CP312 Fall 2018 Assignment 1

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1. By the definition of Θ-notation, for $f(n) \in \Theta(g(n))$ if there exists positive constants c_1, c_2 and n_0 such for all $n \geq n_0$, $c_1 g(n) \leq f(n) \leq c_2 g(n)$

so:

$$(13n+3)(9n+1)(\log(4n^2+100)) \in \Theta(n^2\log n)$$

$$\Downarrow$$

$$c_1 n^2\log n \leq (13n+3)(9n+1)(\log(4n^2+100)) \leq c_2 n^2 \log n$$

$$c_1 n^2\log n \leq (117n^2+40n+3)(\log(4n^2+100)) \leq c_2 n^2 \log n$$

Because the highest power in $117n^2+40n+3$ is 2 so coefficient is 117, so c_2 must greater than 117, c_1 must less than 117

Right side: For $117n^2+40n+3 < 117n^2+40n^2+3n^2 = 160n^2 \Rightarrow c_2 = 160$ for $n \in \mathbb{R}$
and $\log(4n^2+100) \leq c_2 \log n = \log n^{c_2} \Rightarrow c_2 = 10$ for $n \geq 2$
$\therefore c_2 = 1600$ for $n \geq 2$

Left side: $c_1 = 1$ so $n^2\log n \leq (117n^2+40n+3)(\log(4n^2+100))$ for $n > 0$
$\therefore$ There exist $c_1 = 1$ $c_2 = 1600$ and $n_0 \geq 2$ for
$n^2\log n \leq (117n^2+40n+3)(\log(4n^2+100)) \leq 1600\, n^2\log n$,
so $(13n+3)(9n+1)(\log(4n^2+100)) \in \Theta(n^2\log n)$

2. (a) $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \frac{1052n^2+10n^2+10001}{\frac{2}{10000000}n^4+2n}$ $\left(\frac{\frac{1}{n^4}}{\frac{1}{n^4}}\right)$

$$= \frac{\frac{1052}{n^2}+\frac{10}{n^2}+\frac{10001}{n^4}}{\frac{2}{10000000}+\frac{2}{n^3}} \begin{matrix} \leftarrow 0 \\ \leftarrow \frac{1}{5000000} \end{matrix}$$

$= 0$

$\therefore f(n) \in o(g(n))$

(b)

$f(n) = (\log_2 n^{10})^3 = 1000(\log_2 n)^3$ $\quad g(n) = \sqrt{\sqrt{n}} = n^{\frac{1}{4}}$

Because $\log n \le \sqrt{n} \le n$, so $1000(\log n)^3 \le C_1\sqrt{n} \le C_2 n$

$\therefore f(n) \in o(g(n))$

Also if I use $\lim_{n\to\infty} \frac{(\log_2 n^{10})^3}{\sqrt{\sqrt{n}}} = 0$ (By using LH Rule)

(c) $g(n) = 4008n^4\log n$ $\quad f(n) = n^4\log n$

Because $n^4\log n$ in $g(n)$ is equal to $f(n) = n^4\log n$

and 4008 in $g(n)$ is a constant. So the constant in $g(n)$

will depend whether $g(n) \ge f(n)$ or $g(n) \le f(n)$

I write $C_1 g(n) \le f(n) \le C_2 g(n)$

so it is $f(n) \in \Theta(g(n))$

(d)

$f(n) = 16^{\log\sqrt{n}} = \sqrt{n}^{\log 16} = n^{\frac{1}{2}\log_2 16} = n^2$ $\quad g(n) = n^2$

Because $f(n) = g(n)$, so $C_1 g(n) \le f(n) \le C_2 g(n)$

$\therefore f(n) \in \Theta(g(n))$

(e) $f(n) = n^{2+\sin(\frac{n\pi}{2})}$ $\quad g(n) = n^{\frac{5}{3}}$

base of $f(n)$ and $g(n)$ are both n

so compare $2+\sin(\frac{n\pi}{2})$ and $\frac{5}{3}$

the range of sin is between -1 and 1, so $2+(-1)=1$, $2+1=3$

so $n^1 \le f(n) = n^{2+\sin(\frac{n\pi}{2})} \le n^3$ and $1 < \frac{5}{3} < 3$

so whether $f(n) \ge g(n)\cdot C_1$ or $f(n) \le g(n)\cdot C_2$ is about

$2+\sin(\frac{n\pi}{2})$ not because C_1 or C_2

so this can not be decided

3. a) False, For example: $f_1(n) = 3n^2 + n \in \Theta(n^2)$
$f_2(n) = n^2 + 2n \in \Theta(n^2)$
$f_1(n) - f_2(n) = 2n^2 - n \in O(n^2) \neq O(1)$

b) False, For example $f_1(n) = n^2 \in O(g_1(n)) = O(n^3)$
$f_2(n) = n^3 \in O(g_2(n)) = O(n^4)$
$f_1(n) f_2(n) = n^5 < n^3 \cdot n^4$, not greater

c) True
$f_1(n) \in \Theta(g(n))$ so there exist $c_1, c_2 > 0$ and $n_0 > 0$
$f_2(n) \in \Theta(g(n))$ so there exist $c_3, c_4 > 0$ and $n_1 > 0$
Because both are $\Theta(g(n))$ so let c_5 be larger of c_1, c_3
c_6 be larger of c_2 c_4 and n_2 be larger of n_0, n_1.
I got:

$$1 = \frac{c_5\, g(n)}{c_5 (g(n))} \leq \frac{f_1(n)}{f_2(n)} \leq \frac{c_6 \cdot g(n)}{c_6 \cdot g(n)} = 1$$

$\therefore \frac{f_1(n)}{f_2(n)} \in \Theta(1)$

d) False, For example: $f(n) = n^{\frac{1}{3}}$
$g(n) = \frac{1}{n^{\frac{1}{6}}}$
$\log n^{\frac{1}{3}} \in O(\log \frac{1}{n^{\frac{1}{6}}})$ but $n^{\frac{1}{3}} \notin \frac{1}{n^{\frac{1}{6}}}$

$O(\frac{1}{n^{\frac{1}{6}}})$

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number of time: $n+(2n-1)+(3n-2)+(4n-3)+\cdots+[n^2-(n-1)]$ ①

$$\sum_{i=j=1}^{n}(nj-(j-1))=\sum_{i=j=1}^{n} nj-j+1=\frac{n^3+n}{2}=①$$

Because the bigest power is 3

so $\Theta(n^3)$

5、 for $i=1$ to n
  for $j=1$ to $\log i$ } $\log 1+\log 2+\log 3+\cdots+\log i$

$=\log(1\times 2\times 3\times\cdots n)$

By Miscellaneous Formulae

$\log(1\times 2\times 3\times n)=\log n!=\Theta(n\log n)$

$\underbrace{\text{for } i=11n+1 \text{ to } 33n}$

$33n-[11n+1]+1=22n=\Theta(n)$

By maximum rule

$\Theta(\max\{n\log n, n\})=\Theta(n)$

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