Normalisation

$$\int_{-\infty}^{\infty} |\psi(x)|^2 \, dx = 1$$

$$\psi(x) = a \sin\left(\frac{n\pi x}{L}\right) \quad 0 \le x \le L$$

$$= 0 \quad \text{for all other } x$$

$$1 = \int_0^L a^2 \sin^2\left(\frac{n\pi x}{L}\right) dx$$

But $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

$$= \int_0^L a^2 \frac{1}{2}\left(1 - \cos\left(\frac{2n\pi x}{L}\right)\right) dx = \frac{a^2}{2}\left[\int_0^L dx - \int_0^L \cos\left(\frac{2n\pi x}{L}\right) dx\right]$$

$$= \frac{a^2}{2}\left[L - \left[\frac{\sin\left(\frac{2n\pi x}{L}\right)}{\left(\frac{2n\pi}{L}\right)}\right]_0^L\right] \quad (\to 0)$$

$$\Rightarrow 1 = \frac{a^2}{2} L$$

$$a = \sqrt{\frac{2}{L}}$$