Cover-page

PHAS1202

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Atoms, Stars and The Universe

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Answer ALL SIX questions from Section A.

Answer THREE questions from Section B, including AT LEAST ONE question from EACH of Sections B1 and B2.

The following may be assumed if required:

Planck's constant	h	6.63×10^{-34} m^2 kg s^{-1}
	$\hbar$	1.05×10^{-34} m^2 kg s^{-1}
Electron mass	m_e	9.11×10^{-31} kg
Electron charge	e	1.60×10^{-19} C
Bohr radius	a_0	0.53 Å
Rydberg constant	R_H	1.1×10^7 m^{-1}
Proton mass		1.0078 amu
Helium mass		4.0026 amu
Atomic mass unit	amu	1.66×10^{-27} kg
Permittivity of free space	ϵ_0	8.85×10^{-12} F m^{-1}
Speed of light	c	3.0×10^8 m s^{-1}
Solar radius	$R_\odot$	7.0×10^8 m
Solar luminosity	$L_\odot$	3.8×10^{26} W
Solar mass	$M_\odot$	2.0×10^{30} kg
Hubble constant	H_0	75 km s^{-1} Mpc^{-1}
Gravitational constant	G	6.67×10^{-11} m^3 kg^{-1} s^{-2}
1 parsec	pc	3.1×10^{16} m
1 year	yr	3.16×10^7 s
1 Angstrom	Å	10^{-10} m
1 electron Volt	eV	1.60×10^{-19} J

Numbers in square brackets in the right-hand margin indicate a provisional allocation of maximum possible marks for different parts of each question.

Section A – *(Answer ALL SIX questions from this section)*

1. What is the formula for the de Broglie wavelength for a particle of momentum p? Calculate the de Broglie wavelength for the following objects travelling at 10 m s^{-1}, **[8]**

   i) A proton,

   ii) A cricket ball with a mass of 160 g.

   Give an example of an experiment that supports the de Broglie wavelength formula. Why would this kind of evidence not be available in the case of the cricket ball?

2. Which of the following is the correct expression for the expectation value of position $\langle x\rangle$ for a particle with wavefunction $\psi(x)$, **[6]**

   i) $\int_{-\infty}^{\infty} x|\psi(x)|dx$,

   ii) $\int_{-\infty}^{\infty} x|\psi(x)|^2dx$,

   or iii) $\int_{-\infty}^{\infty} x\psi(x)\ dx$?

   Explain in words what $\langle x\rangle$ represents. How do we calculate the probability of finding the particle in a certain interval, e.g. between $x = a$ and $x = b$? What is the name usually given to this formula?

3. Without calculation, describe the process of quantum tunnelling. In your answer, include sketches of the potential and a relevant wavefunction, and discuss the difference in behaviour between a quantum particle and a classical particle experiencing the same potential. **[6]**

4. Explain why such a wide diversity of stars is apparent in the night sky in terms of colour and brightness. **[6]**

5. Explain what is meant by *hydrostatic equilibrium* in a star. **[3]**

   In the spectrum of a star an absorption line due to ionized carbon is detected at 154.79 nm, and the rest wavelength of this line is 154.82 nm. Calculate the radial velocity of the star and state whether it is moving away or towards us. In which waveband does the absorption line appear? **[4]**

6. Describe the Hubble classification scheme for spiral galaxies, using the common nomenclature. **[7]**

Section B – *(Answer THREE questions from this section, including AT LEAST ONE question from EACH of Sections B1 and B2)*

Section B1 – *(Answer AT LEAST ONE question from this section)*

7. (a) Describe Rutherford's planetary atomic model for Hydrogen and state two ways in which Bohr's atomic model differs from it. Give one reason why Bohr's model was superior to the planetary model and one reason why it was inferior to later quantum mechanical models. **[5]**

   (b) An ion is called Hydrogen-like when it consists of a single electron orbiting a nucleus of charge $+Ze$, where Z is the ion's atomic number. The magnitude of the Coulomb force $|F|$ and the potential energy V between two objects with charges q_1 and q_2 separated by distance r are as follows,

$$|F| = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2}\ , \qquad V = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}\ .$$

   Considering the expression for the centripetal acceleration of a particle in a circular orbit, show that the speed v of the electron in a stable circular orbit with radius r in a Hydrogen-like ion satisfies **[3]**

$$v^2 = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{rm_e}\ .$$

   (c) The magnitude of angular momentum l for a particle of mass m in such a circular orbit is $l = mvr$. Use Bohr's angular momentum rule $l = n\hbar$, where n is an integer, to show that radii of electron orbits in the Bohr model are restricted to the values **[3]**

$$r_n = \frac{a_0 n^2}{Z}\ .$$

   Derive an expression for the Bohr radius a_0 in terms of $\hbar$, ϵ_0, m_e and e.

   (d) Hence, show that, in the Bohr model, the energy levels of the electron in a Hydrogen-like ion have energies that depend on integer n and satisfy **[4]**

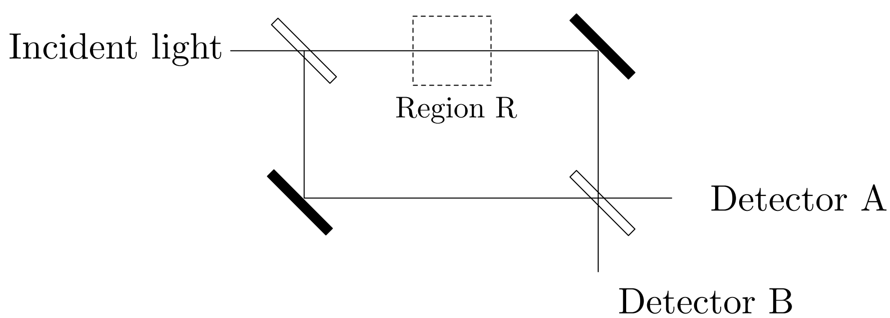
$$E_n = -\frac{Z^2 \times 13.6}{n^2}\ \text{eV}.$$

   (e) In the year 1896, spectral lines from a Hydrogen-like ion were observed in stellar radiation. Two spectral lines, at 10123 Angstroms and 5411 Angstroms, were identified. The 10123 Angstrom line arises in a transition from energy level $n = 5$ to $n = m$, where $m < 5$ is an integer. The 5411 Angstrom light is associated with the transition from $n = 7$ to $n = m$ (the same m as for the other line). Use this information to compute m and Z and hence identify this ion. **[5]**

8. (a) Huygens was famously skeptical of Newton's particle model of light. Imagine you could send a letter to Huygens. Write an excerpt from such a letter (with diagrams if you wish) to tell him of experiments which provide evidence of particle-like properties of light. Include a concise description of *two* different experimental setups and explain what you would expect to observe in the experiments if light were purely wave-like. Explain how the experiments imply that a solely wave-like description of light is incorrect. **[7]**

(b) An interferometer is set up so that all incident light will arrive at one of two detectors, detector A or detector B. Laser light is intense enough that it behaves like a classical wave. A laser beam with power I_0 enters the interferometer. The power detected at the detector A is I_A. Assuming all incident light reaches one of the two detectors, what power is detected at detector B? A single photon is passed through the same interferometer. What would be the probability of detecting it at detector A? (You may assume that your detector will detect any photon incident on it). **[2]**

(c) A Mach-Zehnder interferometer is illustrated in the diagram below. It consists of 50:50 beam splitters (white rectangles), which transmit 50% of incident intensity with zero phase shift and reflect 50% of incident intensity with a $\lambda/4$ phase shift. It also contains perfect mirrors (black rectangles) which reflect without causing a phase shift. Here, λ is the wavelength of the incident light. *[Ignore the marked 'Region R' until part (d).]* **[6]**



If laser light with power I_0 is input into the interferometer, what power will be measured by detectors A and B? Explain your answer with reference to interference and the phase shifts along different paths to the detectors.

(d) A black screen is now placed in 'Region R' (in the diagram) completely absorbing all light entering the region. If laser light with power I_0 enters the interferometer with the screen in place, what power will be measured by detectors A and B? A single photon is now passed through the interferometer. Assuming detectors A and B will detect any incident photon, what are the probabilities of the photon reaching each detector, when 'Region R' is (i) blocked by a screen, or (ii) unblocked (as in part c). Discuss whether this single photon experiment is consistent with a solely particle-like model of light. **[5]**

9. The time-independent Schrödinger equation (TISE) in one-dimension is

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x),$$

where m is the mass of the particle, E its energy, $\psi(x)$ the wavefunction and $V(x)$ the potential.

(a) Consider a particle moving in a constant potential, i.e. $V(x) = V_0$. When $E > V_0$ the following wavefunction, **[3]**

$$\psi(x) = A\sin(\kappa x) + B\cos(\kappa x)\ ,$$

is a solution of the TISE. Determine the relationship between E and κ.

(b) Write down an expression for the potential $V(x)$ for an infinite square well with walls at $x = 0$ and $x = L$ and comment on the form of the wavefunction of a particle in the regions $x < 0$ and $x > L$. **[2]**

(c) What are the boundary conditions which must be satisfied by wavefunctions of a particle in the infinite square well? Identify solutions to the TISE which satisfy these boundary conditions and show that the allowed energies for a particle in the well are **[6]**

$$E_n = \left(\frac{h^2}{8mL^2}\right) n^2\ ,$$

where integer $n > 0$. Why is the $n = 0$ case not an allowed energy?

(d) A quantum particle of mass m in one-dimension experiences a harmonic potential $V_H(x)$

$$V_H(x) = \frac{mx^2}{2}\ .$$

One solution to the TISE with potential $V(x) = V_H(x)$ is the wavefunction $\psi_1(x)$,

$$\psi_1(x) = A_1 x e^{-\gamma^2 x^2/2}\ ,$$

where A_1 is a normalising constant and $\gamma = \sqrt{m/\hbar}$. Determine the energy of a particle with wavefunction $\psi_1(x)$ in the potential $V_H(x)$. Without detailed calculation determine the expectation value $\langle x \rangle$. **[6]**

(e) The particle absorbs a photon. After this has occurred, what physical properties of the particle will have changed? How will our description of the particle change? Fully justify your answer. **[3]**

Section B2 – *(Answer AT LEAST ONE question from this section)*

10. (a) Briefly explain what 'Cepheid variables' are, and why they are useful in determining the distance to galaxies. **[6]**

    (b) Explain the circumstances that give rise to Type Ia supernovae and why they are useful 'standard candles'. What major result about the evolution of the Universe has emerged in recent years from the study of Type Ia supenovae at cosmological distances? **[8]**

    (c) Suppose that luminosity and mass of a star are related by $L \propto M^{3.8}$ on the main sequence; that the Sun's main sequence lifetime is 10^{10} years; and that the same mass fraction is consumed by all stars during the hydrogen-burning phase. Using this information, estimate the main-sequence lifetime, in years, of a 2.5-$M_\odot$ star. **[6]**

11. (a) Define the cosmological density parameter Ω and describe the basic features of the three Universes that result from $\Omega = 0$, 1 and 2 (with zero cosmological constant). **[6]**

    (b) For the case of $\Omega = 1$, calculate the critical density (in kg m^{-3}) for matter in the Universe. **[4]**

    (c) When we observe a quasar with redshift z = 0.15, estimate how far into its past (in years) we are looking. (Assume H_0 is constant in time for this estimate.) **[5]**

    (d) Outline the 'horizon problem' and the 'flatness problem'; and *briefly* describe the accepted solution to these problems within the basic Big Bang model. **[5]**