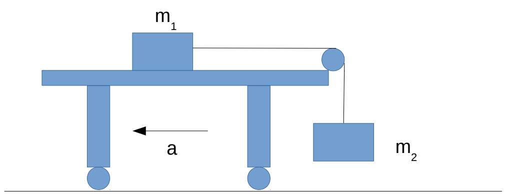
PHAS1247 : Classical Mechanics
In-Course Assessment Test #2 : Mon. 8 January 2018

Answer as many of the questions as you can, in any order. There are 5 questions, and each complete question is worth 6 marks, for a maxmimum mark of 30.

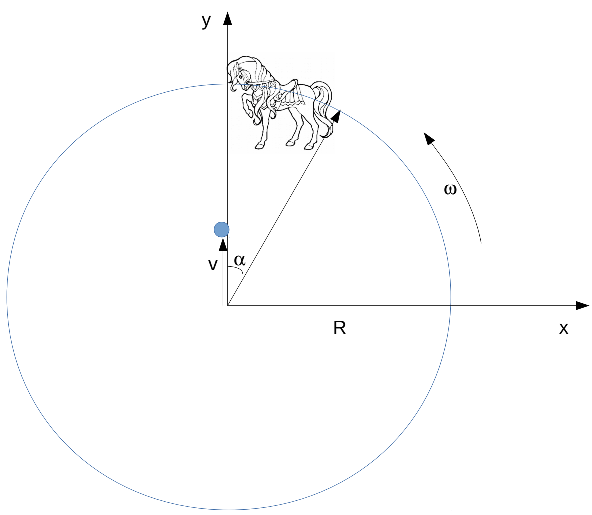
1. A point-like body with mass m_1 is placed on top of a table, that can freely move on wheels with respect to the ground. There is a static friction coefficient of $\mu = 0.3$ between the body and the table. If the body is subject to the gravitational acceleration g, what is the maximal horizontal acceleration that the table can have, before the body start moving with respect to it? **[2]**

   Consider now the case of where the body is attached to a horizontal massless and frictionless rope. This rope turns around a frictionless pulley placed at the side of the table, and below it hangs another body with mass m_2 (see figure). Calculate the forces acting on the system in the reference frame of the table. Calculate the maximal acceleration (in the direction opposite to m_2) that the table can stand before the masses start to move, neglecting the change in the angle of the string supporting m_2.



[4]

2. A carousel of radius R is rotating with angular speed ω in the anti-clockwise direction around the origin of a fixed cartesian system. When the head of a horse placed at the edge of the carrousel along the tangential direction crosses the y axis of the fixed system, a ball is thrown with speed v from the centre of the carrousel parallel to the y axis. What is the minimal angle α between the head and tail of the horse, for the ball to hit it when it reaches the edge of the carousel? Hint: it is convenient to consider the system from the point of view of an external observer, so in an inertial reference frame, where the ball is moving without the influence of external forces, and the carousel rotates behind it.



[6]

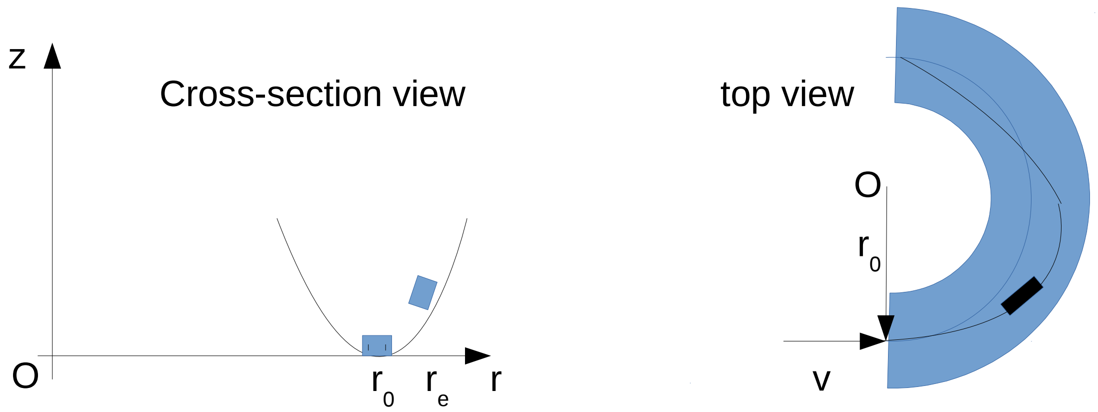
3. Let us now consider again the carousel of the previous problem, and look at the ball in the non-inertial rotating system. Calculate the Coriolis force at the beginning of the motion, both in absolute value and direction. What is the angle between the varying velocity and the Coriolis force? Will the Coriolis force be able to do work on the ball? **[6]**

4. A stick with negligble thickness and length 2L has a density that can be described as $\lambda = \alpha x^2$, where x is the distance from the centre of mass. Calculate the mass and the momentum of inertia of the stick around its centre of mass. **[3]**

   The stick feels a force F for the total duration of 1 second, perpendicular to the length of the stick, and applied at one extreme. Calculate the momentum of the sick, and its angular momentum about its centre of mass, after the application of the force, neglecting the movement of the system during the time the force is applied. **[3]**

5. The section of a bobsleigh racetrack can be described as a parabola with equation $z = \frac{1}{2}\alpha(r - r_0)^2$. The gravitational acceleration $-g$ acts along the z direction (see left side of the figure). A bobsleigh, initially at the lowest point of the section ($r = r_0$, $z = 0$), enters a turn with radius of curvature r_0 (right side of figure). Since it will now also feel the centrifugal force from the turn, its equilibrium position will now be r_e, more external with respect to r_0. Calculate the value of r_e. (hint: calculate the gravitational potential from the shape of the section, then the horizontal component of the resulting force. In the calculation of the centrifugal force, you can assume a constant radius of curvature r_0, and a constant tangential speed v). **[3]**

   Since the bobsleght enters the curve with a smaller radius than the equilibrium position r_e, if seen from its own rotating non-inertial reference frame it will start small oscillations along the r direction around the equilibrium position r_e. Calculate the angular speed ω of these oscillations. (Hint: Work in the reference frame rotating with constant angular velocity v/r_0. The problem is mathematically very similar to that of a ball attached to a spring, rotating in a uniform circular motion, plus small oscillations around its equilibrium position) **[3]**



END OF PAPER