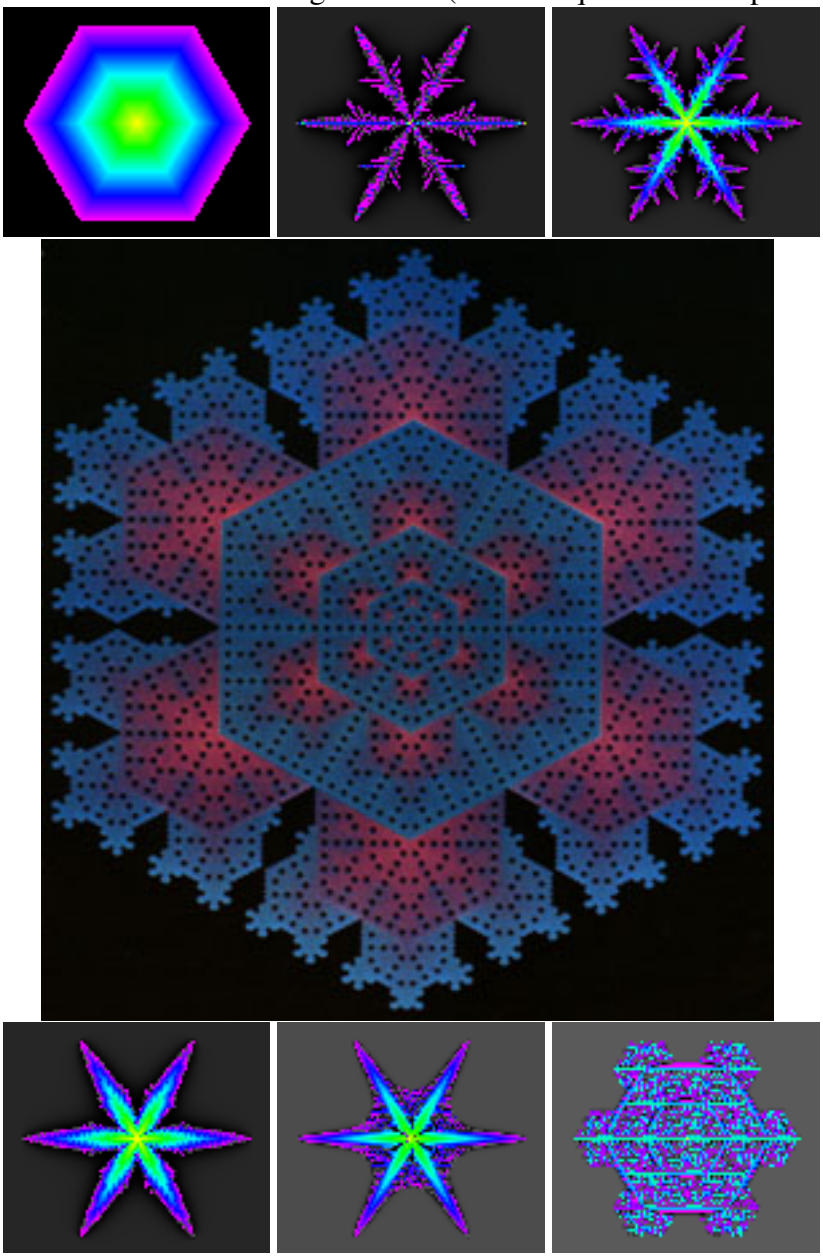
AN AVALANCHE BEGINS WITH A SNOWFLAKE[1]

Introduction

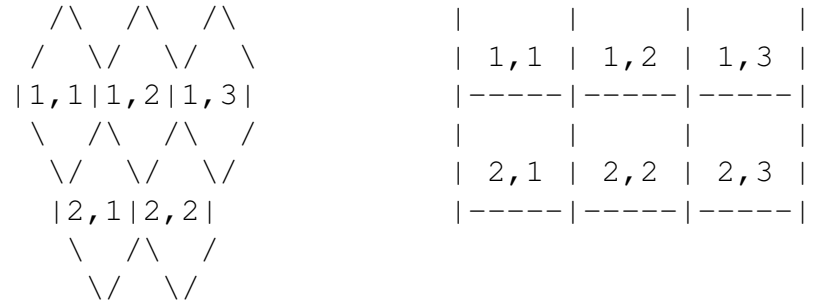
This project uses a cellular automaton to explore the shapes adopted by crystals undergoing dendritic growth, the sort of growth that gives rise to some of the most beautiful snowflake structures (in fact, it is capable of generating dendritic, stellar, sectorial and plate-like flakes). The same basic physics can be used to describe the solidification of some metals, which form tree-like internal patterns. In both cases what is happening depends mainly on the temperature and the degree of saturation of the surrounding material (air and liquid metal respectively).



[1] Joseph D Compton

A simplified model

The basic model goes back to 1611 and Kepler's suggestion that the symmetry of snow crystals is related to the hexagonal packing of spheres[1] – a remarkable insight which happens to lead to similar results to our modern understanding of the hexagonal molecular packing of ice crystals[2]. We can set up our cellular automaton to respect this hexagonal packing (which requires a little thought about identifying neighbouring cells), or use a simpler square array.



One of the simplest models[3, 5] is a Boolean one that evolves on a hexagonal lattice as follows: at each time step, each cell is either ice or not. On the subsequent step, cells that were ice, remain ice, while cells that were not ice, become ice if and only if exactly one of the neighboring cells is ice. This models plates and sectors, but it does not provide global dendrite or stellar growth.

A more realistic model[4] uses an array of real valued (nonnegative) cells, and view the value of any cell as measuring the amount of water at that cellular location. We consider a cell to be solid ice if its value is greater than or equal to one, while lower values are taken to represent water in a form that may possibly move to neighboring cells. Then classify each cell at each step as either receptive or nonreceptive. Cells which are ice or which are immediate neighbors to ice are receptive. The updated value is the sum of two terms. The first term is the value of the receptive sites with a constant γ added; the second term is the average of the nonreceptive sites (with receptive sites contributing values of zero to the averages). The averaging consists of taking the average of the centre value and the average of the immediate neighbors (with zeros as noted above): this represents diffusion of water. On some occasions we take weighted averages which will correspond to changing a parameter α. Ordinarily $\alpha = 1$. We usually begin with a single seed of value one with all other sites initialized at some background level β. Boundary conditions maintain the background level near the edge of the image.

The motivation for the model is that receptive sites are viewed as permanently storing as ice any water that arrives at that point. The water in the unreceptive sites is free to move, and hence moves toward an average value. Lastly, the constant added to receptive sites corresponds to the idea that water may be available from outside the plane of growth. The constant to be added to the receptive sites is one of the parameters that we vary.

The second parameter that we vary is the background level. We will usually begin with a single cell of value one (an ice seed) in a sea of a constant background β. The boundary conditions are taken to be fixed at the background level at a fixed (Euclidean) distance from the initial cell. These boundary conditions attempt to make the boundary conditions as isotropic as possible. Typical array sizes will be a few hundred in each direction, and several thousand steps may be necessary, so the code will have to be written efficiently.

Different diffusion models can be implemented by changing the update rule for cell p to

$$u_p(t+1) = u_p(t) + \frac{\alpha}{12}\left(-6u_p(t) + \sum_{neighbours\ \ q} u_q(t)\right),$$

where the usual value of α is 1.

As well as studying the effects of different values of β and γ, it will be interesting to see what happens when the growth parameters are changed partway through growth. This is motivated by the idea that naturally occurring snow crystals form as they fall through air with different temperature and humidity. We can also start with irregular nuclei rather than the single seed site.

The project

Begin by producing code for the Boolean model, on both hexagonal and square lattices. This will allow you to address the treatment of neighbourhoods in these two geometries.

Next implement the full scheme. As well as studying the effects of different values of β and γ, it will be interesting to see what happens when the growth parameters are changed partway through growth. This is motivated by the idea that naturally occurring snow crystals form as they fall through air with different temperature and humidity.

We can also start with irregular nuclei rather than the single seed site, and explore the effect of modifying the diffusion process through α.

References

[1] Kepler J. *The six sided snowflake* (1611); translation, Oxford University Press; 1966.

[2] Libbrecht K and Rasmussen P. *The snowflake: winter's secret beauty* Stillwater: Voyageur Press; 2003.

[3] Packard N.H. *Lattice models for solidification and aggregation* in *Theory and Applications of Cellular Automata* Wolfram S, ed; Singapore: World Scientific Publishing; 1986. p. 305-310.

[4] Reiter C. *A Local Cellular Model for Snow Crystal Growth*, *Chaos, Solitons and Fractals* **23** 1111-1119 (2005)

[5] Wolfram S. *A new kind of science* Champaign: Wolfram Media; 2002.

A.H. Harker

January 2008