

## Pulleys

Various modelling assumptions are used when solving problems involving pulleys. The connecting string is generally taken to be light and inextensible. The pulley is assumed to be smooth and light.

### Worked Example 1.

A light inextensible string passes over a smooth light pulley. At each end of the string there is a particle. Particle B has a mass of 8 kg and particle C has a mass of 2 kg as shown in the diagram. The particles are released from rest with the string taut. Calculate the tension in the string and the acceleration of the masses.

### Solution

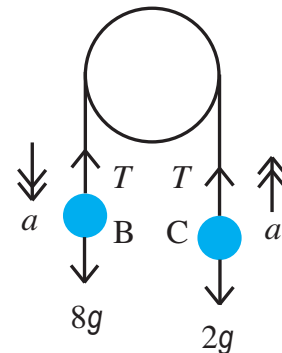
It is important to understand that the magnitude of the tension will be the same throughout the string. This is crucial in problems with connected particles. In addition, the acceleration of each particle is of the same magnitude, but in opposite directions. It is usual to consider each of the particles separately and drawing a separate force diagram for each particle can be beneficial.

Using Newton's second law,  $F = ma$ , on particle B, in the direction of motion ( $\downarrow$ ), gives:

$$8g - T = 8a \quad (1)$$

Using Newton's second law,  $F = ma$ , on particle C, in the direction of motion ( $\uparrow$ ), gives:

$$T - 2g = 2a \quad (2)$$



The simultaneous equations (1) and (2) can now be solved. From (2):  $T = 2a + 2g$  (3)

Substituting (3) into (1) gives:

$$\begin{aligned} 8g - 2a - 2g &= 8a \\ 6g &= 10a \Rightarrow a = \frac{6}{10}g = 5.9 \text{ m s}^{-2} \quad (2 \text{ s.f.}) \end{aligned}$$

It is then a simple matter to substitute this value of  $a$  back into one of the equations, say (3), to obtain

$$T = 2 \times \frac{6}{10}g + 2g = \frac{32}{10}g = 31 \text{ N} \quad (2 \text{ s.f.})$$

### Worked Example 2.

A box, of mass 6 kg, is sliding along a rough horizontal table. It is connected to another box of mass 4 kg, via a light inextensible string, which passes over a smooth light pulley (as shown in the

diagram). Given that the coefficient of sliding friction between the table and the box is 0.3, what is the acceleration of each particle and what is the tension in the string?

### Solution

Firstly consider the box on the table. The magnitude of the normal reaction,  $R$ , is equal to the magnitude of the weight of the box,  $6g$ . As the box is sliding:

$$\text{Friction} = F_r = F_{MAX} = \mu R = 0.3 \times 6g = 1.8g$$

Applying Newton's second law,  $F = ma$ , on this box in the direction of motion ( $\rightarrow$ ), gives:

$$\begin{aligned} T - F_r &= 6a \\ T - \frac{9}{5}g &= 6a \quad (1) \end{aligned}$$

Applying Newton's second law,  $F = ma$ , to the hanging box in the direction of motion ( $\downarrow$ ), gives:

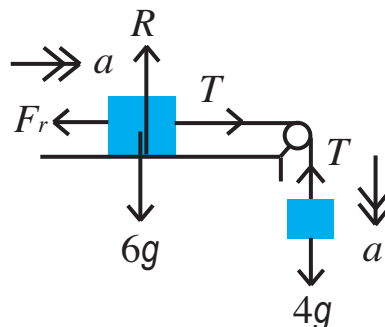
$$4g - T = 4a \quad (2)$$

Rearranging (1) gives:  $T = 6a + \frac{9}{5}g$  (3)

Substituting into (2) gives:  $4g - 6a - \frac{9}{5}g = 4a \Rightarrow 10a = \frac{11}{5}g \Rightarrow a = \frac{11}{50}g = 2.2 \text{ m s}^{-2}$  (2 s.f.)

Substituting this back into one of the equations, say (3) gives:

$$T = 6 \times \frac{11}{50}g + \frac{9}{5}g = \frac{78}{25}g = 31 \text{ N} \quad (2 \text{ s.f.})$$



### Exercises

1. A light, inextensible string passes over a smooth light pulley. At each end of the string there is a particle, as in Worked Example 1. Particle B has a mass of 10 kg and particle C has a mass of 9 kg. The particles are released from rest with the string taut. Calculate the acceleration of the masses and the tension in the string.
2. A light, inextensible string passes over a smooth light pulley. At each end of the string there is a particle, as in Worked Example 1. Particle B has a mass of 11 kg and particle C has a mass of  $x$  kg. Given the acceleration of the masses is  $2.2 \text{ m s}^{-2}$  and particle B is accelerating downwards, calculate the tension in the string and the mass,  $x$ , of particle C.
3. A box, of mass 1.2 kg, is sliding along a rough horizontal table. It is connected, via a light inextensible string passing over a smooth light pulley, to another box of mass 1.2 kg, as in Worked Example 2. Given the coefficient of friction between the table and the box is 0.44, what is the acceleration of each particle and what is the tension in the string?
4. A car, of mass 450 kg, is on a slope which is inclined at  $45^\circ$  to the horizontal. It is attached via a light inextensible string, which passes over a smooth light pulley, to a box of mass 400 kg, which is accelerating downwards (similar to Worked Example 2, only with an inclined slope). Given a resistive force of 450 N acts on the car, what is its acceleration and what is the tension in the string?

**Answers** (All to 2 s.f.)

1.  $a = 0.52 \text{ m s}^{-2}$ ,  $T = 93 \text{ N}$
2.  $T = 84 \text{ N}$ ,  $x = 7.0 \text{ kg}$
3.  $a = 2.7 \text{ m s}^{-2}$ ,  $T = 8.5 \text{ N}$ ,
4.  $a = 0.41 \text{ m s}^{-2}$ ,  $T = 3800 \text{ N}$ .