PHAS1247 Classical Mechanics
Problems for Week 1 of Lectures (2016)

1. A child is travelling in a car with a velocity (as measured by an external observer) of $\mathbf{v}_1 = 10\,\hat{\mathbf{i}}\,\mathrm{m\,s^{-1}}$. The child throws a ball with a velocity (as measured by the child) of $\mathbf{v}_2 = (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})\,\mathrm{m\,s^{-1}}$. What are (a) the velocity and (b) the speed of the ball, as measured by the external observer?

2. Consider the vectors

$$\mathbf{a} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}; \qquad \mathbf{b} = \hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}.$$

   Using the definitions of the unit-vector, scalar and vector products given in the lectures, compute (i) $\hat{\mathbf{a}}$, (ii) $\mathbf{a}\cdot\mathbf{b}$ and (iii) $\mathbf{a}\times\mathbf{b}$.

3. What is the value of the scalar product of two orthogonal vectors?

   Verify that the following three vectors form an 'orthonormal set' (in other words, that they are of unit length and orthogonal to one another):

$$\hat{\mathbf{e}}_1 = \frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}; \quad \hat{\mathbf{e}}_2 = \frac{1}{\sqrt{5}}\begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}; \quad \hat{\mathbf{e}}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

   The force on an object is $\mathbf{F} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$. Find the components of $\mathbf{F}$ along the axes corresponding to $\hat{\mathbf{e}}_1$, $\hat{\mathbf{e}}_2$ and $\hat{\mathbf{e}}_3$. (In other words, find numbers c_1, c_2 and c_3 such that $\mathbf{F}$ can be expressed as $c_1\hat{\mathbf{e}}_1 + c_2\hat{\mathbf{e}}_2 + c_3\hat{\mathbf{e}}_3$.)

4. (a) What is the area of the parallelogram whose sides are formed by the vectors $\mathbf{c} = 2\hat{\mathbf{i}} + 5\hat{\mathbf{j}}$ and $\mathbf{d} = -3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$?

   (b) A particle's position vector is

$$\mathbf{r} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + \hat{\mathbf{k}} = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix}$$

   The particle rotates anti-clockwise through an angle of 0.01 radians about the z-axis. What (approximately) is its new position?