)}{((\pi/\lambda)d\sin\theta)}\right)^2 \rightarrow N^2, \tag{2}$$

and hence intensity $\rightarrow I_0N^2$.

For this grating $d = 75\text{mm}/50,000 = 1.5 \times 10^{-6}$m. For the primary maxima $d\sin\theta = m\lambda$, so if $\lambda = 700$nm

$$\sin\theta = 0.466m, \tag{3}$$

so there are real solutions for θ for $m = 1$ and 2. If $\lambda = 400$nm, then

$$\sin\theta = 0.266m, \tag{4}$$

so this time there are solutions for θ if $m = 1$, 2 or 3.

3. For a circular aperture of diameter D the corresponding minimum angular separtion is $1.22\lambda/D$, and at a distance L the angular separtion of two points a distance d apart is given by $\tan\theta = d/L$ which becomes $\theta = s/L$ if $\theta \ll 1$. Therefore, to resolve the headlights we must have

$$d/L > 1.22\lambda/D \rightarrow L < dD/(1.22\lambda). \tag{5}$$

Inputing the values for this situation

$$L < \frac{1.2 \times 4 \times 10^{-3}}{1.22 \times 5 \times 10^{-7}} = 8000\text{m}. \tag{6}$$

4. Simply making the substitution we obtain

$$\frac{1}{f} = \frac{1}{300\text{cm}} + \frac{1}{150\text{cm}} = \frac{450}{45,000\text{cm}} \rightarrow f = 100\text{cm}. \tag{7}$$

If we the insist that $p = q$ then

$$\frac{1}{p} + \frac{1}{p} = \frac{2}{p} = \frac{1}{f}, \tag{8}$$

and so $p = 2f = 200$cm.