

PHAS1202 –

Astrophysics PST 1 – solutions – Nov. 2017

Q1

If the ionisation potential of the hydrogen atom is 13.6 eV, then $E(1) = 13.6$ eV and

$$E(n) = 13.6\left(1 - \frac{1}{n^2}\right) \text{ eV}$$

$$\text{Thus } E(3) - E(4) = 13.6\left(\frac{1}{3^2} - \frac{1}{4^2}\right) \text{ eV} = 0.661 \text{ eV and}$$

$$\lambda = \frac{c}{\nu} = \frac{hc}{E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{0.661 \times 1.6 \times 10^{-19}} \text{ m} = 1.881 \times 10^{-6} \text{ m}$$

Transitions whose lower level has $n = 3$ are called the *Paschen series*.

The transition between $n = 3$ and $n = 4$ is called $P\alpha$.

Q2.

The spectra of the two stars can be approximated by blackbodies, for which the relation between temperature and wavelength of the emission peak can be written as:

$$\lambda_{\text{max}} \approx \frac{3 \times 10^{-3}}{T} \quad \text{where } \lambda_{\text{max}} \text{ is in m and } T \text{ in K. Thus}$$

HD36956: $T=24100$ K $\rightarrow \lambda_{\text{max}} = 1.24 \times 10^{-7}$ m \rightarrow ultraviolet waveband

Epsilon Indi : $T=2800$ K $\rightarrow \lambda_{\text{max}} = 1.07 \times 10^{-6}$ m \rightarrow infrared waveband

$$\text{Ratio of energies emitted} = (T_{\text{HD36956}})^4 / (T_{\text{eps. Indi}})^4 = 5488$$

Q3.

Brightness, b , depends upon luminosity, L , and distance, d , according to the inverse square law: $b \propto L/d^2$

and so if two stars are at the same distance from Earth then the brightness is determined by the luminosity. The total luminosity is proportional to the energy flux and the surface area:

$$L \propto (\text{Energy Flux}) \times (\text{Surface Area})$$

Since the stars have the same size, the relative luminosity is determined by the stars' respective energy flux, which is in turn, determined from the Stefan- Boltzmann law: $\text{Flux} \propto T^4$

Thus the brighter star will be the hotter one and a blue star is hotter than a red one. Hence the blue star will be brighter in the night sky.

Q4.

$$\text{Luminosity, } L = 4 \pi R^2 \sigma T^4$$

$$\text{For photosphere, } L = 4 \pi (7 \times 10^8)^2 \times (5.7 \times 10^{-8}) (5800)^4$$

$$= 4.0 \times 10^{26} \text{ W}$$

$$\text{For corona, } L = 4 \pi (1.4 \times 10^9)^2 \times (5.7 \times 10^{-8}) (2 \times 10^6)^4$$

$$= 2.2 \times 10^{37} \text{ W}$$

The result for the corona is almost 11 magnitudes greater than the solar luminosity!. This tells you that the blackbody approximation is not valid for the corona. This is the danger of applying blackbody approximations to objects which are not opaque blackbodies.

Q5

The monochromatic intensity is given by the

$$I_{\lambda} = (2 h c^2 / \lambda^5) (1 / [\exp (h c / \lambda k T)] - 1)$$

For large exponentials ($[\exp (h c / \lambda k T)] \gg 1$)

$$I_{\lambda} \sim (2 h c^2 / \lambda^5) \exp (-h c / \lambda k T)$$

$$I_{\text{sunspot}} / I_{\text{photosphere}} = \exp (-h c / \lambda k T_{\text{sunspot}}) / \exp (-h c / \lambda k T_{\text{photosphere}})$$

$$\begin{aligned} &= \exp \{(-h c / \lambda k) (1/T_{\text{sunspot}} - 1/T_{\text{photosphere}})\} \\ &= \exp [-\{ (6.63 \times 10^{-34}) (3.0 \times 10^8) / \lambda (1.38 \times 10^{-23}) \} \times (1/T_{\text{sunspot}} - 1/T_{\text{photosphere}})] \\ &= \exp [-\{ (1.44 \times 10^{-2}) / \lambda \} \times (1/T_{\text{sunspot}} - 1/T_{\text{photosphere}})] \end{aligned}$$

For $\lambda = 550 \text{ nm}$:

$$I_{\text{sunspot}} / I_{\text{photosphere}} = \exp [\{ -1.44 \times 10^{-2} / 5.5 \times 10^{-7} \} \times (1/4000 - 1/5800)]$$

$$= 0.13$$

For $\lambda = 1 \mu\text{m}$:

$$I_{\text{sunspot}} / I_{\text{photosphere}} = \exp [\{ -1.44 \times 10^{-2} / 1 \times 10^{-6} \} \times (1/4000 - 1/5800)]$$

$$= 0.33 \quad (\text{i.e. greater sunspot to photosphere intensity contrast in visible light})$$

Q6

If λ and λ_0 are the observed and emitted wavelengths respectively:

$$\frac{\lambda - \lambda_0}{\lambda_0} = \frac{v}{c}$$

Thus the observed wavelength will be

$$\lambda = 650 \text{ nm} (1 - 300/3 \times 10^5) = 649.35 \text{ nm}$$

(The velocity is negative because the star is travelling towards us)

(ii) Similarly for the cloud of H:

$$= 21 \times 1.00033 \text{ cm} = 21.007 \text{ cm, and}$$

$$\nu = \frac{c}{\lambda} = \frac{300,000 \times 10^5}{21.007} \text{ Hz} = 1.428 \times 10^9 \text{ Hz} = 1428 \text{ MHz}$$