PHAS0012 (1449) Computing for Mathematical Physics 2018/19

Homework 3

Mark for homework 3: 27 /35

(to be competed by your marker)

Feedback from marker:

(to be competed by your marker)

Comments/ explanations are good
-Read questions carefully make sure you are doing what is asked
-Test solutions using equalities or by substituting back in

Which **feedback from your last homework** are you **employing in this homework**?

Marks will be deducted if you do not complete this section. - My homework feedback only talked about learning to use the Limit function correctly, which in this case is not applicable. But I am employing the use of more description in some of my answers and trying to read the question more carefully, while giving more full answers.

OK

Give your answers in the code cells labelled "(*your solution here*)"

1. Solve the simultaneous equations

$x^2 + 2y = 1$

$x^2 - 3y^2 = 10$

Check all the solutions by substituting them back into the original equations and using **Simplify** if necessary. [4 marks]

In[1]:=
```
q1eq1 = x^2 + 2 y == 1
q1eq2 = x^2 - 3 y^2 == 10
```

Out[1]= $x^2 + 2y == 1$

Out[2]= $x^2 - 3y^2 == 10$

In[3]:=
```
Simplify[Solve[{q1eq1, q1eq2}, {x, y}]] // MatrixForm
```

Out[3]//MatrixForm=

$$\begin{pmatrix} x \to -\sqrt{\frac{1}{3}\left(5 - 2\,i\,\sqrt{26}\right)} & y \to \frac{1}{3}\,i\,\left(i + \sqrt{26}\right) \\ x \to \sqrt{\frac{1}{3}\left(5 - 2\,i\,\sqrt{26}\right)} & y \to \frac{1}{3}\,i\,\left(i + \sqrt{26}\right) \\ x \to -\sqrt{\frac{1}{3}\left(5 + 2\,i\,\sqrt{26}\right)} & y \to -\frac{1}{3}\,i\,\left(-i + \sqrt{26}\right) \\ x \to \sqrt{\frac{1}{3}\left(5 + 2\,i\,\sqrt{26}\right)} & y \to -\frac{1}{3}\,i\,\left(-i + \sqrt{26}\right) \end{pmatrix}$$

2/4 -these are the correct solutions but you need to substitute them back in - this can be done using a rule

2. This is an exercise on the Taylor series. You may wish to refresh your memory on what Taylor's series is by looking in a text book or Wikipedia. Proceed as follows: [9 marks]

(i) Construct a list containing $\sin(\pi x)\log(1+x)$ and its first 4 derivatives with respect to x.

In[4]:=
```
Clear[x]
```

In[5]:=
```
q2function1 = Sin[π x] Log[1 + x]
```

Out[5]= Log[1 + x] Sin[π x]

In[6]:=
```
q2table1 = Table[D[q2function1, {x, i}], {i, 0, 4}]
```

Out[6]=

$$\left\{\text{Log}[1+x]\,\text{Sin}[\pi x],\ \pi\,\text{Cos}[\pi x]\,\text{Log}[1+x] + \frac{\text{Sin}[\pi x]}{1+x},\ \frac{2\pi\,\text{Cos}[\pi x]}{1+x} - \frac{\text{Sin}[\pi x]}{(1+x)^2} - \pi^2\,\text{Log}[1+x]\,\text{Sin}[\pi x],\ -\frac{3\pi\,\text{Cos}[\pi x]}{(1+x)^2} - \pi^3\,\text{Cos}[\pi x]\,\text{Log}[1+x] + \frac{2\,\text{Sin}[\pi x]}{(1+x)^3} - \frac{3\pi^2\,\text{Sin}[\pi x]}{1+x},\ \frac{8\pi\,\text{Cos}[\pi x]}{(1+x)^3} - \frac{4\pi^3\,\text{Cos}[\pi x]}{1+x} - \frac{6\,\text{Sin}[\pi x]}{(1+x)^4} + \frac{6\pi^2\,\text{Sin}[\pi x]}{(1+x)^2} + \pi^4\,\text{Log}[1+x]\,\text{Sin}[\pi x]\right\}$$

2 Fine but could be done in one line

(ii) Use a rule to convert this to a second list giving the values of $\sin(\pi x)\log(1+x)$ and its derivatives at x=3.

In[7]:= q2table2 = q2table1 /. x → 3

Out[7]= $\left\{0, -\pi \,\text{Log}[4], -\frac{\pi}{2}, \frac{3\pi}{16} + \pi^3 \,\text{Log}[4], -\frac{\pi}{8} + \pi^3\right\}$ **1 correct**

(iii) Construct another list containing $(x-3)^n/n!$ for n=0,1,..4.

In[8]:= q2function2 = ((x - 3) ^n) / (n!)

Out[8]= $\frac{(-3+x)^n}{n!}$

In[9]:= q2table3 = Table[q2function2, {n, 0, 4}]

Out[9]= $\left\{1, -3+x, \frac{1}{2}(-3+x)^2, \frac{1}{6}(-3+x)^3, \frac{1}{24}(-3+x)^4\right\}$ **2 This is fine**

(iv) Hence find the terms up to x^4 in the expansion of $\sin(\pi x)\log(1+x)$ about x=3.

In[10]:= q2taylor1 = Expand[{q2table2 . q2table3}][[1]]

Why this term only?

Out[10]= $-\frac{225\pi}{64} + \frac{27\pi^3}{8} + \frac{93\pi x}{32} - \frac{9\pi^3 x}{2} - \frac{13\pi x^2}{16} + \frac{9\pi^3 x^2}{4} + \frac{3\pi x^3}{32} - \frac{\pi^3 x^3}{2} - \frac{\pi x^4}{192} + \frac{\pi^3 x^4}{24} + 3\pi\,\text{Log}[4] - \frac{9}{2}\pi^3\,\text{Log}[4] - \pi x\,\text{Log}[4] + \frac{9}{2}\pi^3 x\,\text{Log}[4] - \frac{3}{2}\pi^3 x^2\,\text{Log}[4] + \frac{1}{6}\pi^3 x^3\,\text{Log}[4]$

Don't need to expand here that is done when finding q2table2

(v) Check your result using the built-in *Mathematica* function **Series**.

In[11]:= q2taylor2 = Normal[Series[q2function1, {x, 3, 4}]] // Expand

Out[11]= $-\frac{225\pi}{64} + \frac{27\pi^3}{8} + \frac{93\pi x}{32} - \frac{9\pi^3 x}{2} - \frac{13\pi x^2}{16} + \frac{9\pi^3 x^2}{4} + \frac{3\pi x^3}{32} - \frac{\pi^3 x^3}{2} - \frac{\pi x^4}{192} + \frac{\pi^3 x^4}{24} + 6\pi\,\text{Log}[2] - 9\pi^3\,\text{Log}[2] - 2\pi x\,\text{Log}[2] + 9\pi^3 x\,\text{Log}[2] - 3\pi^3 x^2\,\text{Log}[2] + \frac{1}{3}\pi^3 x^3\,\text{Log}[2]$

Need to show that these are the same

7/9

3. Use **RealDigits[N[Catalan,1000]][[1]]** to generate a list of 1000 digits of the Catalan number. Use a rule with a condition (/;) based on **OddQ** to convert every odd digit in the list to a letter o, and hence find how many odd digits there are in the list. Do the same for even digits, using the letter e. Check your results using the **Cases** function on the original list. [6 marks]

Converting all odd numbers generated into "o"s and then counting the number of "o"s in the list to know how many odd numbers are in the list

In[12]:= q3list1 = RealDigits[N[Catalan, 1000]][[1]] /. x_ /; OddQ[x] → o;

In[13]:= Count[q3list1, o]

Out[13]= 521 **This is correct**

Using the Case function to remove all non-odd numbers and then counting the length of that list to know how many odd numbers their are

In[14]:=
```
Length[Cases[RealDigits[N[Catalan, 1000]][[1]], x_ /; OddQ[x]]]
```
Out[14]= 521

Nicely done

Converting all even numbers generated into “e”s and then counting the number of “e”s in the list to know how many even numbers are in the list

In[15]:=
```
q3list2 = RealDigits[N[Catalan, 1000]][[1]] /. x_ /; EvenQ[x] → e;
```
In[16]:=
```
Count[q3list2, e]
```
Out[16]= 479

Using the Case function to remove all non-even numbers and then counting the length of that list to know how many even numbers their are

In[17]:=
```
Length[Cases[RealDigits[N[Catalan, 1000]][[1]], x_ /; EvenQ[x]]]
```
Out[17]= 479

Again good 6/6

4. Write a rule, or list of rules, that when applied to an object will make all negative numbers positive, but will leave anything else unchanged. Your rule should work with a variety of *Mathematica* constructs (eg. single numbers, lists, nested lists, expressions etc). By writing a selection of tests for your rule, explore its range of operation for different objects. You should construct at least four appropriate test cases. Describe why each test case is useful and distinct. An explanation of a test case should be about one or two lines long. [10 marks]

In[18]:=
```
q4rule1 = {x_ /; x < 0 → -x};
```
OK

This test case uses a single number and shows that the rule is able to be applied to single numbers

In[19]:=
```
-3 /. q4rule1
```
Out[19]= 3

This test case uses a list and shows that the rule is able to be applied to lists

In[20]:=
```
{-3, 1, 2, -10, -12} /. q4rule1
```
Out[20]= {3, 1, 2, 10, 12}

This test case uses a nested list and shows that the rule is able to be applied to nested lists

In[21]:=
```
{{-1, {2, -3}}, {-4, -5, 6}, {-7, 8, 9}} /. q4rule1
```
Out[21]= {{1, {2, 3}}, {4, 5, 6}, {7, 8, 9}}

This test case uses a nested list with expressions and shows that the rule is able to be applied to nested lists with expressions. This assuming that the expressions are positive though.

In[22]:=
```
{{a, {2, -b}}, {-c, -5, d}, {-7, 8, 9}} /. q4rule1
```
Out[22]= {{a, {2, b}}, {c, 5, d}, {7, 8, 9}}

You haven’t checked if this works on Integers which would be a good test

9/10 -good tests and explanations

5. The pair of numbers {E,Q} is used to describe a given energy level of a molecule system. The numbers are defined such that E is a positive number and Q is an integer. Write a single rule that can be used with a list of the form {{E1, Q1}, {E2, Q2}, {E3, Q3}...} to replace any pairs where the E is negative or Q is not an integer with the entry {"Error"}. Demonstrate the use of your rule with the list {{1.0, 1}, {-2.0, 2}, {1.0, 3}, {2.0, 4.1}, {5.0, 5}, {6.0, 6.0}, {7.0, 7}, {-1.0, 8.0}}. Your rule should use two conditions (/;). [6 marks]

In[23]:=
```
q5rule1 = {{x_, y_} /; x < 0 → "Error"} ;
q5rule2 = {{x_, y_} /; Mod[y, 1] ≠ 0 → "Error"};
```

Asked for a single rule

Doesn't work for 1.0,2.0.3.0 etc

In[25]:=
```
q5list = {{1.0, 1}, {-2.0, 2}, {1.0, 3},
   {2.0, 4.1}, {5.0, 5}, {6.0, 6.0}, {7.0, 7}, {-1.0, 8.0}};
```

Doesn't work for integers use IntergerQ -6.0 should give error

In[26]:=
```
q5list /. q5rule1 /. q5rule2
```

Out[26]=
```
{{1., 1}, Error, {1., 3}, Error, {5., 5}, {6., 6.}, {7., 7}, Error}
```

Total marks available: 35

3/6

Solutions are due by **4pm on Monday January 28th.**

Make a copy of your solutions with the output deleted (**`Cell|Delete All Output`**) and upload that file to Moodle.

Please name the file to include your family name and first name, for example I would use **`hw1_Jasvir_Bhamrah`**.

The first thing I shall do when I get the file is to click **`Evaluation|Evaluate Notebook,`** so make sure the file you send me will survive that.

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