

PHAS1245 : Mathematical Methods I—Problem Solving
Tutorial 5

Week 16, starting Monday 13th December

Questions 1–3 are on complex numbers and 4–8 on series.

1. By writing $\pi/12 = \pi/3 - \pi/4$ and considering $e^{i\pi/12}$, evaluate $\cot(\pi/12)$ without using a calculator.
2. Find the roots of

$$\sqrt[4]{i8\sqrt{3} - 8}$$

and sketch them in an Argand diagram.

3. Express $\cos^6 \theta$ in multiple-angle form with single powers of cosine.
4. By grouping and rearranging terms of the absolutely convergent series

$$S = \sum_{n=1}^{\infty} \frac{1}{n^2},$$

show that

$$S_o = \sum_{n \text{ odd}} \frac{1}{n^2} = \frac{3S}{4}.$$

5. A Fabry-Pérot interferometer consists of two parallel heavily silvered glass plates; light enters normally to the plates, and undergoes repeated reflections between them, with a small transmitted fraction emerging at each reflection. Find the intensity $|B|^2$ of the emerging wave, where

$$B = A(1 - R) \sum_{n=0}^{\infty} R^n e^{in\phi},$$

with A, R (the reflection coefficient, $0 < R < 1$) and ϕ real constants.

6. Determine whether the following series are convergent :

$$(a) \sum_{n=1}^{\infty} \frac{n^{1/2}}{(n+1)^{1/2}} \qquad (b) \sum_{n=1}^{\infty} \frac{n^2}{n!}$$

7. Find the Maclaurin series for $\sin^2 x$.
8. Evaluate the following limits

$$\lim_{x \rightarrow 0} \frac{x e^{-x}}{1 - e^{-x}} \qquad \lim_{x \rightarrow 0} \frac{\tan x - x}{\cos x - 1}.$$