Quantum Batteries

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I. INTRODUCTION

As technology becomes increasingly miniaturised, the effects of quantum mechanics become increasingly relevant. Furthermore, theoretical work show the potential advantages due quantum mechanical effects in various applications. One of these are the charging of quantum batteries.

II. QUANTUM BATTERIES

There are many ways to model quantum batteries. We consider the situation where we have N subsystems—which we will model as a collection of qubits [1]. In its natural state, the battery has the Hamiltonian:

$$H_0 = H_B + H_g. \tag{1}$$

We have the external field

$$H_B = B\sum_{i=1}^{N}\sigma_i^z \tag{2}$$

where the subscript i is the ith qubit, and σ_i^k is the Pauli spin operator with $k = x, y, z$: these are simply matrices with form:

$$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{3}$$

where i in σ^y denotes the imaginary number. We also have the term H_g that gives intrinsic interactions between the qubits:

$$H_g = -\sum_{i<j} g_{ij}\left[\sigma_i^z \otimes \sigma_j^z + \alpha\left(\sigma_i^x \otimes \sigma_j^x + \sigma_i^y \otimes \sigma_j^y\right)\right], \tag{4}$$

which makes this system a "XXZ spin chain", with interaction strength g_{ij} between qubit i and qubit j. The tensor product is calculated as:

$$A \otimes B = \begin{pmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{pmatrix} \otimes \begin{pmatrix} b_{00} & b_{01} \\ b_{10} & b_{11} \end{pmatrix} = \begin{pmatrix} a_{00}\begin{pmatrix} b_{00} & b_{01} \\ b_{10} & b_{11} \end{pmatrix} & a_{01}\begin{pmatrix} b_{00} & b_{01} \\ b_{10} & b_{11} \end{pmatrix} \\ a_{10}\begin{pmatrix} b_{00} & b_{01} \\ b_{10} & b_{11} \end{pmatrix} & a_{11}\begin{pmatrix} b_{00} & b_{01} \\ b_{10} & b_{11} \end{pmatrix} \end{pmatrix} = \begin{pmatrix} a_{00}b_{00} & a_{00}b_{01} & a_{01}b_{00} & a_{01}b_{01} \\ a_{00}b_{10} & a_{00}b_{11} & a_{01}b_{10} & a_{01}b_{11} \\ a_{10}b_{00} & a_{10}b_{01} & a_{11}b_{00} & a_{11}b_{01} \\ a_{10}b_{10} & a_{10}b_{11} & a_{11}b_{10} & a_{11}b_{11} \end{pmatrix}. \tag{5}$$

In particular, we consider either nearest-neighbour interactions (NN) or long-range (LR) interactions:

$$g_{ij}^{NN} = g\delta_{i,j-1} \tag{6}$$

$$g_{ij}^{LR} = \frac{g}{|i-j|^p} \tag{7}$$

where $g \in \mathbb{R}$ and $p \geq 0$..

As this is a quantum battery, we would like to charge it up. To do so, we add an external driving field:

$$V = \omega\sum_{i=1}^{N}\sigma_i^x - H_B \tag{8}$$

Then, for a time $t = 0$ to $t = T$, we charge up the battery by applying the full Hamiltonian

$$H(0 < t < T) = H_g + V. \tag{9}$$

We represent the state of the system with the column vector $|\psi(t)\rangle$. Its initial state at time $t = 0$ is $|\psi(0)\rangle$. Then at time T, the state is:

$$|\psi(T)\rangle = e^{-iHT}|\psi(0)\rangle. \tag{10}$$

The total energy (work) we deposit into the battery is the difference between the final energy and the initial energy:

$$W = \langle\psi(T)|H_0|\psi(T)\rangle - \langle\psi(0)|H_0|\psi(0)\rangle\,, \tag{11}$$

where $\langle\psi(t)|$ is a row vector that is the transpose-conjugate of $|\psi(t)\rangle$.

The aim is to optimise either the work W, or the power $P = W/T$, by choosing the best parameters: keeping $B = 1$ fixed, change the parameters g, p, and α in the interaction H_g, and ω in the charging field, as well as the time T.

III. POSSIBLE EXTENSIONS

The main extension would be to compare the quantum battery with its classical equivalent. Beyond this, the interaction H_g can take various different forms to represent different materials. The charging field V can take time-dependence. Noise can be added by adding a jitter to the parameters, and even a heat bath—an environment that sucks away some of the energy—can be modeled using the Lindbald master equation.

[1] T. P. Le, J. Levinsen, K. Modi, M. M. Parish, and F. A. Pollock, Phys. Rev. A **97**, 022106 (2018).