DEPARTMENT OF CSE (ARTIFICIAL INTELLIGENCE AND MACHINE LEARNING)

| **MODULE-01QB: Introduction to Optimization & Mathematical Foundations** | |
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| **Subject:** | NUMERICAL OPTIMIZATION |
| **Sub Code:** | 23CSE616 |
| **Semester:** | VI |
| **Department:** | CSE (AIML) |

| Sl. No | Question | CO | Bloom's Level |
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| 1. | Formally define the mathematical optimization problem in terms of an objective function and constraints. | CO1 | L1 (Remember) |
| 2. | Distinguish between a global minimum and a local minimum in the context of a non-convex objective function. | CO1 | L2 (Understand) |
| 3. | Define a linear optimization problem and state the standard form of a linear programming problem. | CO1 | L1 (Remember) |
| 4. | Mathematically define a convex set and provide one simple geometric example. | CO1 | L1 (Remember) |
| 5. | Write the mathematical definition of the Hessian matrix for a function ��: $\mathbb{R}^{!} \to \mathbb{R}$. | CO1 | L1 (Remember) |
| 6. | Define the gradient of a scalar-valued function and explain its geometric significance regarding the direction of steepest ascent. | CO1 | L2 (Understand) |
| 7. | Calculate the Euclidean (��") norm of the vector �� $= [3, -4]^{\#}$. | CO1 | L3 (Apply) |
| 8. | What is the condition for a symmetric matrix to be considered positive definite? | CO1 | L1 (Remember) |
| 9. | Define the term "feasible region" in the context of constrained optimization. | CO4 | L1 (Remember) |
| 10. | State the multivariable chain rule for a composite function. | CO1 | L1 (Remember) |
| 11. | Briefly explain the role of a cost function (or objective function) in an optimization problem. | CO4 | L2 (Understand) |
| 12. | How do eigenvalues relate to the definiteness of a Hessian matrix? | CO1 | L2 (Understand) |

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| **13.** | Define unconstrained optimization and state the first-order necessary condition for optimality. | **CO4** | **L1 (Remember)** |
| **14.** | Define a partial derivative and explain how it differs from a total derivative. | **CO1** | **L2 (Understand)** |
| **15.** | Briefly define a saddle point in the context of a multivariable function. | **CO1** | **L1 (Remember)** |

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| **1.** | Explain the significance of the determinant and trace of a matrix. How do they relate to the eigenvalues? | **CO1** | **L2 (Understand)** |
| **2.** | Explain the relationship between the gradient of a function and its level sets (contour lines). Illustrate with a diagram. | **CO1** | **L2 (Understand)** |
| **3.** | Discuss the role of the Hessian matrix in determining the curvature of a function. How does it help in classifying stationary points? | **CO1** | **L2 (Understand)** |
| **4.** | Compare and contrast Linear Optimization and Nonlinear Optimization. Provide a mathematical formulation for each. | **CO4** | **L2 (Understand)** |
| **5.** | Describe the ��$, ��", ��% and norms. Explain their geometric interpretations (unit circles) and where each might be used. | **CO1** | **L2 (Understand)** |
| **6.** | Derive or explain the First-Order Necessary Condition (FONC) and Second-Order Sufficient Condition (SOSC) for local minimizers in unconstrained optimization. | **CO1** | **L2 (Understand)** |
| **7.** | Elaborate on the differences between convex and non convex optimization problems. Why is convexity desirable in optimization? | **CO4** | **L4 (Analyze)** |
| **8.** | Define the Jacobian matrix for a vector-valued function. Explain how it differs from the Hessian matrix. | **CO1** | **L2 (Understand)** |
| **9.** | Explain the Taylor series expansion for a multivariable function up to the second order. How is this used in optimization? | **CO1** | **L2 (Understand)** |
| **10.** | Explain the difference between equality and | **CO4** | **L2** |

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| | inequality constraints. How do they alter the search space? | | (Understand) |
| 11. | What does it mean for two vectors to be orthogonal? Explain the concept of an orthonormal basis. | CO1 | L2 (Understand) |
| 12. | Prove that the gradient points in the direction of the steepest ascent of the function. | CO1 | L3 (Apply) |
| 13. | Discuss why Linear Algebra is considered the backbone of modern optimization and Machine Learning. Give specific examples of matrix operations used in ML. | CO5 | L2 (Understand) |
| 14. | Classify stationary points into minima, maxima, and saddle points using eigenvalues of the Hessian. | CO1 | L3 (Apply) |
| 15. | Explain the concept of Eigen decomposition and its relevance in analyzing the geometry of quadratic functions. | CO1 | L2 (Understand) |

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| 1. | A manufacturing company wants to minimize transportation costs while satisfying demand at three different retail locations from two warehouses. Formulate this as a Linear Programming problem, defining decision variables, objective function, and constraints. | CO2 | L3 (Apply) |
| 2. | You are a financial analyst trying to maximize returns on a portfolio of stocks while minimizing risk (variance). Describe how you would formulate this as a constrained optimization problem. Identify the vector/matrix components involved. | CO5 | L4 (Analyze) |
| 3. | A nutritionist needs to design a diet plan that meets minimum daily requirements for Vitamin A, B, and C using three food items, each with different costs and nutrient contents. Formulate the mathematical optimization model to minimize the cost. | CO2 | L3 (Apply) |
| 4. | A cloud computing provider must allocate CPU and RAM to incoming tasks to maximize total throughput without exceeding physical server limits. Formulate this scenario as an integer or linear optimization problem. | CO2 | L3 (Apply) |

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| **5.** | Explain how finding the shortest path in a navigation app (like Google Maps) can be viewed as an optimization problem. Define the nodes, edges, costs, and the objective. | **CO4** | **L4 (Analyze)** |
| **6.** | A factory produces two products, ��$ and ��". ��$ requires 2 hours of labor and 3 units of raw material. ��" requires 4 hours of labor and 1 unit of raw material. Total labor available is 100 hours, and raw material is 80 units. Formulate the profit maximization problem. | **CO2** | **L3 (Apply)** |
| **7.** | Given a set of data points (��&, ��&), formulate the Ordinary Least Squares (OLS) regression problem as an unconstrained optimization problem using vector calculus notation. | **CO5** | **L3 (Apply)** |
| **8.** | An e-commerce warehouse needs to decide how much stock to order to minimize storage and ordering costs while preventing stockouts. Formulate the objective function involving holding costs and shortage penalties. | **CO4** | **L3 (Apply)** |
| **9.** | A marketing team has a fixed budget to spend across Social Media, TV, and Print ads. Each channel has a different "Return on Ad Spend" (ROAS) and saturation point. Formulate an optimization problem to maximize total revenue. | **CO2** | **L3 (Apply)** |
| **10.** | Engineers must design a bridge truss to minimize weight while supporting a specific load without yielding. Describe the decision variables (e.g., cross-sectional areas) and constraints (stress limits). | **CO4** | **L3 (Apply)** |
| **11.** | An electrical grid needs to balance power generation from wind, solar, and coal to match demand at minimum cost. Formulate this as a constrained optimization problem, considering the variability of renewables. | **CO2** | **L4 (Analyze)** |
| **12.** | Describe the geometric intuition of finding the "best" separating hyperplane between two classes of data points. How is this an optimization problem? | **CO5** | **L2 (Understand)** |
| **13.** | A robot arm needs to move from point A to B minimizing energy. Constraints include joint limits and obstacle avoidance. Formulate the path planning optimization problem. | **CO2** | **L3 (Apply)** |

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| **14.** | An airline needs to assign crews to flights to minimize layover costs and ensure legal rest periods. Describe the complexity of this problem and why it is a constrained optimization task. | **CO4** | **L4 (Analyze)** |
| **15.** | A data center wants to minimize cooling energy consumption while maintaining safe server temperatures. Formulate the relationship between fan speed (variable) and temperature (constraint). | **CO4** | **L3 (Apply)** |

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| **Sl. No** | **Question** | **CO** | **Bloom's Level** |
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| **1.** | Imagine a neural network's loss function as a physical terrain. Explain the concepts of "valleys" (minima), "peaks" (maxima), and "flat regions" (plateaus). How does the gradient guide us through this terrain? | **CO5** | **L4 (Analyze)** |
| **2.** | Why is it mathematically easier to train a Linear Regression model (convex loss) compared to a Deep Neural Network (non-convex loss)? Discuss the implications for finding the global minimum. | **CO5** | **L5 (Evaluate)** |
| **3.** | Intuitively explain how adding an norm (Lasso) vs. an norm (Ridge) to the loss function affects the model weights. Why does lead to sparsity? | **CO5** | **L4 (Analyze)** |
| **4.** | In Deep Learning, what is the "exploding gradient" problem? Explain this using the concept of the Chain Rule and repeated matrix multiplication. | **CO5** | **L2 (Understand)** |
| **5.** | How does the curvature of the loss function (represented by the Hessian) affect the speed of learning? Why do we struggle in areas of high curvature? | **CO5** | **L4 (Analyze)** |
| **6.** | Explain how the eigenvalues of the Hessian matrix at a critical point determine the stability of that point in a machine learning model's training process. | **CO1** | **L5 (Evaluate)** |
| **7.** | Discuss the Bias-Variance tradeoff from an optimization perspective. Does minimizing training error (optimization) always lead to lower test error (generalization)? | **CO5** | **L5 (Evaluate)** |
| **8.** | In high-dimensional ML problems, saddle points are more common than local minima. Explain why this happens and why standard gradient descent might get stuck or slow down near them. | **CO5** | **L4 (Analyze)** |

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| **9.** | Why does input feature scaling (normalization) speed up the convergence of gradient descent? Explain using the geometry of the cost function (elliptical vs. spherical contours). | **CO5** | **L4 (Analyze)** |
| **10.** | Optimization algorithms are often treated as black boxes in ML libraries (like scikit-learn). Explain the dangers of this approach without understanding the underlying calculus. | **CO4** | **L5 (Evaluate)** |
| **11.** | What does it mean for a matrix to be ill-conditioned in the context of Linear Regression? How does this affect the stability of the model solution? | **CO1** | **L4 (Analyze)** |
| **12.** | Why can't we initialize all weights in a neural network to zero? Explain using the concept of symmetry breaking and gradients. | **CO5** | **L2 (Understand)** |
| **13.** | Discuss whether getting stuck in a local minimum is always a bad thing in Deep Learning. Are all local minima created equal? | **CO5** | **L5 (Evaluate)** |
| **14.** | Explain how the "Curse of Dimensionality" affects optimization. As we add more features (dimensions), what happens to the volume of the search space? | **CO5** | **L2 (Understand)** |
| **15.** | Explain overfitting as "over-optimizing" on the training set. How do we mathematically constrain the optimization process to prevent this (e.g., early stopping)? | **CO5** | **L4 (Analyze)** |