PHAS1202 – Astrophysics ICA : Solutions – Dec. 2016

1. (Students do *not* use the Stefan-Boltzmann constant)

Luminosity, $L \propto R_*^2\, T_{\rm eff}^4$ **[1]**

$\frac{L_{wd}}{L_\odot} = \frac{R_{wd}^2}{R_\odot^2} \times \frac{70000^4}{5800^4}$

$\frac{R_{wd}^2}{R_\odot^2} = 5 \times 4.713 \times 10^{-5}$ **[2]**

$R_{wd} = 0.015\ R_\odot$ **[1]**

Wien's law (black-body approximation) gives: $\lambda_{\rm max} = \frac{3\times10^{-3}}{T}$ **[1]**

where $\lambda_{\rm max}$ is in m and T is in K.

$\lambda_{\rm max} = \frac{3\times10^{-3}}{70000} = 4.3 \times 10^{-8}$m **[1]**

$\sim$ 43nm corresponds to the ultraviolet waveband . **[1]**

[7 marks total]

2.

Sirius A luminosity is then

$$L = 23.5 \times 3.8 \times 10^{26}\ \text{Watt} = 8.9 \times 10^{27}\ \text{Watt}$$

Since fraction of mass liberated into energy by the fusion of H to He = 0.007 (= efficiency), $E_{total} = 0.007\ mc^2$ *[2]*

The H mass burnt into He per sec will be

$$\frac{L}{\text{efficiency} \times c^2} = \frac{8.9 \times 10^{27}}{0.007 \times (3 \times 10^8)^2} = 1.4 \times 10^{13}\ \text{kg}\,\text{s}^{-1}$$ *[2]*

If we assume that 10% of its H mass burns into He, then, since Sirius A mass is

$$M = 2.3\ \text{solar mass} = 2.3 \times 2 \times 10^{30}\ \text{kg} = 4.6 \times 10^{30}\ \text{kg}$$

Sirius A lifetime will be

$$\frac{0.1 \times M}{\text{Rate of burning}} = \frac{4.6 \times 10^{29}}{1.4 \times 10^{13}}\,\text{s} = 3.3 \times 10^{16}\ \text{s} = \frac{3 \times 10^{16}}{3 \times 10^{7}}\ \text{years} = 1 \times 10^{9}\ \text{years}$$

[2]

while the Sun's lifetime is expected to be $10^{\mathbf{10}}$ years. [1]

[7 marks total]

3. If the ionisation potential of the hydrogen atom is 13.6 eV, then $E(1) = -13.6$ eV and [1]

$$E(n) = -\frac{13.6}{n^2}\,\text{eV}$$

Thus $E(2) - E(3) = -13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right)\text{eV} = -1.89\,\text{eV}$ and

[1]

$$\lambda = \frac{c}{\nu} = \frac{hc}{E} = \frac{6.63\times10^{-34}\times3\times10^{8}}{1.89\times1.6\times10^{-19}}\,\text{m} = 6.577\times10^{-7}\,\text{m} = 657.7\,\text{nm}$$

[2]

Transitions whose lower level has $n = 2$ are called the *Balmer series.*

[1]

The transition between $n = 2$ and $n = 3$ is called Hα.

[1]

[6 marks total]

4.

Population I stars contain

(i) a larger metal abudance, (ii) are distributed closer the the galactic disc, and (iii) are younger than Population II stars. *[3]*

The Sun is a Population I star; Globular clusters are Population II *[1]*

[4 marks total]

5

Velocity = distance/time = circumference/period:

$$T(orbit) = \frac{2\pi 8000\ \text{pc}\ \times\ 3.1\times10^{13}\ \text{km pc}^{-1}}{220\ \text{km s}^{-1}\ \times\ 3.16\times10^{7}\ \text{s yr}^{-1}}$$
$$= 2.24\times10^{8} yr$$

[2]

Approx. age of the solar system is $\sim 4.5\times10^{9}$yr [1]
(Students expected to e.g. know the age of the Sun.)
Thus 4.5×10^{9} / 2.24×10^{8} $\sim$ 20 orbits completed. [1]

The spiral arms cannot be fixed, permanent features, because they would quickly tighten (‘wind up’) after 20 revolutions. They instead represent a density wave phenomenon. [2]

[6 marks total]

6. According to Hubble classification scheme:

SBa galaxy -- galaxy has a large nucleus, with a bar-like structure through it. The spiral arms emerge from the ends of the bar and are tightly wound. [2]

Sc galaxy -- galaxy has a relatively small nucleus, plus loosely wound spiral arms (no central bar). [2]

E6 galaxy -- elliptical galaxy, whose apparent shape is very elongated or flattened. [1]

[5 marks total]

7.

Distance to galaxy:
radial velocity

$$v = \left(\frac{(\lambda - \lambda_0)}{\lambda_0}\right) c$$

[1]

V = [(402.8 – 393.30)/393.30] x 3 x 10^5 = 7246 km s^{-1} [2]

Hubble law, distance d = v / H_o = 7246 / 75 = 96.6 Mpc [2]

[5 marks total]