PHAS1247 Classical Mechanics
In-Course Assessment Test 2
(Thursday 13 December 2012)

Answer as many of the questions as you can, in any order. Calculators may be used. The approximate distribution of marks is given in square brackets on the right of the page; the maximum mark is 30. **Note the final question continues over the page.**

1. A binary star system consists of two stars of masses M_1 and M_2 orbiting their common centre of mass. If the orbits of both stars are observed to be circular and to have period T, find an expression for the distance R between the stars. **[5]**

2. Santa's helpers have their secret headquarters in Lapland at a latitude 70° N. They are flight-testing his sleigh, which has a mass when fully loaded of $m = 1000\,\mathrm{kg}$. Given that the radius of the Earth is approximately 6400 km, find the horizontal component (i.e. the component parallel to the ground) of the centrifugal force on the sleigh arising from the Earth's rotation. **[4]**

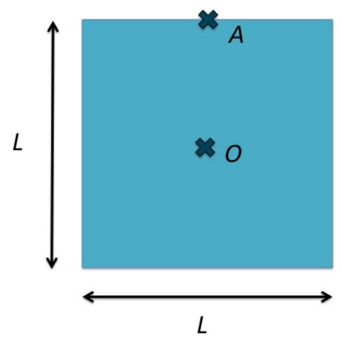
   In the course of the flight test the sleigh is put into a vertical dive at a speed $v = 20\,\mathrm{m\,s^{-1}}$ Find the magnitude and direction of the Coriolis force on the sleigh during the dive. **[4]**

3. Show that the moment of inertia of a uniform square plane of mass M and side L about an axis O passing through its centre and at right angles to the plane is

$$I_0 = \frac{ML^2}{6}.$$

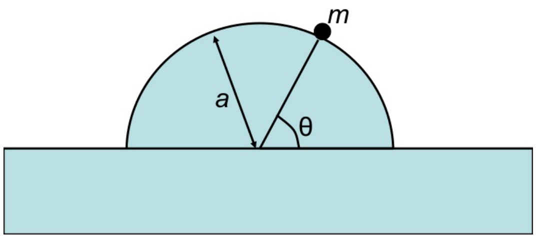
[5]

   Find also its moment of inertia about a parallel axis A through the middle of one side (see diagram). **[2]**



4. A mass m is placed on the top of a smooth hemisphere of radius a such that $\theta = \pi/2$ (see diagram overleaf for the definition of θ). It is given a very small impulse and as a result begins to slide down one side of the hemisphere under the influence of the gravitational acceleration g.

TURN OVER



State the forces acting on the mass, giving their directions, and write down its radial and angular equations of motion in polar coordinates so long as it remains sliding on the sphere. **[4]**

Use conservation of energy to find an expression for the speed of the object at a general angle θ, so long as it remains on the hemisphere. Hence find the reaction force between the mass and the surface of the hemisphere as a function of θ, and show that the mass flies off the surface of the hemisphere when its vertical height has decreased by $a/3$. **[6]**