2013 first attempt

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1. De Broglie postulated that associated with every particle of momentum p is a wave with wavelength $\lambda = h/p$ where h is Planck's constant. Calculate (to 2 s.f.) the de Broglie wavelength for the following particles travelling at 10 m s^{-1}, [8]

   (a) An electron

   (b) A family car with a mass of 1000 kg.

   What experimental evidence is there that de Broglie's postulate is correct for an electron? Why would this kind of evidence not be available in the case of the family car?

$$\lambda = \frac{h}{mv}$$

$a)$

$$m = 9.11 * 10^{-31}, v = 10$$

$$\lambda = \frac{6.63 * 10^{-34}}{9.11 * 10^{-31} * 10} = 7.277 * 10^{-5} m$$

$$= 7.3 * 10^{-5} m$$

$b)$ $m = 1000$

$$\lambda = 6.63 * 10^{-38} = 6.6 * 10^{-38} m$$

Electrons behave as a wave of this calcualted wave lengths when used in experiments such as double slit experiments, forming interference patterns as though they had a wave length equal to de broglie wave length.
The size of a car is far far far larger than the de broglie wave length, and so no such experiment can be conducted.

2. Describe one experiment which implies that light travels as a wave, and one which implies that light has particle characteristics, giving in each case a concise but clear argument for why the observations support a wave or a particle character. [6]

   Youngs double slit experiment showed that when a single source of coherent light passes through 2 slits, the intensity pattern produced shows interference, mimicking the behaviour of waves. The light waves construcivly/destructivly interfere to produce light and dark patches on the screen/detector.

   Einstien photo electric effect gives evidence for light as a particle. Photo-electrons are emitted when light of a small enough wave length is shone on some metals. Electrons are only emmited if the wavelength is below a thershold. Below this point, regardless of intensity, no photo electrons are produced.
   This must mean the light travels as packets of energy, otherwise a high intensity at a low wavelength should produce photoelectrons.

3. Which of the following conditions is fulfilled when a wave-function $\psi(x)$ is normalised? [6]

   i) $\int_{-\infty}^{\infty} \psi(x)dx = 1$

   ii) $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$

   or iii) $\int_{-\infty}^{\infty} |\psi(x)| dx = 1$.

   Name one further condition which $\psi(x)$ must satisfy to be a physical wave-function. How do we calculate the probability of finding a particle with wave-function $\psi(x)$ in a certain interval, e.g. between $x = a$ and $x = b$?

Point ii must be true.
The wave function and its first derivative must me continuous (-first and second derivatives must be finite)

$$< x > = \int_a^b |\psi(x)|^2 \, dx$$

4. Describe the spectral classification scheme of stars in terms of surface temperature and luminosity class. **[7]**

$OBAFGKM$

O has highest surface temperature, M has lowest surface temperature

Luminosity:

Ia = most luminous supergiants

Ib = super giants

II = Luminous giants

III = giants

IV = sub giants

V = main sequence starts

5. In the context of populating excited atomic levels, briefly explain what is meant by (i) *photo-excitation*, and (ii) *recombination*. **[7]**

List *three* examples of astrophysical sources of absorption lines.

Photo-excitation - Increased energy state of electron in atom levels after absorption of photon

Recombination is the time when the universe had cooled enough after the big bang for electrons to bond with nuclei to form atoms.

This caused the universe to be transparent for the first time.

Absorbtion at star atomosphere,

Absorbsion of cool interstella or intergalactic gas clouds

Absorbtion of light through planet atmospheres - allow us to find composition of exoplanet atmospheres.

6. State Hubble's Law. How can the Hubble Constant be used to roughly estimate the age of the Universe? Why is it only a *rough* estimate? **[6]**

$v = H_0 d$

The recession velocity of distant galaxies is proportional to their distance from us.

It is only a rough estimate as all galaxies has some velocity of their own excluding the expansion of the universe - such as due to gravitational interactions.

7. Bohr's model of the Hydrogen atom is based on four postulates: 1) The electron travels in a circular orbit around the nucleus; 2) The electron orbit is stable and does not decay (as classical electromagnetism would predict); 3) Only orbits where the angular momentum $l = n\hbar$ are allowed, where n is an integer and $\hbar$ is the reduced Planck's constant; 4) The electron can move to a higher / lower energy orbit by absorbing / emitting a photon of energy equal to the energy difference of the orbits.

   (a) Describe one way in which Bohr's model improved on previous atomic models, and one way in which it is inferior to later quantum mechanical models. **[2]**

   (b) The Coulomb force between two objects with charges q_1 and q_2 separated by distance r is: **[3]**

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}.$$

   Given that the centripetal acceleration for a particle in a circular orbit is v^2/r, where v is the particle speed and r the orbital radius, show that the speed v of an electron of charge e and mass m_e moving in a stable circular orbit satisfies

$$v^2 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{rm_e}$$

   (c) Show that Bohr's angular momentum rule $l = n\hbar$ restricts the speed of the electron orbit to the values: **[3]**

$$v^2 = \frac{\hbar^2 n^2}{r^2 m_e^2}$$

   (d) Hence show that radii of electron orbits in the Bohr model are restricted to the values **[4]**

$$r_n = a_0 n^2$$

   and derive an expression for the Bohr radius a_0 in terms of $\hbar$, ϵ_0, m_e and e. What is the numerical value (in metres) of a_0?

(e) The energies of orbits in the Bohr model satisfy:

$$E_n = -\frac{13.6}{n^2}\text{eV}.$$

What wavelength of light will a Hydrogen atom emit when its electron transitions from the $n = 3$ to the $n = 2$ orbit? What is the name for this spectral line? **[4]**

(f) In a Helium-3 ion, a single electron orbits a nucleus consisting of two protons and a neutron. If we apply the same 4 postulates of the Bohr model for Hydrogen to describe this ion, what would be the radius of the lowest energy electron orbit? You may neglect any effect due to the differing nuclear mass. **[4]**

A) Bohr forced in the idea of quantised energy states by saying only some orbits were allowed. This isn't ideal as he just forced it in
Later models (quantum) have this result pop out elegantly as a result of the continuous nature of the wave-function.

B)

$$\frac{mv^2}{r} = K\frac{q_1 q_2}{r^2}$$

$$K = \frac{1}{4\pi\epsilon_0}, q_1 = -q_2 \to q_1 q_2 = e^2$$

$$v^2 = K\frac{e^2}{rm} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{rm_e}$$

C) $L = mr^2\omega$

$$v = r\omega \to \omega = \frac{v}{r} \to L = mrv$$

$$mrv = n\hbar$$

$$v^2 = \frac{n^2\hbar^2}{m^2r^2}$$

D)

$$\frac{n^2\hbar^2}{m^2r^2} = K\frac{e^2}{rm_e}$$

$$\to r_n = \frac{n^2\hbar^2}{m_e} * \frac{1}{Ke^2} = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2}n^2$$

$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} = 5.25 * 10^{-11}m$$

E)

$$E = -13.6\left(\frac{1}{3^2} - \frac{1}{2^2}\right)eV = \frac{17}{9} * 1.60 * 10^{-19}J$$
$$= 3.02 * 10^{-19}J$$
$$E = hf$$
$$f\lambda = c \rightarrow \lambda = \frac{c}{f}$$
$$f = \frac{E}{h} \therefore \lambda = \frac{ch}{E} = 6.59 * 10^{-7}$$
$$= 660nm$$

His name is Dave

F)

$$v^2 = K\frac{2e^2}{rm_e} = \frac{\hbar^2}{r^2 m_e^2}n^2$$
$$n = 1$$
$$r_1 = \frac{4\pi\epsilon_0\hbar^2}{m_e 2e^2} = 2.6 * 10^{-11}m$$

8. The time-independent Schrödinger equation (TISE) in one-dimension is:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x).$$

where m is the mass of the particle, E its energy, $\psi(x)$ the wavefunction and $V(x)$ the potential.

(a) Consider a particle moving in a constant potential, i.e. $V(x) = V_0$. Assuming $E > V_0$, show that the following wave-function **[4]**

$$\psi(x) = A\sin(\kappa x) + B\cos(\kappa x)$$

is a solution of the TISE and determine the relationship between E and κ.

(b) Consider an infinite square well with walls at $x = 0$ and $x = L$. The potential is zero inside the well, i.e. $V(x) = 0$ for $0 \leq x \leq L$, and is infinite outside the well, i.e. $V(x) = \infty$ for $x < 0$ and $x > L$. What are the boundary conditions which must be satisfied by physical wave-functions of a particle in this well? **[2]**

(c) By considering boundary conditions determine the allowed values of B and κ for physical wave-functions for a particle in the well. **[5]**

(d) Hence show that the allowed energies for a particle in the well are **[5]**

$$E_n = \left(\frac{h^2}{8mL^2}\right)n^2$$

where integer $n > 0$. Why is the $n = 0$ case not an allowed energy?

(e) If an electron is in an infinite square well as described in part (b) and the well has a width of 1 Angstrom, what is the difference in energy between the lowest and second lowest energy states of the electron? What wavelength of light would be transmitted or absorbed in a transition between these states? **[4]**

A)

$$\psi(x) = Asin(kx) + \ Bcos(kx)$$
$$\frac{d\psi(x)}{dx} = Akcos(kx) - Bksin(kx)$$
$$\frac{d^2\psi(x)}{dx^2} = -Ak^2\sin(kx) - Bk^2\cos(kx)$$
$$= -k^2\psi(x)$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = (E - V_0)\psi(x)$$
$$-\frac{\hbar^2}{2m}(-k^2\psi(x)) = (E - V_0)\psi(x)$$
$$\rightarrow \frac{k^2\hbar^2}{2m} = E - V_0$$

$$E = V_0 + \frac{\hbar^2}{2m}k^2$$

B)

$\psi(0) = 0, \qquad \psi(L) = 0$

This means wave stays continuous

C)

$B = 0$ $(sinodual\ wave?)$

$kL = n\pi$

This means that the wave at point 0 and L is 0

D)

$$E = V_0 + \frac{\hbar^2}{2m}k^2$$

$$V_0 = 0, k = \frac{n\pi}{L}$$

$$E = \frac{\hbar^2}{2m}\frac{n^2\pi^2}{L^2}$$

$$\hbar^2 = \frac{h^2}{4\pi^2}$$

$$E = \frac{h^2\pi^2}{8\pi^2L^2}n^2$$

$$= \frac{h^2}{8mL^2}n^2$$

When n is equal to 0 no wave function exists, only a straight line,

This is not allowed due as it would mean the exact energy would be known for any time (uncertainty principle)

E)

$$E_2 - E_1 = (2^2 - 1^2)\left[\frac{(6.63 * 10^{-34})}{8 * m * (1 * 10^{-10})^2}\right] = 1.81 * 10^{-17}$$

$$\lambda = \frac{ch}{E} = \frac{3 * 10^8 * 6.63 * 10^{-34}}{1.81 * 10^{-17}} = 1.1 * 10^{-8}$$

...?

9. (a) Describe the process of quantum tunnelling. Include a sketch of a relevant potential and the form of the wave-function both inside and outside the barrier. Describe how quantum predictions differ from those of classical physics. [You do not need to derive the exact forms of wave-functions from the Schrödinger equation – a qualitative sketch of their form is sufficient.] **[6]**

(b) Name a physical process in which quantum tunnelling plays an important role. **[2]**

(c) The probability that a quantum particle with mass m and energy E will tunnel through a square barrier of width L and height U is approximately equal to:

$$P = \exp[-2CL]$$

where

$$C = \frac{\sqrt{2m(U-E)}}{\hbar}.$$

In a Scanning Tunnelling Microscope (STM) electrons tunnel from an electrode across a potential barrier to a surface, completing a circuit whose current I is proportional to the tunnelling probability. Draw a labelled diagram of an STM indicating the path of an electron through such a device. Indicate clearly the position of the tunnelling barrier in your diagram. **[4]**

(d) If L represents the distance between electrode and surface, the current I passing through the device will be **[3]**

$$I \propto \exp[-2CL].$$

For a tunnelling barrier with height 5 eV, and an incident electron with energy 0.8 eV, calculate the constant C, justifying its unit.

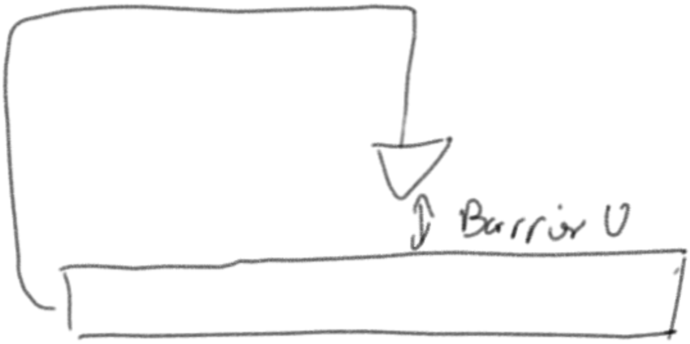
(e) Initially the current in the STM is 0.1 A. If the barrier width decreases by 1 Angstrom, what will the current change to? **[5]**

A) The wave function of a particle must be continuous. This means even when a boundary with potential higher than the energy of the particle is met, there must be a probability of finding the particle within this higher potential.

This means that a particle can travel through a barrier of high potential if there is a barrier of low potential behind it. Classical mechanics dictate this to be impossible, but it is clearly seen in processes such as nuclear fusion in the suns core

B) Fusion in suns core, measuring thickness of thin objects

C)



$E \propto I$

$P = e$?

D)

$$I \propto e^{-2CL}$$

$$C = \frac{\sqrt{2m(U-E)}}{\hbar}$$

$$E = 0.8eV$$

$$U = 5eV$$

$$C = 1.05 * 10^{10}\ m^{-1}$$

E)

$$C = \frac{\sqrt{2m(U-E)}}{\hbar}$$

$$= \frac{\sqrt{2 \times 9.11 \times 10^{-31}(5-0.8) \times 1.6 \times 10^{-19}}}{1.05 \times 10^{-34}}$$

$$= 1.05 \times 10^{10}$$

$$0.1 = ke^{-2CL}$$

$$I_2 = ke^{-2C(L-10^{-11})}$$

$$I_2 = ke^{-2C(L-\ldots)}$$
$$= ke^{-2CL} \, e^{2C \times 10^{-11}}$$
$$= 0.1 \, e^{2 \times 1.05 \times 10^{10} \times 10^{-10}}$$
$$= 0.82$$

10. By outlining the dominant sequence of nuclear reactions, describe the primary energy source in the Sun. Starting from the mass deficit, explain how the Sun can be in balance with this energy source at its centre. **[11]**

A star has a luminosity of 10^{30}W. What is its surface temperature if its radius is 30 times that of the Sun? Calculate the wavelength of the peak of the energy distribution of this star and indicate the main observational waveband this corresponds to. **[6]**

In a spectrum of this star a He absorption line is observed at 471.289 nm; given that the rest wavelength of this line is 471.320 nm, calculate the radial velocity of the star and state whether it is moving toward or away from the observer. **[3]**

A)

Multiple hydrogen nuclei are fused together to (eventually) form a helium nucleus.

H+H=D (duterieum)

H+D = Helium 3 nucleus

He3 + He3 = He4 + 2p

The mass of the helium nucleus is lower than the mass of the 4 hydogen atoms that are used to form it.

This difference in mass results in released energy - E=mc^2

This produces a lot of heat at the suns core. This heat causes radiation pressure which stops the core collapsing.

The intense gravity causes a high pressure and temperature at the core which allows fussion to occur.

The force of gravity is balanced with the radiation pressure, and will for the main sequence of the stars life.

B)

$$L = \sigma T^4 * 4\pi r^2$$
$$r = 7 * 10^8$$
$$T^4 = \frac{L}{\sigma * 4\pi * (30 * 7 * 10^8)^2} = 3.18 * 10^{15}$$
$$T = 7510K$$
$$\lambda = \frac{3 * 10^{-3}}{T} = 400nm$$

C)

$$\frac{\lambda_0 - \lambda}{\lambda_0} = \frac{v}{c}$$
$$\frac{471.320 - 471.289}{471.320} * 3 * 10^8 = 19731 \frac{m}{s}$$

Travelling towards us

11. Describe the main differences between spiral and elliptical galaxies in terms of their stellar properties and content. **[8]**

An active galaxy is thought to have a 10^8 solar mass supermassive black hole at its centre. Calculate the Schwarzchild radius (in units of $R_\odot$) of the black hole. Assuming spherical geometry also determine the average density inside the Schwarzchild radius. **[6]**

Briefly explain two methods for evaluating the masses of clusters of galaxies and hence the dark matter component. **[6]**

A)

Spiral consists of a dense halo of stars around the core - made up of older redder stars
Spiral arms where most star formation occurs - arms are due to density waves/ higher density of stellar gas therefore more star formation
Rotate around galactic core
Almost all stars found within galactic plane, other than core halo which bulges a bit
Eliptical galaxies are made up of older redder stars - very little star formation
Less structure, whole glaxay bulges not just core
Much less star formation

B)

$$\frac{1}{2}mc^2 = \frac{GMm}{r}$$

$$r = \frac{2GM}{c^2} = 2.96 * 10^{11}m$$

$$= 423\ Solar\ radi$$

$$Volume = \frac{4}{3}\pi r^3 = 1.1 * 10^{35}$$

$$Density = \frac{10^8 * 2 * 10^{30}}{1.1 * 10^{35}} = 1820\ kg/m^3$$

C)

Rotation speed of galaxy can be measured - this should give total mass of galaxy as a function of distance from core.
The rotation speed of stars should follow keplers 3rd law
We can measure the apparent mass by counting number of stars and their average size
The measured mass is far lower than the expected mass
This difference in mass is the dark matter