PARKINSON'S LAW

Introduction

C. Northcote Parkinson is best remembered for *Parkinson's Law*, which is expounded in a book of that name but which originate in an article in the Economist magazine (November 1955). His law is summarised as 'work expands to fill the time available for its completion'. His insights, however, were more extensive than this: Parkinson's Law of Triviality suggests that discussions in committees spend more time on trivia than on important issues. He also predicted that the Royal Navy would eventually have more admirals than ships. This project, however, pursues his observations on the sizes of decision-making bodies, in which he found that committees of more than 21 are inefficient. Furthermore, committees of eight members seem to have special problems[1]. The inspiration for this project was an article in *New Scientist*[1].

The Model

According to Klimek, Hanel and Thurner[2] decision-making in small groups can be cast into a *dynamical opinion formation model*. Each of the N members of the committee is represented as a node in a network and holds a binary opinion, say 0/1. Each node has a connectivity k which is the number of undirected links to other nodes, representing a social influence (interactions, such as discussions) two agents exert upon each other. Each node shares k undirected links with other nodes. Thus for $N > k + 1$ the graph is not fully connected and nodes appear which are not directly linked. For this exercise, assume the total network to be connected, so that there are no disjoint subnetworks (You might find it useful to use some of *Mathematica*'s graph-drawing functions from the *Combinatorica* package useful to visualize the links).

According to Parkinson groups such as cabinets are typically highly clustered. Therefore a sensible choice for the fixed network architecture of the model is a small-world network, which has already been shown to be of paramount importance in modelling social interactions. To establish such a network, each node is initially connected with its k nearest neighbors, then with probability e each link is randomly rewired. This network is assumed not to change during the decision-making process.

As a dynamical rule governing the interactions between connected agents use a majority rule with a predefined threshold h in the range $(0.5, 1]$. The update is carried out random-sequentially, i.e. within one iteration each node is updated one after the other in a random order, and a node adopts the state of the majority of its k neighbors only if this majority lies above h, otherwise the node keeps its previous internal state.

The question to be answered is whether the decision-making favours the forming of clusters (neighboring nodes tending to share the same opinion) so that coalitions emerge, and whether the final states consist of agents with the same opinion (consensus) or whether there are at least two factions of opposite opinions (dissensus). We want to know the probabilities that the group evolves into a final state of consensus or dissensus. Define dissensus $D(N)$ as the order parameter

[1] Charles I of England had eight members in his Committee of State, and it didn't do him much good!

of the model: let S_i be the initial population of nodes with internal state 0 and S_f the final population in this state. We then have

$$D(N) = \left\langle \Theta\left(1 - \frac{\max\left(S_f, N - S_f\right)}{N}\right)\right\rangle_{S_i},$$

where $\Theta(x)$ is the Heaviside step function (0 for $x < 0$, 1 for $x > 0$) and $\langle . \rangle_{S_i}$ denotes the average over all possible initial conditions. S_i is drawn with uniform probability from $(0, 1, ...N)$, that is, the opinions are randomly assigned to the individual nodes. $D(N)$ gives the expectation value of a final state without consensus and measures the groups proneness to end up in dispute. Of course, the expectation value averages over initial conditions which involve both the initial random allocation of opinions *and* different network graphs created probabilistically from the rewiring process.

The Project

Implement a model of this decision-making process, and investigate its behaviour as a function of committee size N and the model parameters (k, h, e). Additional experiments to try include making some committee members 'dominant', so that their opinions carry more weight, or including 'non-reciprocal' interactions, so that individual A is influenced by B but B is not influenced by A. As well as showing your results as graphs of dissensus, you should show some examples of the decision-making process in action, using black-and-white block diagrams to show the evolution of opinion.

References

[1] Buchanan M, The curse of the committee, *New Scientist* 10 January 2009, page 38.

[2] Klimek P., Hanel R. and Thurner, S., Parkinson's Law Quantified: Three Investigations on Bureaucratic Inefficiency, `http://arxiv.org/abs/0808.1684`.

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