

# **Beyond $1/f$ scaling laws: A new methodological framework**

## **A Matlab toolbox**

This Matlab toolbox contains the following Matlab codes:

### **Models**

alpha_stable_rand	Creates a series of random numbers with a symmetric alpha-stable distribution
mGn_mBm	Creates multifractional Gaussian noise and Brownian motion
mLn_mLm	Creates multifractional alpha-stable noise and motion (i.e. time dependent Hurst exponent $H(t)$ ) (uses the alpha_stable_rand)
mLn_mLm_STABLE	Creates multifractional alpha-stable noise and motion (i.e. time dependent Hurst exponent $H(t)$ ) (uses STABLE toolbox)

Matlab codes for generation of multiplicative cascading noise and multifractal random walk are available at <http://www.isima.fr/~pchainai/PUB/software.html>

### **Step 1. Non-Gaussian distribution**

Matlab codes for the estimation of alpha-stable distributions are available in the STABLE® toolbox at <http://academic2.american.edu/~jpnolan/stable/stable.html>

### **Step 2. Time-scale decompositions**

Matlab codes for CWT based spectral analyses for one or two variables are available in the wavelet coherence toolbox at <http://www.pol.ac.uk/home/research/waveletcoherence/>

Matlab codes for MODWT are available in the WMTSA toolbox at <http://www.atmos.washington.edu/~wmtsa/>

Matlab codes for EMD are available at <http://perso.ens-lyon.fr/patrick.flandrin/software2.html>

Note. A major portion of the codes for the multifractal analyses in Step 3 below are based on the CWT, MODWT or EMD codes.

### Step 3. Multifractal spectra

MFCWT	Multifractal spectrum estimation based on the cwt contained in the official Wavelet toolbox®
MFCWT_mex	Multifractal spectrum estimation based on the contwtmir.m code contained in the Fraclab® toolbox
MFCWT_comp	Multifractal spectrum estimation based on the wavelet.m code contained in the wavelet coherence toolbox
MFXTWT	Multifractal covariation spectrum estimation based on the wavelet.m code contained in the wavelet coherence toolbox
MFMODWT	Multifractal spectrum estimation based on the modwt.m contained in the wmtsa® toolbox
MFXTMODWT	Multifractal spectrum estimation based on the modwt.m and imodwt_mra.m codes contained in the wmtsa® toolbox
WTMM_local	Direct CWT-based estimation of the multifractal spectrum based on the wavelet.m code contained in the wavelet coherence toolbox
XWTMM_local	Direct CWT-based estimation of the multifractal spectrum of covariation between two variables based on the wavelet.m code contained in the wavelet coherence toolbox
MF DFA	Multifractal spectrum estimated by the multifractal detrended fluctuation analysis with polynomial detrending
MF DFA_modwt	Multifractal spectrum estimated by the multifractal detrended fluctuation analysis with MODWT-based detrending
MF DFA_emd	Multifractal spectrum estimated by the multifractal detrended fluctuation analysis with EMD-based detrending
MF DXA	Multifractal spectrum for covariation between two variables estimated by the multifractal cross-correlation analysis with polynomial detrending.
MF DXA_modwt	Multifractal spectrum for covariation between two variables estimated by the multifractal cross-correlation analysis with MODWT-based detrending.
mod_MF DFA	Multifractal spectrum estimated by the modified version of the multifractal detrended fluctuation analysis with polynomial detrending. This analysis does not assume scale invariance!

mod_MFDFA_modwt	Multifractal spectrum estimated by the modified version of the multifractal detrended fluctuation analysis with MODWT-based detrending. This analysis does not assume scale invariance!
mod_MFDFA_emd	Multifractal spectrum estimated by the modified version of the multifractal detrended fluctuation analysis with EMD-based detrending. This analysis does not assume scale invariance!
mod_MFDXA	Multifractal spectrum for covariation between two variables estimated by the modified version of the multifractal cross-correlation analysis with polynomial detrending. This analysis does not assume scale invariance!
mod_MFDXA_modwt	Multifractal spectrum for covariation between two variables estimated by the modified version of the multifractal cross-correlation analysis with MODWT-based detrending. This analysis does not assume scale invariance!
mGn_dfa_estim	Estimation of the time evolution of the Hurst exponent $H(t)$ by a window version of the detrended fluctuation analysis. This analysis is only valid when the signal variation is similar to multifractional Gaussian noise with a smooth (i.e. slow changing) $H(t)$
mGn_modwt_estim	Estimation of the time evolution of the Hurst exponent $H(t)$ by a MODWT-based analysis. This analysis is only valid when the signal variation is similar to multifractional Gaussian noise with a smooth (i.e. slow changing) $H(t)$

#### **Step 4. Surrogate tests**

Shuffle	Surrogates with identical distribution as the signal variation, but with a log-spectrum with zero slope (i.e. uncorrelated samples).
FTran	Surrogates with identical log-spectrum, but with approximately Gaussian distribution.
laaft	Surrogates with identical distribution and log-spectrum as the signal variation.

The matlab code for the wiaaft surrogates are available at  
<http://www.geog.leeds.ac.uk/people/ckeylock/software.html>

**Note.** The creator, Espen A. F. Ihlen, take no responsibility for any use of the Matlab codes in the present toolbox

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