PHAS2423 - Self Study - Partial Differential Equations - Problems

(1) (a) Verify that any function of p, where $p = x^2 + 2y$, is a solution of

$$\frac{\partial u}{\partial x} = x\frac{\partial u}{\partial y}.$$

Then determine whether $v(x, y)$ is a solution of this PDE if
(b) $v(x, y) = x^4 + 4x^2y + 4y^2$
(c) $v(x, y) = x^4 + 2x^2y + y^2$
(d) $v(x, y) = x^2(x^2 - 4) + 4y(x^2 - 2) + 4(y^2 - 1)$

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(2) Find solutions of the PDE

$$\frac{1}{x}\frac{\partial u}{\partial x} + \frac{1}{y}\frac{\partial u}{\partial y} = 0,$$

for which
  (a) $u(0, y) = y$ (one-dimensional boundary condition);
  (b) $u(1, 1) = 1$ (zero-dimensional boundary condition).
Consider cases (a) and (b) separately.

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(3) Find solutions of the PDE

$$\sin x\frac{\partial u}{\partial x} + \cos x\frac{\partial u}{\partial y} = \cos x,$$

for which
  (a) $u(\pi/2, y) = 0$;
  (b) $u(\pi/2, y) = y(y + 1)$.
Consider cases (a) and (b) separately.

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(4) Find the most general solution of

$$\frac{\partial^2 u}{\partial x^2} - 3\frac{\partial^2 u}{\partial x \partial y} + 2\frac{\partial u}{\partial y^2} = 0,$$

which is consistent with

$$\frac{\partial u}{\partial y} = 1 \qquad\qquad \text{when } y = 0 \text{ for all } x.$$

and evaluate $u(0, 1)$.