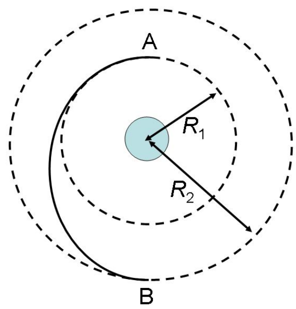
PHAS1247 Classical Mechanics
Problems for Week 6/7 of Lectures (2016)

1. A binary star system consists of two stars of masses M_1 and M_2 orbiting their common centre of mass. If the orbits of both stars are observed to be circular and to have period T, what is the distance R between the stars?

2. A Hohmann transfer orbit is a way of transferring a spacecraft between two planetary orbits (which we shall assume are circular) by using one half of an elliptical orbit about the Sun, shown as the solid line in the diagram.



A spacecraft is initially moving around the Sun with an orbital speed V_1 at a radius R_1, and then just before it reaches point A it fires a booster so that it's orbital speed is increased to v_A causing it to transition to a new orbit of radius R_2 and a orbital speed of v_B.

Write down the conditions on v_A and v_B coming from (i) the conservation of energy and (ii) the conservation of angular momentum, on the assumption that the gravitational fields of the planets have a negligible effect on the spacecraft compared to the gravitational field of the Sun.

Hence show that the velocity boost required to accelerate the spacecraft into the transfer orbit is

$$\Delta v = v_A - V_1 = \sqrt{\frac{GM_\odot}{R_1}} \left[\sqrt{\frac{2R_2}{R_1 + R_2}} - 1 \right],$$

where $M_\odot$ is the mass of the Sun.

Show also, using Kepler's third law i.e. that $\omega^2 = \frac{GM}{a^3}$ where a is the length of the semi-major axis, that the time taken for the transfer is

$$T_{\text{transfer}} = \pi \sqrt{\frac{(R_1 + R_2)^3}{8GM_\odot}}.$$

Evaluate T_{transfer} and Δv for a Hohmann transfer between the (approximately circular) orbits of Earth (radius $R_1 = 1.50 \times 10^{11}$ m) and Jupiter (radius $R_2 = 7.79 \times 10^{11}$ m).

Using, $r(1 + e\cos\theta) = r_o$, derermine the eccentricity of the transfer orbit in this case?

[Mass of Sun: $M_\odot = 1.99\times10^{30}$ kg; gravitational constant $G = 6.67\times10^{-11}\text{m}^3\text{kg}^{-1}\text{s}^{-2}$.]

3. *This is the original form of the question from the 2014 exam, sadly it was vetoed and a tamer version appeared. Here's the original in all its glory*

   Three earth asteriod collisions were averted in Hollywood movies in 1998. In 1999 a real asteroid appeared but tragically Bruce Willis was in rehab following the death of his tortoise and not available to save America and instead, The Land of The Free was put in the hands of a small, but highly skilled monkey, called Gerald. The asteroid was initially in a circular orbit around the earth at a radius of R_o where $R_o = 1.5\,R_E$ and R_E is the radius of the earth. Gerald sings the Star-Spangled Banner, wipes a tear from his eye, and salutes his father, Lionel, who is a Vietnam veteran and climbly, adeptly, into his small craft MKY1, which the aim of being injected into the same orbit as the asteroid but in the same direction. Unfortunately Gerald is distracted at a critical moment by a banana and he puts the craft in the orbit in the wrong direction and he directs MKY1 into a head-on collision with the asteroid. As a result of the collision, the craft, Gerald and his banana coalesce with the asteroid and the combination is put into a new orbit.

   The mass of the earth is m_E, the mass of the asterioid is m_A and the mass of MKY1, monkey and banana is m_{KY}. You need only consider the gravitational interaction between MKY1 and the earth.

   What is the initial orbital speed of the asteroid (v_o) and MKY1 (v_1) ?

   What is the speed of the coalesced asteroid and MKY1 immediately after they collide ? Express your answer in terms of m_A, m_{KY} and v_o.

   After the collision, the coalesced asteroid and MKY1 system is in a new orbit. Use conservation of energy and angular momentum to show that the distance of closest approach (R_X) of the system to the earth satisfies the relation:

$$f^2\frac{R_0^2}{R_X^2} - \frac{2R_0}{R_X} = \left(f^2 - 2\right) \text{ where } f = \frac{m_A - m_{KY}}{m_A + m_{KY}}.$$

   Somewhat disastrously the mass of the space-craft was chosen such that $R_X = R_E$ and the craft crashes into the earth destroying Texas. Lionel who gave little Gerald the banana was stripped of his Vietnam medals and, distraught, is put in a home. He never ate a banana again.

   Show that when $R_X = R_E$, that:

$$m_{KY} = m_A\left(\frac{\sqrt{5}-2}{\sqrt{5}+2}\right).$$