Cover-page

PHAS1202

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Atoms, Stars and The Universe

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Answer ALL SIX questions from Section A and ANY THREE questions from Section B

Numbers in square brackets in the right-hand margin indicate a provisional allocation of maximum possible marks for different parts of each question.

The following may be assumed if required:

Proton mass		1.0078 amu
Helium mass		4.0026 amu
Atomic mass unit	amu	1.66×10^{-27} kg
Speed of light	c	3.0×10^{8} m s^{-1}
Solar radius	$R_\odot$	7.0×10^{8} m
Solar luminosity	$L_\odot$	3.8×10^{26} W
Solar mass	$M_\odot$	2.0×10^{30} kg
1 parsec	pc	3.1×10^{16} m
1 year	yr	3.16×10^{7} s
Hubble constant	H_0	75 km s^{-1} Mpc^{-1}
Planck's constant	h	6.63×10^{-34} m^2 kg s^{-1}
	$\hbar$	1.05×10^{-34} m^2 kg s^{-1}
Electron mass	m_e	9.11×10^{-31} kg
Electron charge	e	1.60×10^{-19} C
1 Angstrom	Å	10^{-10} m
1 electron Volt	eV	1.60×10^{-19} J

Section A
(Answer ALL SIX questions from this section)

1. State the value of the Chandrasekhar limit and explain why there are no white dwarfs with a mass that exceeds it. **[4]**
   State what the 'Schwarzschild radius' is, and give an alternative name for it. **[3]**

2. Briefly explain what 'Cepheid variables' are, and why they are useful in determining distances to galaxies. **[6]**

3. Give a brief account of the origin of the cosmic microwave background radiation in the early Universe and explain how it verifies the Cosmological Principle. **[7]**

4. A street lamp emits light with a power output of 80 W at a wave-length of 589nm. What is the energy of a single photon of emitted light? How many photons are emitted per second by this lamp? **[6]**
   Consider a light source which emits photons with half the energy of the photons given off by the street lamp. Would this light be visible to the human eye? **[2]**

5. Describe an experiment which implies that light travels as a wave, giving a short but clear argument for why the observations support a wave interpretation. Describe a second experiment which implies that light has particle characteristics. How is what is observed in this experiment incompatible with the classical wave theory of light? **[6]**

6. The Higgs boson has received a lot of attention in the news this year. State another example of a boson and an example of a fermion. What is the Pauli exclusion principle? **[6]** Give one example where the Pauli exclusion principle plays an important role in atomic physics.

Section B

(Answer ANY THREE questions from this Section)

7. (a) Draw a labelled Hertzsprung–Russell diagram. Show on it the evolutionary track followed by a 1-$M_\odot$ star, starting from the main sequence. Mark the principal stages of the star's evolution on your sketch, and describe them in a few sentences. **[13]**

   (b) Calculate the main sequence lifetime (in years) of a 10-$M_\odot$ star if it has a luminosity of 10^4 $L_\odot$ and 10% of its mass will be converted from hydrogen to helium in the core. Compare this main sequence lifetime to that of the Sun. What will be the end state of the 10-$M_\odot$ star? **[7]**

8. (a) Sketch a diagram of the 'rotation curve' for our Milky Way Galaxy. Comment on the shape of the rotation curve, and the main inference to be drawn from it. **[6]**

   (b) Suppose that the Sun is in a circular orbit of radius 8.0 kpc about the Galactic Centre, with an orbital velocity of 220 km s^{-1}. Determine the number of orbits completed by the Sun since the birth of the solar system. Based on this result, discuss the nature of the spiral arms in galaxies. **[9]**

   (c) In a galaxy, at what redshift is the Lyman$-\alpha$ line (rest wavelength 121.6 nm) observed at 141.1 nm? Calculate the distance to the galaxy in units of Mpc. **[5]**

9. (a) Name an experiment in which electrons exhibit wave-like properties, describe its setup and what is observed. **[3]**

(b) What is the de Broglie wavelength of i) an electron travelling at 1 metre per second? ii) a football (weighing 0.2 kg) travelling at 1 metre per second? **[3]**

(c) Why do we not see wave-like behaviour in macroscopic objects such as a football? **[2]**

(d) Consider a wavefunction $\psi(x)$ associated with a quantum particle in one-dimension. Which of the following conditions is fulfilled when a wavefunction $\psi(x)$ is normalised? **[1]**

i) $\int_{-\infty}^{\infty} \psi(x)dx = 1$

ii) $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$

or iii) $\int_{-\infty}^{\infty} |\psi(x)| dx = 1$.

(e) Describe one further mathematical condition (in addition to normalisation) which a physical wavefunction must satisfy. **[1]**

(f) Consider a particle in a well with the wavefunction:

$$\psi(x) = \begin{cases} A(9 - x^2) & -3 \leq x \leq 3 \\ 0 & \text{elsewhere} \end{cases}$$

Sketch this wavefunction and calculate (to 2 significant figures) the value of A which normalises $\psi(x)$. **[5]**

(g) What is the probability of finding the particle between $x = 0$ and $x = 2$? **[3]**

(h) By solving an appropriate integral or via another clear argument show that the expectation value for the position of this particle is $\langle x \rangle = 0$. **[2]**

10. (a) Describe Rutherford's planetary model for the hydrogen atom. Describe two ways in which this model is inconsistent with experimentally observed properties of hydrogen. **[4]**

(b) Write down two of the additional postulates that Niels Bohr added to the planetary model to define his model of the atom. **[2]**

(c) Electron orbits in Bohr's model have energy:

$$E_n = \frac{-13.6}{n^2}\text{eV}$$

where n is a positive integer. Show that this is consistent with the Rydberg formula for the wavelengths λ of spectral lines of the hydrogen atom **[6]**

$$\frac{1}{\lambda} = R_H \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$$

(where n and m are positive integers) and hence derive the value of Rydberg's constant R_H in SI units.

(d) Describe one way in which the Bohr model is inconsistent with experimentally observed properties of hydrogen. **[1]**

(e) In a fully quantum mechanical treatment of the hydrogen atom, the wavefunction solutions to the time independent Schrödinger equation for the hydrogen Atom are indexed by three integer *quantum numbers* n, l and m, where

- n is any positive integer, e.g. $1, 2, 3, \ldots$.
- l is any non-negative integer less than n, e.g. $0, 1, 2, \ldots, n-1$.
- m is any integer such that $|m| \leq l$, e.g. $-l, -l+1, \ldots, -1, 0, 1, \ldots, l-1, l$.

Which combinations of quantum numbers correspond to wavefunctions with energy $E = -3.4$eV? **[4]**

(f) At which energy are there 9 distinct quantum numbers? Write out the valid combinations of quantum numbers for this energy. **[3]**

11. The time-independent Schrödinger equation (TISE) in one-dimension is:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x).$$

where m is the mass of the particle, E its energy, $\psi(x)$ the wavefunction and $V(x)$ the potential.

(a) Consider a particle in a region of constant potential, $V(x) = V_0$. Consider the following wavefunctions: **[7]**

$$\psi_1(x) = A\sin(kx)$$
$$\psi_2(x) = Be^{-Kx}$$

Which of these wavefunctions is a solution of the TISE with this potential if $E < V_0$? Which is a solution if $E > V_0$? Determine the relationships between k and E, V_0, m and $\hbar$, and between K and E, V_0, m and $\hbar$.

(b) Without performing any further calculations, describe the process of quantum tunnelling. Include a sketch of a relevant potential and describe how quantum predictions differ from those of classical physics. **[4]**

(c) The probability that a quantum particle with mass m and energy E will tunnel through a square barrier of width L and height U is approximately equal to:

$$P = \exp[-2CL]$$

where

$$C = \frac{\sqrt{2m(U-E)}}{\hbar}.$$

In a Scanning Tunnelling Microscope (STM) electrons tunnel from an electrode across a potential barrier to a surface, completing a circuit whose current I is proportional to the tunnelling probability. Draw a labelled diagram of an STM indicating the path of an electron through the device and how the current is read out. **[4]**

(d) If L represents the distance between electrode and surface in the STM, the current I passing through the device will be given by the expression

$$I \propto \exp[-2CL]\,.$$

Imagine you want to design an STM where a change in L by 1 Angstrom causes the current to change by a factor of 10. What value of constant C will achieve this? If the energy of incident electrons is 1eV, what barrier height U corresponds to the value of C you have just calculated? **[5]**

END OF PAPER