

# Liii STEM 0.12要点速览!

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Liii Network

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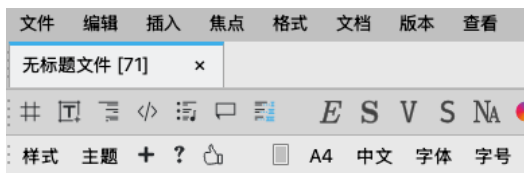
## 需知

- 以下所有演示均在Liii STEM v2025.0.12版本上
- 所有英文的内容均为大模型生成!
- 我们不再维护之前的版本, 请务必下载并安装v2025.0.12使用!

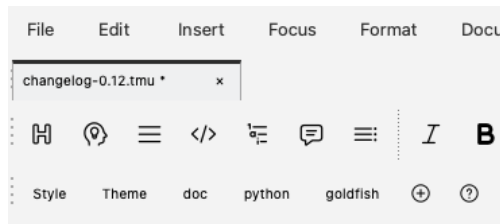
下载地址: <https://liiistem.cn/install.html>

## 1 新的Liii主题!

如图 1所示, 我们现在提供了一个全新的, 更加现代化的主题《Liii 主题》。从v2025.0.12版本开始, 这个主题会成为Liii STEM的默认主题!



(a) 旧默认主题(文本环境)

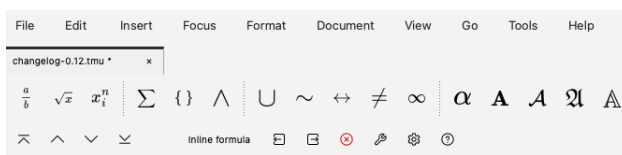


(b) Liii 主题 (文本环境)

图 1. 新的主题 (部分图标)



(a) 旧默认主题 (数学环境)



(b) Liii 主题 (数学环境)

图 2. 新的主题 (部分图标)

注意 1. 老用户可以在 编辑->首选项->用户界面 里恢复之前的主题。

## 2 手写内容（包括公式！）识别和整理

注意 2. 除了OpenAI系列的模型外，其余的大模型都不再需要配置API，开箱即用。

注意 3. 只有名称里带VL的模型和OpenAI的模型支持图片交互。

注意 4. 截图后直接拖进的图片需要嵌入后才可以和AI对话，如图 3所示

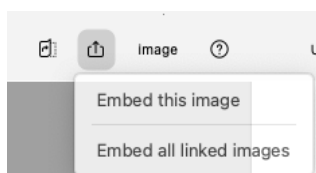


图 3. 嵌入图片

### 2.1 手写公式识别

我们可以使用 Pro-Qwen2.5-VL-7B-Instruct@SiliconFlow这个模型通过%system 关键字设置特定的提示词后当做OCR来用，如下所示：

```
LLM session v20250715 by LiiiLabs
Using Pro/Qwen/Qwen2.5-VL-7B-Instruct provided by siliconflow.cn with max_tokens 4096
Help docs: Help->Plugins->LLM
```

7B> %system Consider yourself as an OCR (math expert), render the contents I sent to you and do not inference!

系统提示词已设置，长度为98

7B&gt;

7B>

32B&gt;

---

### 3. Measure Spaces

A **measure space** is defined as a triple  $(\Omega, \mathcal{F}, \mu)$ , where: -  $(\Omega, \mathcal{F})$  is a measurable space. -  $\mu: \mathcal{F} \rightarrow [0, \infty]$  is a **measure**, satisfying: 1.  $\mu(\emptyset) = 0$ . 2. For any countable collection of pairwise disjoint sets  $\{A_n\} \subset \mathcal{F}$ ,

$$\mu\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} \mu(A_n).$$

**Example:** The **Lebesgue measure** on  $(\mathbb{R}, \mathcal{B}(\mathbb{R}))$ , where  $\mathcal{B}(\mathbb{R})$  is the Borel  $\sigma$ -algebra on  $\mathbb{R}$ .

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### 4. Probability Spaces

A **probability space** is a special case of a measure space  $(\Omega, \mathcal{F}, P)$ , where: -  $P$  is a **probability measure**, satisfying: 1.  $P(\Omega) = 1$ . 2.  $P(A) \geq 0$  for all  $A \in \mathcal{F}$ .

**Random Variables:** - A **random variable**  $X: (\Omega, \mathcal{F}) \rightarrow (\mathbb{R}, \mathcal{B}(\mathbb{R}))$  is a measurable function, i.e., for any  $A \in \mathcal{B}(\mathbb{R})$ ,  $X^{-1}(A) \in \mathcal{F}$ .

---

### 5. Measurable Functions

Let  $(\Omega, \mathcal{F})$  and  $(\Omega', \mathcal{F}')$  be measurable spaces. A function  $f: \Omega \rightarrow \Omega'$  is called **measurable** if:

$$f^{-1}(A) \in \mathcal{F} \quad \text{for all } A \in \mathcal{F}'.$$

**Example:** If  $f: (\Omega, \mathcal{F}) \rightarrow (\mathbb{R}, \mathcal{B}(\mathbb{R}))$  is measurable, then for any  $t \in \mathbb{R}$ ,

$$\{w \in \Omega: f(w) < t\} \in \mathcal{F}.$$

---

### 6. Expectation and $L^2$ Space

Let  $(\Omega, \mathcal{F}, P)$  be a probability space. - The **expectation** of a random variable  $X$  is defined as:

$$\mathbb{E}[X] = \int_{\Omega} X \, dP.$$

- The space  $L^2(\Omega, \mathcal{F}, P)$  consists of square-integrable random variables:

$$L^2(\Omega, \mathcal{F}, P) = \{X: \Omega \rightarrow \mathbb{R} \mid \mathbb{E}[X^2] < \infty\}.$$

Equipped with the inner product:

$$\langle X, Y \rangle = \mathbb{E}[XY],$$

$L^2(\Omega, \mathcal{F}, P)$  becomes a Hilbert space.

**Key Properties:** - If  $\mathbb{E}[X^2] < \infty$ , then  $\mathbb{E}[|X|] < \infty$ . - For a sequence of mutually independent random variables  $\{X_t\}_{t \in T}$ :

$$\mathbb{E}\left[\prod_{t \in T} X_t\right] = \prod_{t \in T} \mathbb{E}[X_t].$$

---

### 7. Hilbert Spaces and Orthogonality

A **Hilbert space** is a complete inner product space. Examples: -  $L^2(\Omega, \mathcal{F}, P)$  (as defined above). -  $\ell^2$ : The space of square-summable sequences.

**Orthogonality:** - Two elements  $x, y$  in a Hilbert space are orthogonal if:

$$\langle x, y \rangle = 0.$$

- For a subspace  $\mathcal{H}_1 \subset \mathcal{H}$ , the orthogonal complement is:

$$\mathcal{H}_1^\perp = \{y \in \mathcal{H}: \langle x, y \rangle = 0 \text{ for all } x \in \mathcal{H}_1\}.$$

---

## 8. Conditional Expectation

Let  $(\Omega, \mathcal{F}, P)$  be a probability space, and let  $\mathcal{G} \subset \mathcal{F}$  be a sub- $\sigma$ -algebra. The **conditional expectation** of a random variable  $X \in L^2(\Omega, \mathcal{F}, P)$  given  $\mathcal{G}$  is a random variable  $\mathbb{E}[X|\mathcal{G}]$  such that: 1.  $\mathbb{E}[X|\mathcal{G}]$  is  $\mathcal{G}$ -measurable. 2. For all  $A \in \mathcal{G}$ ,

$$\int_A \mathbb{E}[X|\mathcal{G}] dP = \int_A X dP.$$

Key Properties: -  $\mathbb{E}[X|\mathcal{G}]$  is the orthogonal projection of  $X$  onto  $L^2(\Omega, \mathcal{G}, P)$ . - If  $X$  is  $\mathcal{G}$ -measurable, then  $\mathbb{E}[X|\mathcal{G}] = X$ .

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## 9. Independence

- Two events  $A, B \in \mathcal{F}$  are **independent** if:

$$P(A \cap B) = P(A)P(B).$$

- Two random variables  $X, Y$  are **independent** if:

$\sigma(X)$  and  $\sigma(Y)$  are independent  $\sigma$ -algebras.

- A sequence of random variables  $\{X_t\}_{t \in T}$  is **mutually independent** if for any finite subset  $\{t_1, \dots, t_n\} \subset T$ ,

$$P\left(\bigcap_{i=1}^n A_{t_i}\right) = \prod_{i=1}^n P(A_{t_i}),$$

for all  $A_{t_i} \in \sigma(X_{t_i})$ .

---

## 10. Linear Contractions and Projections

- A **linear contraction** on a Hilbert space is a linear operator  $T$  such that:

$$\|Tx\| \leq \|x\| \quad \text{for all } x \in \mathcal{H}.$$

- The **projection operator**  $P$  onto a closed subspace  $\mathcal{H}_1 \subset \mathcal{H}$  satisfies:

$$P(x) \in \mathcal{H}_1, \quad \langle x - P(x), y \rangle = 0 \quad \text{for all } y \in \mathcal{H}_1.$$

同之前一样，您可以直接粘贴公式追问！这里我们演示让AI补全上文中红色部分的证明

32B> Let  $(\Omega, \mathcal{F}, P)$  be a probability space, and let  $\mathcal{G} \subset \mathcal{F}$  be a sub- $\sigma$ -algebra. The **conditional expectation** of a random variable  $X \in L^2(\Omega, \mathcal{F}, P)$  given  $\mathcal{G}$  is a random variable  $\mathbb{E}[X|\mathcal{G}]$  such that: 1.  $\mathbb{E}[X|\mathcal{G}]$  is  $\mathcal{G}$ -measurable. 2. For all  $A \in \mathcal{G}$ ,

$$\int_A \mathbb{E}[X|\mathcal{G}] dP = \int_A X dP.$$

Key Properties: -  $\mathbb{E}[X|\mathcal{G}]$  is the orthogonal projection of  $X$  onto  $L^2(\Omega, \mathcal{G}, P)$ . - If  $X$  is  $\mathcal{G}$ -measurable, then  $\mathbb{E}[X|\mathcal{G}] = X$ .

prove the property

We are tasked with proving the property that the conditional expectation  $\mathbb{E}[X|\mathcal{G}]$  is the orthogonal projection of  $X$  onto the subspace  $L^2(\Omega, \mathcal{G}, P)$ . Let us proceed step by step.

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### 2.2.2 Step 1: Understanding the setup

#### 1. Space and sub- $\sigma$ -algebra:

- $(\Omega, \mathcal{F}, P)$  is a probability space.
- $\mathcal{G} \subset \mathcal{F}$  is a sub- $\sigma$ -algebra.
- $X \in L^2(\Omega, \mathcal{F}, P)$ , i.e.,  $X$  is a square-integrable random variable.

## 2. Conditional expectation:

- $\mathbb{E}[X|\mathcal{G}]$  is a random variable that is  $\mathcal{G}$ -measurable and satisfies the following property:

$$\int_A \mathbb{E}[X|\mathcal{G}] dP = \int_A X dP, \quad \forall A \in \mathcal{G}.$$

- This is the defining property of the conditional expectation.

## 3. Orthogonal projection:

- We aim to show that  $\mathbb{E}[X|\mathcal{G}]$  is the orthogonal projection of  $X$  onto the subspace  $L^2(\Omega, \mathcal{G}, P)$ , which is the set of all square-integrable random variables that are  $\mathcal{G}$ -measurable.

### 2.2.3 Step 2: Orthogonality condition

The orthogonal projection of  $X$  onto  $L^2(\Omega, \mathcal{G}, P)$  is characterized by the following condition:

$$X - \mathbb{E}[X|\mathcal{G}] \perp L^2(\Omega, \mathcal{G}, P),$$

which means:

$$\langle X - \mathbb{E}[X|\mathcal{G}], Y \rangle = 0, \quad \forall Y \in L^2(\Omega, \mathcal{G}, P),$$

where the inner product  $\langle \cdot, \cdot \rangle$  is defined as:

$$\langle U, V \rangle = \int_{\Omega} UV dP.$$

Thus, we need to show:

$$\int_{\Omega} (X - \mathbb{E}[X|\mathcal{G}]) Y dP = 0, \quad \forall Y \in L^2(\Omega, \mathcal{G}, P).$$

### 2.2.4 Step 3: Using the defining property of conditional expectation

Let  $Y \in L^2(\Omega, \mathcal{G}, P)$ . Since  $Y$  is  $\mathcal{G}$ -measurable, we can use the defining property of  $\mathbb{E}[X|\mathcal{G}]$  to write:

$$\int_A \mathbb{E}[X|\mathcal{G}] dP = \int_A X dP, \quad \forall A \in \mathcal{G}.$$

Now, consider the integral:

$$\int_{\Omega} (X - \mathbb{E}[X|\mathcal{G}]) Y dP.$$

We can split this into two terms:

$$\int_{\Omega} (X - \mathbb{E}[X|\mathcal{G}]) Y dP = \int_{\Omega} XY dP - \int_{\Omega} \mathbb{E}[X|\mathcal{G}] Y dP.$$

### 2.2.5 Step 4: Simplifying the second term

Since  $Y$  is  $\mathcal{G}$ -measurable, we can use the defining property of conditional expectation. Specifically, for any  $\mathcal{G}$ -measurable function  $Y$ , we have:

$$\int_{\Omega} \mathbb{E}[X|\mathcal{G}] Y dP = \int_{\Omega} XY dP.$$

This follows because we can write  $Y = \sum_i c_i 1_{A_i}$  for some constants  $c_i$  and sets  $A_i \in \mathcal{G}$ , and

the defining property of  $\mathbb{E}[X|\mathcal{G}]$  applies to each indicator function  $1_{A_i}$ .

Thus:

$$\int_{\Omega} \mathbb{E}[X|\mathcal{G}] Y dP = \int_{\Omega} XY dP.$$

### 2.2.6 Step 5: Combining results

Substituting back, we have:

$$\int_{\Omega} (X - \mathbb{E}[X | \mathcal{G}]) Y \, dP = \int_{\Omega} XY \, dP - \int_{\Omega} XY \, dP = 0.$$

This shows that:

$$X - \mathbb{E}[X | \mathcal{G}] \perp L^2(\Omega, \mathcal{G}, P).$$

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### 2.2.7 Step 6: Uniqueness of the orthogonal projection

The orthogonal projection of  $X$  onto  $L^2(\Omega, \mathcal{G}, P)$  is unique. Since  $\mathbb{E}[X | \mathcal{G}]$  satisfies the orthogonality condition and is  $\mathcal{G}$ -measurable, it must be the orthogonal projection of  $X$  onto  $L^2(\Omega, \mathcal{G}, P)$ .

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### 2.2.8 Final Answer

$\mathbb{E}[X | \mathcal{G}]$  is the orthogonal projection of  $X$  onto  $L^2(\Omega, \mathcal{G}, P)$ .

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## 3 预览编号

在文本模式下如图 4 的选项里选择 `preview-ref` 便可以预览编号。Liii STEM 的 0.12 版本对这个功能做了些优化，增大了预览框的大小。当鼠标悬停在编号上会显示如图 5 的效果。

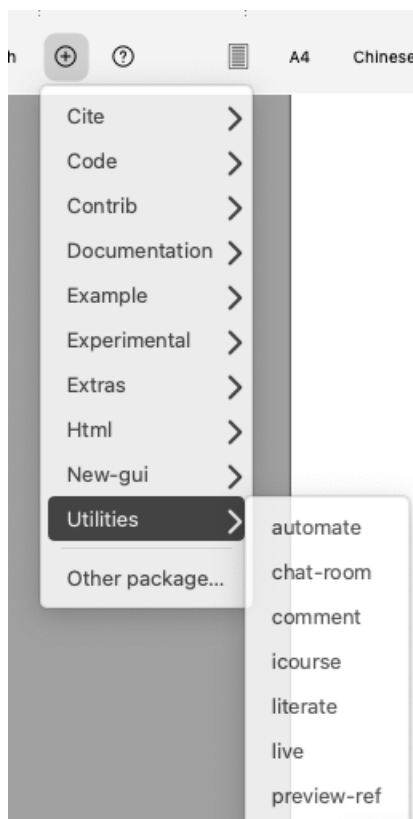


图 4. 编号预览

**定理 5.** 实数集合  $\mathbb{R}$  不可数

根据定理 5.

**Theorem 1.** *The set of real number  $\mathbb{R}$  is uncountable.*

By Theorem [1](#).

The set of real number  $\mathbb{R}$  is uncountable.

图 5. 例子：定理编号预览

## A 部分Bug 修复

- 修正了自定义宏的bug，现在默认导出 $\text{\LaTeX}$ 不会加入 $\text{\TeX}_{\text{MACS}}$ 自定义宏 例如粗体之前导出代码为 $\text{\tm-textbf}$ ，现在为  $\text{\textbf}$ 。
- 切换标签页现在会停留在之前光标所在的位置，不会回到文档开头。
- 在 Windows 上解决使用 LLM 插件显示黑色弹出窗口的问题

## B 部分新特性

- 除了OpenAI系列的模型外，其余的大模型都不再需要配置API，开箱即用。
- macOS和Linux用户现在不再需要安装pandoc就可以使用LLM了。
- macOS平台现在支持Cmd + 数字 (1-6) 切换标签页
- 现在支持Ctrl/Cmd + T 新增空标签页
- 新增大量新模型
- 数学环境下 $\times$ 优先输出 $\times$ 而不是空格。