$$\langle x \rangle = \int_{-\infty}^{\infty} x\,|\psi(x)|^2\,dx \qquad \psi(x) = \sin(\pi x) \quad 0 \le x \le 2$$
$$= 0 \quad \text{elsewhere.}$$

$$= \int_0^2 x \sin^2(\pi x)\,dx = \int_0^2 x\left(\frac{1 - \cos(2\pi x)}{2}\right)dx$$

$$= \int_0^2 \frac{x}{2}\,dx - \underbrace{\frac{1}{2}\int_0^2 x \cos(2\pi x)\,dx}_{=0}$$

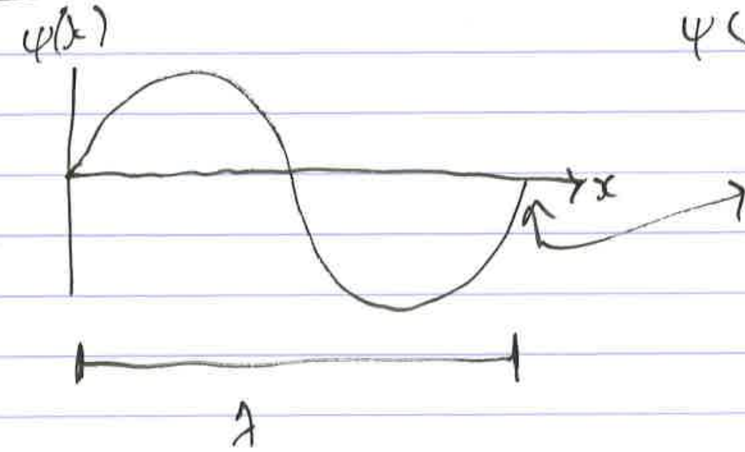
Integration by parts: $\int_a^b u(x)\,v'(x)\,dx = \Big[u(x)\,v(x)\Big]_{x=a}^{x=b} - \int_a^b v(x)\,u'(x)\,dx$

$$u(x) = x \qquad u'(x) = \frac{du}{dx} = 1 \qquad v'(x) = \cos(2\pi x) \qquad v(x) = \frac{1}{2\pi}\sin(2\pi x)$$

$$\int_0^2 x\cos(2\pi x)\,dx = \left[x\,\frac{\sin(2\pi x)}{2\pi}\right]_0^2 - \int_a^b \frac{1}{2\pi}\sin(2\pi x)\,dx$$

$$= 0 - 0 + \frac{1}{2\pi}\frac{1}{2\pi}\Big[\cos(2\pi x)\Big]_0^2 = \frac{1}{(2\pi)^2}\big(1 - 1\big) = 0$$

$$= \int_0^2 \frac{x}{2}\,dx = \frac{1}{2}\frac{(4-0)}{2} = \frac{4}{4} = 1$$



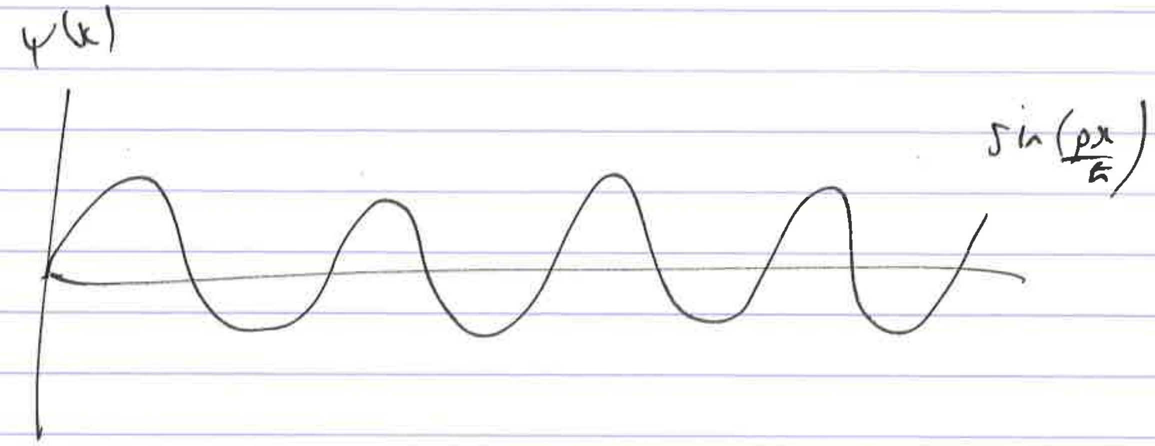
$$\psi(x) = \sin\left(\frac{px}{\hbar}\right)$$

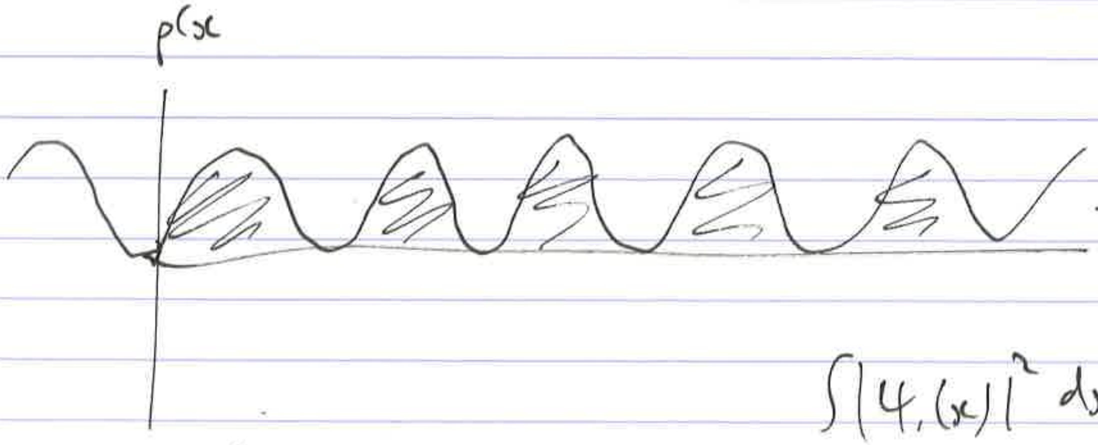
$$\frac{px}{\hbar} = 2\pi$$

$$\frac{px}{h}\cancel{2\pi} = \cancel{2\pi}$$

$$\boxed{\lambda = \frac{h}{p}}$$

$$\int_{-\infty}^{\infty} |\psi(x)|^2 \, dx = 1 \qquad \psi(x) = \sin\left(\frac{px}{\hbar}\right)$$





$$\int |\psi_1(x)|^2 \, dx = N$$

$$\int_{-\infty}^{\infty} p(x) \, dx = \infty \qquad \psi_2(x) = \frac{\psi_1(x)}{\sqrt{N}}$$

$$\frac{1}{\sqrt{\infty}} = 0$$