2013 first attempt

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1. State Newton's first and second laws of motion. Why is the first law a special case of the second? **[3]**

   Define the *impulse* exerted by the force **F** acting on an object, and show how it is related to the momentum of the object. **[3]**

1) 1st law – an object will remain at rest / constant velocity if no force is applied.

2nd – The force applied is equal to the rate of change of momentum.

$$\text{2nd: } F = \frac{dp}{dt} \rightarrow F = 0 \rightarrow \frac{dp}{dt} = 0 \rightarrow \frac{dv}{dt} = 0$$

first law.

Impulse $I = Ft$

$$F = \frac{dp}{dt} \rightarrow Ft = \Delta p \rightarrow \underline{I = \Delta p}$$

2. An object of mass m is moving with velocity $\mathbf{v}$ and is acted on by a force $\mathbf{F}$. Give expressions for (i) the kinetic energy of the object and (ii) the power developed by the force. **[4]**

   Show that the power is equal to the rate of change of the kinetic energy. **[2]**

i) $KE = \frac{1}{2}mv^2$

$P = \underline{F} \cdot \underline{v}$

$$KE = \frac{1}{2} m v^2$$

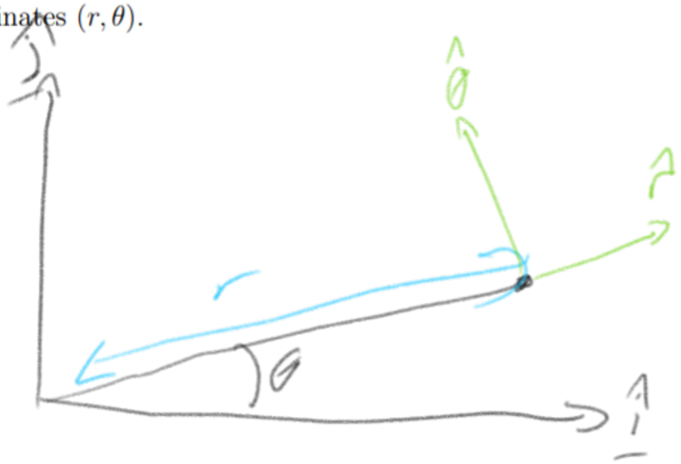
$$\frac{d(KE)}{dt} = \frac{1}{2} m \frac{d}{dt}(v^2)$$

$$= \frac{1}{2} m (2 \underline{v} \cdot \underline{a})$$

$$= \underline{F} \cdot \underline{v} = P$$

3. Sketch a diagram showing the polar coordinates r and θ for a particle moving in the xy-plane. Show on your diagram the unit vectors $\hat{\boldsymbol{r}}$ and $\hat{\boldsymbol{\theta}}$ and express each in terms of the Cartesian unit vectors $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$. **[4]**

   Derive an expression for the velocity vector $\mathbf{v}$ of a particle in terms of its polar coordinates (r, θ). **[2]**



$$\hat{r} = \cos(\theta)\,\hat{i} - \sin(\theta)\,\hat{j}$$

$$\hat{\theta} = \sin(\theta)\,\hat{i} + \cos(\theta)\,\hat{j}$$

$$\underline{r} = r\hat{r}$$

$$\underline{v} = \frac{d\underline{r}}{dt} = \frac{d}{dt}(r\hat{r})$$

$$= \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

4. Define the *reduced mass* of two masses m_1 and m_2. **[2]**

Show that the momentum of particle 1, as viewed in the centre-of-mass frame of the two objects, can be written

$$\mathbf{p}_1' = \mu \mathbf{v},$$

where $\mathbf{v} = \mathbf{v}_1 - \mathbf{v}_2$ is the velocity of particle 1 relative to particle 2. What is the momentum of particle 2 in the centre-of-mass frame? **[4]**

Show also that the total angular momentum of both the particles, viewed in the centre-of-mass frame, is

$$\mathbf{L} = \mu \mathbf{r} \times \mathbf{v},$$

where $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ is the displacement of particle 1 relative to particle 2. **[2]**

The mass that describes the system in CM reference frame: $\mu = \frac{m_1 m_2}{m_1 + m_2}$

$$\begin{aligned}
p_1' &= m_1 v_1 - m_1 v_{cm} \\
&= m_1 v_1 - m_1 \left(\frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \right) \\
&= m_1 \left[\frac{(m_1 + m_2) v_1 - m_1 v_1 - m_2 v_2}{m_1 + m_2} \right] \\
&= m_1 \left[\frac{m_2 (v_1 - v_2)}{m_1 + m_2} \right] \\
&= \mu (v_1 - v_2) \\
&= \mu \underline{v}
\end{aligned}$$

$$p_2' + p_1' = 0$$

$$\rightarrow p_2' = -\mu \underline{v}$$
