Subtracting

$$\frac{v_B' - v_A'}{v_A - v_B} \quad \text{———} \quad 1$$

If e is known, you can use equation together with the equation of conservation of linear momentum to determine V_A' and V_B'

If $e = 0$, this implies that $V_B' = V_A'$ i.e the objects remain together after the impact, the impact is perfectly plastic. If $e = 1$, it can be shown that the total kinetic energy is the same before and after the impact

$$\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m_A (v_A')^2 + \frac{1}{2} m_B (v_B')^2$$

Sol Continue

Using the equation

$$m_A v_{Ax} + m_B v_{Bx} = m_A v_{Ax}' + m_B v_{Bx}'$$

$$(18)(0.2) = 18 v_{Ax}' + 6.6 v_{Bx}' \quad \text{———} \quad (2)$$

Since $e = 0.95$, thus

$$e = \frac{v_{Bx}' - v_{Ax}'}{v_{Ax} - v_{Bx}}$$

$$0.95 = \frac{v_{Bx}' - v_{Ax}'}{0.2 - 0} \quad \text{———} \quad (3)$$

$v_{Ax}' = 0.095 m/s$
$v_{Bx} = 0.285 m/s$

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$$V_A' = (0.095i + 0.103j - 0.082k)\ m/s$$

$$V_B = 0.285i\ m/s.$$

Suppose that A and B approach with arbitrary velocities v_A and v_B and that the forces they exert on each other during their impact are parallel to the x axis and point towards their centre of mass. No forces are exerted on them in the y ~~axis~~ z direction, so their velocities in these directions are unchanged by the impact

$$\left.\begin{aligned} (v_A')_y &= (v_A)_y \quad (v_B')_y = (v_B)_y \\ (v_A')_z &= (v_A)_z \quad (v_B')_z = (v_B)_z \end{aligned}\right\} \text{———}$$

In the x direction, linear momentum is conserved

$$m_A (v_A)_x + m_B (v_B)_x = m_A (v_A')_x + m_B (v_B')_x \text{ —}$$

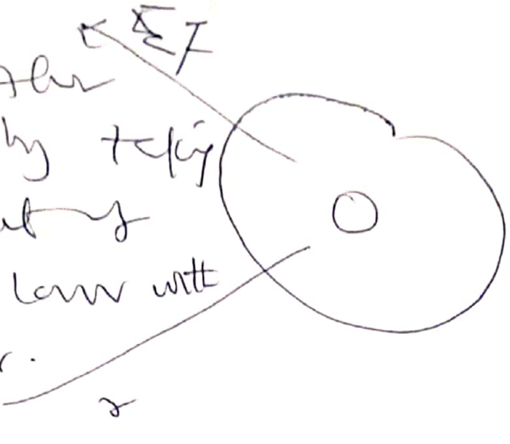
and by the same analysis we ~~use~~ to ~~assume~~ the x component of velocity satisfy the relation

$$e = \frac{(v_B')_x - (v_A')_x}{(v_A)_x - (v_B)_x} \text{ ———}$$

6.0 Angular Momentum

Given the position of an object by the position vector of its centre of mass relative to a reference point O. and recall that we obtained the useful principle of work and energy by taking the dot product of Newton's second law with the velocity. as shown below

We obtain another useful result by taking the cross product of Newton's second law with the position vector.



This procedure gives a relation between the moment of the external forces about O and the object's motion. Then

$$r \times \sum F = r \times ma = r \times m\frac{dv}{dt} \qquad (1)$$

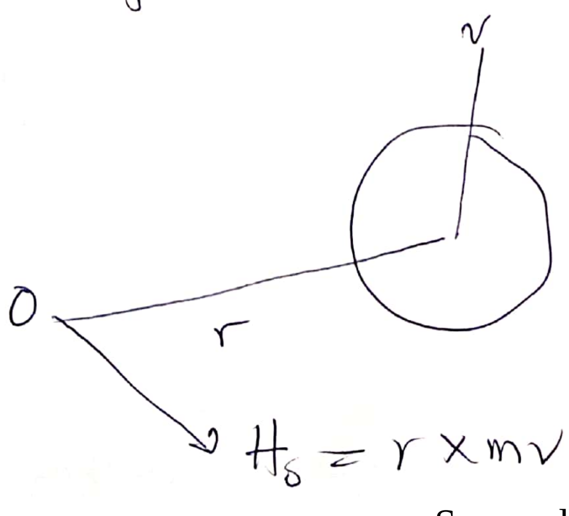
Notice that the time derivative

$$\frac{d}{dt}(r \times mv) = \frac{dr}{dt} \times mv + \left(r \times m\frac{dv}{dt}\right) \qquad (2)$$

where $\frac{dr}{dt} = v$, and the cross product of parallel vectors is zero. We can write equation (1) as

$$r \times \sum F = \frac{dH_O}{dt} \quad \text{where } H_O = r \times mv \qquad (33)$$

This is called the angular momentum about O as shown below



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Integrating (2)

$$\int_{t_1}^{t_2} (r \times \Sigma F)\, dt = (H_O)_2 - (H_O)_1 \quad \text{—— (4)}$$

The integral on the left is the called the angular impulse, and the equation is called the principle of angular impulse and momentum.

If the total force acting on an object remains directed towards a point that is fixed relative to an inertial reference frame, the object is said to be central force motion. The fixed point is called the centre of motion.

If the position vector r is parallel to the total force, so $r \times \Sigma F$ equals zero. Therefore eq (3) indicates that in central-force motion, an object's angular momentum is conserved.

$$H_O = \text{Constant} \quad \text{—— (5)}$$

In a plane central-force motion, we can express r and v in cylindrical coordinates:

$$r = r e_r \qquad v = v_r e_r + v_\theta e_\theta$$

Substituting these expression into eq (3) we obtain the angular

$$H_O = (r e_r) \times m(v_r e_r + v_\theta e_\theta) \quad \text{—— (6)}$$

$$= m r v_\theta e_z$$

From this expression, we see that in plane central-force motion, the product of radial distance from the centre of the motion and the transverse xample A disc of mass m attached to a string slides on a smooth horizontal table under the action of a constant transverse force F. The string is drawn through a hole in the table at O at constant velocity v_0. At $t=0$, $r=r_0$ and the transverse velocity of the disc is zero. What is the disc velocity as a function of time?

Component of the velocity is constant

$$r v_0 = \text{Constant} \quad \text{——— } 6$$

Example. When an earth satellite is at perigee (the point at which it is nearest to the earth) the magnitude of its velocity is $v_B = 7000$ m/s and its distance from the centre of the earth is $r_B = 10000$ km. What are the magnitude of its velocity v_A and its ~~distance~~ distance r_A from the earth at apogee (the point at which it is farthest from the earth). The radius of the earth is $R_E = 6370$ km

Sol

From the equation of conservation of angular momentum, we have

$$r_A v_A = r_B v_B$$

~~that sum of kinetic & potential energies at apogee and perigee must be equal to~~

$$\frac{1}{2} m v_A^2 -$$

equation is the satellite potential energy in terms of distance from the centre of the earth is

$$V = -\frac{mgR_E^2}{r}$$

The sum of kinetic & potential energies of apogee and perigee must be equal to

$$\frac{1}{2}mv_A^2 - \frac{mgR_E^2}{r_A} = \frac{1}{2}mv_B^2 - \frac{mgR_E^2}{r_B}.$$

Substituting $r_A = \frac{r_B v_B}{v_A}$ into this equation, we obtain

$$(v_A - v_B)\left(v_A + v_B - \frac{2gR_E^2}{r_B v_B}\right) = 0$$

$$v_A = v_A \quad \text{or} \quad v_A = \frac{2gR_E^2}{r_B v_B} - v_B$$

Substituting the value of g, R_E, r_B, v_B we obtain $v_A = 4373$ m/s $r_A = 16007$ km

Principle of Conservation of Energy

In this section, we state the principle of work and energy as a conservation law: ~~is~~ the sum of the kinetic and potential energies is constant.

From the newton's second law, we have

$$U = \int_{r_1}^{r_2} \Sigma F \cdot dr = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \quad \text{——} (1)$$

Suppose we could determine a scalar function ~~of~~ position V such that

$$dV = -\Sigma F \cdot dr \quad \text{——} (2)$$

then, we have

$$U = \int_{r_1}^{r_2} \Sigma F \cdot dr = \int_{V_1}^{V_2} -dV = V_1 - V_2 \quad \text{—} (3)$$

where V_1 and V_2 are the values of V at the positions r_1 and r_2. The principle of work and energy is of the form

$$\frac{1}{2}mv_1^2 + V_1 = \frac{1}{2}mv_2^2 + V_2 \quad \text{——} (4)$$

which means that the sum of the kinetic energy and the function V is constant

$$\frac{1}{2}mv^2 + V = \text{constant} \quad \text{——} (5)$$

Example 1 ~~A~~ A spacecraft at a distance $r_0 = 2R_E$ from the centre of the earth is moving outwards with initial velocity $v_0 = \sqrt{2gR_E/3}$. ~~D~~ Determine its velocity as a function of its distance from

Given $V = -\frac{mgR_E^2}{r}$

By applying the principle conservation of energy

Let v be the magnitude of the spacecraft velocity at an arbitrary distance r, the sum of the potential and kinetic energies at r_0 and r must be equal

$$-\frac{mgR_E^2}{r_0} + \frac{1}{2}mv_0^2 = -\frac{mgR_E^2}{r} + \frac{1}{2}mv^2$$

$$-\frac{mgR_E^2}{2R_E} + \frac{1}{2}m\left(\frac{2}{3}gR_E\right) = -\frac{mgR_E^2}{r} + \frac{1}{2}mv^2$$

Solving for v, as a function of r

$$v = \sqrt{gR_E\left(\frac{2R_E}{r} - \frac{1}{3}\right)}.$$