UNIVERSITY COLLEGE LONDON

EXAMINATION FOR INTERNAL STUDENTS

MODULE CODE	:	**PHAS1202**
ASSESSMENT PATTERN	:	**PHAS1202A**
MODULE NAME	:	**Atoms, Stars and the Universe**
DATE	:	**06 May 2016**
TIME	:	**10:00 am**
TIME ALLOWED	:	**2 hours 30 mins**

This paper is suitable for candidates who attended classes for this module in the following academic year(s):

2014/15 and 2015/16

2015/16-PHAS1202A-001-EXAM-209

TURN OVER

Answer ALL SIX questions from Section A.

Answer THREE questions from Section B, including AT LEAST ONE question from EACH of Sections B1 and B2.

The following may be assumed if required:

Planck's constant	h	6.63×10^{-34} m^2 kg s^{-1}
	$\hbar$	1.05×10^{-34} m^2 kg s^{-1}
Electron mass	m_e	9.11×10^{-31} kg
Electron charge	e	1.60×10^{-19} C
Proton mass		1.0078 amu
Helium mass		4.0026 amu
Atomic mass unit	amu	1.66×10^{-27} kg
Bohr radius	a_0	5.3×10^{-11} m
Permittivity of free space	ϵ_0	8.85×10^{-12} F m^{-1}
Speed of light	c	3.0×10^8 m s^{-1}
Solar radius	$R_\odot$	7.0×10^8 m
Solar luminosity	$L_\odot$	3.8×10^{26} W
Solar mass	$M_\odot$	2.0×10^{30} kg
Hubble constant	H_0	75 km s^{-1} Mpc^{-1}
Gravitational constant	G	6.67×10^{-11} m^3 kg^{-1} s^{-2}
1 parsec	pc	3.1×10^{16} m
1 year	yr	3.16×10^7 s
1 Angstrom	Å	10^{-10} m
1 electron Volt	eV	1.60×10^{-19} J

Numbers in square brackets in the right-hand margin indicate a provisional allocation of maximum possible marks for different parts of each question.

Section A – *(Answer ALL SIX questions from this section)*

1. Describe the process that leads to the formation of absorption lines in the spectrum of a hot object seen through a cooler gas, and give three astrophysical examples of this phenomenon. **[7]**

2. Aldebaran is a K5 III star, Sirius is a A1 V star and Rigel is a B8 Ia star. Explain what is meant by these classification terms, including indications of approximate surface temperature. **[6]**

3. Sketch a diagram of the 'rotation curve' for our Milky Way Galaxy, carefully labelling the axes. Comment on the shape of the rotation curve, and the main inference to be drawn from it. **[7]**

4. A laser pointer emits green light (wavelength 500nm) with a power output of 1mW. Compute the energy of a single photon at this frequency. How many photons per second are emitted by the laser pointer? **[6]**

5. Describe an experiment which implies that light travels as a wave, giving a short but clear argument for why the observations support a wave interpretation. Describe a second experiment which implies that light has particle characteristics, explaining clearly how the observations made in the experiment are incompatible with a wave theory of light. **[8]**

6. Which of the following conditions is fulfilled when a wavefunction $\psi(x)$ is normalised? **[6]**

   i) $\int_{-\infty}^{\infty} \psi(x)dx = 1$

   ii) $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$

   or iii) $\int_{-\infty}^{\infty} |\psi(x)| dx = 1$.

   Name one further condition which $\psi(x)$ must satisfy to be a physical wavefunction. How do we calculate the probability of finding a particle with a normalised wavefunction $\psi(x)$ in a certain interval, e.g. between $x = a$ and $x = b$?

Section B – *(Answer THREE questions from this section, including AT LEAST ONE question from EACH of Sections B1 and B2)*

Section B1 – *(Answer AT LEAST ONE question from this section)*

7. The time-independent Schrödinger equation (TISE) in one-dimension is:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x).$$

where m is the mass of the particle, E its energy, $\psi(x)$ the wavefunction and $V(x)$ the potential.

(a) Consider a particle moving in a constant potential, i.e. $V(x) = V_0$. Assuming $E > V_0$, show that the following wavefunction

$$\psi(x) = A\sin(\kappa x) + B\cos(\kappa x)$$

is a solution of the TISE and find an expression for E in terms of κ, $\hbar$, m and V_0. **[4]**

Write down a wavefunction that is a solution to the TISE for the same potential, but when $E < V_0$. **[2]**

(b) Consider an infinite square well with walls at $x = 0$ and $x = L$. The potential is zero inside the well, i.e. $V(x) = 0$ for $0 \leq x \leq L$, and is infinite outside the well, i.e. $V(x) = \infty$ for $x < 0$ and $x > L$. By identifying the form of the wavefunction in each of these regions and considering boundary conditions, show that the allowed energies for a particle in the well are **[10]**

$$E_n = \left(\frac{h^2}{8mL^2}\right) n^2$$

for integer n. Why is the $n = 0$ case not an allowed energy?

(c) Consider an electron trapped in an infinite square well in one dimension. How wide must the well be such that a transition from the $n = 2$ state to the ground state will emit red light at 620nm? **[4]**

8. (a) Describe the process of quantum tunnelling. Include a sketch of a relevant potential and the form of the wavefunction both inside and outside the barrier. Describe how quantum predictions differ from those of classical physics. [You do not need to derive the exact forms of wavefunctions from the Schrödinger equation – a qualitative sketch of their form is sufficient.] **[6]**

(b) The Scanning Tunnelling Microscope (STM) is a device which depends crucially on quantum tunnelling. Name a physical process or technology (other than the STM) in which quantum tunnelling plays an important role. **[1]**

(c) The probability that a quantum particle with mass m and energy E will tunnel through a square barrier of width L and height U is approximately equal to:

$$P = \exp[-2CL]$$

where

$$C = \frac{\sqrt{2m(U-E)}}{\hbar}.$$

In an STM, electrons tunnel from an electrode across a potential barrier to a surface, completing a circuit whose current I is proportional to the tunnelling probability. Draw a labelled diagram of an STM indicating the path of an electron through such a device. Indicate clearly the position of the tunnelling barrier in your diagram. **[4]**

(d) If L represents the distance between electrode and surface, the current I passing through the device will be **[3]**

$$I \propto \exp[-2CL]\,.$$

A microscope is set up so that the tunnelling barrier height is 10eV and incident electrons have energy 2eV. Calculate the constant C for this microscope, justifying its unit.

(e) This microscope is positioned over a surface and its ammeter records a current of 0.1A. It then scans over a surface feature and the current rises to 7.7A before falling to 0.1A again. What is the height of this surface feature? **[6]**

9. (a) Describe Rutherford's planetary atomic model for Hydrogen and explain two ways in which the model was incompatible with the known properties of Hydrogen at the beginning of the 20th century. [3]

(b) Building on Rutherford's work, Niels Bohr proposed his own atomic model. In Bohr's model, how is the angular momentum of the electron restricted? [1]

Describe one way in which Bohr's model is inferior to a fully quantum mechanical model of the atom. [1]

(c) An ion is called Hydrogen-like when it consists of a single electron orbiting a nucleus of charge $+Ze$, where Z is the ion's atomic number. The magnitude of the Coulomb force $|F|$ and the potential energy V between two objects with charges q_1 and q_2 separated by distance r are as follows,

$$|F| = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2}, \qquad V = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}.$$

Given that a particle in a circular orbit of radius r and speed v undergoes centripetal acceleration v^2/r and has angular momentum $l = mvr$, show that, in the Bohr model for a Hydrogen-like ion, the radii of electron orbits are restricted to the values [6]

$$r_n = \frac{a_0 n^2}{Z}.$$

Derive an expression for the Bohr radius a_0 in terms of $\hbar$, ϵ_0, m_e and e.

(d) Hence, show that, in the Bohr model, the energy levels of the electron in a Hydrogen-like ion have energies that depend on integer n and satisfy [4]

$$E_n = -\frac{Z^2 \times 13.6}{n^2}\ \text{eV}.$$

(e) Consider the nth energy level of the Hydrogen atom and the mth energy level of a Helium+ ion. How must m and n be related for the energies to coincide? Hence identify the lines in the spectrum of Helium+ that coincide with a visible spectral line of Hydrogen. [5]

Section B2 – *(Answer AT LEAST ONE question from this section)*

10. (a) List the stages of the dominant chain reaction that describes the primary energy source in the Sun. **[6]**

(b) By first outlining the basic physical properties of neutrinos, describe the solar neutrino problem and how it has been solved. **[8]**

(c) Sirius A has a luminosity 23.5 times that of the Sun and burns H into He. How many kg of H does Sirius A burn into He each second? If Sirius A has 2.3 times the mass of the Sun and 10% of its mass converts from H to He, calculate the lifetime (in years) of Sirius A. **[6]**

11. (a) Describe the main differences between spiral and elliptical galaxies in terms of content and the properties and motion of stars. **[9]**

(b) Calculate the critical density of the Universe for a Hubble constant of 75 km s^{-1} Mpc^{-1}. **[4]**

(c) Calculate the density of visible mass in the Local Group of galaxies, assuming that it contains 4×10^{12} $M_{\odot}$ of matter within a spherical volume of 3 Mpc. If the Universe has a similar density as the Local Group value, state whether the Universe is open or closed. Explain which fundamental principle is invoked in this estimate. **[7]**