http://zuserver2.star.ucl.ac.uk/~idh/PHAS1102 & /PHAS1423

Problem Paper 3

Solutions to be handed in on Wednesday Dec 2, at the PHAS 1102 lecture, or to the Undergraduate Office (E2) *not later than 16:45.* Don't email solutions, and don't leave answers in my pigeon-hole (if you do, they'll be treated as having been submitted when I find them, not when you say you left them there, and late penalties will apply).

solar mass	$M_\odot$	1.989×10^{30} kg
parsec	pc	3.0857×10^{16} m
hydrogen mass	m_H	1.66×10^{-27} kg

Question 1 [marks]

The Schwarzschild radius marks the 'event horizon' of a nonrotating black hole, such as might be found in the centre of a galaxy; classically, we can extract no information from inside this radius, which in some sense represents 'the' radius of a black hole. Although it requires a full relativistic treatment to derive correctly an expression for the Schwarzschild radius, the same result comes out just by finding the radius at which the escape velocity from a body of mass M equals the speed of light. Derive an expression for the Schwarzschild radius in this way (by equating the kinetic energy required to eject a particle at the speed of light with that particle's potential energy). [3]

Question 2

Suppose, for the purposes of a rough calculation, that the Milky Way galaxy contains 10^{11} stars (and nothing else), each weighing one solar mass on average, and each composed entirely of hydrogen (not unreasonable for an order-of-magnitude estimate). Suppose further that the Milky Way is a typical galaxy, and that each cubic megaparsec of space contains 1 galaxy. On average, what volume of space is occupied by a single hydrogen atom? (Express your answer in units of cubic metres.) [6]

Question 3

Another rough calculation: if the age of the Universe is 1.4×10^{10} years, estimate the volume of the observable universe. (Remember, this is intended to be a *rough* calculation, so the only extra information you should need to make this estimate is the speed of light. Express your answer in units of cubic megaparsecs.) [4]

Suppose the European Standard Beach is 1km long, 10m across, and 1m deep; suppose also that the European Standard Sandgrain occupies 1 cubic mm. Which is the larger number: the number of stars in the observable universe, or the number of grains of sand on a beach? What about the number of galaxies compared to the number of sand grains? (You will need to re-use some information from Question 2.) [5]

Continued

Question 4
Suppose that the Sun's distance from the centre of the Galaxy is 8.0 kpc, and its orbital velocity about the centre is 220 km/s. Calculate the length of the Galactic 'year' (the time taken for the Sun to complete one orbit around the Galaxy), expressing your answer in (Earth) years. [3]

Calculate how many times the Sun has orbited the Galaxy. (Assume that the age of the Sun is 0.5×10^{10} yr.) [2]

Question 5
The redshift, z, of distant galaxies can be measured from the doppler displacement of features in their spectra. The redshift can be directly converted to a velocity of recession (using the relativistic doppler formula), and thence to a distance (since there exists a proportionality between distance and velocity; we will discuss this in lectures shortly). Finally, because light has a finite velocity, and so takes a finite time to reach us from distant galaxies, the redshift tells us the age of universe at the time when the galaxy emitted the light we now see.

Using Ned Wright's 'cosmology calculator',[1]
`http://www.astro.ucla.edu/~wright/CosmoCalc.html`,
tabulate the distances[2] of galaxies at redshifts of 0.1, 0.5, 1.0, 2.0, 4.0, 6.0, 10.0 and 100.0, together with their ages (the time since the Big Bang). [5]

[In fact, there are certainly no galaxies at $z = 100$, and it's unclear if there are any at $z = 10$, but let's not worry about that little detail...
Diligent students might consider computing results for some intermediate redshifts, and presenting the results graphically, but this isn't required. If you *do* do this, you might find it easier to take logs of all quantities before plotting.]

[1] Accept the default parameters of $H_0 = 71, \Omega_{\rm M} = 0.27, \Omega_{\rm vac} = 0.73$, and adopt a 'General' calculation.

[2] The calculator returns several different 'distances'; you can follow the links from the web page if you're interested in the details, but here we want the 'comoving radial distance'.