2017 first attempt

07 April 2019 16:39

1. Describe the spectral classification scheme of stars in terms of surface temperature and luminosity class. [6]

Surface temperature:

O B A F G K M

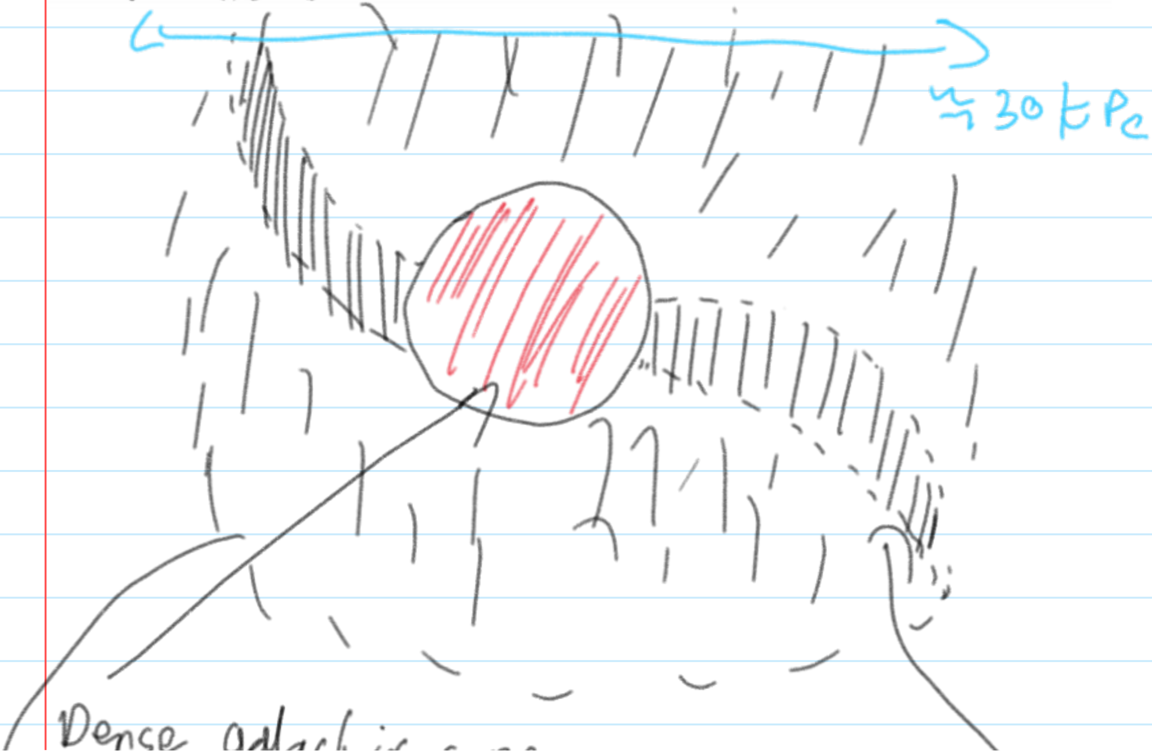
hottest (+30,000K) Blue/violet

ours in

coolest (<4000K) Red/orange

Luminosity: Ia - Most luminous super giant
Ib - Super giant
II - Luminous giant
III - giants
IV - Sub giants
V - Main sequence.

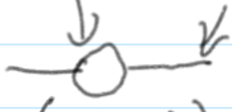
2. Using a labelled diagram, outline the main structural components of the Milky Way Galaxy, giving an indication of dimensions and content. [7]



Dence galac

Dense galactic core -
Mostly population $\underline{II}$ stars -
old & cool - metal poor
Stars have tilted & elliptical
orbits
Not all stars in galactic plane
Very little star
formation

Bulge/halo



galactic plane

(side view)

Spiral arms -
in galactic plane. large amount of star
formation. Density waves cause arms -
higher amount of gas - more
star formation

rest of galactic plane - little star
formation

3. We might suppose the rate of expansion of the Universe to be slowing down, because of the mutual gravitational attraction of galaxies. Explain why supernovae may be used to measure the rate of change of the expansion, and the results that have emerged. [7]

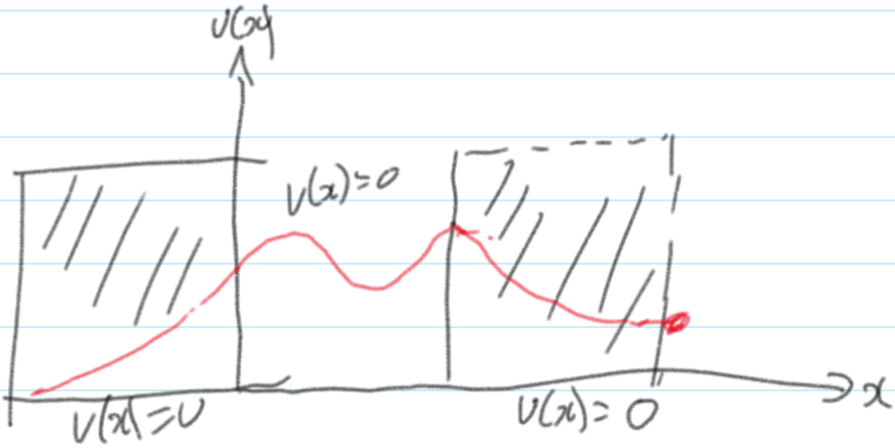
- some super nova have very consistent luminosity (white dwarf / type Ia)
- apparent brightness $\propto \frac{L}{r^2}$ where L is luminosity.
- we can measure brightness, and approximately know the luminosity $\therefore$ can find distance.
- we can measure red shift to find recession velocity.

recession velocity.

- we do this for many super novae – we find that larger r → larger velocity
- Hubble's law: $\underline{v = H_0 d}$

- distant galaxies travel faster – universe is expanding

4. Describe the phenomenon of quantum tunnelling and contrast the behaviour of a quantum particle encountering a finite potential barrier with that of a classical particle. Your answer should not include detailed calculations, but should include sketches of the potential and the relevant wavefunction. [7]



- wave function must be continuous at boundary.
- ∴ probability of finding particle in area when $V(x) > E$
- ∴ can pass through potential.
- Quantum tunneling

- In classical mechanics, a particle can only pass a barrier if $E > V(x)$

7. (a) The time-independent Schrödinger equation for a particle in one-dimension is:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

where m is the mass of the particle, E its energy, $\psi(x)$ the wavefunction and $V(x)$ the potential. [3]

For a constant potential show that

$$\psi(x) = A\sin(\kappa x) + B\cos(\kappa x)$$

is a general solution. Determine the relationship between κ and E, and state the energy regime in which this is a general solution.

(b) Many simple quantum systems can be described as a "particle-in-a-box", in which a particle is confined by some potential. We will consider the case of a particle confined in a infinite well, where $V(x) = 0$ for $0 \leq x \leq L$ and $V(x) = \infty$ for all other x. [3]

Explain what the wavefunction must be in the regions where $V(x) = \infty$. If instead of infinite the potential in these regions was finite (but still larger than the energy), what form would the wavefunction have in the region $x < 0$?

(c) Apply boundary and normalisation conditions to show that the wavefunction and allowed energy levels of the particle inside the infinite well are [6]

$$\psi(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right) \quad \text{and} \quad E_n = \left(\frac{h^2}{8mL^2}\right)n^2.$$

(d) Determine the probability that the particle is found in the first (closest to 0) 10% of the well. How does this probability change with n and how does it compare to the classical picture? [4]

(e) The nuclear binding force in atoms is sometimes described as an infinite square well potential. Assume a proton is bound in an infinite well 10 fm wide. Sketch the wavefunctions of the lowest three energy levels and determine the wavelength of a photon that would need be absorbed to excite the proton from the first to third level. [4]

$V(x) \to V_0$

a) $$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = (E - V_0)\psi(x)$$

$$\psi(x) = A\sin(kx) + B\cos(kx)$$

$$\frac{d}{dx}\psi(x) = Ak\cos(kx) - Bk\sin(kx)$$

$$\frac{d^2}{dx^2}\psi(x) = -Ak^2\sin(kx) - Bk^2\cos(kx)$$

$$= -k^2\psi(x)$$

$$\rightarrow -\frac{\hbar^2}{2m}(-k^2\psi(x)) = (E - V_0)\psi(x)$$

$$\rightarrow \frac{\hbar^2}{2m}k^2 = E - V_0$$

$$\rightarrow \frac{\hbar^2}{2m} k^2 = E - V_0$$

solution valid.

$$k = \frac{\sqrt{2m(E-V_0)}}{\hbar}$$

valid if $\underline{E > V_0}$

b)

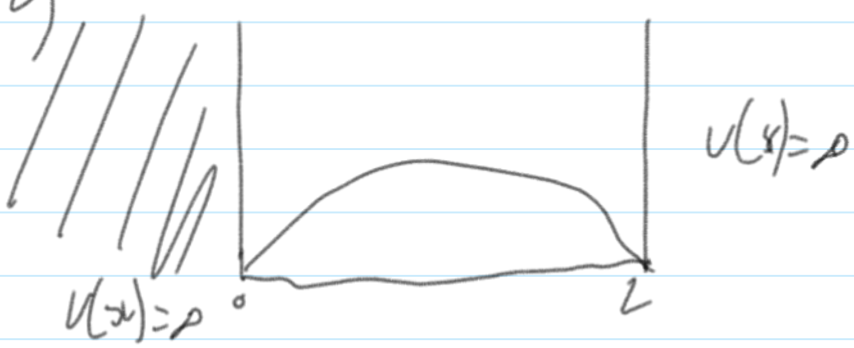
The wave function must be 0 if $V(x) = \infty$ as it is impossible to find particle there.

If $V(x) \neq \infty \rightarrow$

$$\psi(x) = e^{-kx} = e^{kx}$$

must decay, but wave form must be continuous.

c)



$$\psi(x) = A\sin(kx) + B\cos(kx)$$

$x = 0$, $\psi(0) = 0$

$\rightarrow B = 0$

$x = L$, $\psi(L) = 0$

$\rightarrow kL = M\pi$

$\rightarrow k = M\pi$

$$\rightarrow kL = \pi n$$
$$\rightarrow k = \frac{\pi n}{L}$$
$$\rightarrow \psi(x) = A\sin\left(\frac{\pi n x}{L}\right)$$

$$\int_{-\infty}^{\infty} |\psi(x)|^2 \, dx = 1$$

$$\Rightarrow \int_0^L \sin^2\left(\frac{\pi n x}{L}\right) dx = \frac{1}{A^2}$$

$$\cos(2x) = c^2 - s^2$$
$$c^2 + s^2 = 1 \rightarrow c^2 = 1 - s^2$$
$$\cos(2x) = 1 - 2s^2$$
$$\sin^2(\theta) = \frac{1 - \cos(2x)}{2}$$

$$\int_0^L \frac{1}{2} - \cos\left(\frac{2\pi n x}{L}\right) dx = \frac{1}{A^2}$$

$$\left[\frac{1}{2}x - \frac{L}{2\pi n}\sin\left(\frac{2\pi n x}{L}\right)\right]_0^L = \frac{1}{A^2}$$

$$\frac{L}{2} = \frac{1}{A^2} \rightarrow A = \sqrt{\frac{2}{L}}$$

$$\psi(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$$

$$k = \frac{\sqrt{2m(E - V_0)}}{\hbar}$$

$$V_0 = 0$$

$$\frac{\hbar^2 \pi^2 n^2}{2mL^2} = E$$

$$\hbar = \frac{h}{2\pi} \Rightarrow \hbar^2 = \frac{h^2}{4\pi^2}$$

$$\frac{h^2 \cancel{\pi^2}}{8\cancel{\pi^2} mL^2} n^2 = E$$

$$E = \frac{h^2}{8mL^2} n^2$$

d) $\frac{2}{L}\int_0^{\frac{L}{10}} \sin^2\left(\frac{n\pi x}{L}\right) dx$

$$\frac{2}{L}\int_0^{\frac{L}{10}} \frac{1}{2} - \cos\left(\frac{2\pi n x}{L}\right) dx$$

$$= \frac{2}{L}\left[\frac{1}{2}x - \frac{L}{2\pi n}\sin\left(\frac{2\pi n x}{L}\right)\right]_0^{\frac{L}{10}}$$

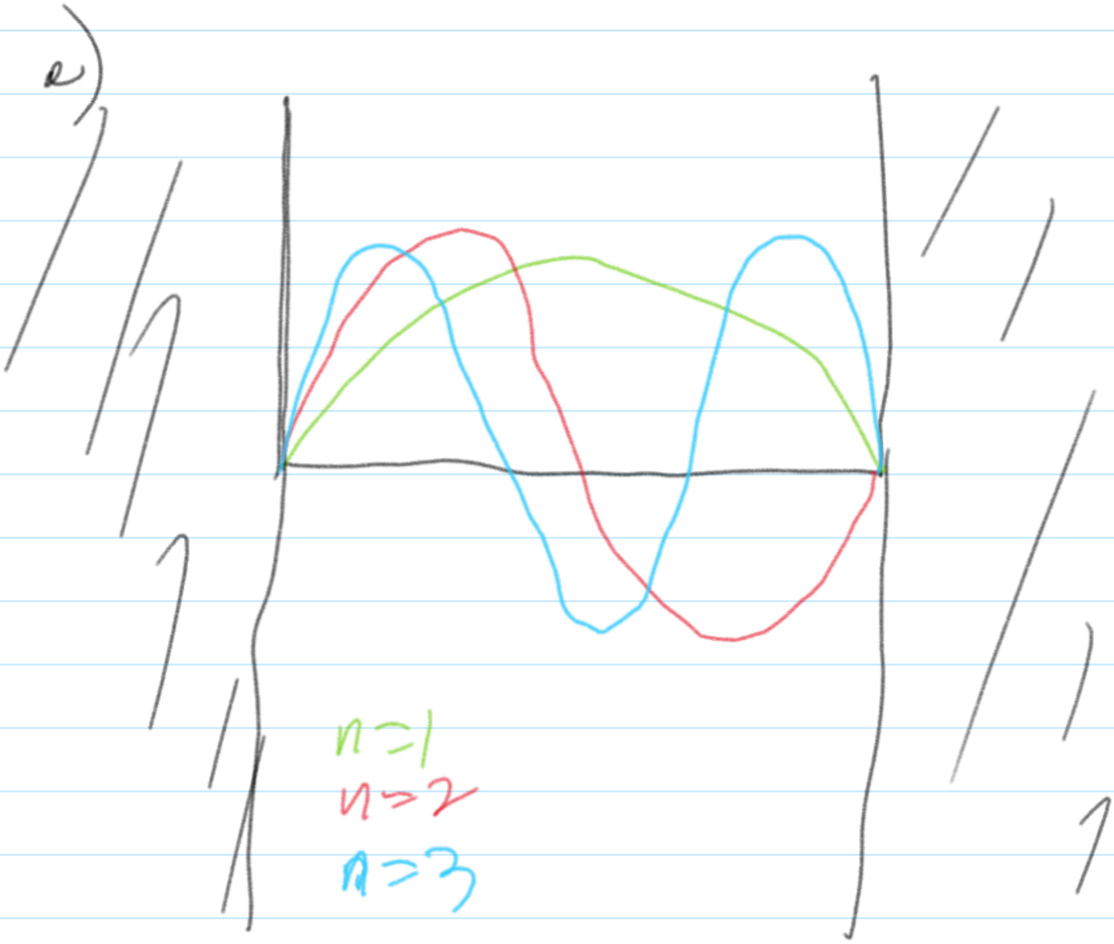
$$= \frac{2}{L}\left(\frac{L}{20} - \frac{L}{2\pi n}\sin\left(\frac{\pi n}{5}\right)\right)$$

?

$$= \frac{1}{10} - \frac{\sin\left(\frac{\pi}{5}n\right)}{\pi n}$$

Larger $n \Rightarrow$ lower probability. ?

e)



$$E = \frac{h^2}{8mL^2} n^2$$

$$E_{3-1} = \left(\frac{h^2}{8mL^2}\right)(3^2 - 1^2)$$

$$= \frac{h^2}{mL^2}$$

$$L = 10^{-14} m$$

$$E = \frac{(6.63 \times 10^{-34})^2}{1.0078 \times 1.66 \times 10^{-27} \times (10^{-14})^2}$$

$$= 2.63 \times 10^{-12}$$

$$\lambda = \frac{hc}{E}$$

$$= 7.56 \times 10^{-14} m$$

$$= 76 \text{ fm}$$

8. (a) Consider a particle with the wavefunction $\psi(x)$

$$\psi(x) = \begin{cases} Ae^{-x}\left(1 - e^{-x}\right) & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$$

Sketch this wavefunction and show that $A = \sqrt{12}$ normalises the wavefunction $\psi(x)$. [5]

(b) Calculate the expectation value $\langle x \rangle$ for the position of the particle . [5]

(c) Use the probability distribution function to determine the most probable position of the particle. Explain the difference between the most probable position and the expectation value. [4]

(d) Sketch a potential which could give rise to a wavefunction with the form above. [2]

(e) Considering only its motion around the Sun estimate the de Broglie wavelength of the Earth. [4]

a) $\psi(x) = Ae^{-x}\left(1-e^{-x}\right)$

$$\int_0^\infty |\psi(x)|^2 \, dx = 1$$

$$\int_0^\infty e^{-2x}\left(1-e^{-x}\right)^2 dx = \frac{1}{A^2}$$

$$\int_0^\infty e^{-2x}\left(1-2e^{-x}+e^{-2x}\right) dx = \frac{1}{A^2}$$

$$\int_0^\infty e^{-2x} - 2e^{-3x} + e^{-4x}\,dx = \frac{1}{A^2}$$

$$\rightarrow \left[-\frac{e^{-2x}}{2} + \frac{2e^{-3x}}{3} - \frac{e^{-4x}}{4}\right]_0^\infty$$

$$= \frac{1}{12}$$

$$\rightarrow A = \sqrt{12}$$

b)

$$\langle x \rangle = 12 \times \int_0^\infty x\left[e^{-2x} - 2e^{-3x} + e^{-4x}\right] dx$$

$u = x$ $\qquad dv = e^{-2x} - 2e^{-3x} + e^{-4x}$

$du = 1$ $\qquad v = -\frac{1}{2}e^{-2x} + \frac{2}{3}e^{-3x} - \frac{1}{4}e^{-4x}$

$$= -\frac{1}{12}e^{-4x}\left(6e^{2x} - 8e^{x} + 3\right)$$

$$I = -\frac{1}{12}\left[xe^{-4x}\left(6e^{2x} - 8e^{x} + 3\right)\right]_0^\infty - \int \frac{du}{dx} v\,dx$$

$$= -\frac{1}{12}(0) + \int_0^\infty \frac{e^{-2x}}{2} - \frac{2e^{-3x}}{3} + \frac{e^{-4x}}{4}\,dx$$

$$= \left[-\frac{e^{-2x}}{4} + \frac{2e^{-3x}}{9} - \frac{e^{-4x}}{16}\right]_0^\infty$$

$$= (0) - \left(\frac{2}{9} - \frac{1}{4} - \frac{1}{16}\right)$$

$$= (0) - \left(\frac{4}{9} - \frac{1}{4} - \frac{1}{16} \right)$$

$$= \frac{13}{144}$$

$$cx = 12 \Rightarrow \frac{13}{144} = \frac{13}{12}$$

c) $$\frac{d}{dx}\left(\sqrt{12}\, e^{-x} \left(1 - e^{-x}\right)\right) = 0$$

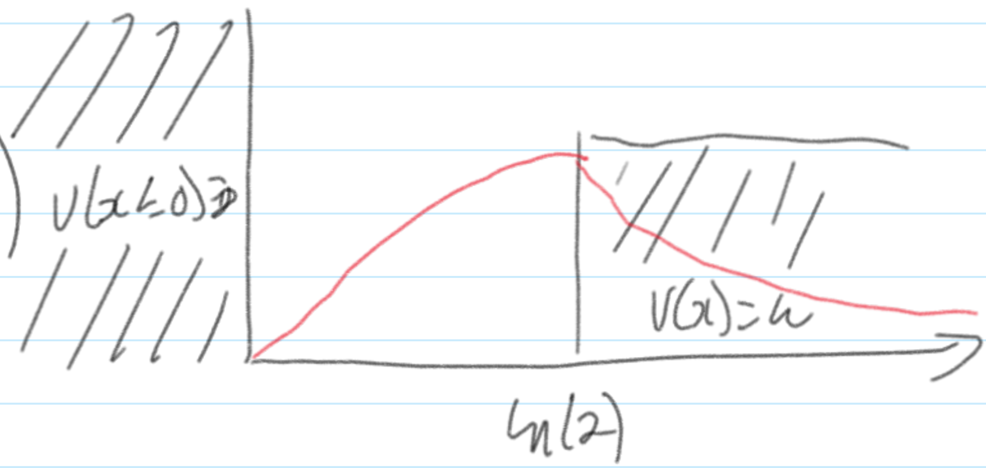
$$u = \sqrt{12}\, e^{-x} \qquad v = 1 - e^{-x}$$
$$du = -\sqrt{12}\, e^{-x} \qquad dv = e^{-x}$$

$$\frac{d}{dx} = \sqrt{12}\, e^{-2x} - \sqrt{12}\, e^{-x}\left(1 - e^{-x}\right)$$
$$= \sqrt{12}\left(e^{-2x} + e^{-2x} - e^{-x}\right)$$
$$= \sqrt{12}\left(2e^{-2x} - e^{-x}\right) = 0$$

$$\Rightarrow 2e^{-2x} = e^{-x}$$
$$2e^{-x} = 1$$
$$e^{-x} = \frac{1}{2}$$
$$-x = \ln\left(\frac{1}{2}\right)$$
$$x = \ln(2)$$

2...

d)



e)

$$\lambda = \frac{h}{p}$$

$$p = mv$$

$$m = 5.972\times10^{24}\,\text{kg}$$

v: $\text{dist} = 2\pi r$

$$= 2\pi \times 1.496\times10^{11}\,\text{m}$$

$$= 9.4\times10^{11}\,\text{m}$$

$$\frac{9.4\times10^{11}}{60\times60\times24\times365} = 29800\ \text{m/s}$$

$$\lambda = \frac{6.63\times10^{-34}}{5.972\times10^{24}\times 29800} = 3.73\times10^{-63}$$

9. (a) Describe Rutherford's planetary model of the hydrogen atom. Explain two ways in which this model is inconsistent with experimental data. [4]

(b) Describe two of the postulates of Bohr's model of the hydrogen atom, which improved upon Rutherford's planetary model. [2]

(c) The energies of the orbits in the Bohr model satisfy:

$$E_n = -\frac{13.6}{n^2}\text{eV}$$

where n is a positive integer. What wavelength of light is required to excite an electron from the $n = 1$ to $n = 4$ orbit? [3]

(d) Describe two ways in which a fully quantum-mechanical model improves upon Bohr's model of the atom. [2]

(e) In a fully quantum-mechanical treatment of the hydrogen atom, there are two additional quantum numbers that describe the system:

- l is any integer between 0 and $n - 1$.
- m is any integer such that $|m| \leq l$

List all of the quantum number combinations that correspond to wavefunctions with energy $E = -0.85$eV. [5]

(f) Find a formula which describes the the number of different wavefunctions for a general n. [Hint: Write out the possible allowed combinations of quantum numbers for n =1, 2, 3, 4 etc. and look for a pattern.] [4]

A) Dense positively charged nucleus. Electrons orbit like planets do.
Theory implies EM radiation would be produced due to accelerating electron - not measured
Theory implies electron would spiral down into nucleus after time - not seen

B) Only orbits of specifc radius allowed
Quantisation based on angular momentum

C)
$$E_{4-1} = -13.6 \times 1.6 \times 10^{-19} \times \left(\frac{1}{4^2} - \frac{1}{1^2}\right)$$
$$= 2.04 \times 10^{-18}\,\text{J}$$

$$\lambda = \frac{hc}{E} = 9.75 \times 10^{-8}\,\text{m}$$
$$= 97.5\,\text{nm}$$

d) Quantised states not forced - came out as a result of boundary conditions.
Doesn't treat electron as classical particle

?

10. (a) Briefly contrast how the key evolutionary stages, up to the end-state, of a $30\mathcal{M}_\odot$ star differ from those of $1\mathcal{M}_\odot$ star. [9]

(b) Calculate the main-sequence lifetime (in years) of the $30\mathcal{M}_\odot$ star if it has a luminosity of $8 \times 10^5\ \mathcal{L}_\odot$ and 10% of its mass will be converted from hydrogen to helium in the core. Compare this main-sequence lifetime to that of the Sun. [7]

(c) After the $1\mathcal{M}_\odot$ star has used up its supply of hydrogen in fusion, its temperature decreases by a factor of 5 and the luminosity increases by a factor of 2. Determine the factor by which the radius increases. State the physical mechanism that carries energy into the expanding envelope of this evolved star. [4]

A) Both stars start in the same way

Collection of gas starts to attract each other and move towards a central point. Gravity pulls the gas in closer to each other. There is no pressure - particles are in free fall. This is a proto-star

The density of this gas cloud increases, until friction between the particles cause heat. The particles help trap this heat in. The heat causes the collapse to slow. This is a pre-main sequence star.

As the core gets denser it gets hotter due to the high pressure. It eventually gets hot and dense enough to start fusing hydrogen to produce energy. This is the main sequence of the star

When the hydrogen in the core is burned, the outer core starts to collapse in. This causes extra pressure and heat which causes helium to be fused
In a 1 solar mass star, this extra heat causes the outer layers of the star to start expanding as they also start fusion. The star turns into a red giant, and eventually the outer layers are blown away to produce a white dwarf.

In a 30 solar mass star, there is enough mass and so pressure to start fussing heavier elements. This keeps occuring until Iron is produced. Iron cannot be fused as there would be an energy deficit.
The outer layers collapse, at which point they bounce off the iron core, producing a super nova. The core then turns to a black hole.

B)

$$M = (4 \times 1.0078 - 4.0026)\ \text{amu}$$
$$= 0.0286\ \text{amu}$$
$$= 4.7476\ \text{kg}$$

$$E = mc^2$$
$$= 4.27 \times 10^{-12}\ \text{J}$$

$$\frac{4.27 \times 10^{-12}}{(4 \times 1.0078 \times 1.66 \times 10^{-27})\ \text{kg}} = 6.38 \times 10^{14}\ \text{J/kg}$$

$$30 M_\odot \times 10\% = 3 M_\odot = 3 \times 1.989 \times 10^{30}$$
$$\approx 5.97 \times 10^{30}\ \text{kg}$$

5.97×10^{30} kg

$$8 \times 10^5 L_\odot = 8 \times 10^5 \times 3.828 \times 10^{26}$$

$$= 3.1 \times 10^{32} \frac{J}{s}$$

$$\frac{3.1 \times 10^{32} J/s}{6.39 \times 10^{14} J/kg} = 4.8 \times 10^{17} \frac{kg}{s}$$

$$\frac{5.97 \times 10^{30} kg}{4.8 \times 10^{17} kg/s} = 1.25 \times 10^{13} s$$

$$\div (60 \times 60 \times 24 \times 365) = 3.95 \times 10^5 yr$$

c) $L_\odot = \sigma T_\odot^4 4\pi r_\odot^2$

$L_1 = 2L_\odot$

$T_1 = \left(\frac{T_\odot}{5}\right)$

$$r_\odot^2 = \frac{L_\odot}{4\pi\sigma T_\odot^4}$$

$$r_1^2 = \frac{2L_\odot}{4\pi\sigma \frac{T^4}{625}}$$

$$r_1^2 = 1250 \, r_\odot^2$$

$$r_1 = 35.4 \, r_\odot$$

radiation?

' ' ' 0

radiation?

11. (a) The galaxy NGC5754 is observed with the Hubble Space Telescope and measured to have a redshift z = 0.14. Estimate how far in the past (in years) are we looking toward this galaxy. State why this estimate is not exact. [6]

(b) In this galaxy, at what wavelength would the Lyman α line (rest wavelength 121.6 nm) be observed? [3]

(c) NGC5754 is thought to have a $5 \times 10^6 \mathcal{M}_\odot$ black hole at its centre. Calculate the Schwarzschild radius (in units of $\mathcal{R}_\odot$) of the black hole. Assuming spherical geometry also determine the average density (in kg m^{-3}) inside the Schwarzschild radius. [5]

(d) This galaxy resides in a cluster; briefly explain two methods for evaluating the masses of clusters of galaxies and hence the dark matter component. [6]

a) $z = 0.14$

$\rightarrow v = cz$

$= 4.2 \times 10^7 \, m/s$

$v = H_0 d$

$d = \frac{v}{H_0} = \frac{4.2 \times 10^4 \, km/s}{68}$

$= 618 \, Mpc$

$\doteq 1.9 \times 10^{25} \, m$

Light year $= 3.16 \times 10^7 \times 3 \times 10^8$

$= 9.48 \times 10^{15} \, m$

$\rightarrow 2 \times 10^9$ years –

Not accurate due to expansion of universe

b) $\frac{\lambda - 121.6nm}{121.6nm} = 0.14$

$\lambda = 0.14 \times 121.6 \times 10^{-9} + 121.6 \times 10^{-9}$

$$\lambda = 0.14 \times 121.6 \times 10^{-9} + 121.6 \times 10^{-9}$$
$$= 138.6\ nm$$

c)
$$R_0 = \frac{2GM}{c^2}$$
$$= 1.47 \times 10^{10}\ m$$

$$\rho = \frac{M}{\frac{4}{3}\pi r^3}$$
$$= \frac{5 \times 10^6 \times 1.989 \times 10^{30}}{\frac{4}{3}\pi \times (1.47 \times 10^{10})^3}$$
$$= 7.47 \times 10^5\ kg/m^3$$

d) Mass of cluster;
relative velocity between galaxies.
The velocity between will be affected by gravity