PHAS1247 Classical Mechanics
Problems for Week 3 of Lectures (2016)

1. A particle moving in two dimensions with position coordinates (x, y) has a potential energy given by
$$V(x, y) = Ax^2y - By^3,$$
where A and B are constants. Find the force acting on the particle at position (x, y).

2. A particle of mass m has a position vector at time t of:
$$\mathbf{r} = [a\cos(\omega t)\hat{\mathbf{i}} + b\sin(\omega t)\hat{\mathbf{j}} + ct\hat{\mathbf{k}}].$$
By differentiating twice, find the force acting on the particle. Hence find the power P developed by the force acting on the particle. Determine the kinetic energy of the particle and show that P is equal to the rate of change of the kinetic energy with respect to time, as you would expect from the work-kinetic-energy theorem.

3. A particle moving in three dimensions moves in a straight line from the origin to the point with position vector $3\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$. Find the work done during the displacement by (a) a constant force $\mathbf{F} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ and (b) by the position-dependent force $\mathbf{F} = xy^2\hat{\mathbf{i}} + x^2y\hat{\mathbf{j}}$.

   Would the work done for each force depend on the path taken between the start and end points i.e. are these forces conservative ?

4. A particle of mass $m_1 = 2\,\text{kg}$ and initial velocity $\mathbf{u}_1 = 2\hat{\mathbf{i}}\,\text{m}\,\text{s}^{-1}$ collides with another of mass $m_2 = 4\,\text{kg}$ and initial velocity $\mathbf{u}_2 = (-\hat{\mathbf{i}} + \hat{\mathbf{j}})\,\text{m}\,\text{s}^{-1}$. After the collision the first particle has final velocity $\mathbf{v}_1 = (\frac{4}{3}\hat{\mathbf{i}} + \frac{2}{3}\hat{\mathbf{j}})\,\text{m}\,\text{s}^{-1}$. Assuming that only internal forces act during the collision, find

   (a) the final velocity $\mathbf{v}_2$ of particle 2;

   (b) the impulse on each particle

   (c) the change in total kinetic energy during the collision; and

   (d) $\mathbf{u}_1'$ and $\mathbf{u}_2'$ i.e. the values of $\mathbf{u}_1$ and $\mathbf{u}_2$ in the centre of mass frame. Hence show that the sum of the momenta of the two particles in this frame is zero.