PHAS1202
Atoms, Stars and The Universe
Exam 2017

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Answer ALL SIX questions from Section A.

Answer THREE questions from Section B, including AT LEAST ONE question from EACH of Sections B1 and B2.

The numbers in square brackets in the right-hand margin indicate a provisional allocation of maximum possible marks for different parts of each question.

The following may be assumed, if required:

Speed of light *in vacuo*	c	2.998×10^{8} m s^{-1}
Universal gravitational constant	G	6.674×10^{-11} m^{3} kg^{-1} s^{-2} (= N m^{2} kg^{-2})
Planck's constant	h	6.626×10^{-34} m^{2} kg s^{-1} (=J s)
Boltzmann's constant	k	1.381×10^{-23} m^{2} kg s^{-2} K^{-1} (=J K^{-1})
Stefan-Boltzmann constant	σ	5.670×10^{-8} W m^{-2} K^{-4}
Radiation constant	$a = 4\sigma/c$	7.566×10^{-16} J m^{-3} K^{-4}
Proton mass		1.0078 amu
Helium mass		4.0026 amu
Atomic mass unit	amu	1.66×10^{-27} kg
Electron mass	m_e	9.109×10^{-31} kg
Electron charge	e	1.602×10^{-19} C
Electron volt	eV	1.602×10^{-19} J
Solar radius	$\mathcal{R}_\odot$	6.957×10^{8} m
Solar luminosity	$\mathcal{L}_\odot$	3.828×10^{26} W
Solar mass	$\mathcal{M}_\odot$	1.989×10^{30} kg
Solar effective temperature	$\mathcal{T}_{\rm eff}(\odot)$	5772 K
Astronomical unit	au	1.496×10^{11} m
Parsec	pc	3.086×10^{16} m
Hubble constant	H_0	68 km s^{-1} Mpc^{-1}
1 year	yr	3.16×10^{7} s
Mass of Earth		5.972×10^{24} kg

Section A

(Answer ALL SIX questions from this section)

1. Describe the spectral classification scheme of stars in terms of surface temperature and luminosity class. **[6]**

2. Using a labelled diagram, outline the main structural components of the Milky Way Galaxy, giving an indication of dimensions and content. **[7]**

3. We might suppose the rate of expansion of the Universe to be slowing down, because of the mutual gravitational attraction of galaxies. Explain why supernovae may be used to measure the rate of change of the expansion, and the results that have emerged. **[7]**

4. Describe the phenomenon of quantum tunnelling and contrast the behaviour of a quantum particle encountering a finite potential barrier with that of a classical particle. Your answer should not include detailed calculations, but should include sketches of the potential and the relevant wavefunction. **[7]**

5. Briefly describe the main features of Compton's scattering experiment. How does this experiment provide evidence of the particle-like nature of light? **[4]**

   Describe an experiment which implies that light has wave-like characteristics. **[3]**

6. Many modern lighthouses utilise 1kW bulbs coupled with rotating mirror and lens assemblies. Assuming the wavelength of the light is 570nm calculate the energy of a single photon and the number of photons emitted per second. **[6]**

Section B

(Answer THREE questions from this section, including AT LEAST ONE question from EACH of Sections B1 and B2)

Section B1 – *(Answer AT LEAST ONE question from this section)*

7. (a) The time-independent Schrödinger equation for a particle in one-dimension is:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

where m is the mass of the particle, E its energy, $\psi(x)$ the wavefunction and $V(x)$ the potential. **[3]**

For a constant potential show that

$$\psi(x) = A\sin(\kappa x) + B\cos(\kappa x)$$

is a general solution. Determine the relationship between κ and E, and state the energy regime in which this is a general solution.

(b) Many simple quantum systems can be described as a "particle-in-a-box", in which a particle is confined by some potential. We will consider the case of a particle confined in a infinite well, where $V(x) = 0$ for $0 \leq x \leq L$ and $V(x) = \infty$ for all other x. **[3]**

Explain what the wavefunction must be in the regions where $V(x) = \infty$. If instead of infinite the potential in these regions was finite (but still larger than the energy), what form would the wavefunction have in the region $x < 0$?

(c) Apply boundary and normalisation conditions to show that the wavefunction and allowed energy levels of the particle inside the infinite well are **[6]**

$$\psi(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right) \quad \text{and} \quad E_n = \left(\frac{h^2}{8mL^2}\right)n^2.$$

(d) Determine the probability that the particle is found in the first (closest to 0) 10% of the well. How does this probability change with n and how does it compare to the classical picture? **[4]**

(e) The nuclear binding force in atoms is sometimes described as an infinite square well potential. Assume a proton is bound in an infinite well 10 fm wide. Sketch the wavefunctions of the lowest three energy levels and determine the wavelength of a photon that would need be absorbed to excite the proton from the first to third level. **[4]**

8. (a) Consider a particle with the wavefunction $\psi(x)$

$$\psi(x) = \begin{cases} Ae^{-x}\left(1 - e^{-x}\right) & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$$

Sketch this wavefunction and show that $A = \sqrt{12}$ normalises the wavefunction $\psi(x)$. . [5]

(b) Calculate the expectation value $\langle x \rangle$ for the position of the particle . [5]

(c) Use the probability distribution function to determine the most probable position of the particle. Explain the difference between the most probable position and the expectation value. [4]

(d) Sketch a potential which could give rise to a wavefunction with the form above. [2]

(e) Considering only its motion around the Sun estimate the de Broglie wavelength of the Earth. [4]

9. (a) Describe Rutherford's planetary model of the hydrogen atom. Explain two ways in which this model is inconsistent with experimental data. [4]

(b) Describe two of the postulates of Bohr's model of the hydrogen atom, which improved upon Rutherford's planetary model. [2]

(c) The energies of the orbits in the Bohr model satisfy:

$$E_n = -\frac{13.6}{n^2}\text{e}V$$

where n is a positive integer. What wavelength of light is required to excite an electron from the $n = 1$ to $n = 4$ orbit? [3]

(d) Describe two ways in which a fully quantum-mechanical model improves upon Bohr's model of the atom. [2]

(e) In a fully quantum-mechanical treatment of the hydrogen atom, there are two additional quantum numbers that describe the system:

- l is any integer between 0 and $n - 1$.
- m is any integer such that $|m| \leq l$

List all of the quantum number combinations that correspond to wavefunctions with energy $E = -0.85$eV. [5]

(f) Find a formula which describes the the number of different wavefunctions for a general n. [Hint: Write out the possible allowed combinations of quantum numbers for n =1, 2, 3, 4 etc. and look for a pattern.] [4]

Section B2 – *(Answer AT LEAST ONE question from this section)*

10. (a) Briefly contrast how the key evolutionary stages, up to the end-state, of a $30\mathcal{M}_\odot$ star differ from those of $1\mathcal{M}_\odot$ star. **[9]**

(b) Calculate the main-sequence lifetime (in years) of the $30\mathcal{M}_\odot$ star if it has a luminosity of $8 \times 10^5\ \mathcal{L}_\odot$ and 10% of its mass will be converted from hydrogen to helium in the core. Compare this main-sequence lifetime to that of the Sun. **[7]**

(c) After the $1\mathcal{M}_\odot$ star has used up its supply of hydrogen in fusion, its temperature decreases by a factor of 5 and the luminosity increases by a factor of 2. Determine the factor by which the radius increases. State the physical mechanism that carries energy into the expanding envelope of this evolved star. **[4]**

11. (a) The galaxy NGC5754 is observed with the Hubble Space Telescope and measured to have a redshift z = 0.14. Estimate how far in the past (in years) are we looking toward this galaxy. State why this estimate is not exact. **[6]**

(b) In this galaxy, at what wavelength would the Lyman α line (rest wavelength 121.6 nm) be observed? **[3]**

(c) NGC5754 is thought to have a $5 \times 10^6\mathcal{M}_\odot$ black hole at its centre. Calculate the Schwarzschild radius (in units of $\mathcal{R}_\odot$) of the black hole. Assuming spherical geometry also determine the average density (in kg m^{-3}) inside the Schwarzschild radius. **[5]**

(d) This galaxy resides in a cluster; briefly explain two methods for evaluating the masses of clusters of galaxies and hence the dark matter component. **[6]**

END OF PAPER