Similarly, the lower tail, that is, the shaded area below P, has an area

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{-1.96-(B/\sigma)} e^{-t^2/2}\,dt$$

From the form of the integrals it is clear that the amount of disturbance depends solely on the ratio of the bias to the standard deviation. The results are shown in Table 1.1.

TABLE 1.1

EFFECT OF A BIAS B ON THE PROBABILITY OF AN ERROR GREATER THAN 1.96σ

B/σ	Probability of Error		Total
	$< -1.96\sigma$	$>1.96\sigma$	
0.02	0.0238	0.0262	0.0500
0.04	0.0228	0.0274	0.0502
0.06	0.0217	0.0287	0.0504
0.08	0.0207	0.0301	0.0508
0.10	0.0197	0.0314	0.0511
0.20	0.0154	0.0392	0.0546
0.40	0.0091	0.0594	0.0685
0.60	0.0052	0.0869	0.0921
0.80	0.0029	0.1230	0.1259
1.00	0.0015	0.1685	0.1700
1.50	0.0003	0.3228	0.3231

For the total probability of an error of more than 1.96σ, the bias has little effect provided that it is less than one tenth of the standard deviation. At this point the total probability is 0.0511 instead of the 0.05 that we think it is. As the bias increases further, the disturbance becomes more serious. At $B=\sigma$, the total probability of error is 0.17, more than three times the presumed value.

The two tails are affected differently. With a positive bias, as in this example, the probability of an underestimate by more than 1.96σ shrinks rapidly from the presumed 0.025 to become negligible when $B=\sigma$. The probability of the corresponding overestimate mounts steadily. In most applications the total error is the primary interest, but occasionally we are particularly interested in errors in one direction.

As a working rule, the effect of bias on the accuracy of an estimate is negligible if the bias is less than one tenth of the standard deviation of the estimate. If we have a biased method of estimation for which $B/\sigma<0.1$, where B is the absolute value of the bias, it can be claimed that the bias is not an appreciable disadvantage of the