CP 312, Fall 2018
Assignment 1 (5% of the final grade)
(due Friday, September 28, at 11:50 pm)

There are five questions on this assignment. Note that all logarithms in this assignment are base 2, i.e. $\log f(n) \equiv \log_2 f(n)$.

1. [2 marks] Using the definition of Θ-notation, prove that $(13n+3)(9n+1)(\log(4n^2+100)) \in \Theta(n^2 \log n)$.

2. [10 marks] For each of the following pairs of functions $f(n)$ and $g(n)$, determine the **most appropriate symbol** in the set $\{O, o, \Theta, \Omega, \omega\}$ to complete the statement $f(n) \in$ __ $(g(n))$ (if one of the symbols applies at all).
   Remark: Although $n \in O(2^n)$ is technically correct, $n \in o(2^n)$ is more appropriate as it gives more information.
   Justify your answer.

   You may use the following information, where $f(n) \ll g(n)$ is shorthand for $f(n) \in o(g(n)$:

   $$1 \ll \log\log n \ll \log n \ll \log^2 n \ll \sqrt{n} \ll n \ll n \log n \ll n^2 \ll 2^n \ll n!$$

   Furthermore, for all positive real a and b, $n^a \in o(n^b)$ if $a < b$, $\log^a n \in o(n^b)$, and $n^a \in o(2^n)$.

   (a) $f(n) = 1052n^3 + 10n^2 + 10001, \quad g(n) = \frac{2}{10000000}n^4 + 2n;$
   (b) $f(n) = \log^3(n^{10}), \quad g(n) = \sqrt{\sqrt{n}};$
   (c) $f(n) = n^4 \log n, \quad g(n) = 2n^4 \log n^{2004};$
   (d) $f(n) = 16^{\log\sqrt{n}}, \quad g(n) = n^2$;
   (e) $f(n) = n^{2+\sin\frac{n\pi}{2}}, \quad g(n) = n^{5/3}.$

3. [10 marks] Consider each of the following statements, assuming that all functions are non-negative:

   a) if $f_1(n) \in \Theta(g(n))$ and $f_2(n) \in \Theta(g(n))$, then $f_1(n) - f_2(n) \in O(1)$;
   b) if $f_1(n) \in O(g_1(n))$ and $f_2(n) \in O(g_2(n))$, then $f_1(n)f_2(n) \in \Theta(g_1(n)g_2(n))$;
   c) if $f_1(n) \in \Theta(g(n))$ and $f_2(n) \in \Theta(g(n))$, then $f_1(n)/f_2(n) \in \Theta(1)$;
   d) if $\log(f(n)) \in O(\log(g(n)))$ then $f(n) \in O(g(n))$.

   For each statement: if the statement is true then provide a proof that starts with the formal definition of the order notation utilized in the statement. If the statement is false then provide a counter example and demonstrate why the statement is false.

4. [4 marks] Analyze the following pseudocode and give a tight (Θ) bound on the running time as a function of n. You can assume that all individual instructions are elementary. Show your work.

```
l := 0;
for i = 1 to n do
  for j = i to n·i do
     l := l + 2*i + 3*j
  od
od.
```

5. [4 marks] Analyze the following pseudocode and give a tight (Θ) bound on the running time as a function of n. You can assume that all individual instructions are elementary. Show your work.

```
k := n; s := 0;
for i = 1 to k do
   for j = 1 to ⌈log i⌉ do
       s := s + i*j
   od
od
for i = 11k + 1 to 33k do
   s := s + 1
od
```

Submission. Submit **single PDF file!!!** with your answers to A1 drop-box on `MyLearningSpace`. To typeset your submission and to produce PDF you can use LaTeX, Microsoft Word with Equation Editor, or any other reasonable software that allows you to produce readable text with formulae.

You can also submit scan of handwritten solution converted **to PDF format!!!**. In this case readability of your solution will be subject of judgement by the assignment marker, and you can loose some marks if your solution is deemed to be unreadable.

Any submission that does not comply with above requirements will automatically receive zero grade.