Work, power, and kinetic energy

15 January 2019 17:46

When a force is applied to a body, work is done.

Given a small distance, $\delta \boldsymbol{r}$ we can say the work done by a force to be:

$$\delta W = \boldsymbol{F} \cdot \delta \boldsymbol{r} = F(cos(\theta))\delta \boldsymbol{r}|$$

The work done is only in the direction the force acts. This result means it is a scalar quantity

This also means:

$$W = \int_{r_1}^{r_2} \boldsymbol{F} \cdot d\boldsymbol{r} = \int_{\boldsymbol{r_1}}^{r_2} (F_x dx + F_y dy + F_z dz)$$

Centre of mass and collision problems

13 January 2019 15:42

The centre of mass is defined as the weighted average position of all objects:

$$\boldsymbol{R} = \frac{\sum_i m_i \boldsymbol{r_i}}{M}$$

The total momentum of the system can therefore be found by:

$$\boldsymbol{P} = M\frac{d\boldsymbol{R}}{dt} = \sum_{\boldsymbol{i}} m_i \frac{d\boldsymbol{r_i}}{dt} = \sum_{\boldsymbol{i}} \boldsymbol{p_i}$$

This means the centre of mass only accelerates if there is an external force

In collision problems, it is often easiest to look at the problem from the centre of mass reference frame.

The velocity of the centre of mass can be described as the weighted average velocity of all objects. For 2 objects this can be written as:

$$\boldsymbol{V} = \frac{m_1\boldsymbol{v_1} + m_2\boldsymbol{v_2}}{m_1 + m_2}$$

The new relative position velocity and of each object is found by:

$$\boldsymbol{r_i'} = \boldsymbol{r_i} - \boldsymbol{R}$$
$$\boldsymbol{v_i'} = \boldsymbol{v_i} - \boldsymbol{V}$$

We can also then find the momentum of the centre of mass:

$$\boldsymbol{P} = \sum_i m_i\boldsymbol{v_i'} = \sum_i m_i(\boldsymbol{v_i} - \boldsymbol{V}) = \sum_i (m_i\boldsymbol{v_i}) - M\boldsymbol{V} = 0$$

Because it is 0, this is one of the important things to use in collision problems

The kinetic energy of the system can be found as well:

$$K = \frac{1}{2}mv^2 = \sum_i \frac{m_i v_i^2}{2} = \sum_i m_i \frac{(\boldsymbol{v_i'} + \boldsymbol{V})^2}{2}$$

$$K = \frac{1}{2}\sum_i [m_i {v_i'}^2 + 2(m_i v_i')V + m_i V^2]$$

However we already have shown that:

$$\sum_i m_i v_i' = 0$$

So

$$K = \frac{1}{2}\sum_i m_i v_i'^2 + \frac{1}{2}\sum_i m_i V^2 = K_{rel} + K_{cm}$$

This means the total kinetic energy is the Kinetic energy of the Centre of mass and the relative KE of each body relative to the centre of mass

When working in the centre of mass reference frame with only 2 objects, its easier to define relative position and velocity:

$$\boldsymbol{r} = \boldsymbol{r_1} - \boldsymbol{r_2}$$
$$\boldsymbol{v} = \boldsymbol{v_1} - \boldsymbol{v_2}$$

The relative position and velocity can also be found by:

$$\boldsymbol{r} = \boldsymbol{r_1'} - \boldsymbol{r_2'}$$
$$\boldsymbol{v} = \boldsymbol{v_1'} - \boldsymbol{v_2'}$$

Because we know the total momentum of the system in the centre of mass reference frame is 0, we can find $\boldsymbol{r_1'}$ and $\boldsymbol{r_2'}$ knowing just $\boldsymbol{r}$ and the masses:

$$m_1\boldsymbol{r_1'} + m_2\boldsymbol{r_2'} = 0$$

$$\boldsymbol{r_1'} = \boldsymbol{r} + \boldsymbol{r_2'}, \rightarrow \boldsymbol{r_2'} = -\frac{m_1\boldsymbol{r_1'}}{m_2} \rightarrow \boldsymbol{r_1'} = \boldsymbol{r} - \frac{m_1\boldsymbol{r_1'}}{m_2}$$

$$m_2\boldsymbol{r_1'} + m_1\boldsymbol{r_1'} = m_2\boldsymbol{r} \rightarrow \boldsymbol{r_1} = \frac{m_2}{m_1 + m_2}\boldsymbol{r} = \frac{m_2}{M}\boldsymbol{r}$$

$$\boldsymbol{r}_2' = \boldsymbol{r}_1' - \boldsymbol{r} = -\frac{m_2}{m_1}\boldsymbol{r}_2' - \boldsymbol{r} \rightarrow m_1\boldsymbol{r}_2' + m_2\boldsymbol{r}_2' = -m_1\boldsymbol{r}$$

$$\boldsymbol{r}_2' = -\frac{m_1}{m_1 + m_2}\boldsymbol{r} = -\frac{m_1}{M}\boldsymbol{r}$$

The same can be done with the velocities:

$$m_1\boldsymbol{v}_1' + m_2\boldsymbol{v}_2' = 0$$

$$\boldsymbol{v}_1' = \boldsymbol{v} + \boldsymbol{v}_2' = \boldsymbol{v} - \frac{m_1\boldsymbol{v}_1'}{m_2} \rightarrow \boldsymbol{v}_1 = \frac{m_2}{M}\boldsymbol{v}$$

$$\boldsymbol{v}_2' = \boldsymbol{v}_1' - \boldsymbol{v} = -\frac{m_2\boldsymbol{v}_2'}{m_1} - \boldsymbol{v} \rightarrow \boldsymbol{v}_2 = -\frac{m_1}{M}\boldsymbol{v}$$

Momentum of each object in the centre of mass reference frame can be found:

$$\boldsymbol{p}_1' = m_1\boldsymbol{v}_1' = \frac{m_1 m_2}{m_1 + m_2}\boldsymbol{v} = \mu\boldsymbol{v}$$

$$\boldsymbol{p}_2' = -\boldsymbol{p}_1' = -\mu\boldsymbol{v}$$

We call μ the reduced mass

Newtons laws of motion, and constant acceleration

01 April 2019 21:30

Newton's 3 laws:

1 - If there is no external force an objects velocity will be constant

2 - The force applied is equal to the rate of change of momentum:

$$\boldsymbol{F} = \frac{d\boldsymbol{p}}{dt}$$

This can be re-arranged such that:

$$\Delta\boldsymbol{p} = \boldsymbol{F}t$$

The R.H.S quantity is known as an impulse.

As momentum is the produce of mass and velocity, we can also find:

$$\boldsymbol{F} = m\frac{d\boldsymbol{v}}{dt} = m\boldsymbol{a}$$

3 - Every force has an equal and opposite reaction

If I push something with 10N of force with my hands, my hands (and so my whole body) will feel 10N in response.

If an object has constant mass and feels a constant force, then it will have a constant acceleration.

For constant acceleration we can use SUVAT equations.

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$

$$\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$$

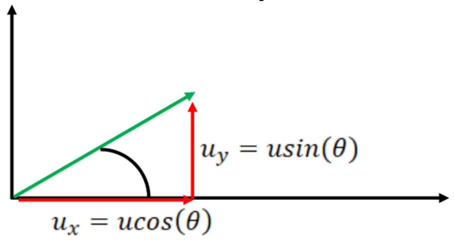
$$v^2 = u^2 + 2\boldsymbol{as}$$

$$\boldsymbol{s} = \frac{1}{2}(\boldsymbol{u} + \boldsymbol{v})t$$

When we have problems in multiple dimensions, we can look at each dimension separately. For example, in projectile motion we separate the motion into x and y components.

Imagine a projectile that is launched at an angle θ and an initial velocity of u. We can break this into the x and y components:

$$u_x = ucos(\theta), \qquad u_y = usin(\theta)$$



If we assume there is no air resistance, then the acceleration in the x direction will be 0, and so the velocity will be constant.

The only acceleration will be from gravity in the y direction.

We can therefore find the x and y positions as a function of time:

$$s_x = u_x t = ucos(\theta)t$$

$$s_y = u_y t + \frac{1}{2}a_y t^2$$

$$a = -g$$

$$s_y = usin(\theta)t - \frac{gt^2}{2}$$

As the time is the same for each dimension, we can re-arrange to find the projectiles y position as a function of its x position:

$$s_x = x \rightarrow t = \frac{x}{ucos(\theta)}$$

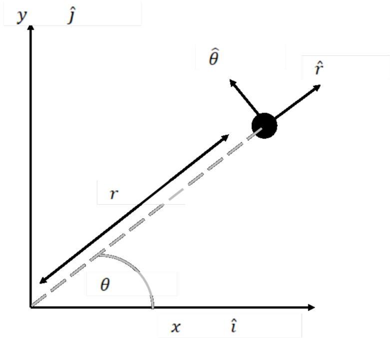
$$y = usin(\theta) * \left[\frac{x}{ucos(\theta)}\right] - \frac{g}{2}\left[\frac{x}{ucos(\theta)}\right]^2$$

$$y = x\tan(\theta) - \frac{gx^2\sec^2(\theta)}{2u^2}$$

Polar Coordinates, angular momentum, and central force

12 January 2019 21:32

It is often easiest to use polar coordinates rather than cartesian coordinates for objects moving in a circular like motion.



With the above diagram we can find the position of the particle in terms of cartesian and polar coordinates:

$$P = x\,\hat{\imath} + y\,\hat{\jmath}$$
$$P = \hat{r}r$$

We must also find the directions $\hat{r}$ and $\hat{\theta}$

$$\hat{r} = \cos(\theta)\,\hat{\imath} + \sin(\theta)\,\hat{\jmath}$$
$$\hat{\theta} = -\sin(\theta)\,\hat{\imath} + \cos(\theta)\,\hat{\jmath}$$

It's also easy to see that:

$$\frac{d}{d\theta}(\hat{r}) = \hat{\theta}$$

We now want to find the velocity and acceleration of the particle:

$$v = \frac{d}{dt}(P) = \frac{d}{dt}(\hat{r}r) = \frac{d}{dt}(\hat{r}) \cdot r + \frac{d}{dt}(r) \cdot \hat{r}$$

As $\hat{r}$ is in terms of θ, we use the chain rule to find $\dot{\hat{r}}$

$$\frac{d\hat{r}}{dt} = \frac{d\hat{r}}{d\theta}\frac{d\theta}{dt} = \hat{\theta}\dot{\theta}$$

This means velocity is found as:

$$v = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

We then differentiate again to find acceleration:

$$a = \frac{dv}{dt} = \frac{d}{dt}(\dot{r}\hat{r} + r\dot{\theta}\hat{\theta}) = \ddot{r}\hat{r} + \dot{r}\dot{\theta}\hat{\theta} + \dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} + r\dot{\theta}\frac{d\theta}{dt}$$

$$\frac{d\hat{\theta}}{dt} = \frac{d\hat{\theta}}{d\theta}\frac{d\theta}{dt} = -\hat{r}\dot{\theta}$$

And so:

$$a = \ddot{r}\hat{r} + \dot{r}\dot{\theta}\hat{\theta} + \dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} - r\dot{\theta}^2\hat{r}$$
$$a = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$

For both velocity and acceleration, the terms relating to $\hat{r}$ and $\hat{\theta}$ are radial and transverse components

When talking about angular movement, we need to look at angular momentum and torques.

Angular momentum is defined by:

$$\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p} = m(\boldsymbol{r} \times \boldsymbol{v})$$

Angular momentum is the equivalent of linear momentum for a rotating body, and the equivalent to force is torque

$$(Linear)\boldsymbol{F} = \frac{d\boldsymbol{p}}{dt}$$
$$(Angular)\boldsymbol{\tau} = \frac{d\boldsymbol{L}}{dt}$$

If a force acts on a body to rotate it, the torque can be found by:

$$\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F}$$

Where $\boldsymbol{r}$ is the vector from the point of rotation to the point that force is applied

When talking about angular motion, we often use angular velocity and angular acceleration:

$$(linear) v \to \omega = \frac{\boldsymbol{v}}{\boldsymbol{r}}$$

$$(linear) a \to \alpha = \frac{I}{\boldsymbol{\tau}}$$

Torque can also be written in an equivalent way to force:

$$(linear) \boldsymbol{F} = m\boldsymbol{a}$$

$$(angular) \boldsymbol{\tau} = I\boldsymbol{\alpha}$$

Where I is the moment of inertia of the body

When we have only a central force, we can write:

$$\boldsymbol{F}(\boldsymbol{r}) = F(r)\hat{r}$$

When we equate this to angular acceleration found earlier:

$$F(r)\hat{r} = m\boldsymbol{a} = m\left[(\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}\right]$$

We can see that only a radial acceleration is present, and so:

$$F(r) = m(\ddot{r} - r\dot{\theta}^2)$$

$$2\dot{r}\dot{\theta} + r\ddot{\theta} = 0$$

When there is only a central force, angular momentum $\boldsymbol{L}$ is conserved.

For a central force, the $\hat{\theta}$ term is 0:

$$m\left[2\dot{r}\dot{\theta} + r\ddot{\theta}\right] = 0$$

$$L = mr^2\dot{\theta}$$

$$\frac{dL}{dt} = m\left[\dot{\theta}\frac{d}{dt}(r^2) + r^2\frac{d}{dt}(\dot{\theta})\right]$$

$$= m\left[2r\dot{r}\dot{\theta} + r^2\ddot{\theta}\right]$$

$$= r * m\left[2\dot{r}\dot{\theta} + r\ddot{\theta}\right] = r * 0 = 0$$

The rate of change of angular momentum is 0, and so it is constant

SHM

02 January 2019 22:29

In SHM, an objects acceleration is directly proportional to the negative displacement:

$$m\ddot{x} = -kx$$
$$\ddot{x} = -\frac{k}{m}x$$

The simplest example is a stretched spring, which follows hooks law of $F = kx$

If a spring is vertical, there are two extensions. One which takes it to the new equilibrium position, and a second which is the displacement for the oscillation.

Gravity acts negatively down.

kx acts against the dispalcement, and is positivly up

x_0 is the streching of the string to account for gravity

x_1 is the stretching past the new equilibrum point.

$$ma = mg - k(x_0 + x_1)$$
$$kx_0 = mg$$
$$m\ddot{x} = -kx_1$$

Giving SHM about equilibrium point.

If we put in the form:

$$\ddot{x} = -\frac{k}{m}x$$

We can call $\frac{k}{m} = \omega^2$

Period of oscillation can be found by:

$$P = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

To solve the equation we re-arrange and have a second order differential:

$$\ddot{x} + \omega^2 x = 0$$

This has a general solution of:

$$x(t) = Acos(\omega t) + Bsin(\omega t)$$

This can be proved easily by differentiation.:

$$\dot{x}(t) = -A\omega\sin(\omega t) + B\omega\cos(\omega t)$$
$$\ddot{x}(t) = -\omega^2 A(cos(\omega t) + Bsin(\omega t)) = -\omega^2 x(t)$$

The values for A and B can be found by equation the displacement and velocity with t=0

$$t = 0, x(t) = x, A = x$$
$$t = 0, \dot{x}(t) = 0, \omega B = 0, B = 0$$

For SHM solution is $x(t) = x_{max}\cos(\omega t)$

In SHM total energy is constant. Potential can be found by integrating the force

$$V(x) = \frac{1}{2}kx^2 + V_0$$

We take $V(x = 0) = 0 \; so \; V_0 = 0$

$$\therefore V(x) = \frac{1}{2}ka^2\cos^2\omega t$$

Where a is max displacement

The kinetic energy can also be found:

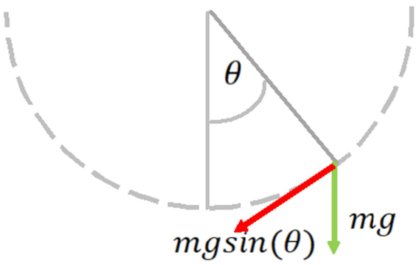
$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\,\dot{x}(t)^2 = \frac{1}{2}ma^2\omega^2\sin^2\omega t$$

And so total energy is:

$$E = KE + V = \frac{1}{2}a^2(kcos^2(\omega t) + m\omega^2\sin^2(\omega t))$$
$$\omega^2 = \frac{k}{m}$$
$$E = \frac{1}{2}ka^2$$

Constant as we can see

A simple pendulum can be approximated to SHM



In this example we ignore tension as it is balanced by $mgcos(\theta)$
We call the distance travelled along the arc s.

$$s = l\theta, and\ so\ \ddot{s} = l\ddot{\theta}$$

The force making the pendulum move is gravity, it accelerates it towards $\theta = 0$ in the s direction

$$m\ddot{s} = -mgsin(\theta)$$
$$l\ddot{\theta} = -gsin(\theta)$$

For small angles $\sin(\theta) = \theta$ (Taylor expansion)

$$\therefore \ddot{\theta} = -\frac{g}{l}\theta$$

A pendulum will exhibit SHM for small theta, with $\omega = \sqrt{\frac{g}{l}}$, and so Period $T = 2\pi\sqrt{\frac{l}{g}}$

See end of this document for other methods of this derivation (Using energy)

In the real world, oscillations have some damping force acting against them. This force opposes the motion of the particle. For this course we only look at forces proportional to the velocity of the particle:

We use λ, a positive constant to describe the size of a damping force.
It opposes the motion of the particle, and so we can set up an equation of motion:

$$m\ddot{x} = -kx - \lambda\dot{x}$$
$$m\ddot{x} + \lambda\dot{x} + kx = 0$$

This is a second order differential, we need to solve it.

We introduce $\gamma = \frac{\lambda}{2m}$ and so

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = 0$$

We solve as a quadratic:

$$q^2 + 2\gamma q + \omega_0^2 = 0$$

With solutions:

$$q = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2}$$
$$q = -\gamma \pm i\omega$$

Where $\omega = \sqrt{\omega_0^2 - \gamma^2}$

And so the solution to the equation of motion is:

$$x(t) = Ae^{q_1 t} + Be^{q_2 t}$$
$$x(t) = Ae^{(-\gamma+i\omega)t} + Be^{(-\gamma-i\omega)t}$$
$$x(t) = e^{-\gamma t}(Ae^{i\omega t} + Be^{-i\omega t})$$

This means motion is dependant on size of damping constant:

$\gamma = 0$

No damping, SHM, simplifies via complex to:

$$x(t) = Acos(\omega t) + Bsin(\omega t)$$

$\gamma < \omega_0$

$\therefore \omega\ is\ real$

$$x(t) = e^{-\gamma t}(Ae^{i\omega t} + Be^{-i\omega t})$$

The first term exponentially decays, the second is SHM equation.
Gives exponentially decaying synodal graph.
This is **Under Damping/Light Damping**

$\gamma = \omega_0$

$$\omega = 0 \therefore x(t) = e^{-\gamma t}(A + Bt)$$

There is no oscillation. This oscillation stops is the quickest time.
This is **Critical Damping**

$\gamma > \omega_0$

$\omega = \sqrt{\omega_0^2 - \gamma^2} \therefore \omega\ is\ imaginary$

As it is imaginary, the of the equation are real:

$e^{-\gamma t}(Ae^{\omega t} + Be^{-\omega t})$

This means there is no oscillation. This takes longer to reach 0 than critical damping.

This is known as **Over Damping/Heavy Damping**

In a Force Damped Oscillator, an oscillating force acts on a system.

Our equation is in a similar form as before, but now with a periodic force on the RHS

$m\ddot{x} + \lambda\dot{x} + kx = F_0 \cos(\omega_f t)$

The driving force has a maximum value of F_0 and a period of ω_f

We ignore the stage of transitioning from no motion, and simply look at the solution when the system is already in motion.

To solve this, we assume the equation takes the form:

$x(t) = R\cos(\omega_f + \phi)$

This is easiest to solve in the complex form:

$R\cos(\omega_f + \phi) = Real(z(t)), z(t) = Re^{i(\omega_f + \phi)}$

$\dot{x}(t) = i\omega_f x(t)$

$\ddot{x} = -\omega_f^2 x(t)$

$m\left(-\omega_f^2 x(t)\right) + \lambda\left(i\omega_f x(t)\right) + kx(t) = F_0 e^{i\omega_f}$

$x(t)\left[-\omega_f^2 + 2i\gamma\omega_f + \omega_0^2\right] = f_0 e^{i\omega_f}$

$(f_0 = F_0/m)$

Divide by $e^{i\omega_f}$

$Re^{i\phi}\left[-\omega_f^2 + 2i\gamma\omega_f + \omega_0^2\right] = f_0$

$$Re^{i\phi} = \frac{f_0}{-\omega_f^2 + 2i\gamma\omega_f + \omega_0^2}$$

Take modulus squared for real result.

$$R^2 = \frac{f_0^2}{\left(\omega_0^2 - \omega_f^2\right)^2 + 4\gamma^2\omega_f^2} \rightarrow R = \frac{F_0}{m\sqrt{\left(\omega_0^2 - \omega_f^2\right)^2 + 4\gamma^2\omega_f^2}}$$

We can see that this means R is maximum when $\omega_0 = \omega_f$

This is called resonance. The amplitude of a forced oscillation is largest when the driving force has an equal frequency to the natural frequency of the system.

Through complex numbers, we can eventually find:

$$\tan(\phi) = \frac{2\gamma\omega_f}{\omega_f^2 - \omega_0^2}$$

At the point of resonance, $\phi = \frac{\pi}{2}$

This means the system lags behind by $\frac{\pi}{2}$

This means the driving force always acts to accelerate the system

No energy is lost fighting the driving force

Hence maximum amplitude

Orbits

15 January 2019 17:46

In an orbit there is only 1 central force, which can be written as:

$m(\ddot{r} - r\dot{\theta}^2) = F(r)$

As there is only a central force, the angular momentum is constant:

$$L = mr^2\dot{\theta} \rightarrow \dot{\theta} = \frac{L}{mr^2}$$

$$\rightarrow m\ddot{r} = F(r) + \frac{L^2}{mr^3}$$

The second term here is known as the centrifugal force. It is the fictitious force needed to balance the radial force.

In circular motion, the radius is constant, and so $\ddot{r} = 0$, meaning the centrifugal force (acting radially outwards) must have equal magnitude to the central force (acting radially inwards).

We can write centrifugal potential as:

$$V_C = \frac{1}{2}\frac{L^2}{mr^2}$$

The most import thing with orbits is to know the first 2 equations.

Angular momentum is constant with only a radial force

We can imagine an orbit without a constant radius.

We first look at the circular orbit with a radius of r_0. As it is circular, the $m\ddot{r}$ term is equal to 0. If we let the attractive force be an attractive power force it will take the form

$$F(r) = -Kr^n \rightarrow mr_0\dot{\theta}^2 = Kr_0^n$$

The period is then:

$$\tau = \frac{2\pi}{\dot{\theta}} = 2\pi\sqrt{\frac{m}{Kr_0^{n-1}}}$$

As we have a nearly circular orbit we can write the actual radius as a point close to r_0:

$$r = r_0 + x$$

We can then taylor expand each term which results in:

$$m\ddot{x} = -xKr_0^{n-1}[n+3]$$

For $n \succ -3$ this gives symple harmonic motion, with the period as:

$$\mathrm{T} = \frac{\tau}{\sqrt{n+3}}$$

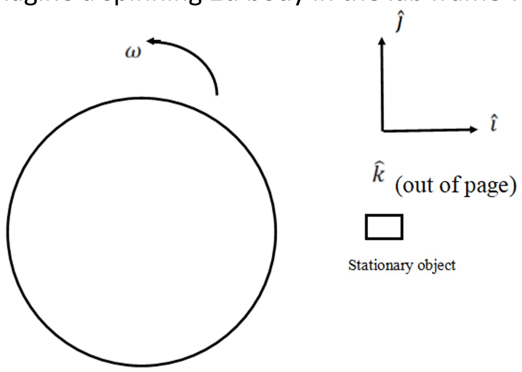
If n is -2, as with gravity for example, the period is the same, and the orbit forms an elliptical orbit.

Accelerating and rotating reference frames

31 December 2018 12:29

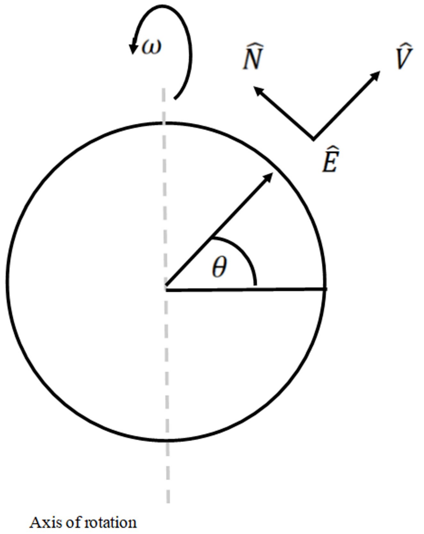
Fictitious forces are required to balance real forces when in a frame with non constant velocity
- This is because Newton's Laws are the same in all inertial frames
- There are two fictitious forces (that we cover)
  - Centrifugal Force

  - Coriolis Force
    - When on a rotating body, a body travelling in a straight line will follow a curved path.
    - The fictitious force which makes it follow this path is the Coriolis force.
    - Imagine a spinning 2d body in the lab frame with an object near it:



    - From the lab frame, there are no forces (as expected)
    - However, from the reference frame of an object on the turning table:
      - The stationary object is spinning around the object in the opposite direction.
      - ω acts in the $\hat{k}$ direction
    - There is 0 net force on the object so:
      - $F_{real} = F_{fict}$ to balance
      - $F_{real} = m\omega^2 R\ (-\hat{r})$
      - We know the fictitious forces include centrifugal:
      - $F_{fict} = m\omega^2 R\hat{r}$
      - But this doesn't balance, to balance we need another fictitious force:
      - $F_{fict} = m\omega^2 R\ \hat{r} + (-)2m\omega^2 R(\hat{r})$
      - The second fictitious force is the Coriolis force
    - $2m\omega^2 R = 2m\boldsymbol{\omega}\ \times \boldsymbol{v}$
    - $\boldsymbol{\omega} = \omega\hat{\omega} = \omega\hat{k}, \boldsymbol{v} = v \times (-\hat{\jmath})$
    - And so:
      - $F_{cor} = -2m\ \omega v\ (\hat{\imath}) = -2m\omega v\hat{r}$
    - And so the Coriolis force is found

  - A common example in an object moving on earth:



At a given latitude (θ) the angular velocity is $\boldsymbol{\omega} = \omega\hat{\omega}$
Going from $\hat{\omega}$ to our vector system:

$$\hat{\omega} = \sin(\theta)\,\hat{V} + \cos(\theta)\,\hat{N}$$

Suppose we have an object travelling directly east:

$$\boldsymbol{v} = v\hat{E}$$

And so the Coriolis force:

$$F_{cor} = -2m\boldsymbol{\omega} \times \boldsymbol{v} = -2m\omega v\left(\sin(\theta)\left(\hat{V} \times \hat{E}\right) + \cos(\theta)\left(\hat{N} \times \hat{E}\right)\right)$$

$$= -2m\omega v\left(\sin(\theta)\,\hat{N} + \cos(\theta)\left(-\hat{V}\right)\right)$$

This means the force will act south and vertically up for an object travelling east.

Rigid bodies

15 January 2019 17:47

A rigid body can be described as many small points rigidly joined together.

For a large number of particles we can describe this by their density as a function of position.

The kinetic energy of rotation of a Rigid body is similar to normal kinetic energy, with velocity being replaced with angular velocity, and the mass being replaced by the moment of inertia, I

$Ke_{rot} = \frac{1}{2} I\omega^2$

The moment of inertia describes the resistance to rotating an object, similarly to how mass describes the resistance to moving an object.

The moment of inertia can be found by summing the distance to each infinitesimal point by the mass of the point, for all points in the object.

This means the moment of inertia is dependent on the axis of rotation.

If we have a 2 dimensional flat object, the total moment of inertia is equal to the sum of the moment of inertia of each axis perpendicular to the axis of rotation.

$I_z = I_x + I_y$

This is the theory of perpendicular axes

If we known the moment of inertia at a point, the theorem of parallel axes allows us to find the moment of inertia about another point that is in a direction parallel to one axis.

$I_a = I_0 + Ma^2$

Where a is the distance from the initial point to the new point