

# THERMAL PROPERTIES OF COMPOSITE MATERIALS

## Introduction

Many modern materials are composites, with their structures designed to optimise their properties for specific applications. For example, many high-strength materials consist of a host material (the ‘matrix’) reinforced with fibres or particles of a strong but brittle material such as silicon carbide or carbon. As the sketch below shows, the fibres might be identical and regularly arranged, or they might be variable and randomly positioned. The aim of this project is to study the thermal properties of such composites. We shall consider only two-dimensional systems (imagine these as slices through composites in which the structure does not change in the third dimension, corresponding to very long fibres, and we shall look at thermal conduction transverse to the fibre direction).

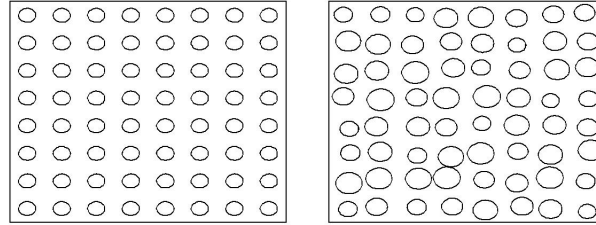


Figure 1: Regular and irregular fibre composites.

## Thermal Conductivity: Analytic Bounds

There have been several analytical models of thermal conductivity. Gurtmann *et al.* based their model on a regular structure, and found that the transverse conductivity was

$$K = V_f K_f (1 + V_m A) + V_m K_m (1 - V_f A),$$

where

$$A = \frac{K_m - K_f}{K_m + K_f + V_f(K_m + K_f)},$$

where the subscript  $m$  denotes matrix,  $f$  denotes fibre, and  $V_i$  and  $K_i$  are respectively the volume fraction and the conductivity of component  $i$ . Hatta and Taya used a slightly different approach, which led to

$$K = K_m + \frac{V_f(K_f - K_m)K_m}{(1 - V_f)(K_f - K_m)/2 + K_m}.$$

It is also possible to derive rigorous upper and lower bounds on the transverse conductivity:

$$K_U = V_f K_f + V_m K_m$$

and

$$K_L = \frac{1}{V_f/K_f + V_m/K_m}.$$

The project will explore how well these approximations and bounds work for various fibre configurations.

## Numerical Models

The first task will be to set up a version of the numerical solver for steady-state heat transport that can cope with inhomogeneous material, that is, material in which the properties change from place to place. This code will then be tested for situations in which the solution is known: for example, a slab of lower-conductivity material between two slabs of higher-conductivity material. In each case the thermal conductivity will be computed from the temperature difference between two faces of a block and the heat flux across the block.

Once the code is working satisfactorily, it will be applied to models of composite materials. Regular and irregular arrangements of identical and variable fibres will be used. Experiments will be conducted to see whether, for regular fibre arrays, the thermal conductivity varies with the direction of heat flow through the array. Comparisons will be made with the analytical estimates, with the aim of determining which estimate is best for each type of fibre distribution.

## Extensions

If time permits, some numerical experiments will be made on three-dimensional systems. It is expected that these will prove too computationally expensive to be used in a Mathematica© code for large enough systems to allow composites to be handled, but the feasibility will be assessed.

## References

- Gurtman, G.A., Rice, M.H. and Maewal, A. *Thermomechanical analysis of graphite/metal matrix composites*, Final Report (SSS-R-81-4862) to DARPA (# 3788), February 1981.
- Hatta, H. and Taya, M., *International Journal of Engineering Science* **24**(7), 1159-1172 (1986)

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January 2007