CP 312, Fall 2018
Assignment 3 (7% of the final grade)
(due Friday, November 2, at 22:30)

There are four questions in this assignment. Note that all logarithms in this assignment are base 2, i.e. $\log f(n) \equiv \log_2 f(n)$.

Note! "Giving" an algorithm means: describe the algorithm briefly in words, give high-level pseudocode, justify correctness, and analyze worst case running time.

1. [10 marks] **Divide-and-conquer** One popular way to rank researchers is by their "h-index". A researcher's h-index is the maximum integer k such that the researcher has at least k papers that have been cited at least k times each. Suppose a researcher X has written n papers and paper i has been cited c_i times. Suppose you have these sorted in an array C with $c_1 \geq c_2 \geq \ldots \geq c_n$. Give a divide-and-conquer algorithm to find researcher X's h-index. Your algorithm **must** run in time $\Theta(\log n)$.

2. [10 marks] **Divide-and-conquer** The input for this problem consists of n radar stations where station i is given by its integer coordinates x_i and y_i in the plane. We say that station i can transmit to station j if station j is south-west of station i, i.e., $x_i \geq x_j$ and $y_i \geq y_j$. The *load factor* of station i is defined to be the number of stations it can transmit to (not counting itself). The goal is to compute the load factor of each station.
   Give a divide and conquer algorithm for this problem. You algorithm has to have worst case running time in $o(n^2)$ (i.e. it must be asymptotically faster than quadratic time direct algorithm).

3. [10 marks] Suppose we would like to buy n items from SuperCheapStore (SCS); all items are currently priced at 1 CAD. Unfortunately there is no delivery and we have to deliver them home. The bad news are:
   – we can fit only 1 item in our truck;
   – it takes one day to drive home and back.
   There is even worse news – SCS charges us for the storage of undelivered items and the charge for storage of item i growth exponentially as original price times factor $c_i > 1$ each day. It means that if item i is picked up d days from now, the charge will be $1 \cdot c_i^d$ CAD. In which order should we pick up our items from SCS so that total amount of charges is as small as possible.

   Develop a greedy algorithm to solve this problem assuming that $c_i \neq c_j$ for $i \neq j$. Prove that your algorithm gives an optimal solution. What is the running time of your algorithm?

4. [20 marks] **Divide-and-conquer; programming question.** Write a Java program that uses your algorithm developed for question 2 of this assignment to compute load factor of qiven radars. You program will read multiple lines from the standard input and print the results to the standard output.

Input Specification
The input consists of $n+1$ lines. The first line has n, the number of stations. The following n lines have three numbers each: the ID number of station i, and its x_i and y_i coordinates. All station IDs are different. We will not be testing whether your program detects input errors.

Output Specification
The output must have n lines, each giving a station ID number and the load factor for that station. Your output must be sorted by station ID (in increasing order).

Sample input

```
4
2 3 3
10 4 2
13 5 6
7 2 1
```

Sample output

```
2 1
7 0
10 1
13 3
```

Submit single Java file named `radar.java`.