Answer ALL SIX questions from section A and THREE questions from section B.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

You may need these:

Law of cosines:

$$a^2 = b^2 + c^2 - 2bc\cos\alpha,$$

where α is the angle across from the side of length a.

Law of sines:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma},$$

where the sides of length a, b and c are across from the angles α, β and γ, respectively.

Addition of cosines:

$$\cos A + \cos B = 2\cos\left[\frac{1}{2}(A+B)\right]\cos\left[\frac{1}{2}(A-B)\right].$$

Addition of sines:

$$\sin A + \sin B = 2\sin\left[\frac{1}{2}(A+B)\right]\cos\left[\frac{1}{2}(A-B)\right].$$

SECTION A

[Part marks]

1. (a) State the changes of phase which occur when light is reflected **[2]**
   i. in air from an air-glass interface; and,
   ii. in glass from a glass-air interface.

   (b) Two flat rectangular glass sheets touch at one edge and are separated by a hair 100 mm from, and parallel to, that edge to form a wedge-shaped air gap. When illuminated at near-normal incidence with light with a wavelength of 600 nm, straight dark fringes are observed in reflection with a spacing of 2 mm. **[5]** Sketch this arrangement, showing typical rays, and calculate the thickness of the hair.

2. (a) Write down the partial differential equation describing wave motion in one dimension. Define your variables. **[2]**

   (b) Show that this equation is satisfied by a disturbance of the form **[2]**

$$y_1(x,t) = A\sin(k_1 x - \omega_1 t).$$

   (c) This wave is superposed on another wave, $y_2(x,t) = A\sin(k_2 x - \omega_2 t)$, with the frequencies almost equal. Briefly describe the resulting disturbance. **[2]**

3. (a) Write down expressions for the phase velocity v_{p} and group velocity v_{g} of a wave in terms of the angular frequency ω and the wavevector k. **[2]**

   (b) At what speed does energy propagate in the wave? **[1]**

   (c) The dispersion relation for transverse waves on an elastic beam is

$$\omega = \frac{a}{\lambda^2},$$

   where λ is the wavelength and a is a constant depending on the material and dimensions of the beam. Show that for these waves the group velocity is twice the phase velocity. **[4]**

4. A diffraction grating has 10 slits each of width 0.1 mm. If the intensity pattern created by this grating has the fifth principal maximum missing (due to the first diffraction minimum):

(a) How many subsidiary maxima should be found between the principal maxima of the diffracted intensity pattern? **[2]**

(b) How far apart are the slits? **[2]**

(c) If the wavelength of the incident light changes from λ_1 to λ_2 where $\lambda_2 < \lambda_1$, do the observed maxima move to a smaller, or greater, angle? Why? **[2]**

5. (a) Draw a diagram of a Michelson interferometer, and label its main features. **[4]**

(b) Under what conditions will circular fringes be observed with the apparatus you have drawn? **[2]**

(c) Explain why a compensator plate is necessary if a Michelson interferometer is to be used with non-monochromatic light. **[2]**

6. Unpolarized light is incident upon an interface between a medium with index of refraction n_{inc} and one with n_{trans}, where $n_{\text{inc}} > n_{\text{trans}}$. Sketch the graph of reflected intensity for the perpendicular and parallel components of the light as a function of incident angle, θ_i. Indicate any special angles on your sketch. **[6]**

7. (a) In 1976, the Who set a record for the loudest concert: the sound level 46 m in front of the speaker system was $\beta_2 = 120$ dB. What is the ratio of the intensity I_2 of the band at that spot to the intensity I_1 of a jackhammer operating at sound level $\beta_1 = 92$ dB? **[6]**

(b) i. A source moving at speed v_s towards a stationary detector emits sound waves at a frequency f. Show that the detector measures a frequency f' given by **[6]**

$$f' = f\frac{1}{1-\frac{v_s}{v}},$$

where v is the speed of sound in air.

ii. A rocket moves at a speed of 242 m s^{-1} directly toward a stationary pole (through stationary air) while emitting sound waves at frequency $f = 1250$ Hz. Take the speed of sound to be 343 m s^{-1}.

A. What frequency f' is measured by a detector that is attached to the pole? **[3]**

B. Some of the sound reaching the pole reflects back to the rocket as an echo. What frequency f'' does a detector on the rocket detect for the echo? **[5]**

8. (a) The lens-makers' equation is given by

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

Explain the significance of the terms in this formula. **[4]**

(b) A lens for a microwave system is made by embedding metal spheres in expanded polystyrene to give a material with a refractive index of 12. If a symmetrical biconvex lens made from this material is used to produce a parallel beam from a source placed on the axis of the lens and two metres from it, what is the radius of curvature of the two surfaces? **[5]**

(c) The near point of a certain hyperopic eye is 100 cm in front of the eye. To see clearly an object that is 25 cm in front of the eye, what type of contact lens is required (converging or diverging), and what would its focal length be? **[6]**

(d) You have two lenses, one bi-convex and the other bi-concave, each with a focal length with absolute value 5.0 cm. Which lens can you use as a simple magnifier, and why? What is its angular magnification? **[5]**

9. (a) Write down an expression for the speed of a wave on a stretched string, defining your symbols. **[4]**

(b) The equation for standing waves for a string of length l with fixed ends can be written as

$$y(x,t) = 2A\sin kx\,\cos\omega t$$

Show that the resonant frequencies f_n are given by $f_n = (v/2l)n$ for $n = 1, 2, 3, ...$, where v is the wave speed in the string. **[4]**

(c) A string, tied to a sinusoidal vibrator at a point P and running over a support at Q, is stretched by a block of mass m. The separation L between P and Q is 1.2 m, the linear density of the string is 1.6 g m^{-1}, and the frequency f of the vibrator is fixed at 120 Hz. P and Q can be considered to be nodes.

What mass m allows the vibrator to set up the fourth harmonic on the string? **[4]**

(d) A small loudspeaker is driven by an audio oscillator and amplifier, adjustable in frequency from 1000 to 2000 Hz only. A cylindrical piece of sheet-metal pipe makes a tube 45 cm long and open at both ends. The loudspeaker is used to drive the air in the tube into resonance.

i. If the speed of sound in air is 340 m s^{-1} at the existing temperature, at what frequencies will resonance occur in the pipe when the frequency emitted by the speaker is varied from 1000 to 2000 Hz? **[4]**

ii. Describe with a sketch the characteristics of the standing wave with the lowest frequency found in 9(d)i above. **[4]**

10. (a) Describe the difference between the conditions under which Fraunhöfer and Fresnel diffraction may be observed. [2]

(b) The intensity distribution in the Fraunhöfer pattern of a slit of width w illuminated with light of wavelength λ is

$$I(\theta) = I(0)\left[\frac{\sin(\beta)}{\beta}\right]^2,$$

where

$$\beta = \frac{\pi w}{\lambda}\sin(\theta).$$

i. Sketch the intensity pattern for a single slit. [3]

ii. Describe Rayleigh's criterion for the resolution of images formed by a slit. [4]

iii. Deduce from the above formulae for the diffraction pattern that the minimum angular separation between two images which can just be resolved, at wavelength λ, by a slit of width w, is λ/w. [5]

iv. State how this expression is modified for a circular aperture of diameter D. [2]

v. Use this result to calculate the smallest separation between two objects that can be resolved by a human eye with a pupil diameter of 2.5 mm at a distance of 250 mm, assuming a wavelength of 500 nm. [4]

11. (a) Two sinusoidal waves travel in the same direction along a string and interfere to produce a resultant wave given by

$$y_{\mathrm{R}}(x,t) = (3.0\ \mathrm{mm})\sin(20x - 4.0t + 0.820\ \mathrm{rad}),$$

with x in meters and t in seconds. The two waves are identical, except for phase; one of the waves has zero phase. What are

i. the wavelength λ of the two waves; **[3]**

ii. the phase difference between them; and **[3]**

iii. their amplitude y_0? **[4]**

(b) Two parallel light waves have wavelength 550.0 nm before entering media 1 and 2 separately. They also have equal amplitudes and are in phase. Medium 1 is just air, and medium 2 is a transparent plastic layer of index of refraction 1.600 and thickness 2.600 μm.

i. What is the phase difference of the emerging waves in wavelengths, radians and degrees? What is their effective phase difference (in wavelengths)? **[7]**

ii. If the rays of the waves were angled slightly toward each other so that the waves reached the same point on a distant viewing screen after travelling equal distances, what type of interference would the waves produce at that point? **[3]**

END OF PAPER