$$\frac{2\pi}{\lambda} = \left(\frac{2\pi}{h}\right)\sqrt{2mE} \rightarrow \frac{1}{\lambda} = \frac{\sqrt{2mE}}{h}$$

Let $\frac{1}{\lambda} = \frac{p}{h}$ $\quad \frac{p}{\cancel{h}} = \frac{\sqrt{2mE}}{\cancel{h}}$

$$\frac{p^2}{2m} = E$$

Replace λ by $\frac{h}{p}$

Set $\lambda = \frac{h}{p}$

$$\psi(x) = A\sin\left(\frac{2\pi(x-\phi)}{\lambda}\right) = A\sin\left(\frac{2\pi p}{h}(x-\phi)\right)$$

$$= A\sin\left(\frac{px}{\hbar} - \frac{\phi p}{\hbar}\right)$$

$$c = -\frac{\phi p}{\hbar}$$

$$= A\sin\left(\frac{px}{\hbar} + c\right)$$

$$\psi(x) = A \sin\left(\frac{2\pi(x-\phi)}{\lambda}\right)$$

$$\frac{1}{\lambda} = \frac{p}{h} \qquad \frac{p^2}{2m} = E$$

$$\psi(x) = A \sin\left(\frac{2\pi}{h} p x - \frac{2\pi \phi p}{h}\right)$$

$\frac{1}{\hbar}$

$+C.$

Energies in the ∞-square well.

Subst. $\psi(x) = a \sin\left(\frac{n\pi x}{L}\right)$ $n = 1, 2, 3, \ldots$

into TISE: $$\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi(x).$$

$$\frac{d\psi}{dx} = a\cos\left(\frac{n\pi x}{L}\right)\left(\frac{n\pi}{L}\right) \quad \Bigg| \quad \frac{d^2\psi}{dx^2} = a\left(-\sin\left(\frac{n\pi x}{L}\right)\right)\left(\frac{n\pi}{L}\right)^2$$

$$\frac{d^2\psi}{dx^2} = -\left(\frac{n\pi}{L}\right)^2 \underbrace{a\sin\left(\frac{n\pi x}{L}\right)}_{\psi(x)} = -\left(\frac{n\pi}{L}\right)^2 \psi(x)$$

subst. into TISE:
$$\left(\cancel{-}\frac{\hbar^2}{2m}\right)\left(\cancel{-}\left(\frac{n\pi}{L}\right)^2\right)\cancel{\psi(x)} = E\cancel{\psi(x)}$$

$$E = \frac{\hbar^2 n^2 \pi^2}{2mL^2} = \frac{h^2}{4\cancel{\pi^2}}\frac{n^2\cancel{\pi^2}}{2mL^2} = \frac{h^2 n^2}{8mL^2}$$

We want: $\int_{-\infty}^{\infty} |\psi(x)|^2 \, dx = 1$

$$= \int_0^L a^2 \sin^2\left(\frac{n\pi x}{L}\right) dx$$

Use: $\sin^2(x) = \dfrac{1-\cos(2x)}{2}$

$$= a^2 \int_0^L \frac{1 - \cos\left(\cancel{2x}\ \frac{2n\pi x}{L}\right)}{2} \, dx$$

$$= \frac{a^2}{2}\left[\int_0^L 1 \, dx - \int_0^L \cos\left(\cancel{2x}\ \frac{2n\pi x}{L}\right) dx\right] = \frac{a^2}{2}(L-0) - \left[\frac{\sin\left(\frac{2n\pi x}{L}\right)}{\left(\frac{2n\pi}{L}\right)}\right]_{x=0}^{x=L}$$

$$= \frac{a^2}{2}\Big((L-0) - (0-0)\Big) = \frac{a^2 L}{2}$$

Set $\frac{a^2 L}{2} = 1$ $\quad a^2 = \frac{2}{L}$ $\quad \boxed{a = \sqrt{\frac{2}{L}}}$ (ignore -ve solution)