PHAS1247 Classical Mechanics
Problems for Week 4 of Lectures (2016)

1. A person is standing at the left-hand end, labelled A, of a uniform log of length 5 m floating at rest in still water. If the person has a mass of 80 kg and the mass of the log is 1000 kg, determine the distance from A to the centre of mass of the total system (man plus log).

   The person then walks to the other (right-hand) end of the log and stops. Determine the distance from A to the new centre of mass of the total system. Hence, ignoring friction between the log and the water, determine the displacement (magnitude and direction) of the log relative to the water, explaining clearly the principle on which your reasoning is based.

2. A skier of mass m slides down a slope at an angle α to the horizontal. The skier's initial speed down the hill is u. Unfortunately the skier has neglected to wax their skis and consequently the coefficient of sliding friction μ between their skis and the snow is greater than $\tan\alpha$.

   (a) Draw a diagram showing the forces acting on the skier and find an equation for the rate of change of the skier's speed v;

   (b) Show that the skier will come to rest and that the distance travelled before stopping is:

   $$s = \frac{u^2}{2g(\mu - \tan\alpha)\cos\alpha};$$

   (c) Find (i) the work done by gravity and (ii) the work done by the frictional force on the skier up to the point where the skier stops. Show that the total work done is equal to the change in his kinetic energy.

3. A mass $m_1 = 5\,\mathrm{kg}$ has an initial velocity in the laboratory frame (i.e. relative to a stationary observer) $\mathbf{u}_1 = 5\hat{\mathbf{i}}\,\mathrm{ms}^{-1}$, while a second mass $m_2 = 3\,\mathrm{kg}$ has velocity $\mathbf{u}_2 = -3\hat{\mathbf{i}}\,\mathrm{ms}^{-1}$. Find (i) the total momentum of the system, (ii) the velocity of the centre of mass, and (iii) the relative velocity of particle 1 relative to particle 2.

   The two particles undergo a head-on inelastic collision in which the coefficient of restitution is $e = 0.75$. Assuming they continue to move along the x-axis, find the velocities of both particles after the collision in the lab frame.

4. A nucleus of mass $20\,m_u$ (where m_u is the atomic mass unit $1.66 \times 10^{-27}\,\mathrm{kg}$) is moving with a velocity of $3 \times 10^6\,\mathrm{ms}^{-1}$ when it breaks into two fragments. In the course of this process internal energy is released from within the nucleus and the kinetic energy consequently increases by an amount $\Delta E = 10^{-12}\,\mathrm{J}$. The heavier fragment has mass $16\,m_u$ and is emitted at 90° to the original line of flight (as viewed in the lab frame). What is the speed of the lighter fragment? [Since the energy released is small compared with the rest-mass energy of the nucleus you may assume that mass is conserved and that both fragments remain non-relativistic.]