Problem Set 7

ASSIGNED: February 24, 2023
DUE: **Friday, March 3, 2023** at the beginning of class. No extensions to the deadline will be granted.

Problem set guidelines:

1. Each solution must be your own work.
2. All problems that involve python must be completed in Jupyter Notebooks**.** Some problems may not require python and may be better completed with pen and paper.
3. Highlight your final answer when providing numerical results. Provide plots, graphs, and tables of your results when appropriate.
4. **Submission Instructions:** This quarter we will use Gradescope to collect your submissions and grade them. We will only accept a single PDF file with your compiled solutions. Please follow the instructions below to obtain a PDF file from a Jupyter Notebook:
   i. Make sure all your code runs without error, and all figures (if any) show up where intended. We will not be running your code, therefore it is essential that your solutions output and highlight your results. Please be mindful of your line length so that it fits into the PDF layout and your results are clearly shown.
   ii. Select `File->Download as->HTML (.html)`. This will download an HTML version `your_homework.html` of your notebook to your computer, typically in your `Download` folder.
   iii. Open `your_homework.html` in your web browser (just double-click on it). ***Use the `File->Print` command of your browser to produce a PDF file.*** *(Do not produce the PDF in other ways, such as "Export to PDF" or other. Alternative methods will usually produce poor page margins, format headers differently, fail to incorporate figures, and so forth.)*
   iv. **Submit your PDF file** `your_homework.pdf` to Gradescope. Do *not* submit your HTML file.
5. Problem sets are due at the beginning of the class period on the due date listed. Late problem sets will not be accepted.

Note that you can wrap lines of python code using the "\" operator to ensure that all your code is visible within the width of the page. See the following examples:

```
In [1]:
1 print("demonstra\
2 tion")

demonstration
```

```
In [2]:
1 def factorial(n):
2     if n==1:
3         return(1)
4     return(n*\
5         factorial(n-1))
6
7 factorial(4)

Out[2]: 24
```

```
In [3]:
1 factorial\
2 (4)

Out[3]: 24
```

Problem 1 – Ignoring safety considerations…

Adapted from Chapra and Canale, Problem 13.22.

Use the golden-section search method to determine the length of the shortest ladder that reaches from the ground over the fence to touch the building's wall (see figure at right).

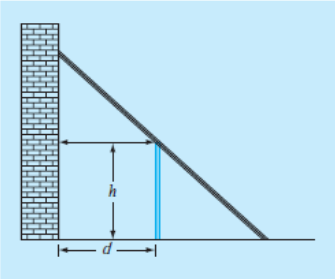

FIGURE P13.22
A ladder leaning against a fence and just touching a wall.

(a) Write your own code for the golden-section search method. Demonstrate your code to determine the shortest ladder possible for the case where $h = d = 6\ m$.

(b) Demonstrate the application of the `scipy.optimize.golden` function in python to find the minimum.

(c) Note that the `golden` function is a 'legacy' function still available in Python, but is no longer recommended for use. Instead, please demonstrate the application of the `scipy.optimize.minimize_scalar` function.

(d) Prepare a plot of the length of the shortest ladder versus d for the case where $h = 6\ m$. Consider distances between the fence and wall up to $20\ m$ (*i.e.*, $0 \leq d \leq 20\ m$).

Problem 2 – Steepest descent

Adapted from Chapra and Canale, Problem 16.15.

A finite-element model of a cantilever beam subject to loading and moments is given by optimizing

$$f(x, y) = 5x^2 - 5xy + 2.5y^2 - x - 1.5y$$

Figure P16.15
A cantilever beam.

where x = end displacement and y = end moment. Find the values of x and y that minimize $f(x, y)$.

(a) Starting with an initial guess of $x = y = 1$, compute by hand (or in a markdown box of Jupyter notebooks) the first steps taken using 1) the steepest descent optimization method and 2) the optimal steepest descent optimization method.

(b) Now implement in python your own versions of the 1) steepest descent and 2) optimal steepest descent optimization methods. Apply to find the minimize of $f(x, y)$.

(c) Construct a plot comparing the path taken by each method in their search for the minimum position.

Problem 3 – The cost of water

Adapted from Chapra and Canale, Problem 16.21.

The Splash River has a flow rate of 2×10^6 m^3/d, of which up to 70% can be diverted into two channels where it flows through Splish County. These channels are used for transportation, irrigation, and electric power generation, with the latter two being sources of revenue. The transportation use requires a minimum diverted flow rate of 0.3×10^6 m^3/d for Channel 1 and 0.2×10^6 m^3/d for Channel 2. For political reasons it has been decided that the absolute

difference between the flow rates in the two channels cannot exceed 40% of the total flow diverted into the channels.

The Splish County Water Management Board has also limited maintenance costs for the channel system to be no more than $\$1.8 \times 10^6$ per year. Annual maintenance costs are estimated based on the daily flow rate. Channel 1 costs per year are estimated by multiplying \$1.1 times the m^3/d of flow; while for Channel 2 the multiplication factor is \$1.4 per m^3/d. Electric power production revenue is also estimated based on daily flow rate. For Channel 1 this is \$4.0 per m^3/d, while for Channel 2 it is \$3.0 per m^3/d. Annual revenue from irrigation is also estimated based on daily flow rate, but the flow rates must first be corrected for water loss in the channels previous to delivery for irrigation. This loss is 30% in Channel 1 and 20% in Channel 2. In both channels the revenue is \$3.2 per m^3/d.

(a) Set up a linear programming problem that specifies the flows in the channels that maximize profit.

(b) Solve the linear programming problem with the Simplex method (i.e., by hand, using pen and paper and calculator, or equivalent; do not code).

(c) Solve the problem using the `linprog` function in python.

(d) If the Splish County Water Management Board wanted to increase the profit further, what would you advise them to do? Justify your response.

(e) Consider the challenges being faced by the southwestern states and US government as they decide how to manage the rapidly diminishing water resources provided by the Colorado River. Reviewing the recent news, create a list of some of the key parameters and resources that are highlighted as key to the decision-making process and part of the optimization problem. Good starting points for learning more about this issue are:
https://www.washingtonpost.com/climate-environment/2023/02/05/colorado-river-drought-explained/
https://www.latimes.com/environment/story/colorado-river-in-crisis
https://www.watereducationcolorado.org/

Problem 4 – A basic approach to global optimization

Please see attached Jupyter notebook template.