# COMP90042 Web Search & Text Analysis

## Workshop Week 4

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March 26, 2019

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# Road Map

## Indexing

- Data Structure
  - Document-Term Matrix
  - Inverted Index
- Compression
  - Variable Byte Compression
  - OptPFor Delta Compression
- Index Construction
  - Invert Batch Indexing
  - Auxiliary Indexing
  - Logarithmic Indexing

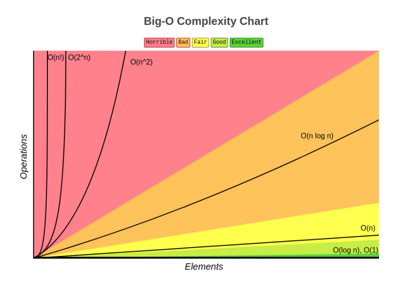
## Road Map

### Search

- Vector Space Models
    - TF-IDF
    - BM25
- Efficient Query Processing
    - Operation GEQ
    - WAND
- Query Completion
    - Prefix Trie
    - Range Maximum Query
- Query Expansion
    - Relevance Feedback
    - Semantic-Based Methods
- Phrase Search
    - Inverted Index + Positional Information
    - Suffix Array
- Evaluation and Re-rank

# Warm Up - Complexity

## Time Complexity




*Big O cheat sheet*

## Notation

- $T(n) = O(f(n)) \Leftrightarrow \exists\ c, n_o, \forall\ n > n_o, T(n) \leq c \cdot O(f(n))$
- $T(n) = \Omega(f(n)) \Leftrightarrow \exists\ c, n_o, \forall\ n > n_o, T(n) \geq c \cdot \Omega(f(n))$
- $T(n) = \Theta(f(n)) \Leftrightarrow \exists\ c_1, c_2, n_o, \forall\ n > n_o,$
  $c_1 \cdot \Theta(f(n)) \leq T(n) \leq c_2 \cdot \Theta(f(n))$

# Warm Up - Complexity

## Space Complexity

- Amount of auxiliary space the algorithm need in the function of input size.

## Example - Merge Sort

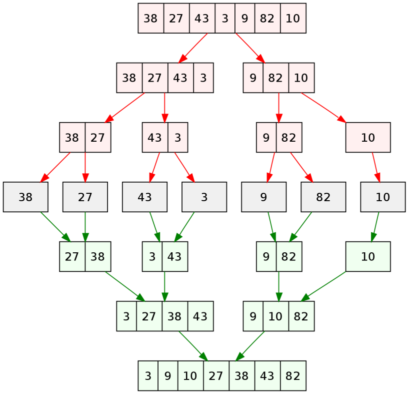



*Fig from Wikipedia.*

# Outline

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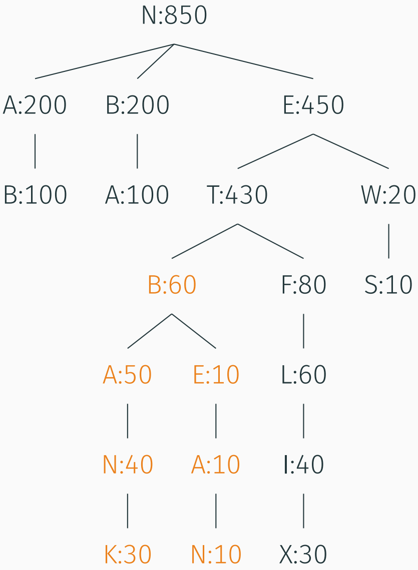
- Re-rank
  - Point-wise learning
  - Pair-wise learning

# Prefix Trie - Data Structure




- Each edge a character in a prefix.
- Each node store the frequency of the prefix being searched.
- Children of nodes are ordered.
- Traverse the tree to generate an array.
- Sub-tries are continuous sub-arrays.

| | | | | | | |
|---|---|---|---|---|---|---|
| 60 | 50 | 40 | 30 | 10 | 10 | 10 |

*Array for sub-trie in red.*

# Range Maximum Query (RMQ)

How to get top-$k$ most frequent items have the given prefix?

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 9 | 10 | 5 | 32 | 7 | 13 | 21 | 1 |

- Sort the corresponding sub-array and take the first-$k$ elements.
- Less time complexity?
- Less space complexity?

# RMQ - Reduced Time Complexity

Pre-compute max-value for all sub-arrays with $O(N^2)$ space.$M = [m_{ij}]_{N\times N}$
$RMQ(i,j)$ can be done by accessing the value $m_{ij}$ in matrix.

```
arr ← Array[0...k − 1];
heap ← emptyMaxHeap();
heap.insert(left, right);
for i in [0...k-1] do
    node ← heap.pop();
    maxPos ← RMQ(node.left, node.right);
    arr[i] ← maxPos;
    if node.left ≠ null then
        heap.insert(node.left, maxPos − 1);
    end
    if node.right ≠ null then
        heap.insert(maxPos + 1, node.right);
    end
end
```

# RMQ - Reduced Space Complexity

- Store max-value of every $A[i, i + 2^n]$ only, instead of all $A[i, j]$.
- Use 2 overlapping region $A[i, P]$ and $A[Q, j]$ to cover the region in query $A[i, j]$.
- Take $max(RMQ(i, P), RMQ(Q, j))$.

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# Query Expansion

Feedback-based

- User Relevance Feedback
- Pseudo Relevance Feedback
- Indirect Relevance Feedback

Questions:

- Can we expand query without feedback?

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# Positional Inverted Index

| | *DocID* | *Frequency* | *Position* |
|---|---|---|---|
| big | < 1, 3, 5> | < 1, 2, 1> | < <23>, <43, 65>, <31> > |
| brother | < 2, 3, 6> | < 1, 1, 1> | < <2>, <42>, <67> > |
| ... | | | |

Query: big brother

- Intersect *DocID* first, then intersect *Position*.
- Sort list by length, starting from the smallest.

| | *DocID* | *Frequency* | *Position* |
|---|---|---|---|
| big brother | <3> | <1> | < <42, 43> > |
| ... | | | |

# String Matching and Suffix Array

Trivial string matching takes $O(|n| \cdot |m|)$ for matching $m$ in $n$.

Suffix arrays of $T$ (abrac$).

| id | start | suffix array |
|---|---|---|
| 0 | 5 | $abrac |
| 1 | 0 | abrac$ |
| 2 | 3 | ac$abr |
| 3 | 1 | brac$a |
| 4 | 4 | c$abra |
| 5 | 2 | rac$ab |

- Store suffix array
  - Complete arrays $O(n^2 log\sigma)$.
  - Start index only $O(nlogn)$
- Sort $n$ array of length $n$ takes
  - $O(n^2 logn)$ for quick sort
  - $O(n)$ for radix sort
- Perform binary search
  - $T(n) = O(m \cdot logn)$
  - $S(n) = O(|T| + nlogn)$

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# Evaluation Metric

Recap:

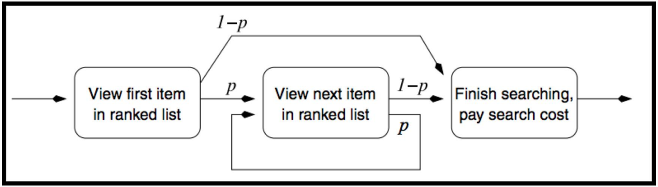
- Precision: $\frac{TP}{TP+FP}$
- Recall: $\frac{TP}{TP+FN}$
- $F_1$ measure: $\frac{2 \cdot P \cdot R}{P+R}$

IR measurements:

- Based on relevance vector
- Precision@$k$: only first $k$ element in relevance vector.
- AP: average P@$k$ for all $k$, $r_k = 1$.
- MAP: average AP for all queries.

# Rank-biased Precision

Introducing patience factor $p$ to average precision.




$$RBP = \sum_{i=1}^{d} r_i \times p^{i-1} \times (1-p)$$

- $r_i$: the $i^{th}$ element in relevance vector.
- $p^{i-1}$: probability of the reader reaches the $i^{th}$ element.
- $(p-1)$: probability of the reader stops at this element.
- Assumption: $P(Stop)$ independent to $i$.

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# Learning to Rank

## Point-wise

- Predict relevance factor $P(r_i|x_i)$ (e.g. [-2, 2]).
- Sort documents by relevance factor.

## Pair-wise

- Predict which document is more relevant $P(y_{i,j}|x_i, x_j)$
- Sort by comparing documents.

How to convert documents to $X$?