Expectation value

$\psi(x) = \sin(\pi x) \quad 0 \leq x \leq 2$
$\quad\;\; = 0 \quad$ for all other x

$$\langle x \rangle = \int_{-\infty}^{\infty} |\psi(x)|^2 x \, dx$$

$$\langle x \rangle = \int_0^2 \sin^2(\pi x)\, x \, dx$$

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\langle x \rangle = \int_0^2 \frac{x}{2}(1 - \cos 2\pi x)\, dx$$

$$\langle x \rangle = \int_0^2 \frac{x}{2}\, dx - \underbrace{\int_0^2 \frac{x \cos 2\pi x}{2}\, dx}_{\text{Integrate by parts.}}$$

$$\int_a^b u(x) \frac{dv(x)}{dx}\, dx = \Big[u(x)v(x)\Big]_a^b - \int \frac{du(x)}{dx} v(x)\, dx = \underbrace{\cancel{\left[x \frac{\sin 2\pi x}{2\pi}\right]_0^2}}_{0} - \int_0^2 \frac{\sin(2\pi x)}{2\pi}\, dx$$

$u(x) = x \qquad \frac{du(x)}{dx} = 1$

$\frac{dv(x)}{dx} = \cos 2\pi x \qquad v = \frac{\sin 2\pi x}{2\pi}$

$$= -\underbrace{\cancel{\left[-\frac{\cos(2\pi x)}{4\pi^2}\right]_0^2}}_{0}$$

$$\langle x \rangle = \int_0^2 \frac{x}{2}\,dx = \left[\frac{x^2}{4}\right]_0^2 = 1$$

$|\psi|^2$