meng 21200 pset 6

init

In [1]:
```
import numpy as np
from time import time
import matplotlib.pyplot as plt
import warnings
from scipy.stats import linregress

warnings.filterwarnings('ignore')
```

problem 1

a

In [15]:
```
def generate(n):
    '''
    :param n: size
    :return: [matrix, vector]
    '''

    return [np.random.uniform(-10, 10, size=(n, n)), np.random.uniform(-10,
```

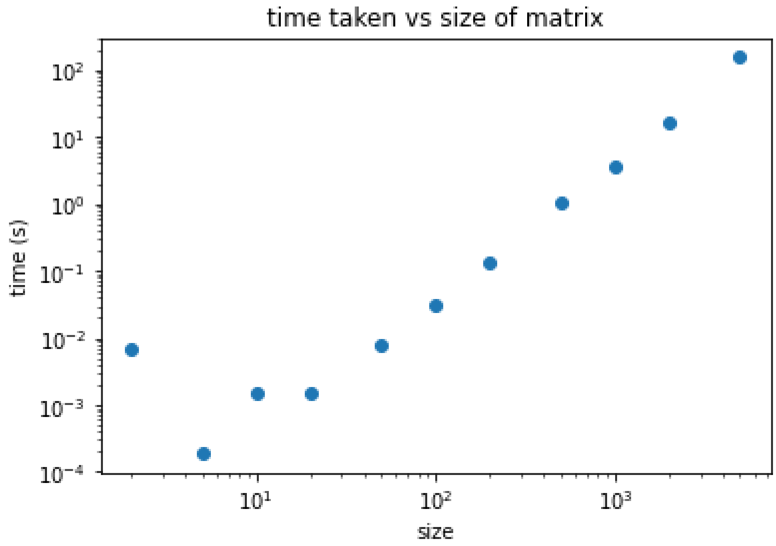
b

In [16]:
```
sizes = [2, 5, 10, 20, 50, 100, 200, 500, 1000, 2000, 5000]
times = []
for n in sizes:
    [A, b] = generate(n)
    x = np.zeros(n)
    startTime = time()

    # Gaussian elimination
    for i in range(n - 1):
        for j in range(i + 1, n):
            iRow = A[i]
            jRow = A[j]

            factor = iRow[i] / jRow[i]
            row = jRow - factor * iRow
            b[j] -= factor * b[i]
            A[j] = row

    # substitution
    for i in range(n - 1, -1, -1):
        x[i] = b[i]
        for j in range(i + 1, n):
            x[i] -= x[j] * A[i, j]
        x[i] /= A[i, i]

    finishTime = time()
    times.append(finishTime - startTime)
```

In [17]:
```
plt.scatter(sizes, times)
plt.xscale('log')
```

```
plt.yscale('log')
plt.xlabel('size')
plt.ylabel('time (s)')
plt.title('time taken vs size of matrix')
```

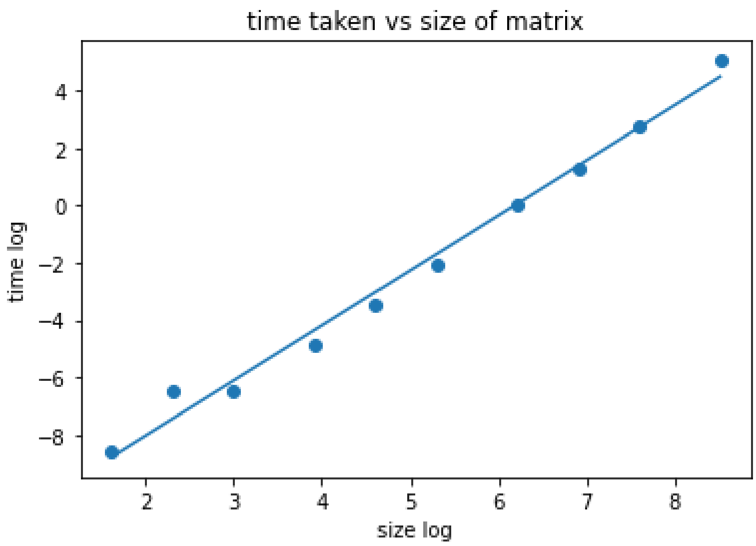
Out[17]: Text(0.5, 1.0, 'time taken vs size of matrix')



c

In [22]:
```
sizesLog = np.log(sizes[1:])
timesLog = np.log(times[1:])
m, b, r, _, _ = linregress(sizesLog, timesLog)

xlist = np.linspace(sizesLog[0], sizesLog[-1])
plt.scatter(sizesLog, timesLog)

plt.xlabel('size log')
plt.ylabel('time log')
plt.title('time taken vs size of matrix')
plt.plot(xlist, xlist * m + b)
print(f'slope: {round(m, 4)}')
print(f'r-value: {round(r, 4)}')
```

```
slope: 1.9176
r-value: 0.9945
```



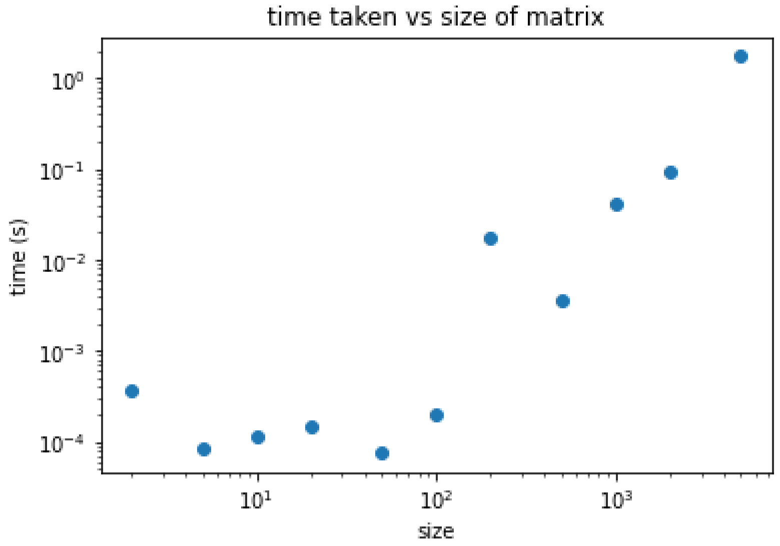
The complexity of Gaussian elimination is $O(N^3)$. However, the slope of the graph is 1.98, with an r-value of 0.9985, which implies a complexity of $O(N^2)$. These do not match.

d

In [41]:
```
times_ = []
for n in sizes:
    [A, b] = generate(n)
    startTime = time()
    x = np.linalg.solve(A, b)
    finishTime = time()
    times_.append(finishTime - startTime)
```

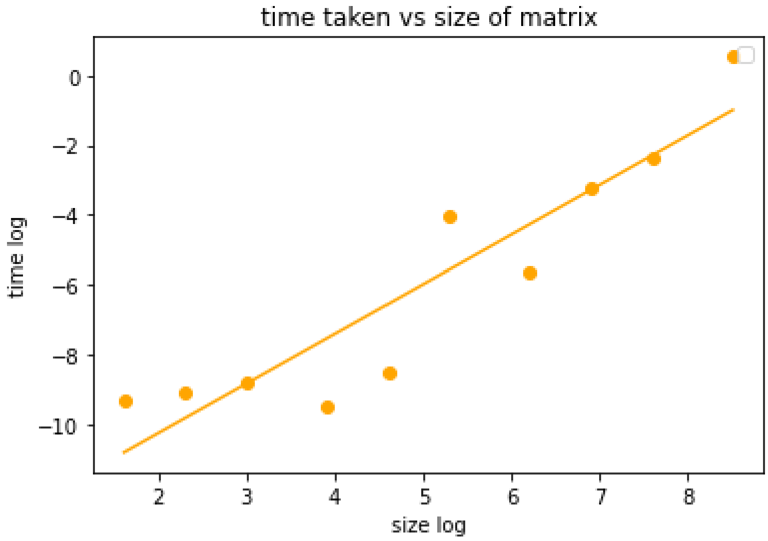
In [42]:
```
plt.scatter(sizes, times_, label='library code')

plt.xscale('log')
plt.yscale('log')
plt.xlabel('size')
plt.ylabel('time (s)')
plt.title('time taken vs size of matrix')
```

Out[42]: `Text(0.5, 1.0, 'time taken vs size of matrix')`



In [45]:
```
plt.legend()
sizesLog = np.log(sizes[1:])
timesLog_ = np.log(times_[1:])
m_, b_, r_, _, _ = linregress(sizesLog, timesLog_)

xlist = np.linspace(sizesLog[0], sizesLog[-1])
plt.scatter(sizesLog, timesLog_, color='orange')

plt.xlabel('size log')
plt.ylabel('time log')
plt.title('time taken vs size of matrix')
plt.plot(xlist, xlist * m_ + b_, color='orange')
print(f'slope: {round(m, 4)}')
print(f'r-value: {round(r, 4)}')
```

```
No artists with labels found to put in legend.  Note that artists whose labe
l start with an underscore are ignored when legend() is called with no argum
ent.
slope: 1.9176
r-value: 0.9945
```



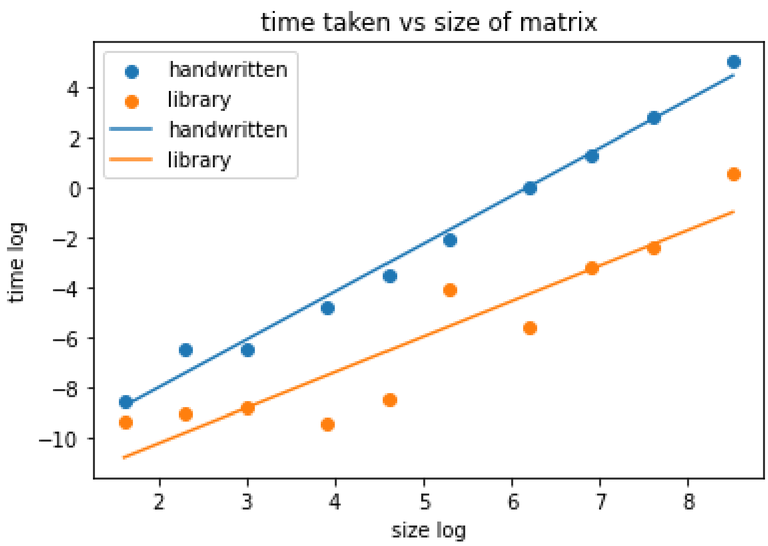
In [47]:

```
sizesLog = np.log(sizes[1:])
timesLog = np.log(times[1:])
timesLog_ = np.log(times_[1:])
xlist = np.linspace(sizesLog[0], sizesLog[-1])
plt.xlabel('size log')
plt.ylabel('time log')
plt.title('time taken vs size of matrix')

plt.scatter(sizesLog, timesLog, label='handwritten')
plt.scatter(sizesLog, timesLog_, label='library')

m, b, r, _, _ = linregress(sizesLog, timesLog)
plt.plot(xlist, xlist * m + b, label='handwritten')
print(f'handwritten slope: {round(m, 4)}')
print(f'handwritten r-value: {round(r, 4)}')

m_, b_, r_, _, _ = linregress(sizesLog, timesLog_)
plt.plot(xlist, xlist * m_ + b_, label='library')
print(f'library slope: {round(m_, 4)}')
print(f'library r-value: {round(r_, 4)}')

plt.legend()
```

```
handwritten slope: 1.9176
handwritten r-value: 0.9945
library slope: 1.4228
library r-value: 0.9235
```

Out[47]:

```
<matplotlib.legend.Legend at 0x7f79ba3b35e0>
```



The complexity of Gaussian elimination is $O(N^3)$. However, the slope of the graph is 1.42, with an r-value of 0.9235, which implies a complexity of $O(N^1.5)$. These do not match.

problem 2

a

In [58]:
```
def LU(matrix):
    l = len(matrix)
    L = np.identity(l)
    U = np.zeros((l, l))

    for j in range(l):
        U[0][j] = matrix[0][j]

    for i in range(1, l):
        L[i][0] = matrix[i][0] / U[0][0]

    for i in range(1, l):
        for j in range(i, l):
            s = 0
            for k in range(i):
                s += L[i][k] * U[k][j]
            U[i][j] = matrix[i][j] - s

        for j in range(i + 1, l):
            s = 0
            for k in range(i):
                s += L[j][k] * U[k][i]
            L[j][i] = (matrix[j][i] - s) / U[i][i]

    return L, U
```

b

In [68]:
```
def invert(L, U):
    l = len(L)
    matrix = np.identity(l)

    for k in range(l):
        x = np.zeros(l)
        y = np.zeros(l)
        y[0] = matrix[0][k] / L[0][0]

        for i in range(1, l):
            s = 0
            for j in range(i):
                s += L[i][j] * y[j]
            y[i] = (matrix[i][k] - s) / L[i][i]

        x[l - 1] = y[l - 1] / U[l - 1][l - 1]

        for i in range(l - 2, -1, -1):
            s = 0
            for j in range(i + 1, l):
                s += U[i][j] * x[j]
            x[i] = (y[i] - s) / U[i][i]

        matrix[:, k] = x
```

```
    return matrix
```

c

In [70]:
```
A = np.array([[6, 1, -5, 3], [2, 4, 5, 6], [2, 3, 7, 1], [1, 2, 0, -2]])
L, U = LU(A)
inv = invert(L, U)


print('L:')
print(L, '\n')
print('U:')
print(U, '\n')
print('Inverse: ')
print(inv, '\n')
```

```
L:
[[ 1.          0.          0.          0.        ]
 [ 0.33333333  1.          0.          0.        ]
 [ 0.33333333  0.72727273  1.          0.        ]
 [ 0.16666667  0.5        -0.6547619   1.        ]]

U:
[[ 6.          1.         -5.          3.        ]
 [ 0.          3.66666667  6.66666667  5.        ]
 [ 0.          0.          3.81818182 -3.63636364]
 [ 0.          0.          0.         -7.38095238]]

Test Inverse:
[[ 0.16129032 -0.15483871  0.22580645 -0.10967742]
 [-0.07258065  0.20967742 -0.2016129   0.41935484]
 [-0.01612903 -0.06451613  0.17741935 -0.12903226]
 [ 0.00806452  0.13225806 -0.08870968 -0.13548387]]
```

d

In [73]:
```
print('linalg.inv:')
print(np.linalg.inv(A))
```

```
linalg.inv
[[ 0.16129032 -0.15483871  0.22580645 -0.10967742]
 [-0.07258065  0.20967742 -0.2016129   0.41935484]
 [-0.01612903 -0.06451613  0.17741935 -0.12903226]
 [ 0.00806452  0.13225806 -0.08870968 -0.13548387]]
```

Function produces same result as library.

e

In [75]:
```
def det(L, U):
    detL = np.prod(np.diag(L))
    detU = np.prod(np.diag(U))
    return detL * detU
```

In [76]:
```
L, U = LU(A)
print(f'det(A) = {round(det(L, U), 1)}')
```

```
det(A) = -620.0
```

problem 3

a

In [78]:
```
A = np.array([[1, 2 , 3, -5],
           [2, 5, 4, -1],
           [1, -1, 10, 2],
           [3, -2, 5., -3]])
b = np.array([-44, 8., 44, -16])

print(np.linalg.solve(A, b))
```

```
[ 3.  1.  2. 11.]
```

b

In [116...
```
def guess(a0, b0):
    l = len(a0)
    a = []
    b = []

    for i in range(l):
        r = (0, -80)
        for j in range(l):
            x = abs(a0[j][i])
            if x > r[0]:
                r = (x, j)
        a.append(a0[r[1]])
        b.append(b0[r[1]])
    return a, b
```

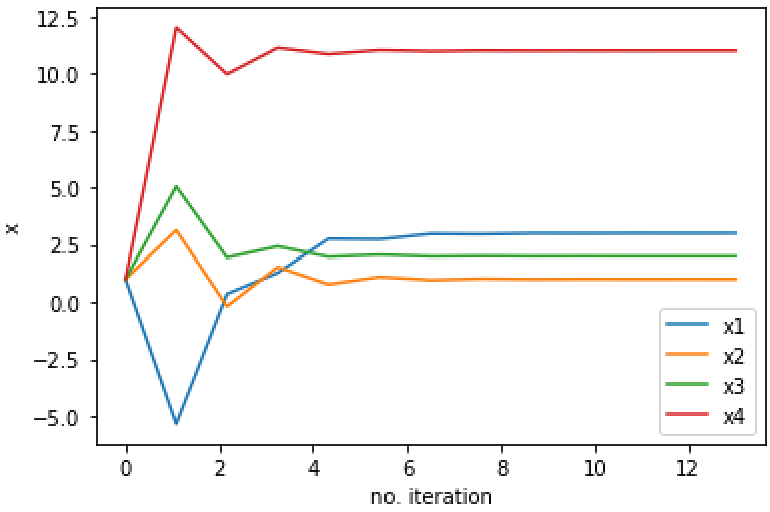
In [125...
```
def f(a0, b0, omega=1):
    tol = 0.001
    a, b = guess(a0, b0)
    x = [np.ones(len(b))]
    maxError = 1
    while maxError > tol:
        maxError = 0
        c = []
        for i in range(len(a)):
            sum = 0
            d = b[i]
            for j in range(len(c)):
                sum += (a[i][j] * c[j])
            for j in range(len(c), len(a[i])):
                sum += (a[i][j] * x[-1][j])

            c.append(x[-1][i] + omega * (d - sum) / (a[i][i]))
            error = abs((c[i] - x[-1][i]) / x[-1][i])
            if error > maxError:
                maxError = error
        x.append(c)
    return x
```

In [128...
```
soln = f(A, b)
nlist = np.linspace(0, len(soln), len(soln))
```

In [129...
```
x1 = [i[0] for i in soln]
x2 = [i[1] for i in soln]
x3 = [i[2] for i in soln]
x4 = [i[3] for i in soln]

plt.plot(nlist, x1,label = 'x1')
```

```
plt.plot(nlist, x2, label = 'x2')
plt.plot(nlist, x3,label = 'x3')
plt.plot(nlist, x4,label = 'x4')
plt.xlabel('no. iteration')
plt.ylabel('x')
plt.legend()
```

Out[129]: <matplotlib.legend.Legend at 0x7f79480f4df0>

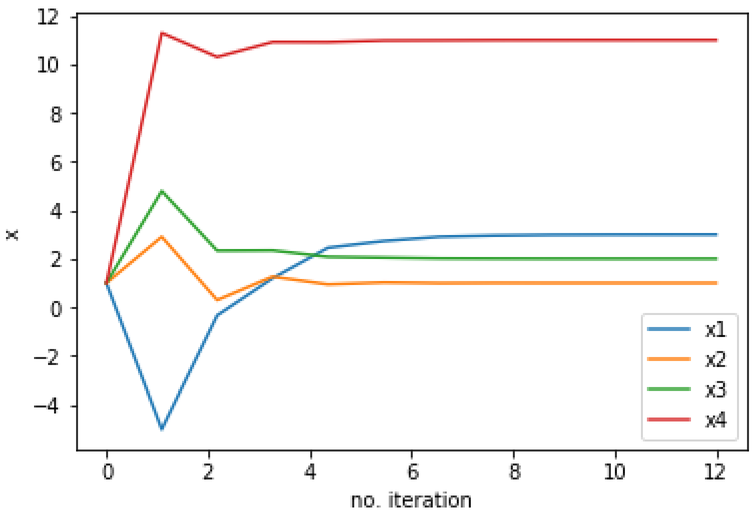


c

In [130…
```
soln = f(A, b, omega=0.95)
nlist = np.linspace(0, len(soln), len(soln))
```

In [131…
```
x1 = [i[0] for i in soln]
x2 = [i[1] for i in soln]
x3 = [i[2] for i in soln]
x4 = [i[3] for i in soln]

plt.plot(nlist, x1,label = 'x1')
plt.plot(nlist, x2, label = 'x2')
plt.plot(nlist, x3,label = 'x3')
plt.plot(nlist, x4,label = 'x4')
plt.xlabel('no. iteration')
plt.ylabel('x')
plt.legend()
```

Out[131]: <matplotlib.legend.Legend at 0x7f79582836d0>



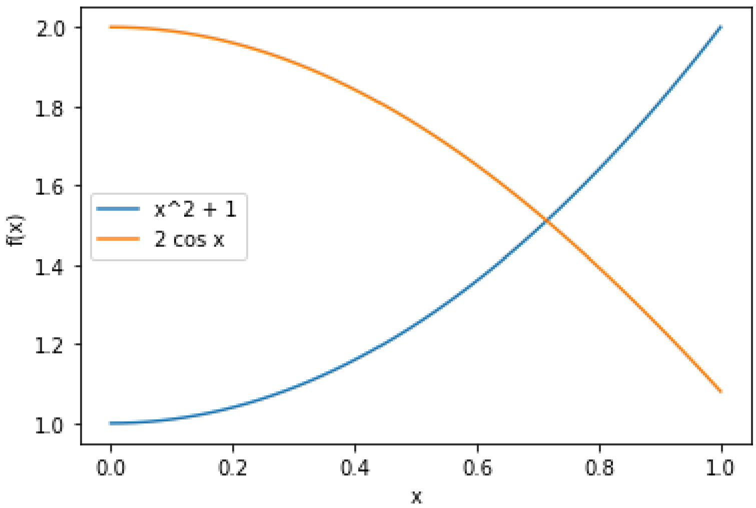
problem 4

a

In [101…

```
def f1(x):
    return x ** 2 + 1
def f2(x):
    return 2 * np.cos(x)

xlist = np.linspace(0, 1)

plt.plot(xlist, f1(xlist),label='x^2 + 1')
plt.plot(xlist, f2(xlist),label='2 cos x')
plt.xlabel('x')
plt.ylabel('f(x)')
plt.legend()
```

Out[101]: <matplotlib.legend.Legend at 0x7f79a918fdf0>



b

In [102…

```
# guesses
x = 0.7
y = 1.5

while abs(f1(x) - f2(x)) > 0.001:
    x = np.sqrt(y - 1)
    y = np.sqrt(2 * np.cos(x) + y ** 2 - y)

print(f'the solution is {round(x, 4)}, {round(y, 4)}')
```

```
the solution is 0.7143, 1.5105
```

c

In [98]:

```
def jacobian(x1, x2):
    return np.array([[2 * x1, -1],
                     [-2 * np.sin(x1), -1]])
def diff(x1, x2):
    a = f(x1, x2)
    return abs(a[0]) + abs(a[1])
def f(x1, x2):
    return np.array([x1 ** 2 + 1 - x2,
                     2 * np.cos(x1) - x2])
```

In [99]:
```
x1, x2 = [0.7, 1.5]
tol = 0.001
while diff(x1, x2) > tol:
    x1, x2 = np.array([x1, x2]) - np.matmul(np.linalg.inv(jacobian(x1, x2)),

print(f'the solution is {round(x1, 4)}, {round(x2, 4)}')
```

```
the solution is 0.7148, 1.5107
```

problem 5

a

In [96]:
```
def jacobian(x1, x2, x3):
    return np.array([[3 * x2, 1, -1],
                     [1 - 2 * x1 * x2, -x1 ** 2, 1],
                     [1, -1, -1]])
def diff(x1, x2, x3):
    a = f(x1, x2, x3)
    return abs(a[0]) + abs(a[1]) + abs(a[2])
def f(x1, x2, x3):
    return np.array([3 * x1 * x2 + x2 - x3 - 12,
                     x1 - x1 ** 2 * x2 + x3 - 12,
                     x1 - x2 - x3 + 2])
```

In [97]:
```
x1, x2, x3 = [0, 0, 0]
tol = 0.0001
while diff(x1, x2, x3) > tol:
    x1, x2, x3 = np.array([x1, x2, x3]) - np.matmul(np.linalg.inv(jacobian(x

print(f'the solution is {round(x1, 4)}, {round(x2, 4)}, {round(x3, 4)}')
```

```
the solution is -2.6535, -1.9036, 1.2502
```

b

Need to set the initial guess very close to the answer for the method to converge

In [10]:
```
beta = 0.0001
x1, x2, x3 = [-2.65, -1.9, 1.25]
tol = 0.01
while diff(x1, x2, x3) > tol:
    x1, x2, x3 = np.array([x1, x2, x3]) + beta * f(x1, x2, x3)

print(f'the solution is {round(x1, 4)}, {round(x2, 4)}, {round(x3, 4)}')
```

```
the solution is -2.6528, -1.9037, 1.2501
```