

# Final Project Report

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## Abstract

This report is about a re-implementation of the paper "Automatic Extraction of Generic Focal Features on 3D Shapes via Random Forest Regression Analysis of Geodesics-in-Heat" [1]. The paper attempts to extract focal features defined by user instead of geometric features. Based on the assumption that the distance between the distribution of focal features and the distribution of geometric features is consistent, a Random Forest Regression method is used to learn the mapping from geometric feature descriptors to this distance. Then the distance can be predicted and applied to localize focal features on new 3D shapes with the generalization ability of the learning method.

*Keywords:* feature extraction, 3D shapes, Computer Graphics

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## 1. Overview

In this section, the idea of the paper will be explained as well as the roles of different steps of the method play in this idea.

### 1.1. General Idea

Let the distribution of geometric features on a category of 3D shapes be  $P_g$  and the distribution of focal features be  $P_f$ . Assume the distance  $d(P_g, P_f)$  be consistent. The general idea is to reconstruct  $P_f$  from  $P_g$  and  $d(P_g, P_f)$ .

In this re-implemented paper, let  $P_g^u = \delta_u$  and  $P_f^v = \delta_v$ , the descriptor  $\phi$  be SIHKS(scale invariant heat kernel signature) which maps vertices  $w \in R^3$  to vector feature space  $F \in R^n$  and the distance  $d(P_g^u, P_f^v)$  be HGD( $u, v$ )(heat geodesic distance). Then the RFR(random forest regression) will learn the regression  $f$  from  $\phi(u)$  to HGD( $u, v$ ). On new 3D shapes,  $P_f^v = \delta_v$  can be reconstructed by localization using predicted HGD with  $f$ ,  $\phi$  and samples of  $P_g^u = \delta_u$ .

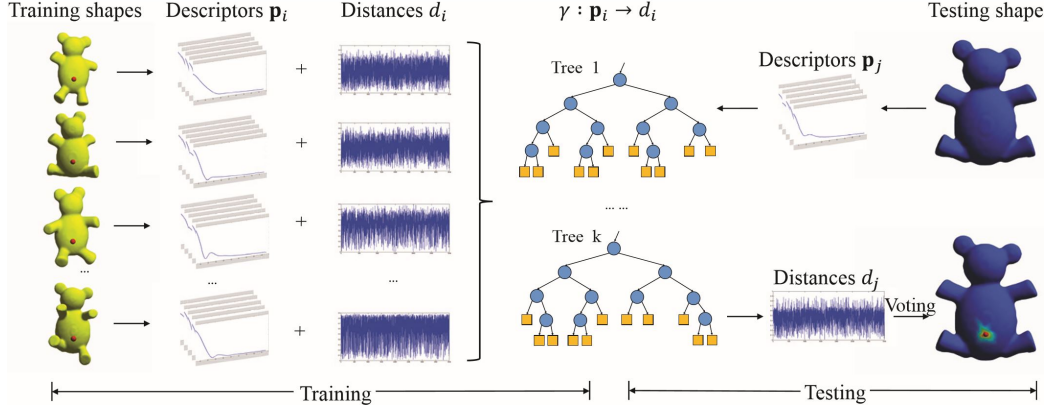


Figure 1: Pipeline of the method (picture captured from the paper).

### 1.2. Method Overview

The content in this section is not much different from the paper.

For the training step:

1. Random Samples Description: Randomly sample points on training shapes, compute their shape descriptors  $\mathbf{p}_i$ .
2. Geodesic Distance Metric: For each sampled point, compute its distance to focal feature points  $d_i$ .
3. Random Forest Construction: Build the Random Forest with these descriptors and distances which learns the regression function from descriptors to distances.

For the testing step:

1. Random Samples Description: Randomly sample points on testing shapes and compute their shape descriptors .
2. Geodesic Distance Prediction: Using the built random forest to predict the distances corresponding to these testing shape descriptors.
3. Feature Point Localization: With predicted distances from testing samples to focal feature points, estimate the focal feature distribution via voting strategy.

## 2. Implementation

Details of implementation are discussed in this section. Based on the overview of the method, there are three essential parts of the implementation: SIHKS descriptors, Heat Geodesic Distances and Random Forest Regression.

### 2.1. SIHKS

Heat kernel  $K_t(x, z)$  is the solution  $u(x, t)$  of heat diffusion equation 1 with initial condition  $u_0(x) = u(x, 0) = \delta(x - z)$ .

$$(\Delta + \frac{\partial}{\partial t})u = 0. \quad (1)$$

On compact manifolds, based on eigenvalue decomposition of Laplace-Beltrami operator, the heat kernel can be represented as

$$K_t(x, z) = \sum_k e^{-\lambda_k t} \phi_k(x) \phi_k(z). \quad (2)$$

Heat kernel signature  $h(x, t)$  is  $K_t(x, x)$ . As local shape descriptors, the HKS is intrinsic and multi-scale based on  $t$ . The HKS is widely used in feature extraction, however, the HKS is not scale-invariant. When a scale factor  $\beta$  is applied,

$$h'(x, t) = \beta^2 h(x, \beta^2 t). \quad (3)$$

From [2], scale invariance can be achieved by several transformations. First let  $t = \alpha^\tau$  and  $h_\tau = h(x, \alpha^\tau)$ ; second, let  $s = 2 \log_\alpha \beta$ , equation 3 converts to  $h'_\tau = \beta^2 h_{\tau+s}$ ; third, let  $d(h'_\tau) = \log h_{\tau+1} - \log h_\tau$  then  $d(h'_\tau) = d(h_{\tau+s})$ ; Finally, with discrete-time Fourier transform on  $d(h')$  and  $d(h)$  we have

$$H'(w) = H(w) e^{2\pi w s}, |H'(w)| = |H(w)|. \quad (4)$$

In implementation, the time  $t$  is sampled logarithmically with base 2, whose exponent ranges from 1 to 20 with increments of 0.2. The first 20 lowest discrete frequencies are used. Laplace-Beltrami operator is discretized with cotangent weight matrix. During the eigen-decomposition, 10 eigenfunctions are used for the efficiency also because the eigenvalues become extremely large compared to first 10.

### 2.2. HGD

Heat geodesic distance introduced in [3] is a kind of diffusion distance like biharmonic distance. But it can overcome computational issues since it doesn't need eigen-decomposition.

$$X = -\nabla u / |\nabla u|. \quad (5)$$

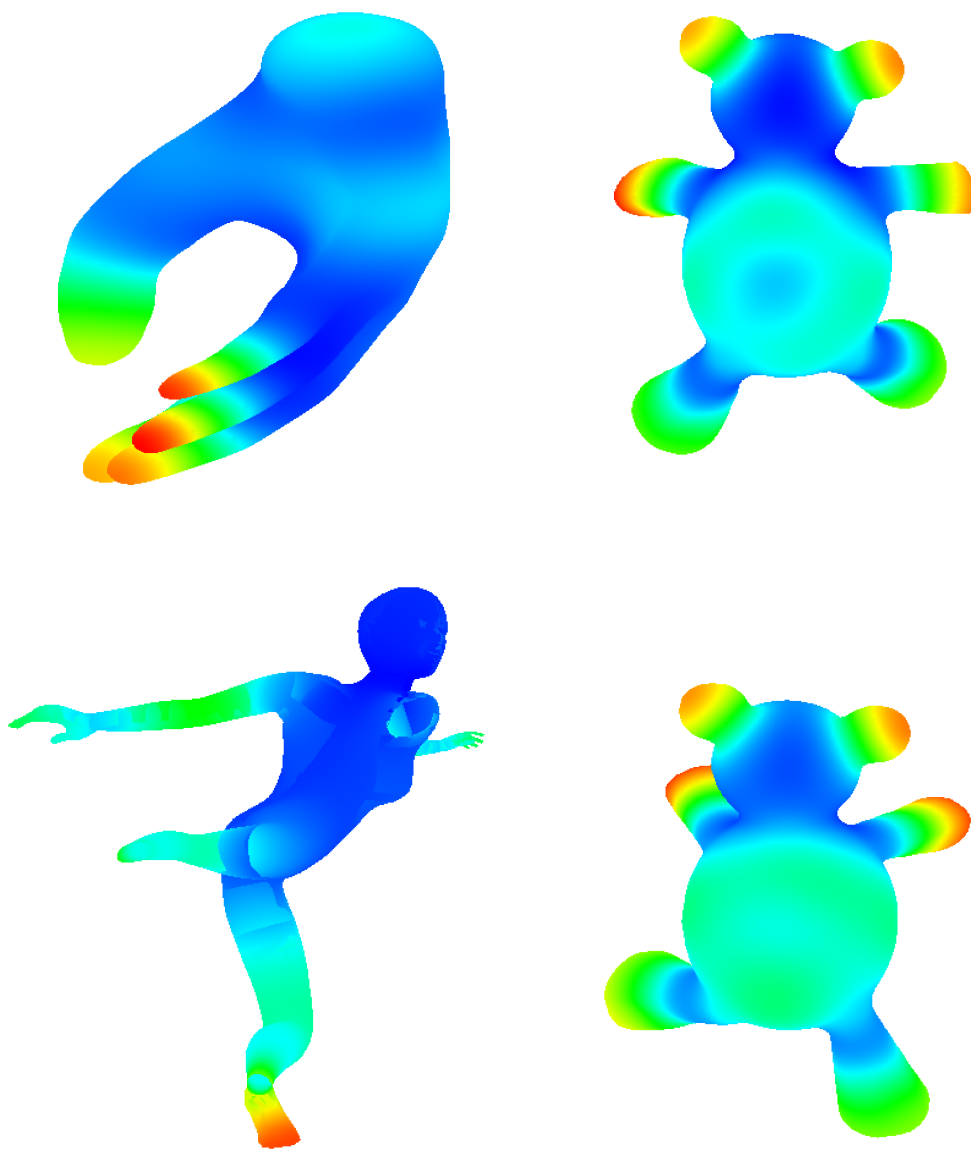


Figure 2: SIHKS descriptors on 3D models.

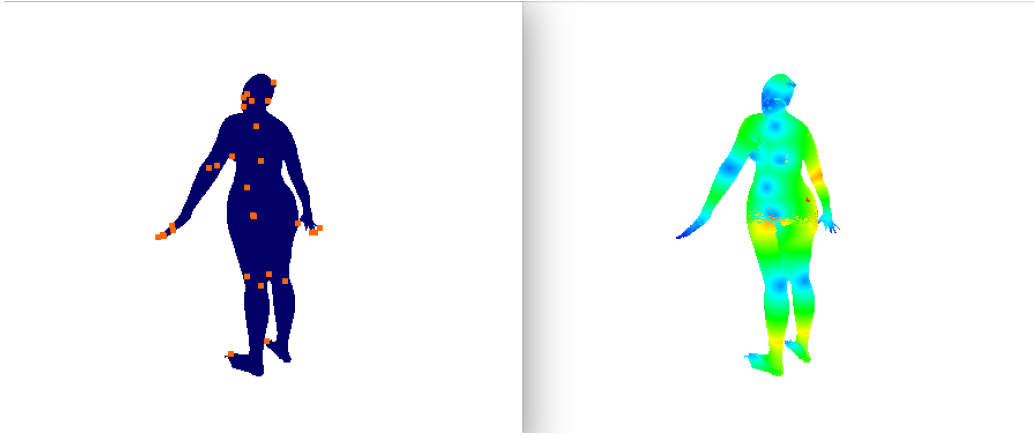


Figure 3: HGD field on human(left: focal feature points, right: heat geodesic distance).

$$\Delta\phi = \nabla \cdot X. \quad (6)$$

Heat geodesics can be computed in three steps.

1. diffuse  $u$  on 3D shapes based on equation 1 in a numerical way,
2. normalize the gradient field in first step to satisfy the constraints of geodesics  $|\nabla\phi| = 1$  as showed in equation 5,
3. optimize the gradient field approximation  $\min_{\phi} ||\nabla\phi - X||_2$  problem by solving a linear system of Poisson equation 6.

In implementation, diffusion time is set as 0.001, sources of diffusions are set as user defined focal feature points.

### 2.3. RFR

Random Forest can learn the regression from SIHKS to HGD. In implementation, the parameters of random forest training are chosen as following:

1. The number of trees in random forest is 30.
2. The number of random features used to branch at nodes is 5.
3. The minimal size of nodes is 5.

### 2.4. Other

Aside from three main parts, there are other details of the implementation:

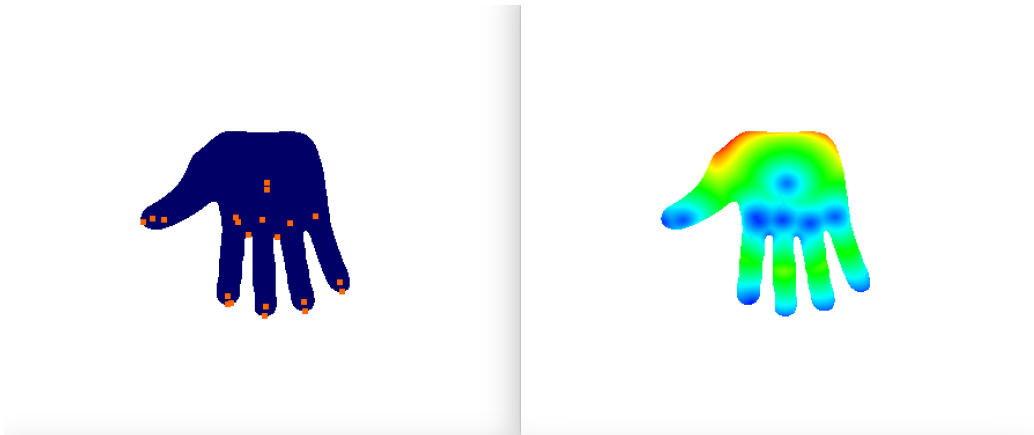


Figure 4: HGD Field on hand(left: focal feature points, right: heat geodesic distance).

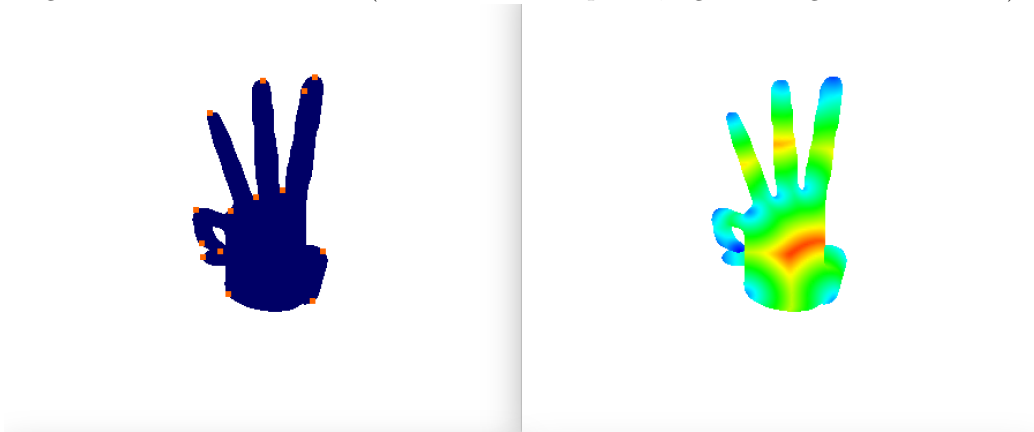


Figure 5: HGD Field on hand(left: focal feature points, right: heat geodesic distance).

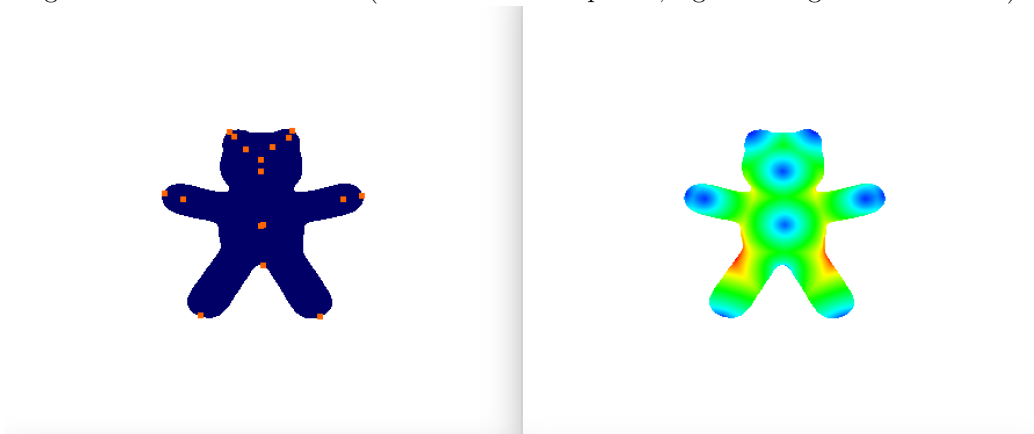


Figure 6: HGD Field on bear(left: focal feature points, right: heat geodesic distance).

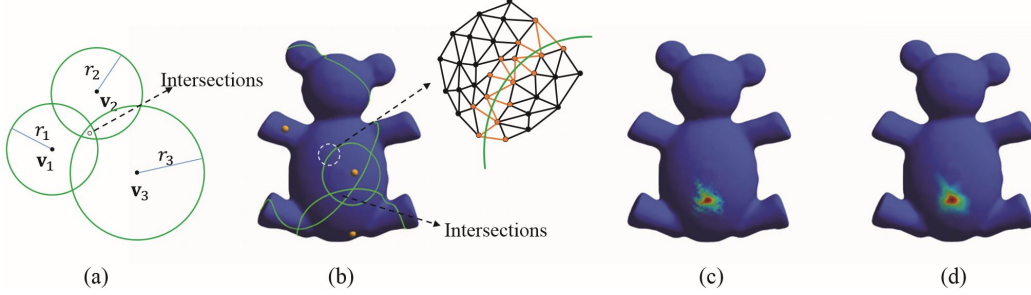


Figure 7: Voting for focal feature localization(picture captured from the paper).

1. The 3D shapes in Schelling Dataset are normalized that the surface area is equal to 1.
2. Feature points are localized by a GPS way with voting as shown in 7.
3. During localization, the weight coefficient  $\mu$  is set as 5 and the neighbor of a point is defined as points with geodesic distance less than 2 percent of the predicted distance.

### 3. Results and Conclusion

Here some results are shown in figures. Results of SIHKS descriptors are shown in 2 on human, hand and bear models. Results of HGD with original sources are shown in 3, 4, 5 and 6 on human, hand and bear models. Predicted feature distributions are shown with user defined focal features in 8, 9, 10 and 11 on human, hand and bear models.

**Conclusion.** From the result we can see that the predicted feature distribution can capture the user defined focal features. However it is too fuzzy to get sharp results. Future work can focus on using more advanced learning techniques or try to extract global structures from local features.

Actually, from the general idea that we want to reconstruct  $P_f$  from  $P_g$ , one possibility is to use the idea of GAN. Consider images instead of 3D shapes, for each image  $I$  a corresponding feature distribution image can be generated as  $g_\theta(I)$ . To make  $g_\theta(I)$  close to user defined feature distribution  $J$  on  $I$ , a discriminator  $D$  can be used. Generalized to 3D shapes, this adversarial training framework actually learns how to transport from  $P_g$  to  $P_f$  with the assumption that  $d(P_g, P_f)$  is Wasserstein distance in the literature of WGAN. This idea is similar to Apple’s first research paper ”Learning from Simulated and Unsupervised Images through Adversarial Training” in which

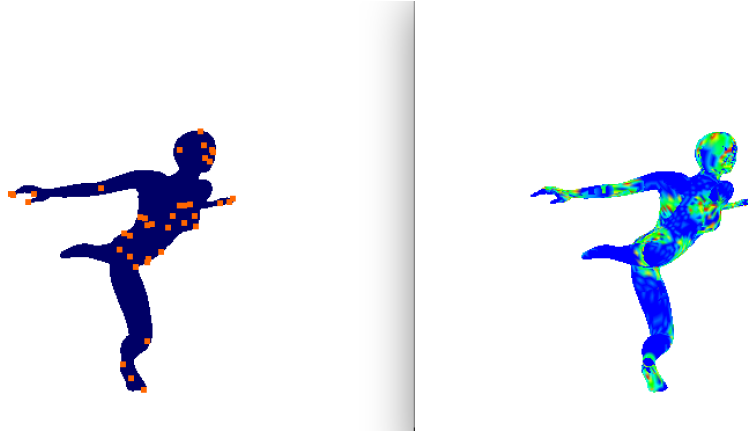


Figure 8: Testing on human(left: focal features, right: predicted feature distribution).

it is applied to learn how to refine a synthetic image. The good news is it means this idea is good and applicable, but the generalization from image to 3D shapes is not trivial, so this idea still needs to be further explored.

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- [2] M. M. Bronstein, I. Kokkinos, Scale-invariant heat kernel signatures for non-rigid shape recognition, in: *The Twenty-Third IEEE Conference on Computer Vision and Pattern Recognition, CVPR 2010, San Francisco, CA, USA, 13-18 June 2010, IEEE Computer Society, 2010*, pp. 1704–1711.
- [3] K. Crane, C. Weischedel, M. Wardetzky, Geodesics in heat: A new approach to computing distance based on heat flow, *ACM Trans. Graph.* 32 (2013) 152:1–152:11.



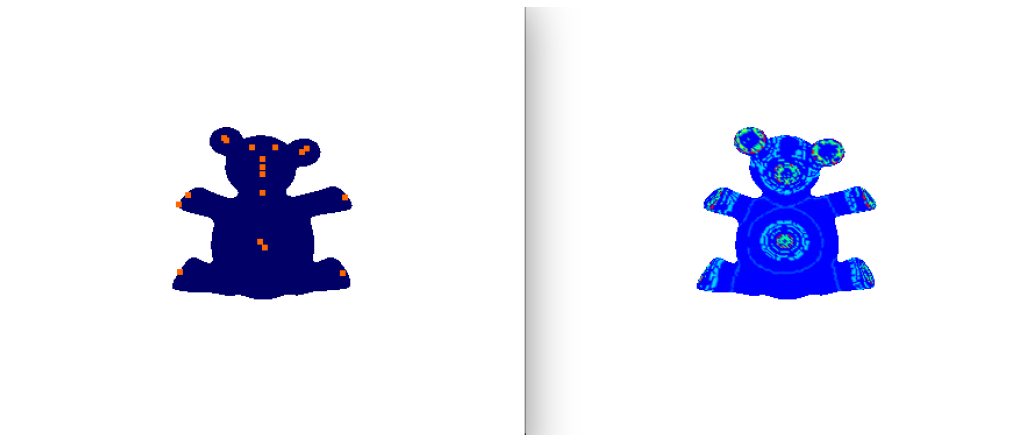


Figure 9: Testing on bear(left: focal features, right: predicted feature distribution).

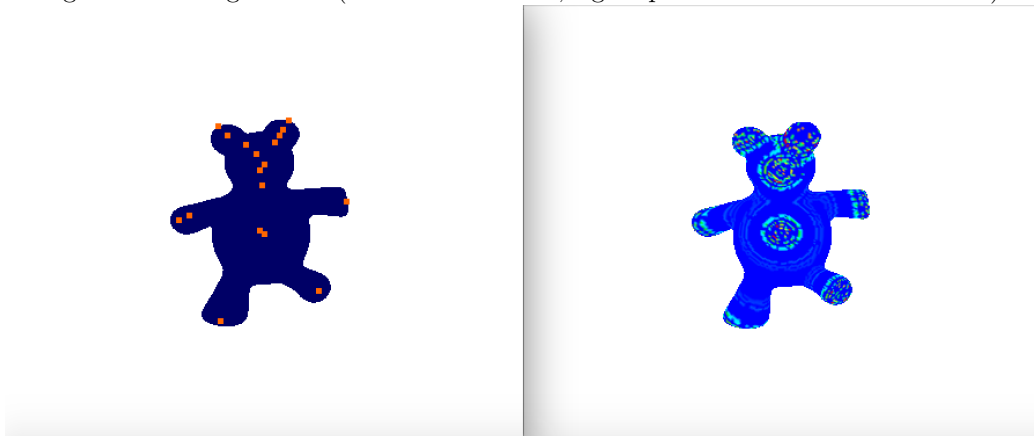


Figure 10: Testing on bear(left: focal features, right: predicted feature distribution).

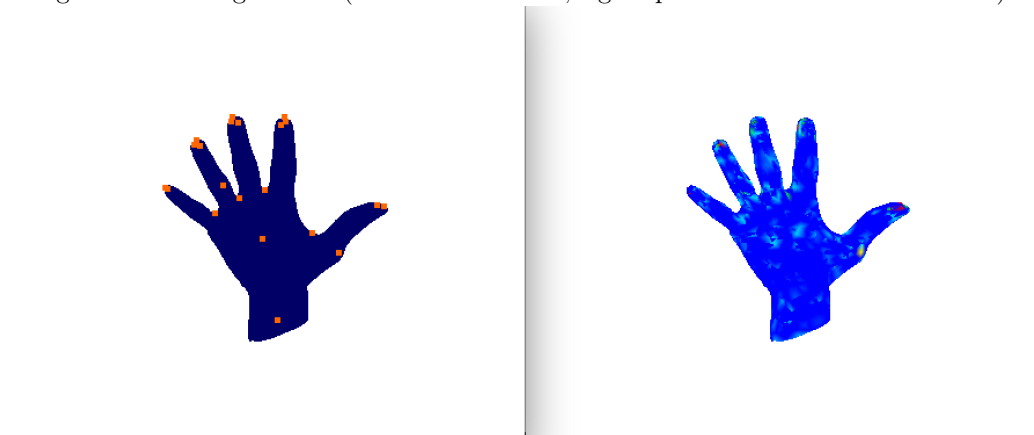


Figure 11: Testing on hand(left: focal features, right: predicted feature distribution).