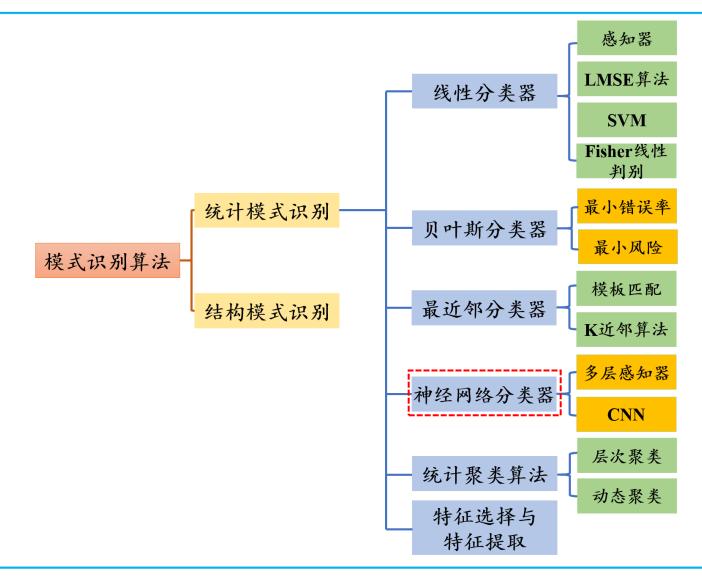
神经网络分类器张俊超

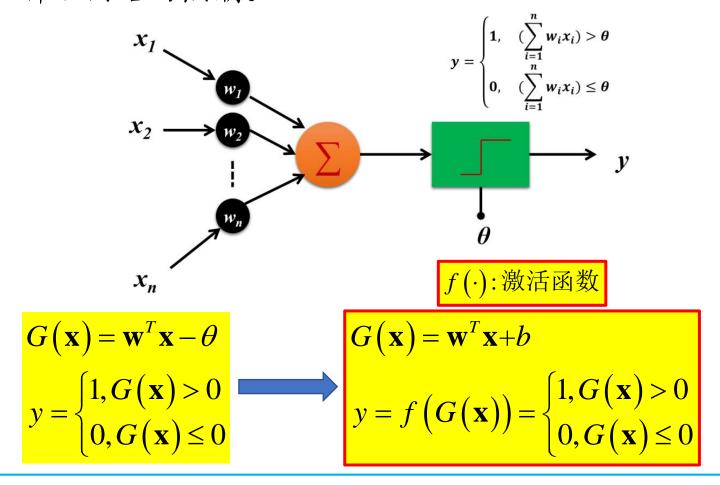








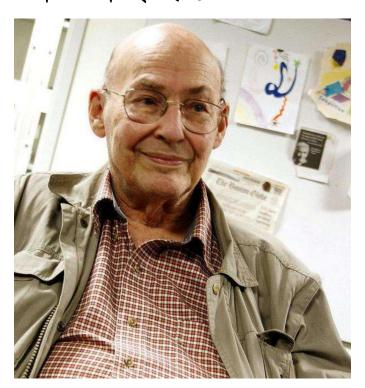
**感知器:** 1957年罗森布拉特提出的感知器,激起了第一次人工神经网络的热潮。

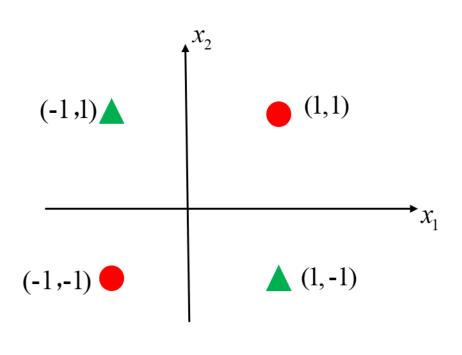


模式识别



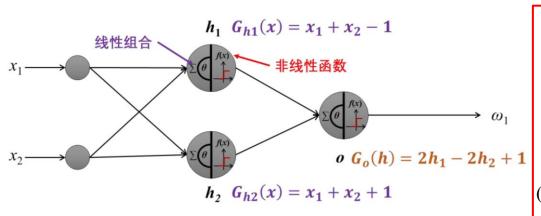
1969年明斯基等人无情的批判,导致人工神经网络研究跌入第一个寒冬。

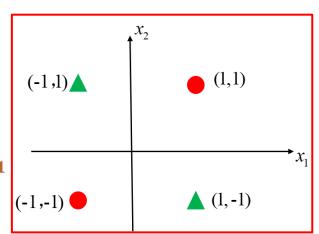




异或问题能够用感知器解决吗?







$$G_{h_1}(1,1) = 1 + 1 - 1 = 1, y_{h_1} = 1$$

$$G_{h_2}(1,1) = 1 + 1 + 1 = 3, y_{h_2} = 1$$

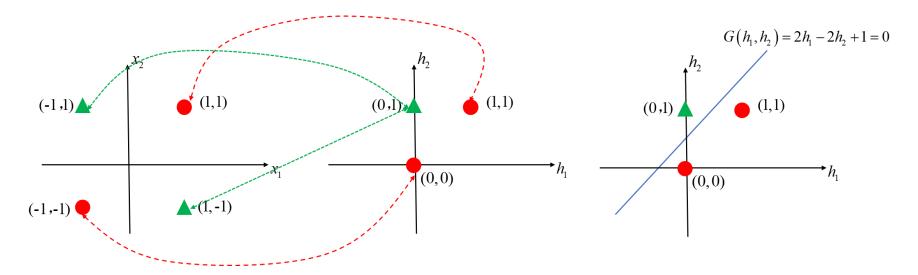
$$\rightarrow G_o(1,1) = 2 * 1 - 2 * 1 + 1 = 1, y_o = 1$$

$$G_{h_1}(1,-1) = 1 - 1 - 1 = -1, y_{h_1} = 0$$

$$G_{h_2}(1,-1) = 1 - 1 + 1 = 1, y_{h_2} = 1$$

$$\rightarrow G_o(0,1) = 2 * 0 - 2 * 1 + 1 = -1, y_o = 0$$





两层感知器并联,再与一个串联,解决了异或问题。可是,多次感知器是如何训练的呢?

罗森布拉特:固定前面的权重,用感知器算法只更新最后一层的权重。



**多层感知权重的更新**:固定前面的权重,用感知器算 法只更新最后一层的权重

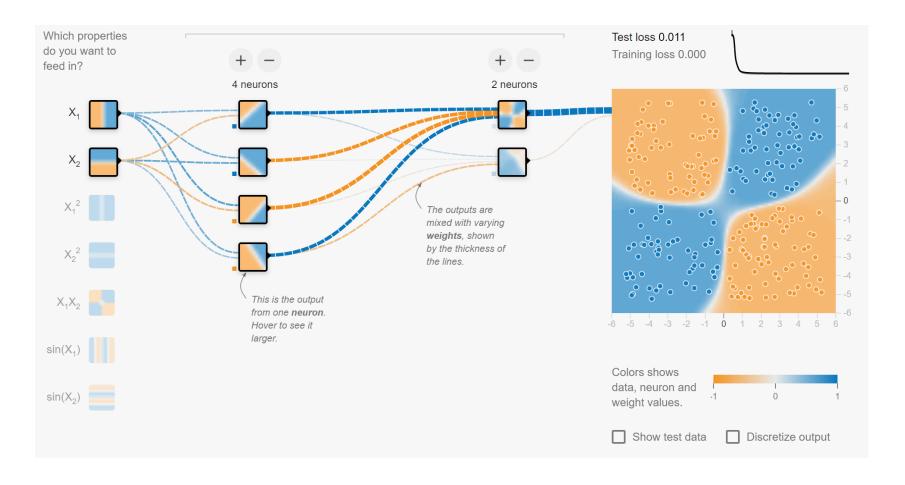
- 户前面的权重需要事先人为设定, 导致模型的性能很大程度上取决于事先选择的权重。
- ▶未能找到能够针对任意问题的前面权重赋值的方法。



1986年, Rumelhart 和Hinton 等人提出误差反向传播算法 (BP, back propagation)

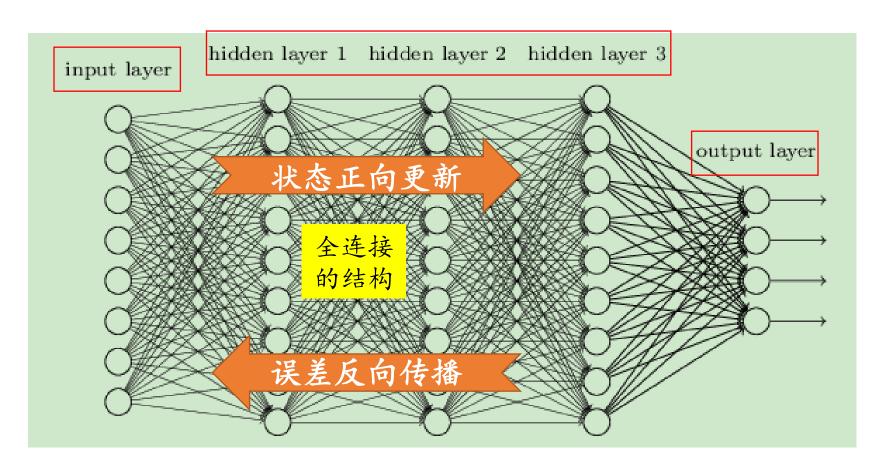


### http://playground.tensorflow.org





### 人工神经网络(ANN, Artificial Neural Network)





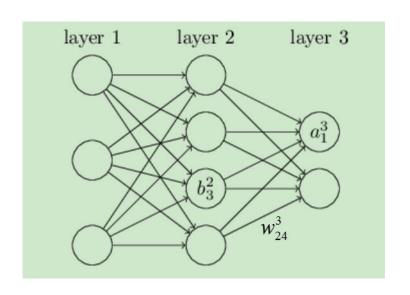
#### 【记号】

 $w_{ik}^{l}$ :第l-1层的第k个神经元到第l层的第j个神经元的权重

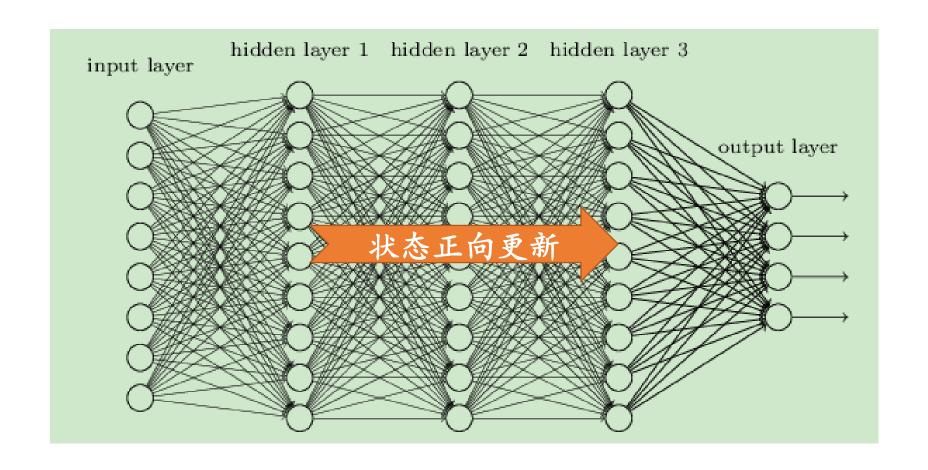
 $b_i^l$ :第l层的第j个神经元对应的偏置

需要考虑 前一层

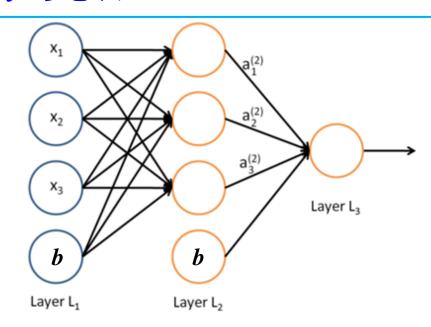
 $a_i^l$ :第l层的第j个神经元对应的输出











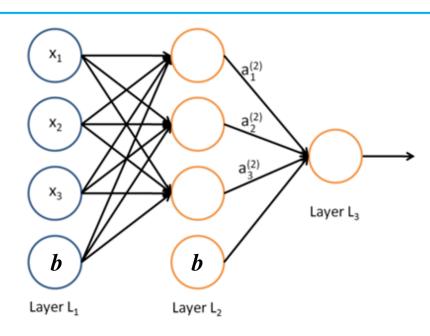
对于第二层的输出 $a_1^2, a_2^2, a_3^2$ ,我们有:

$$a_1^2 = \sigma(z_1^2) = \sigma(w_{11}^2 x_1 + w_{12}^2 x_2 + w_{13}^2 x_3 + b_1^2)$$

$$a_2^2 = \sigma(z_2^2) = \sigma(w_{21}^2 x_1 + w_{22}^2 x_2 + w_{23}^2 x_3 + b_2^2)$$

$$a_3^2 = \sigma(z_3^2) = \sigma(w_{31}^2 x_1 + w_{32}^2 x_2 + w_{33}^2 x_3 + b_3^2)$$
其中, $\sigma(\cdot)$ 是激活函数

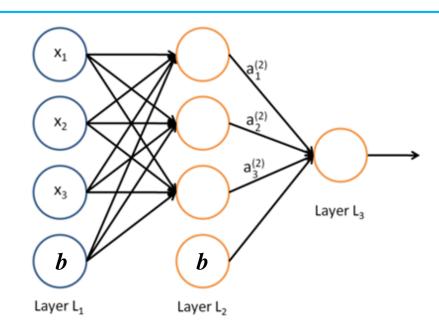




对于第三层的输出a<sub>1</sub>, 我们有:

$$a_1^3 = \sigma(z_1^3) = \sigma(w_{11}^3 a_1^2 + w_{12}^3 a_2^2 + w_{13}^3 a_3^2 + b_1^3)$$





将上面的式子一般化,设l-1层有m个神经元,则对第l层的第j个神经元的输出 $a_i^l$ ,我们有:

$$a_{j}^{l} = \sigma(\mathbf{z}_{j}^{l}) = \sigma(\sum_{k=1}^{m} w_{jk}^{l} a_{k}^{l-1} + b_{j}^{l})$$



#### 矩阵表示:

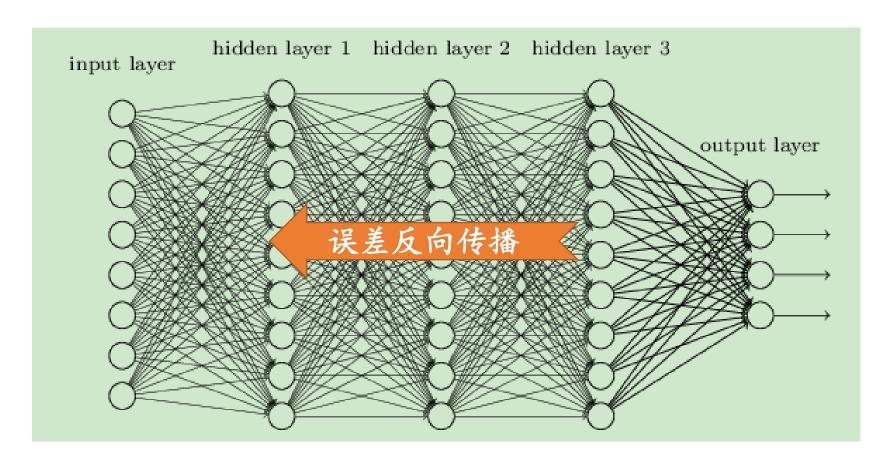
假设l-1层有m个神经元,l层有n个神经元,则l层的权重w就组成了一个 $n\times m$ 的矩阵  $\mathbf{W}^l$ ,偏置就组成了一个 $n\times 1$ 的向量  $\mathbf{b}^l$ ,第l-1的输出就组成了一个 $m\times 1$ 的向量  $\mathbf{a}^{l-1}$ ,第l层的未激活前的输出  $\mathbf{z}^l$ ,是一个 $n\times 1$ 的向量。第l的输出就组成了一个 $n\times 1$ 的向量  $\mathbf{a}^l$ 。

$$\mathbf{a}^{l} = \sigma(\mathbf{z}^{l}) = \sigma(\mathbf{W}^{l}\mathbf{a}^{l-1} + \mathbf{b}^{l})$$

$$\begin{bmatrix} a_{1}^{l} \\ a_{2}^{l} \\ \vdots \\ a_{n}^{l} \end{bmatrix} = \sigma \begin{pmatrix} \begin{bmatrix} w_{11}^{l} & w_{12}^{l} & \cdots & w_{1m}^{l} \\ w_{21}^{l} & w_{22}^{l} & \cdots & w_{2m}^{l} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n1}^{l} & w_{n2}^{l} & \cdots & w_{nm}^{l} \end{bmatrix} \begin{bmatrix} a_{1}^{l-1} \\ a_{2}^{l-1} \\ \vdots \\ a_{m}^{l-1} \end{bmatrix} + \begin{bmatrix} b_{1}^{l} \\ b_{2}^{l} \\ \vdots \\ b_{n}^{l} \end{bmatrix}$$



### 更新权重:





### 一般步骤:

- 选择损失函数
- 最小化损失函数
- 梯度下降法更新权重

均方差损失函数:  $J(\mathbf{W}, \mathbf{b}, \mathbf{x}, \mathbf{y}) = \frac{1}{2} \|\mathbf{a}^L - \mathbf{y}\|_2^2$ 

x:输入

y:期望的输出

W,b:权重

L:代表输出层



首先是输出层: 第 L 层, 输出层满足如下关系:

$$\mathbf{a}^{L} = \sigma(\mathbf{z}^{L}) = \sigma(\mathbf{W}^{L}\mathbf{a}^{L-1} + \mathbf{b}^{L})$$

对于输出层,我们的损失函数变为:

$$J(\mathbf{W}, \mathbf{b}, \mathbf{x}, \mathbf{y}) = \frac{1}{2} \|\mathbf{a}^{L} - \mathbf{y}\|_{2}^{2} = \frac{1}{2} \|\sigma(\mathbf{W}^{L} \mathbf{a}^{L-1} + \mathbf{b}^{L}) - \mathbf{y}\|_{2}^{2}$$

这样求解  $\mathbf{W}$ , $\mathbf{b}$  的梯度为:

$$\frac{\partial J(\mathbf{W}, \mathbf{b}, \mathbf{x}, \mathbf{y})}{\partial \mathbf{W}^{L}} = \frac{\partial J(\mathbf{W}, \mathbf{b}, \mathbf{x}, \mathbf{y})}{\partial \mathbf{z}^{L}} \cdot \frac{\partial \mathbf{z}^{L}}{\partial \mathbf{W}^{L}} = (\mathbf{a}^{L} - \mathbf{y}) \odot \sigma'(\mathbf{z}^{L})(\mathbf{a}^{L-1})^{T}$$

$$\frac{\partial J(\mathbf{W}, \mathbf{b}, \mathbf{x}, \mathbf{y})}{\partial \mathbf{b}^{L}} = \frac{\partial J(\mathbf{W}, \mathbf{b}, \mathbf{x}, \mathbf{y})}{\partial \mathbf{z}^{L}} \cdot \frac{\partial \mathbf{z}^{L}}{\partial \mathbf{b}^{L}} = (\mathbf{a}^{L} - \mathbf{y}) \odot \sigma'(\mathbf{z}^{L})$$

从上式我们可以看到,存在公共部分,记为:

$$\boldsymbol{\delta}^{L} = \frac{\partial J(\mathbf{W}, \mathbf{b}, \mathbf{x}, \mathbf{y})}{\partial \mathbf{z}^{L}} = (\mathbf{a}^{L} - \mathbf{y}) \odot \sigma'(\mathbf{z}^{L})$$

# 矩阵求导过程参考链接:

https://zhuanlan.zh ihu.com/p/247097 48



#### 其它层的计算:

我们注意到第1层的未激活输出为z<sup>1</sup>,它的梯度可以表示为:

$$\boldsymbol{\delta}^{l} = \frac{\partial J(\mathbf{W}, \mathbf{b}, \mathbf{x}, \mathbf{y})}{\partial \mathbf{z}^{l}} = \frac{\partial J(\mathbf{W}, \mathbf{b}, \mathbf{x}, \mathbf{y})}{\partial \mathbf{z}^{L}} \cdot \frac{\partial \mathbf{z}^{L}}{\partial \mathbf{z}^{L-1}} \cdot \frac{\partial \mathbf{z}^{L-1}}{\partial \mathbf{z}^{L-2}} \cdot \dots \cdot \frac{\partial \mathbf{z}^{l+1}}{\partial \mathbf{z}^{l}}$$

如果我们依次获得 $\delta^l$ , 根据 $\mathbf{z}^l = \mathbf{W}^l \mathbf{a}^{l-1} + \mathbf{b}^l$ , 则:

$$\frac{\partial J(\mathbf{W}, \mathbf{b}, \mathbf{x}, \mathbf{y})}{\partial \mathbf{W}^{l}} = \frac{\partial J(\mathbf{W}, \mathbf{b}, \mathbf{x}, \mathbf{y})}{\partial \mathbf{z}^{l}} \cdot \frac{\partial \mathbf{z}^{l}}{\partial \mathbf{W}^{l}} = \boldsymbol{\delta}^{l} (\mathbf{a}^{l-1})^{T}$$

$$\frac{\partial J(\mathbf{W}, \mathbf{b}, \mathbf{x}, \mathbf{y})}{\partial \mathbf{b}^{l}} = \frac{\partial J(\mathbf{W}, \mathbf{b}, \mathbf{x}, \mathbf{y})}{\partial \mathbf{z}^{l}} \cdot \frac{\partial \mathbf{z}^{l}}{\partial \mathbf{b}^{l}} = \mathbf{\delta}^{l}$$

用数学归纳法,可得 $\boldsymbol{\delta}^l = (\mathbf{W}^{l+1})^T \boldsymbol{\delta}^{l+1} \odot \sigma'(\mathbf{z}^l)$ 



$$\boldsymbol{\delta}^{l} = \frac{\partial J(\mathbf{W}, \mathbf{b}, \mathbf{x}, \mathbf{y})}{\partial \mathbf{z}^{l}} = \frac{\partial J(\mathbf{W}, \mathbf{b}, \mathbf{x}, \mathbf{y})}{\partial \mathbf{z}^{l+1}} \cdot \frac{\partial \mathbf{z}^{l+1}}{\partial \mathbf{z}^{l}} = \boldsymbol{\delta}^{l+1} \frac{\partial \mathbf{z}^{l+1}}{\partial \mathbf{z}^{l}}$$

可见,用归纳法递推 $\boldsymbol{\delta}^{l+1}$ 和 $\boldsymbol{\delta}^{l}$ 的关键在于求解 $\frac{\partial \mathbf{z}^{l+1}}{\partial \mathbf{z}^{l}}$ 。

而 $\mathbf{z}^{l+1}$ 和 $\mathbf{z}^{l}$ 的关系是:

$$\mathbf{z}^{l+1} = \mathbf{W}^{l+1}\mathbf{a}^l + \mathbf{b}^{l+1} = \mathbf{W}^{l+1}\sigma(\mathbf{z}^l) + \mathbf{b}^{l+1}$$

很容易得到:

$$\frac{\partial \mathbf{z}^{l+1}}{\partial \mathbf{z}^{l}} = (\mathbf{W}^{l+1})^{T} \odot \underbrace{(\sigma'(\mathbf{z}^{l}), \dots, \sigma'(\mathbf{z}^{l}))}_{n_{l+1}}$$

将上式带入 $\delta^{l}$ 中,可得:

$$\boldsymbol{\delta}^{l} = \boldsymbol{\delta}^{l+1} \frac{\partial \mathbf{z}^{l+1}}{\partial \mathbf{z}^{l}} = (\mathbf{W}^{l+1})^{T} \boldsymbol{\delta}^{l+1} \odot \sigma'(\mathbf{z}^{l})$$



### 梯度下降法更新权重:

$$\mathbf{W}^{l}(t+1) = \mathbf{W}^{l}(t) - \eta \cdot \nabla_{\mathbf{W}^{l}} = \mathbf{W}^{l}(t) - \eta \delta^{l}(t) (\mathbf{a}^{l-1}(t))^{T}$$

$$\mathbf{b}^{l}(t+1) = \mathbf{b}^{l}(t) - \eta \cdot \nabla_{\mathbf{b}^{l}} = \mathbf{b}^{l}(t) - \eta \delta^{l}(t)$$

Mini-batch SGD(随机梯度下降法)

m个样本

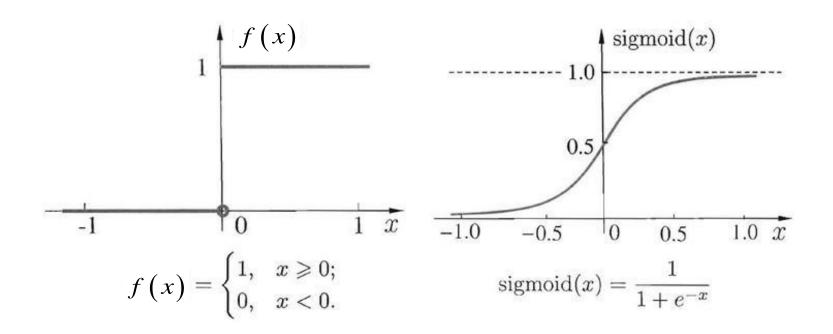
$$\mathbf{W}^{l}\left(t+1\right) = \mathbf{W}^{l}\left(t\right) - \eta \cdot \nabla_{\mathbf{W}^{l}} = \mathbf{W}^{l}\left(t\right) - \eta \sum_{i=1}^{m} \boldsymbol{\delta}^{i,l}\left(t\right) \left(\mathbf{a}^{i,l-1}\left(t\right)\right)^{T}$$

$$\mathbf{b}^{l}\left(t+1\right) = \mathbf{b}^{l}\left(t\right) - \eta \cdot \nabla_{\mathbf{b}^{l}} = \mathbf{b}^{l}\left(t\right) - \eta \sum_{i=1}^{m} \delta^{i,l}\left(t\right)$$



$$\mathbf{a}^{l} = \sigma(\mathbf{z}^{l}) = \sigma(\mathbf{W}^{l}\mathbf{a}^{l-1} + \mathbf{b}^{l})$$

激活函数如何选择?





### ▶均方差损失函数+Sigmoid 激活函数

均方差损失函数:  $J(\mathbf{W}, \mathbf{b}, \mathbf{x}, \mathbf{y}) = \frac{1}{2} \|\mathbf{a}^L - \mathbf{y}\|_2^2$ 

$$\mathbf{a}^{l} = \sigma(\mathbf{z}^{l}) = \sigma(\mathbf{W}^{l}\mathbf{a}^{l-1} + \mathbf{b}^{l}) = \frac{1}{1 + e^{-(\mathbf{W}^{l}\mathbf{a}^{l-1} + \mathbf{b}^{l})}}$$

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

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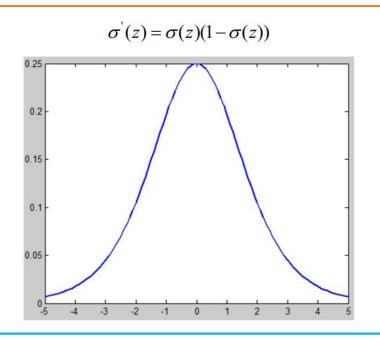
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模式识别

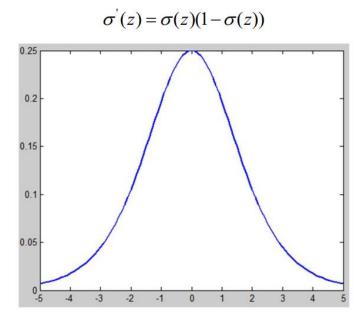
中南大学航空航天学院



$$\frac{\partial J(\mathbf{W}, \mathbf{b}, \mathbf{x}, \mathbf{y})}{\partial \mathbf{W}^{l}} = \frac{\partial J(\mathbf{W}, \mathbf{b}, \mathbf{x}, \mathbf{y})}{\partial \mathbf{z}^{l}} \cdot \frac{\partial \mathbf{z}^{l}}{\partial \mathbf{W}^{l}} = \boldsymbol{\delta}^{l} (\mathbf{a}^{l-1})^{T}$$

$$\frac{\partial J(\mathbf{W}, \mathbf{b}, \mathbf{x}, \mathbf{y})}{\partial \mathbf{b}^{l}} = \frac{\partial J(\mathbf{W}, \mathbf{b}, \mathbf{x}, \mathbf{y})}{\partial \mathbf{z}^{l}} \cdot \frac{\partial \mathbf{z}^{l}}{\partial \mathbf{b}^{l}} = \boldsymbol{\delta}^{l}$$

$$\boldsymbol{\delta}^{l} = (\mathbf{W}^{l+1})^{T} \boldsymbol{\delta}^{l+1} \odot \boldsymbol{\sigma}'(\mathbf{z}^{l})$$



从上面的图像可以看出,g'(z)在z=0时取得最大值 0.25, 且是小于 1 的。

从 BP 算法中,我们知道,每一层向前递推时都要乘以 $\sigma'(z)$ ,得到梯度变化值。 若采用 Sigmoid 函数作为激活函数,将导致权重更新较慢,网络收敛较慢。



### ▶交叉熵损失函数+Sigmoid 激活函数

我们知道 Sigmoid 函数的输出表征了当前样本标签为 1 的概率:

$$\hat{y} = P(y = 1 \mid x)$$

则, 当前样本标签为0的概率就可以表达成:

$$1 - \hat{y} = P(y = 0 \mid x)$$

上面的式子可以合在一起,表示为:

$$P(y \mid x) = \hat{y}^{y} (1 - \hat{y})^{1-y}$$

从极大似然角度出发,我们希望P(y|x)越大越好,因  $\log$  函数不影响函数单调

性, 我们对P(y|x)求 log:

$$\log P(y \mid x) = y \log \hat{y} + (1 - y) \log(1 - \hat{y})$$

因此, 损失函数(-log)可以定义成:

$$L = -[y \log \hat{y} + (1 - y) \log(1 - \hat{y})]$$



我们计算输出层 $\delta^L$ :

$$\delta^{L} = \frac{\partial J(W, b, a^{L}, y)}{\partial z^{L}}$$

$$= -y \frac{1}{a^{L}} (a^{L}) (1 - a^{L}) + (1 - y) \frac{1}{1 - a^{L}} (a^{L}) (1 - a^{L})$$

$$= -y (1 - a^{L}) + (1 - y) (a^{L})$$

$$= a^{L} - y$$

可见此时, $\delta^L$  表达式中已经没有了 $\sigma'(z)$ ,因此避免了反向传播收敛速度慢的问题。

通常情况下,若使用了 Sigmoid 函数做激活函数,交叉 熵损失函数肯定比均方差损失函数好用。



### 多分类问题的交叉熵损失函数怎么定义呢?

Softmax 函数:

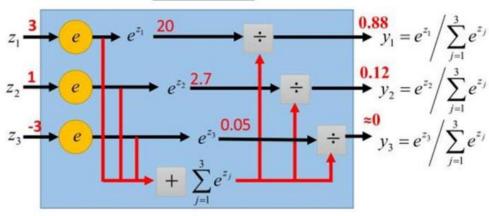
$$a_{i}^{L} = \frac{e^{z_{i}^{L}}}{\sum_{j=1}^{n_{L}} e^{z_{j}^{L}}}$$

· Softmax layer as the output layer

#### Probability:

- $\blacksquare 1 > y_i > 0$
- $\blacksquare \sum_i y_i = 1$

#### Softmax Layer





对应的损失函数(对数似然函数):

$$J(\mathbf{W}, \mathbf{b}, \mathbf{a}^L, \mathbf{y}) = -\sum_k y_k \ln a_k^L$$

其中 $y_k$ 的取值为 0 或者 1。如果某一类样本的输出为第i类,则 $y_i$  = 1,其余的  $j \neq i$ 都有 $y_i$  = 0。由于每个样本属于一个类别,所以这个似然函数可以简写为:

$$J(\mathbf{W}, \mathbf{b}, \mathbf{a}^L, \mathbf{y}) = -\ln a_i^L$$



### Matlab神经网络工具:

https://blog.csdn.net/weixin\_42296976/article/details/81123843

# Talk is cheap, Show me the code.



```
%% Demo for ANN
clc:
close all:
clear all:
%%
load mnist uint8;
train_x = double(train_x) / 255;
test x = double(test x) / 255;
train_y = double(train y);
test y = double(test y);
% Normalize
[train x, mu, sigma] = Whiting(train x);
test_x = Normalize(test_x, mu, sigma);
```



```
%% Network setup
nn = AnnSetup([784 100 10]);
opts.MaxEpoch = 1;
opts.BatchSize = 200;

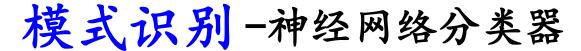
%% Train
[nn, L] = AnnTrain(nn, train_x, train_y, opts);
figure, plot(1:length(L), L, 'r-', 'LineWidth', 2);
xlabel('Number of Iteration'); ylabel('Loss');
```



```
function nn = AnnSetup(architecture)
 nn. size = architecture:
 nn. n = numel(nn. size);
 nn.activation_function = 'sigm';
 nn.learningRate = 2;
 nn.loss_function = 'cross_entropy'; %cross_entropy % mse
 %%
\exists for i = 2 : nn. n
     nn. W\{i-1\} = (rand(nn. size(i), nn. size(i-1)) - 0.5):
     nn. b\{i-1\} = zeros(nn. size(i), 1);
 end
 end
```



```
☐ function [nn, L] = AnnTrain(nn, train_x, train_y, opts)
 m = size(train x, 1);
 BatchSize = opts.BatchSize;
 MaxEpoch = opts. MaxEpoch:
 NumBatches = ceil(m / BatchSize):
 N = NumBatches*BatchSize:
 train x = [train x; train x(1:N-m,:)];
 train y = [train y; train y(1:N-m,:)];
 L = zeros (MaxEpoch*NumBatches, 1);
 n = 1;
for i = 1:MaxEpoch
     kk = randperm(N):
     for 1 = 1: NumBatches
         batch x = train x(kk((1-1) * BatchSize + 1 : 1 * BatchSize), :);
         batch y = train y(kk((1-1) * BatchSize + 1 : 1 * BatchSize), :);
         nn = Annforward(nn, batch x, batch y);
         nn = Annbp(nn);
         L(n) = nn. L;
         n = n + 1;
     end
 end
```





```
\Box function nn = Annforward (nn, x, y)
  n = nn.n;
 m = size(x, 1):
  nn. a\{1\} = x:
\triangle for i = 2 : n
      tmp = nn. a\{i - 1\} * nn. W\{i - 1\}';
      tmp1 = repmat(nn.b{i-1}', [size(tmp, 1), 1]);
      switch nn. activation function
          case 'sigm'
               nn. a\{i\} = sigm(tmp+tmp1);
           case 'tanh opt'
               nn. a\{i\} = tanh opt(tmp+tmp1);
      end
  end
  nn. e = y - nn. a\{n\};
  switch nn.loss_function
      case 'mse'
          nn.L = 1/2 * sum(sum(nn.e.^2)) / m;
      case 'cross entropy'
          nn. L = -sum(sum(y .* log(nn. a{n}))) / m;
  end
  end
```



```
]function nn = Annbp(nn)
n = nn.n;
 switch nn. loss function
     case 'mse'
          d\{n\} = - \text{ nn. e } .* (\text{nn. a}\{n\} .* (1 - \text{nn. a}\{n\}));
     case 'cross entropy'
          d\{n\} = - nn. e:
 end
] for i = (n - 1) : -1 : 2
      switch nn. activation function
          case 'sigm'
               d act = nn. a\{i\} .* (1 - nn. a\{i\});
          case 'tanh opt'
               d act = 1.7159 * 2/3 * (1 - 1/(1.7159)^2 * nn. a{i}.^2);
     end
     d\{i\} = (d\{i+1\} * nn, W\{i\}) .* d act:
- end
```



```
for i = 1 : (n - 1)
     nn. dW\{i\} = (d\{i + 1\}' * nn. a\{i\}) / size(d\{i + 1\}, 1);
     nn. db\{i\} = d\{i+1\}'*ones\{size(d\{i+1\}, 1), 1\}/ size\{d\{i+1\}, 1\};
 end
 %% Gradient Descent
 for i = 1 : (n - 1)
     dW = nn. learningRate * nn. dW{i};
     nn. W\{i\} = nn. W\{i\} - dW;
     db = nn. learningRate * nn. db{i};
     nn. b\{i\} = nn. b\{i\} - db:
 end
 end
```

模式识别



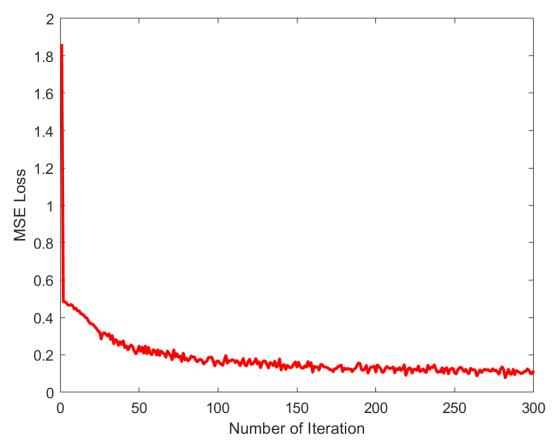
#### %% Test

```
nn = Annforward(nn, test_x, test_y);
[~, labels] = max(nn.a{end},[],2);
[~, expected] = max(test_y,[],2);
bad = find(labels ~= expected);
er = numel(bad) / size(test_x, 1);
acr = 1-er;
fprintf('Accuracy ratio is %f.\n', acr);
```



#### MSE loss

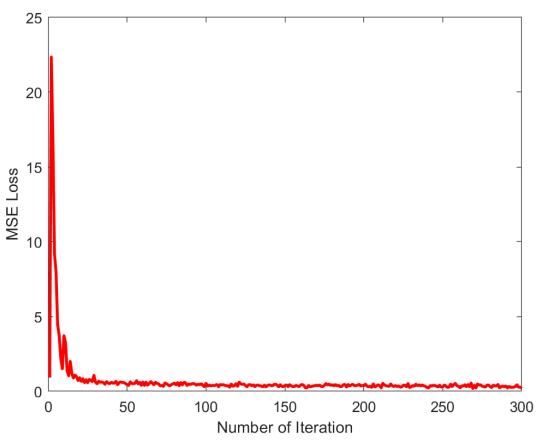
准确率: 0.8798





### Cross\_entropy loss

准确率: 0.9277





### 思考与讨论:

- >激活函数的作用是什么?没有激活函数可以吗?
- ▶Sigmoid激活函数有什么缺点?
- ▶全连接网络有什么不足?







