Cover-page

PHAS1202

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Atoms, Stars and The Universe

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Answer ALL SIX questions from Section A and ANY THREE questions from Section B

Numbers in square brackets in the right-hand margin indicate a provisional allocation of maximum possible marks for different parts of each question.

The following may be assumed if required:

Speed of light, c	=	3.0×10^{8} m s^{-1}
Constant in Wien's law	=	3×10^{-3} m.K
Stefan-Boltzmann constant, σ	=	5.7×10^{-8} W m^{-2} K^{-4}
Mass of the Sun, $M_{\odot}$	=	2.0×10^{30} kg
Effective temperature of the Sun	=	5800 K
Radius of the Sun, $R_{\odot}$	=	7.0×10^{8} m
Solar Luminosity, $L_{\odot}$	=	3.8×10^{26} W
Gravitational constant, G	=	6.67×10^{-11} m^{3} kg-1 s^{-2}
Planck's constant, h	=	6.63×10^{-34} m^{2} kg s^{-1}
$\hbar$	=	1.05×10^{-34} m^{2} kg s^{-1}
Electron mass, m_e	=	9.11×10^{-31} kg
Electron charge, e	=	1.60×10^{-19} C
1 Angstrom	=	10^{-10} m
Permittivity of free space, ϵ_0	=	8.85×10^{-12} F m^{-1}
1 electron Volt, eV	=	1.60×10^{-19} J

Section A

(Answer ALL SIX questions from this section)

1. De Broglie postulated that associated with every particle of momentum p is a wave with wavelength $\lambda = h/p$ where h is Planck's constant. Calculate (to 2 s.f.) the de Broglie wavelength for the following particles travelling at 10 m s^{-1}, **[8]**

   (a) An electron

   (b) A family car with a mass of 1000 kg.

   What experimental evidence is there that de Broglie's postulate is correct for an electron? Why would this kind of evidence not be available in the case of the family car?

2. Describe one experiment which implies that light travels as a wave, and one which implies that light has particle characteristics, giving in each case a concise but clear argument for why the observations support a wave or a particle character. **[6]**

3. Which of the following conditions is fulfilled when a wave-function $\psi(x)$ is normalised? **[6]**

   i) $\int_{-\infty}^{\infty} \psi(x)dx = 1$

   ii) $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$

   or iii) $\int_{-\infty}^{\infty} |\psi(x)| dx = 1$.

   Name one further condition which $\psi(x)$ must satisfy to be a physical wave-function. How do we calculate the probability of finding a particle with wave-function $\psi(x)$ in a certain interval, e.g. between $x = a$ and $x = b$?

4. Describe the spectral classification scheme of stars in terms of surface temperature and luminosity class. **[7]**

5. In the context of populating excited atomic levels, briefly explain what is meant by (i) *photo-excitation*, and (ii) *recombination*. **[7]**

   List *three* examples of astrophysical sources of absorption lines.

6. State Hubble's Law. How can the Hubble Constant be used to roughly estimate the age of the Universe? Why is it only a *rough* estimate? **[6]**

Section B

(Answer ANY THREE questions from this Section)

7. Bohr's model of the Hydrogen atom is based on four postulates: 1) The electron travels in a circular orbit around the nucleus; 2) The electron orbit is stable and does not decay (as classical electromagnetism would predict); 3) Only orbits where the angular momentum $l = n\hbar$ are allowed, where n is an integer and $\hbar$ is the reduced Planck's constant; 4) The electron can move to a higher / lower energy orbit by absorbing / emitting a photon of energy equal to the energy difference of the orbits.

   (a) Describe one way in which Bohr's model improved on previous atomic models, and one way in which it is inferior to later quantum mechanical models. **[2]**

   (b) The Coulomb force between two objects with charges q_1 and q_2 separated by distance r is: **[3]**

   $$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}.$$

   Given that the centripetal acceleration for a particle in a circular orbit is v^2/r, where v is the particle speed and r the orbital radius, show that the speed v of an electron of charge e and mass m_e moving in a stable circular orbit satisfies

   $$v^2 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r m_e}$$

   (c) Show that Bohr's angular momentum rule $l = n\hbar$ restricts the speed of the electron orbit to the values: **[3]**

   $$v^2 = \frac{\hbar^2 n^2}{r^2 m_e^2}$$

   (d) Hence show that radii of electron orbits in the Bohr model are restricted to the values **[4]**

   $$r_n = a_0 n^2$$

   and derive an expression for the Bohr radius a_0 in terms of $\hbar$, ϵ_0, m_e and e. What is the numerical value (in metres) of a_0?

   (e) The energies of orbits in the Bohr model satisfy:

   $$E_n = -\frac{13.6}{n^2}\text{eV}.$$

   What wavelength of light will a Hydrogen atom emit when its electron transitions from the $n = 3$ to the $n = 2$ orbit? What is the name for this spectral line? **[4]**

   (f) In a Helium-3 ion, a single electron orbits a nucleus consisting of two protons and a neutron. If we apply the same 4 postulates of the Bohr model for Hydrogen to describe this ion, what would be the radius of the lowest energy electron orbit? You may neglect any effect due to the differing nuclear mass. **[4]**

8. The time-independent Schrödinger equation (TISE) in one-dimension is:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x).$$

where m is the mass of the particle, E its energy, $\psi(x)$ the wavefunction and $V(x)$ the potential.

(a) Consider a particle moving in a constant potential, i.e. $V(x) = V_0$. Assuming $E > V_0$, show that the following wave-function **[4]**

$$\psi(x) = A\sin(\kappa x) + B\cos(\kappa x)$$

is a solution of the TISE and determine the relationship between E and κ.

(b) Consider an infinite square well with walls at $x = 0$ and $x = L$. The potential is zero inside the well, i.e. $V(x) = 0$ for $0 \leq x \leq L$, and is infinite outside the well, i.e. $V(x) = \infty$ for $x < 0$ and $x > L$. What are the boundary conditions which must be satisfied by physical wave-functions of a particle in this well? **[2]**

(c) By considering boundary conditions determine the allowed values of B and κ for physical wave-functions for a particle in the well. **[5]**

(d) Hence show that the allowed energies for a particle in the well are **[5]**

$$E_n = \left(\frac{h^2}{8mL^2}\right) n^2$$

where integer $n > 0$. Why is the $n = 0$ case not an allowed energy?

(e) If an electron is in an infinite square well as described in part (b) and the well has a width of 1 Angstrom, what is the difference in energy between the lowest and second lowest energy states of the electron? What wavelength of light would be transmitted or absorbed in a transition between these states? **[4]**

9. (a) Describe the process of quantum tunnelling. Include a sketch of a relevant potential and the form of the wave-function both inside and outside the barrier. Describe how quantum predictions differ from those of classical physics. [You do not need to derive the exact forms of wave-functions from the Schrödinger equation – a qualitative sketch of their form is sufficient.] **[6]**

(b) Name a physical process in which quantum tunnelling plays an important role. **[2]**

(c) The probability that a quantum particle with mass m and energy E will tunnel through a square barrier of width L and height U is approximately equal to:

$$P = \exp[-2CL]$$

where

$$C = \frac{\sqrt{2m(U-E)}}{\hbar}.$$

In a Scanning Tunnelling Microscope (STM) electrons tunnel from an electrode across a potential barrier to a surface, completing a circuit whose current I is proportional to the tunnelling probability. Draw a labelled diagram of an STM indicating the path of an electron through such a device. Indicate clearly the position of the tunnelling barrier in your diagram. **[4]**

(d) If L represents the distance between electrode and surface, the current I passing through the device will be **[3]**

$$I \propto \exp[-2CL]\,.$$

For a tunnelling barrier with height 5 eV, and an incident electron with energy 0.8 eV, calculate the constant C, justifying its unit.

(e) Initially the current in the STM is 0.1 A. If the barrier width decreases by 1 Angstrom, what will the current change to? **[5]**

10. By outlining the dominant sequence of nuclear reactions, describe the primary energy source in the Sun. Starting from the mass deficit, explain how the Sun can be in balance with this energy source at its centre. [**11**]

A star has a luminosity of 10^{30}W. What is its surface temperature if its radius is 30 times that of the Sun? Calculate the wavelength of the peak of the energy distribution of this star and indicate the main observational waveband this corresponds to. [**6**]

In a spectrum of this star a He absorption line is observed at 471.289 nm; given that the rest wavelength of this line is 471.320 nm, calculate the radial velocity of the star and state whether it is moving toward or away from the observer. [**3**]

11. Describe the main differences between spiral and elliptical galaxies in terms of their stellar properties and content. [**8**]

An active galaxy is thought to have a 10^8 solar mass supermassive black hole at its centre. Calculate the Schwarzchild radius (in units of $R_\odot$) of the black hole. Assuming spherical geometry also determine the average density inside the Schwarzchild radius. [**6**]

Briefly explain two methods for evaluating the masses of clusters of galaxies and hence the dark matter component. [**6**]

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