PHAS1247 : Classical Mechanics
Solutions for In-Course Assessment Test #1 : Mon. 6 November 2017

1.

$$\underline{\mathbf{r}}' = -\sin(t)\,\hat{\underline{\mathbf{i}}} + \sqrt{3}\cos(t)\,\hat{\underline{\mathbf{j}}} + (\mathrm{e}^{10t} + 10t\mathrm{e}^{10t}\,\hat{\underline{\mathbf{k}}}\ .$$

$$\underline{\mathbf{r}}'' = -\cos(t)\,\hat{\underline{\mathbf{i}}} - 3sin(t)\,\hat{\underline{\mathbf{j}}} + (10\mathrm{e}^{10t} + 10\mathrm{e}^{10t} + 100t\mathrm{e}^{10t}\,\hat{\underline{\mathbf{k}}}\ .$$

Acceleration for $t = 0$ is $(-1, 0, 120)$, so the vector of the force is $(-1, 0, 120)$.

2. The vertical force of gravity is 1 g, with a component $g\cos\alpha$ perpendicular to the plane, and a component $g\sin\alpha$ parallel to it. For the body to start moving, the parallel component has to be larger than 0.2 times the perpendicular one, so $\tan\alpha_m > 0.2$, $\alpha_m = 0.197$ rad. When the mass starts moving, the friction will be 0.1, so along the parallel component the force will be

$$F = g\sin\alpha_m - 0.1 * g\cos\alpha_m = 0.957N$$

and the acceleration $0.957m/s^2$.

3. Potential energy is $V(x, y, z) = mg\cos(x)$, in exact case, $V(x, y, z) = \frac{1}{2}mgx^2$ in parabolic approximation. The force acting on the ball is $F = -mg\sin(x) \approx -mgx$, so the system is mathematically identical to a pendulum with length 1 meter. When the approximation is made, the motion will be a harmonic oscillation with parameter $\omega = \sqrt{g} = 3.1/s$. Motion is described by

$$x(t) = x_0\cos(\omega t + \phi)$$

. To calculate the paramters, we know that $x(0) = 0$, $x'(0) = 1m/s$. From the initial position,

$$x_0\cos(\phi) = 0$$

so $\phi = \pi/2 + n\pi$. Taking the derivative of the velocity:

$$x'(t) = -x_a\omega\sin(\omega t + \phi)$$

so

$$1 = -x_a * 3.1$$

$$x_0 = -0.32$$

so maybe the approximation is not so good after all...

4. The force acting on the particle will be

$$F = (2x, 12y^2 - 2yz^4, -4y^2z^4)$$

Initial potential is $V(1, 1, 1) = 4$, final potential is $V(0, 0, 0) = 0$, so its kinetic energy will be 4. Applying gravity, the potential becomes

$$V(x, y, z) = x^2 + 4y^3 + 10z - y^2z^4$$

and the force

$$F = (2x, 12y^2 - 2yz^3, -4y^2z^4 - 10)$$

5. The position of m_2 as a function of time will be

$$y_2(t) = y_0 - \frac{1}{2}gt^2$$

whre y_0 is the initial height of 5 meterd. It will reach position $y = 0$ in time

$$t_0 = \sqrt{2y_0/g} = 1s$$

with speed $v_0 = t_0 g = \sqrt{2y_0 g} = 10m/s^2$. The mass m_1 will be free-falling from the time t_0 (when m_2 reaches $y = 0$), so its position as a function ot time will be

$$y_1(t) = -\frac{1}{2}g(t - t_0)^2 = -\frac{1}{2}gt^2 + g\sqrt{\frac{2y_0}{g}}t - y_0$$

The position of the centre of mass will be the weighted average of the positions of the two masses, where the weights are the values of their masses, divided byt the sum of the two:

$$y_{CM}(t) = \frac{m_2}{m_1 + m_2}y_2(t) + \frac{m_1}{m_1 + m_2}y_1(t) =$$

$$\frac{m_2}{m_1 + m_2}y_0 - \frac{1}{2}\frac{m_2}{m_1 + m_2}gt^2 - \frac{1}{2}\frac{m_1}{m_1 + m_2}gt^2 + \frac{m_1}{m_1 + m_2}\sqrt{2gy_0}t - \frac{m_1}{m_1 + m_2}y_0$$

$$= -\frac{1}{2}gt^2 + \frac{m_1}{m_1 + m_2}\sqrt{2gy_0}t - \frac{m_2 - m_1}{m_1 + m_2}y_0$$

In case of collision, the total momentum before the collision will be $P_1 = v_0 m_2 = \sqrt{2y_0 g}m_2 = 50kgm/s$. Since momentum is anyway conserved, $P_1 = (m_1 + m_2)v_{com}$, where v_{com} is the speed of the combined system; so

$$v_{com} = \frac{m_2}{m_1 + m_2}\sqrt{2y_0 g} = 500/15 = 33.3m/s$$

The two-mass system will start free-falling at time t_0, but with an initial velocity v_{com}, so its equation of motion will be

$$y_{com} = -\frac{1}{2}g(t - t_0)^2 - v_{com}(t - t_0) = -\frac{1}{2}gt^2 + \sqrt{2y_0 g}t - y_0 - \frac{m_2}{m_1 + m_2}\sqrt{2y_0 g}t + \frac{m_2}{m_1 + m_2}2y_0$$

$$= -\frac{1}{2}gt^2 + \frac{m_1}{m_1 + m_2}\sqrt{2y_0 g}t - \frac{m_2 - m_1}{m_1 + m_2}y_0$$

it is not different from the previous case because momentum is conserved, and total momentum is the sum of those of the two masses (for the correct answer, no need to to the whole calculation, the momentum conservation argument is sufficient).