PART I: Waves

Simple harmonic oscillator

Spring-block system:

$$
\begin{aligned}
F &= -kx \\
&= ma = m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} \\
m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} &= -kx \\
x(t) &= A\cos(\omega_0 t + \phi) \quad \text{with} \quad \omega = \sqrt{k/m} \\
v(t) &= \frac{\mathrm{d}x}{\mathrm{d}t} = -A\omega_0 \sin(\omega_0 t + \phi) \\
a(t) &= \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -A\omega_0^2 \cos(\omega_0 t + \phi) \\
K &= \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega_0^2 \sin^2(\omega_0 t + \phi) \\
U &= \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \cos^2(\omega_0 t + \phi) \\
E &= \frac{1}{2}mA^2\omega_0^2
\end{aligned}
$$

Phasors, complex exponential notation, addition

$$
\begin{aligned}
x + iy &= a\cos(\omega_0 t + \phi) + ia\sin(\omega_0 t + \phi) \\
&= ae^{i(\omega_0 t + \phi)}.
\end{aligned}
$$

Beats

$$A\cos 2\pi f_1 + A\cos 2\pi f_2 = 2A\cos 2\pi\frac{f_1 - f_2}{2}\cos 2\pi\frac{f_1 + f_2}{2}$$

$$f_{\mathrm{beat}} = |f_1 - f_2|$$

Wave equation

$$\frac{\partial^2\psi(x,t)}{\partial t^2} = v^2\frac{\partial^2\psi(x,t)}{\partial x^2}$$

Solutions

$$\psi(x,t) = h(x - ct) + g(x + ct)$$

$$f(x,t) = A\cos(kx - \omega t + \phi)$$

Speed

$$v = \sqrt{\frac{T}{\mu}}$$
$$v = f\lambda$$

Linearity and principle of superposition

Travelling Waves

Energy transfer – reflection and transmission

Phase changes at boundaries

$$K_\lambda = \frac{1}{4}\mu\omega^2 A^2 \lambda$$
$$E_\lambda = U_\lambda + K_\lambda = \frac{1}{2}\mu\omega^2 A^2 \lambda.$$
$$P = \frac{1}{2}\mu\omega^2 A^2 v$$

Longitudinal and transverse waves

Carrier and envelope

$$\begin{aligned}
\psi_1(x,t) + \psi_2(x,t) &= a\cos(\omega_1 t - k_1 x) + a\cos(\omega_2 t - k_2 x) \\
&= 2a\cos\left[\frac{\omega_1 + \omega_2}{2}t - \frac{k_1 + k_2}{2}x\right]\cos\left[\frac{\omega_1 - \omega_2}{2}t - \frac{k_1 - k_2}{2}x\right]
\end{aligned}$$

Phase and group velocities (dispersion)

$$v_{\mathrm{p}} = \frac{\omega}{k}$$
$$v_g = \frac{\partial\omega}{\partial k} = \frac{\partial(v_p k)}{\partial k} = v_p + k\frac{\partial v_p}{\partial k}.$$

Standing waves

$$\begin{aligned}
y_1 + y_2 &= Ae^{i(kx - \omega t + \pi)} + Ae^{i(kx + \omega t)} \\
&= -2A\sin kx \sin\omega t
\end{aligned}$$

Harmonics

On a fixed string:

$$f_n = n\frac{v}{2L}, \quad n = 1, 2, 3, \ldots$$

Pipe open at both ends:

$$f_n = n\frac{v}{2L}, \quad n = 1, 2, 3, \ldots$$

Pipe closed at one end:

$$f_n = n\frac{v}{4L}, \quad n = 1, 3, 5, \ldots$$

Nodes and antinodes

On a fixed string:

$$x = n\frac{\lambda}{2},\ n = 0, 1, 2, \ldots \quad \text{nodes}$$

$$x = n\frac{(2n+1)\lambda}{4},\ n = 0, 1, 2, \ldots. \quad \text{antinodes}$$

Sound

$$s(x,t) = s_{\text{max}} \cos(kx - \omega t)\Delta P = \Delta P_{\text{max}} \sin(kx - \omega t)$$

$$v_{\text{sound in air}} \approx 340\,\text{ms}^{-1}$$

Measurement of sound

The *sound level*, β, is measured in decibels (dB).

$$\beta = 10 \log \frac{I}{I_0}$$

Doppler effect

Observer moving with respect to stationary source

$$f' = \left(\frac{v \pm v_0}{v}\right) f$$

Source moving towards stationary observer

$$f' = \left(\frac{v}{v \mp v_s}\right) f$$

Both source/observer moving

$$f' = \left(\frac{v \pm v_{\text{O}}}{v \mp v_{\text{S}}}\right) f$$

Relativistic Doppler effect

$$f_{\text{obs}} = \frac{\sqrt{1 + v/c}}{\sqrt{1 - v/c}} f_{\text{source}}$$

For $v \ll c$ this is equivalent to the "normal" Doppler effect.

PART II: Light

General nature

$c = f\lambda$

electromagnetic spectrum, especially visible light

$$
\begin{aligned}
\mathbf{E} &= \mathbf{E}_0 e^{i(\omega t - \mathbf{k}.\mathbf{x})} \\
\mathbf{H} &= \mathbf{H}_0 e^{i(\omega t - \mathbf{k}.\mathbf{x})}
\end{aligned}
$$

$$
\begin{aligned}
\text{Optical Path Length} &= n_\alpha d \\
&= \text{index of refraction} \times \text{distance travelled} \\
\text{Associated phase} &= k n_\alpha d \quad \text{where} \quad k = \frac{2\pi}{\lambda}
\end{aligned}
$$

$$
\begin{aligned}
n_\alpha &= \frac{\text{speed of light in vacuum}}{\text{speed of light in medium } \alpha} \\
&= \frac{c}{v_\alpha}
\end{aligned}
$$

$$\lambda_n = \frac{\lambda_0}{n}$$

Polarization

Types of polarization

Methods of producing polarizations

Brewster angle

$$\tan\theta_p = \frac{n_2}{n_1}$$

Malus' law

$$I = I_0 \cos^2\theta$$

Propagation

Fermat's and Huygens' principles

Reflection

$$\theta_i = \theta_r$$

Refraction

$$n_i \sin\theta_i = n_t \sin\theta_t$$

Total Internal Reflection and the Critical Angle

$$\sin\theta_c = \frac{n_1}{n_2}$$

Interference

Coherence

Fraunhöfer and Fresnel limits

Constructive and destructive interference

$$d_2 - d_1 = n\lambda, \qquad n = 0, 1, 2, 3, \ldots \qquad \text{(constructive interference)}$$

Young's double slit experiment

$$d\sin\theta = m\lambda$$

$$y_m = m\frac{\lambda D}{d}$$

Thin films

Change of phase upon reflection

Soap bubble

$$2nd\cos\beta = \left(m - \frac{1}{2}\right)\lambda \quad \text{(constructive interference)}$$

Oil film

$$2nd\cos\beta = \left(m - \frac{1}{2}\right)\lambda \quad \text{(constructive interference)}$$

Anti-reflection coating

Thin film wedge

$$(p + \frac{1}{2})\lambda = 2\alpha x \qquad - \text{bright fringes}$$

Newton's rings

$$2t = (p + \frac{1}{2})\lambda \quad \text{(constructive interference)}$$

$$r^2 = (p + \frac{1}{2})\lambda R$$

Interferometers

Michelson interferometer

The distance d associated with m fringes is $d = m\lambda/2$.

Dark fringes

$$\cos\theta = p\frac{\lambda}{2d}$$

Doublet source

$$\Delta\lambda = \lambda_2 - \lambda_1 = \frac{\lambda_1\lambda_2}{2\Delta d} \approx \frac{\lambda^2}{2\Delta d}$$

Fabry-Perot interferometer

Etalon

Diffraction

Single-slit diffraction

$$a\sin\theta = n\lambda$$

Multiple slits

$$I = I_0\frac{\sin^2\alpha}{\alpha^2} \times \frac{\sin^2 N\beta}{\sin^2\beta}$$

$$\alpha = \frac{\pi}{\lambda}a\sin\theta \quad \text{and} \quad \beta = \frac{\pi}{\lambda}d\sin\theta$$

Interference Maxima

Maxima when $\frac{\sin N\beta}{\sin\beta} = N$ or when $\beta = 0, \pm\pi, \pm 2\pi, \ldots$

Interference Minima

Minima when $\frac{\sin N\beta}{\sin\beta} = 0$ or

$$\beta = \pm\frac{\pi}{N}, \pm\frac{2\pi}{N}, \ldots, \pm\frac{(N-1)\pi}{N}, \pm\frac{(N+1)\pi}{N}, \ldots.$$

$N - 2$ subsidiary maxima.

Subsidiary Interference Maxima

$$\beta = \pm\frac{3\pi}{2N}, \pm\frac{5\pi}{2N}, \ldots$$

Missing peaks

$$\frac{d}{a} = \frac{m}{n}$$

Instrumental broadening

$$\Delta\theta = \frac{2\lambda}{ND\cos\theta_{\mathrm{m}}}$$

Circular apertures

Rayleigh criterion

$$\sin\theta \approx \theta \geq \frac{\lambda}{a}$$

$$\sin\theta \approx \theta \geq 1.22\frac{\lambda}{D}$$

Gratings

Dispersion relation

$$\frac{\mathrm{d}\theta}{\mathrm{d}\lambda} = \frac{m}{d\cos\theta} \approx \frac{m}{d}$$

Resolving power

$$R = \frac{\lambda}{\Delta\lambda} = Nm$$

PART III: Optics

Real and virtual images

Mirrors

Magnification

$$M = -\frac{q}{p} = \frac{h_\mathrm{i}}{h_\mathrm{o}}$$

Plane mirrors

Mirror equation

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Sign conventions

Convex mirrors

$f < 0$
p anywhere

Concave mirrors

$f > 0$
$p > f$
$p < f$

Ray tracing

Lenses

Refraction at a spherical surface

Lens-makers' equation

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Thin lenses

Thin-lens equation

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Magnification

$$M = -\frac{q}{p} = \frac{h_\mathrm{i}}{h_\mathrm{o}}$$

Sign conventions

Positive, converging, bi-convex lens

$f > 0$
$p > f$
$p < f$

Negative, diverging, bi-concave lens

$f < 0$
p anywhere

Ray tracing

Aberrations

Spherical
Chromatic

Compound lenses

$$M_T = M_1 M_2 \ldots M_N$$

Optical systems and instruments

Camera

f-number

$$f - \text{number} \equiv \frac{f}{D}$$

The human eye

Near point $\equiv$ 25 cm
Far point $\equiv \infty$

The simple magnifier

$$m_\mathrm{min} = \frac{\theta}{\theta_0} = \frac{25 \text{ cm}}{f}$$

$$m_\mathrm{max} = 1 + \frac{25 \text{ cm}}{f}$$

The compound microscope

$$M = M_1 m_e = -\frac{L}{f_o}\left(\frac{25\ \mathrm{cm}}{f_e}\right)$$

The refracting telescope

The astronomical telescope

$$L \approx f_o + f_e$$

$$m = -\frac{f_o}{f_e}$$

The terrestrial telescope

$$L = f_o - |f_e|$$

$$m = -\frac{f_o}{f_e}$$

The reflecting telescope

Why?

The Hubble space telescope

COSTAR