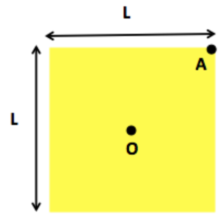
PHAS1247 : Classical Mechanics
In-Course Assessment Test #2 : Thurs. 12 December 2013

Answer as many of the questions as you can, in any order. Approved calculators may be used. The approximate distribution of marks is given in square brackets on the right of the page. There are 4 questions: they continue **ON THE OTHER SIDE OF THE PAGE**. *The maximum mark is 32.*

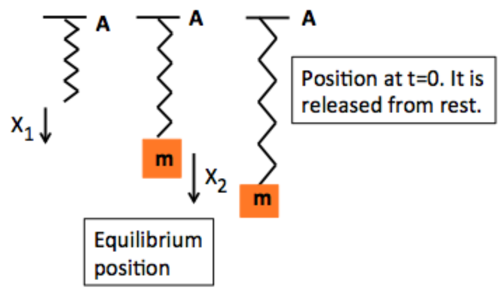
1. (a) Show that the moment of inertia of a uniform square plane of mass M and with sides of length L about an axis, O, passing through its centre and at right angles to the plane is: **[4]**

$$I_O = \frac{ML^2}{6}.$$

   (b) Determine also the moment of inertia about a parallel axis, A, through the corner of the square (see diagram below). **[2]**



2. A mass, m, is attached to a spring of negligible mass with a spring constant k. The spring is vertical and attached to a fixed point A and the mass is subject to a gravitational force. The acceleration due to gravity is g. The mass is released from rest at time $t = 0$ with a displacement x_2 (away from point A) from the equilibrium position (see figure).



   (a) Write down the differential equation satisfied by the displacement x from the equilibrium position as long as the mass remains moving and show that the mass executes simple harmonic motion with angular frequency $\omega = \sqrt{\frac{k}{m}}$ about the equilibrium position. **[3]**

   (b) Verify that it is satisfied by a solution of the form:

$$x(t) = A\cos(\omega t) + B\sin(\omega t)$$

   and find the values of the constants A and B for the data given. **[3]**

   (c) For the range of displacements, x, the mass is now also immersed in a viscous fluid that provides a retarding force: $-\beta\dot{x}$. Illustrate with a sketch of x as a function of time, how the solution for x found in (b) changes if $\frac{\beta}{2m} < \sqrt{\frac{k}{m}}$. **[3]**

3. Alastair Cook, the England cricket captain, mass $m = 80\,\text{kg}$, is stood alone and stationary on the Brisbane cricket pitch (latitude 27° S) lamenting his team's woeful batting in the second Ashes test.

   (a) Given the radius of the earth is approximately 6400 km, find the horizontal component (i.e. the component parallel to the pitch) of the centrifugal force acting on the out-of-form Mr. Cook. **[3]**

   (b) Mr. Cook then practises his coin toss for the Perth test in front of a stationary, watching umpire. He holds out his arm horizontally at a height 1.7 m above the pitch and releases a coin from rest. Find the magnitude of the Coriolis force on the coin (mass = 0.01 kg) at the moment immediately before it hits the ground (i.e. after it has travelled 1.7 m). You may take the acceleration due to gravity to be $g = 9.81\,\text{ms}^{-2}$. **[5]**

   (*Assume you can neglect air resistance, the small corrections to the coin's vertical velocity arising from the fictitious forces, that the motion of the coin is vertical as viewed by the watching umpire and that the coin is subject to the earth's rotation in the same way as Mr. Cook.*).

4. In spherical polar coordinates the formula for the velocity, $\underline{v}$, and acceleration, $\underline{a}$ of a particle are:

$$\underline{v} = \dot{r}\,\hat{\underline{r}} + r\dot{\theta}\,\hat{\underline{\theta}} \qquad \text{and}$$
$$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{\underline{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{\underline{\theta}}.$$

   A particle of mass, m, is rotating with an angular frequency, $\dot{\theta}$, which depends on time and has a radial velocity component that decreases with time at a constant rate, α, i.e. $\dot{r} = -\alpha$. At time $t = 0$ the particle has a radial position, R_o and $\dot{\theta} = \alpha$.

   (a) Assuming angular momentum is conserved, show that $\dot{\theta}$ at time $t > 0$ is given by:

$$\dot{\theta}(t) = \frac{R_o^2\,\alpha}{(R_o - \alpha t)^2}.$$

   [3]

   (b) Show that the angular component of the acceleration is zero. **[2]**

   (c) Determine the kinetic energy of the particle, K, and show that:

$$\frac{dK}{dt} = \frac{m\,R_o^4\,\alpha^3}{(R_o - \alpha t)^3}.$$

   [2]

   (d) Determine the power, $\underline{F} \cdot \underline{v}$ and compare your expression to $\frac{dK}{dt}$. **[2]**

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