CM CHEATSHEET:

Forces and Energy:

$\underline{F} = -\underline{\nabla} V \qquad W = \int \underline{F}\cdot d\underline{r}$

$|F_{fric}| = |\mu N| \qquad F_F = -\lambda v$ or $-\lambda' v^2$

Surface ↗ ↖ Fast in gas

CF are conservative: $\oint \underline{F}\cdot d\underline{r} = 0$

" act only in $\hat{\underline{r}}$ and conserve $\underline{L}$ ($\tau = 0$)

$\underline{F}(r) = m(\ddot{r} - r\dot{\theta}^2)\hat{\underline{r}}$

CM-Frames (Non-Inertial):

Linearly Accelerating: $F' = F - ma$

Of body ↗ ↖ Of frame

Rotating: $F' = F + F_{COR} + F_{CF}$

$-2m\omega\times v$ ↗ ↖ $m\omega\times(\omega\times r)$

In equilibrium: $L = 0$ so $\tau = 0$ ($dL/dt = 0$)

$\tau = \sum m_i(r_i - R)\times g = 0 \Rightarrow$ CoM = CoG

SHM:

Damping ↓ Driving Force ↓ | $T = \frac{2\pi}{\omega}$

$$m\frac{d^2x}{dt^2} + \lambda\frac{dx}{dt} + kx = F_0\cos\omega_f t$$

↖ Acceleration ↖ Restoring Force

$\gamma = \frac{\lambda}{2m} \qquad \omega^2 = \sqrt{\omega_0^2 - \gamma^2}$

For low damping $\begin{cases} dE/dt = -\lambda v^2 \approx -2\gamma E \\ Q = \frac{\omega}{2\gamma} \approx \frac{\omega_0}{2\gamma}, \quad \Delta\omega = 2\gamma = \frac{\omega_0}{Q} \end{cases}$

$\bar{P} = m\gamma v_{max}^2 = 2K_{max}\gamma$

Rigid Bodies:

For common moments of inertia, see corresponding table.

$I = \int r^2 dm = \int r^2 \rho\, dV \leftarrow$ (or dA, dL)

$L = I\omega \qquad \tau = I\ddot{\theta} \qquad KE_{ROT} = \frac{1}{2}I\omega^2$

$KE_{TOT} = KE_{ROT} + \frac{1}{2}mv^2$

Perpendicular Axis Theorem: If I_x, I_y, I_z are orthogonal and I_y, I_x lie in a plane containing lamina, then $\rightarrow I_z = I_x + I_y$ (I about z = etc).

Parallel Axis Theorem: $I = I_{cm} + ma^2$

About a point → ; About CM of body ; Total Mass ; Distance from CM to point

Angular Preliminaries:

$\hat{\underline{r}} = \begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix} \qquad \hat{\underline{\theta}} = \begin{pmatrix}-\sin\theta\\ \cos\theta\end{pmatrix}$

$\dot{\hat{\underline{r}}} = \dot{\theta}\hat{\underline{\theta}} \qquad \dot{\hat{\underline{\theta}}} = -\dot{\theta}\hat{\underline{r}} \qquad \underline{r} = r\hat{\underline{r}}$

From this derive $\underline{r}'$ and $\underline{r}''$

$v = \dot{\theta}r \qquad |a| = |v^2/r|$

$L = r\times p \qquad \tau = r\times F \qquad dL/dt = \tau$

$L = m\omega r^2$

CM-Frames (Inertial):

$\left.\begin{aligned} \underline{r}_{cm} &= \frac{1}{M}\sum m_i\underline{r}_i \\ \underline{r}_i &= \underline{r}_i' + \underline{r}_{cm} \\ \underline{r} &= \underline{r}_1 - \underline{r}_2 \end{aligned}\right\}$ All true for $\underline{v}_i$ as well.

$\underline{r}_1' = \frac{m_2}{M}\underline{r} \qquad \underline{r}_2' = -\frac{m_1}{M}\underline{r} \qquad \sum p_i' = 0$

$v = eu$ KE conserved in Elastic.

Uvast:

$v = u + at$

$v^2 = u^2 + 2as$

$s = ut + \frac{1}{2}at^2 = vt - \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

Reduced Mass:

2-Body reduced to 1-Body of mass μ and position $\underline{r}$: $\underline{r} = \underline{r}_1 - \underline{r}_2$, $\mu = \frac{m_1 m_2}{m_1 + m_2}$

$p_1' = m_1\underline{v}_1' = -m_2\underline{v}_2' = \mu\underline{v}$

Undamped SHM Common Systems:

Point Pendulum: $I = ml^2 \qquad \tau = I\ddot{\theta} \qquad \omega = \sqrt{g/L}$

Spring: $F = -kx \qquad V = \frac{1}{2}kx^2 + V_0 \qquad \omega = \sqrt{k/m}$

Compound Pendulum: $F = \tau = -mg\bar{l} \qquad \bar{l} = l\sin\theta \approx l\theta$

$\tau = I\ddot{\theta} = -mgl\theta \Rightarrow \ddot{\theta} = -\frac{mgl}{I}\theta$

$\omega = \sqrt{mgl/I}$