PHAS1247 Classical Mechanics
Problem-Solving Tutorial 1, 16–20 October 2017
Vectors, Newton's laws, momentum

Objectives: To gain problem-solving experience in

- The application of vectors and calculus to physical problems;
- Application of the concepts of momentum, impulse and energy.

Take the gravitational acceleration $g = 9.81\,\mathrm{ms}^{-2}$ throughout. You may neglect air resistance in all the problems.

Section A—calculus, vectors and motion

1. A particle of mass $m = 2\,\mathrm{kg}$ has an initial velocity $\mathbf{v} = 2\hat{\mathbf{i}}\,\mathrm{m\,s}^{-1}$. It experiences a constant force of $2\hat{\mathbf{j}}\,\mathrm{N}$ for a time of 5 s. What is its final velocity?

2. A particle's position vector $\mathbf{r}$ as a function of time t is

$$\mathbf{r}(t) = \alpha t\hat{\mathbf{i}} + (\beta t^3 + \gamma)\hat{\mathbf{j}} + \eta t^2\hat{\mathbf{k}}.$$

   Find (i) its velocity and (ii) its acceleration at a general time t in terms of the constants α, β, γ and η. In which direction is the force on the particle at time $t = 0$?

3. A gardener holds a hose so the water emerges horizontally at a height of 1 m above the ground. It lands on a flowerbed 2 m in front of her; what is the speed of the water (a) when it emerges from the hose and (b) just before it hits the ground?

Problem for general discussion

A firework of mass M is launched into the sky at an angle, so that it reaches its highest point when it is $d = 10\,\mathrm{m}$ horizontally away from its launch point. When it is precisely at this highest point the firework explodes, splitting into two fragments of equal mass $M/2$. The two fragments are observed to fall back to Earth at exactly the same time. One lands at the launch point; where does the other fragment land?

[You may neglect air resistance and assume that forces from the firework's propulsion systems operate only at the instant of launch and the instant of the explosion.]

Section B—Newton's laws and momentum

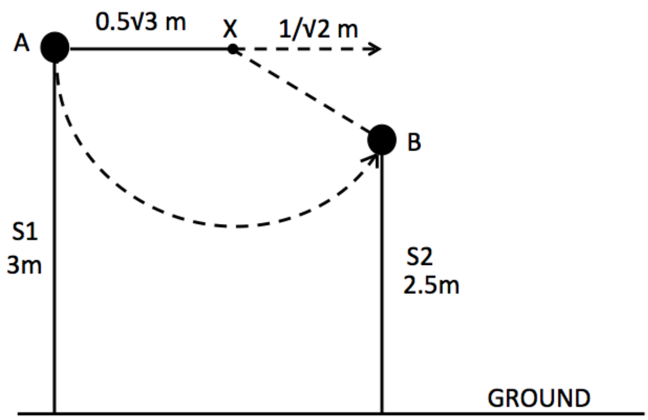
1. A boy kicks a football into the air at a speed of $20\,\mathrm{ms}^{-1}$ and at an angle of $30°$ to the horizontal. What are (a) the minimum distance and (b) the maximum distance he can be from the foot of a thin wall, 3 m high, in order for the ball to clear the top of the wall without bouncing first?

2. A rifle bullet of mass $m = 0.03\,\mathrm{kg}$ is fired horizontally at a speed of $v = 400\,\mathrm{ms}^{-1}$ into a sand-bag of mass $M = 2\,\mathrm{kg}$ (considered as a point particle), suspended by a light inextensible string of length $l = 1.8\,\mathrm{m}$ from a beam. The bullet remains within the sand-bag and no sand falls out.

   What can you say about the horizontal momentum of the system (bullet + sand-bag) during the collision? Hence determine the (i) the velocity of the sand-bag with the embedded bullet after the collision, (ii) the initial kinetic energy, and (iii) the final kinetic energy. Explain any difference between your answers to (ii) and (iii).

   Also determine the maximum height h_{max} attained by the sand-bag after the collision.

3. As shown in the figure below a mass (A) of 5 kg is fixed to point X via an inextensible string of negligible mass and length $\frac{\sqrt{3}}{2}$ m. The mass is stationary on a support (S1) that is is 3 m above the ground and $-\frac{\sqrt{3}}{2}$ m horizontally from X. S1 is removed and the mass falls freely under gravity with the string remaining under tension. A second mass (B) of 2 kg is stationary on a support (S2) at a height of 2.5 m above the ground and a horizontal distance of $+\frac{1}{\sqrt{2}}$ m from X.



   At the instant A collides with B the string detaches from A and the masses coalesce to form a single object that falls freely under gravity. Neglecting the size of the masses and any forces due to the earth's rotation:

   (a) Determine the angle $A\hat{X}B$ at which the balls collide.

   (b) Determine the maximum height above the ground attained by the coalesced masses after the collision.

   (c) How far from the base of S2 does the coalesced mass hit the ground?