PHAS0004

Astrophysics PST1 – revised solutions – Dec 2018

Q1

If the ionisation potential of the hydrogen atom is 13.6 eV, then $E(1) = 13.6$ eV and

$$E(n) = 13.6(1 - \frac{1}{n^2})\,\mathrm{eV}$$

Thus $E(3) - E(4) = 13.6\left(\frac{1}{3^2} - \frac{1}{4^2}\right)\mathrm{eV} = 0.661\,\mathrm{eV}$ and

$$\lambda = \frac{c}{\nu} = \frac{hc}{E} = \frac{6.63\times10^{-34}\times3\times10^{8}}{0.661\times1.6\times10^{-19}}\,\mathrm{m} = 1.881\times10^{-6}\,\mathrm{m}$$

Transitions whose lower level has $n = 3$ are called the *Paschen series*.

The transition between $n = 3$ and $n = 4$ is called Pα.

Q2.

The spectra of the two stars can be approximated as blackbodies,
for which the relation between temperature and wavelength of the
emission peak (when the blackbody curve is expressed as a function
of wavelength, see Q7) can be written as:

$$\lambda_{\max} \approx \frac{3\times10^{-3}}{T}$$ where $\lambda_{\max}$ is in m and T in K. Thus

HD36956: T=24100 K → $\lambda_{\max} = 1.24 \times 10^{-7}$ m → ultraviolet waveband
Epsilon Indi : T=2800 K → $\lambda_{\max} = 1.07 \times 10^{-6}$ m → infrared waveband

Ratio of energies emitted = $(T_{HD36956})^4/(T_{\text{eps. Indi}})^4 = 5488$

Q3.

Brightness, b, depends upon luminosity, L, and distance, d, according to the inverse square law: $b \propto L/d^2$
and so if two stars are at the same distance from Earth then the brightness is determined by the luminosity. The total luminosity is proportional to the energy flux and the surface area:
$L \propto$ (Energy Flux) x (Surface Area)
Since the stars have the same size, the relative luminosity is determined by the stars' respective energy flux, which is in turn, determined from the Stefan- Boltzmann law: Flux $\propto T^4$
Thus the brighter star will be the hotter one and a blue star is hotter than a red one. Hence the blue star will be brighter in the night sky.

Q4.

Luminosity $L = 4 \pi R^2 \sigma T^4$

For photosphere, $L = 4 \pi (7 \times 10^8)^2 \times (5.7 \times 10^{-8}) (5800)^4$

$= 4.0 \times 10^{26}$ W

For corona, $L = 4 \pi (1.4 \times 10^9)^2 \times (5.7 \times 10^{-8}) (2 \times 10^6)^4$

$= 2.2 \times 10^{37}$ W

The result for the corona is almost 11 magnitudes greater than the solar luminosity! This tells you that the Blackbody approximation is not valid for the corona. This is the danger of applying Blackbody approximations to objects which are not opaque blackbodies.

Q5

The monochromatic intensity is given by the

$I_\lambda = (2\, h\, c^2 / \lambda^5)\, (1 / [\exp(h\, c / \lambda\, k\, T)] - 1)$

For large exponentials ($[\exp(h\, c / \lambda\, k\, T)] >> 1$)

$I_\lambda \sim (2\, h\, c^2 / \lambda^5) \exp(-h\, c / \lambda\, k\, T)$

$I_{sunspot} / I_{photosphere} = \exp(-h\, c / \lambda\, k\, T_{sunspot}) / \exp(-h\, c / \lambda\, k\, T_{photosphere})$

$= \exp\{(-h\, c / \lambda\, k)\, (1/T_{sunspot} - 1/ T_{photosphere})\}$
$= \exp[-\{ (6.63 \times 10^{-34})\, (3.0 \times 10^8) / \lambda\, (1.38 \times 10^{-23}) \} \times$
$(1/T_{sunspot} - 1/ T_{photosphere})]$
$= \exp[-\{ (1.44 \times 10^{-2}) / \lambda \} \times (1/T_{sunspot} - 1/ T_{photosphere})]$

For $\lambda = 550$ nm:
$I_{sunspot}/ I_{photosphere} = \exp[\{-1.44 \times 10^{-2}/ 5.5 \times 10^{-7} \} \times (1/4000-1/ 5800)]$

$= 0.13$

For $\lambda = 1$ µm:

$I_{sunspot} / I_{photosphere} = \exp[\{ -1.44 \times 10^{-2} / 1 \times 10^{-6} \} \times (1/4000-1/ 5800)]$

$= 0.33$ (i.e. greater sunspot to photosphere intensity contrast in visible light)

Q6
If λ and λ_o are the observed and emitted wavelengths respectively:

$$\frac{\lambda - \lambda_0}{\lambda_0} = \frac{v}{c}$$

Thus the observed wavelength will be

650 nm $(1 - 200/3 \times 10^5)$= 649.57 nm
(The velocity is negative because the star is travelling towards us)

(ii) Similarly for the cloud of H:

21 x 1.0005 cm = 21.0105,
and
v=c/ λ = 300000*100000/(21.0105) = 1427.9 MHz

Q7
(i)

$$\boxed{I(\lambda)=\frac{2hc^2}{\lambda^5}\left[\frac{1}{e^{hc/\lambda kT}-1}\right]}$$

I(λ) d λ = I(v) dv

So I(v) = I(λ) (d λ/dv)

But v =c/ λ, so |d λ/dv| =(1/c) λ^2

Hence

$$\boxed{I(\nu)=\frac{2h\nu^3}{c^2}\left[\frac{1}{e^{h\nu/kT}-1}\right]}$$

(ii)

$$\boxed{\frac{d}{d\lambda}I(\lambda,T)=0 \;\Rightarrow\; \frac{hc}{\lambda k_B T}\frac{e^{\frac{h\,c}{\lambda\,k_B\,T}}}{e^{\frac{h\,c}{\lambda\,k_B\,T}}-1}-5=0 \;\Rightarrow\; \lambda_{max}=\frac{hc}{x}\frac{1}{k_B T}=\frac{b}{T}}$$