Do you like Texas hold 'em

STAT230 Real World Assignment 2024W

Eason Li, Johnson Ji

Faculty of Math
University of Waterloo
Canada
April 8, 2024

Starting Hands (with 2 players)

Out of the $\binom{52}{2}$ possible starting hands, there are 13×13 distinct types of starting hands since there are 13 ranks, there are 13 distinct pocket pairs (e.g. AA), $\binom{13}{2} = 78$ distinct suited 2-card combinations (e.g. AKs) and $\binom{13}{2} = 78$ distinct unsuited 2-card combinations (e.g. AKo). Notice that

$$13 \times 13 = 13 + \binom{13}{2} + \binom{13}{2}$$

It is intended to calculate the pre-flop (before any community cards are dealt) win rate for each distinct 2-card combination. For each distinct combination, there are

$$\left[\prod_{i=1}^{n-1} \binom{52-2i}{2} \right] \times \binom{52-2n}{5}$$

possible outcomes for n players. A simulation (Appendix) size of 100 000 is chosen for this section:

```
Pocket Pair:   5976

Suited:   23167

Unsuited:   70857

Contains Ace or King:   28643

Does Not Contain Ace or King:   71357
```

Recall that we consider in a game consisting two players. In theory, we know that the probability of getting pocket pair in a deck of 52 cards is

$$\frac{52}{52} \cdot \frac{3}{51} \approx \boxed{0.058823}$$

the probability for suited pair is

$$\frac{52}{52} \cdot \frac{12}{51} \approx \boxed{0.235294}$$

while the probability for unsuited pair is

$$\frac{52}{52} \cdot \frac{39}{51} - \frac{52}{52} \cdot \frac{3}{51} \approx \boxed{0.707014}$$

Lmao our simulations are pretty close to our theoretical values.

Poker Hands

The chance of making, winning and tying with each poker hand is investigated. The total number of possible outcomes for a hand of poker between n players is given by:

$$\left[\prod_{i=0}^{n-1} \binom{52-2i}{2} \right] \cdot \binom{52-2n}{2}$$

which is 2.781×10^{12} for just 2 players. Since it isn't realistic to go through all possible outcomes, a simulation (Appendix) size of 100 000 is chosen (with 2 players):

Player 1:		Player 2:	
Straight Flush:	4	Straight Flush:	5
Quad:	15	Quad:	11
Full House:	250	Full House:	259
Flush:	307	Flush:	274
Straight:	447	Straight:	439
Triple:	536	Triple:	461
Two Pairs:	2341	Two Pairs:	2368
Pair:	4355	Pair:	4395
High Card:	1745	High Card:	1788

We can also calculate the theoretical expression of the absolute frequency of the occurences of each hands:

1. **Straight flush (including royal straight flush):**

$$\binom{10}{1}\binom{4}{1}\binom{46}{2} = 4,324$$

2. **Quad:**

$$\binom{13}{1}\binom{48}{3} = 224,848$$

3. **Full house:**

$$\left[\binom{13}{2}\binom{4}{3}^2\binom{44}{1}\right] + \left[\binom{13}{1}\binom{12}{2}\binom{4}{3}\binom{4}{2}^2\right] + \left[\binom{13}{1}\binom{12}{1}\binom{11}{2}\binom{4}{3}\binom{4}{2}\binom{4}{1}^2\right] \approx 3,473,184$$

4. **Flush:**

$$\left[\binom{4}{1} \times \left[\binom{13}{7} - 217\right]\right] + \left[\binom{4}{1} \times \left[\binom{13}{6} - 71\right] \times 39\right] + \left[\binom{4}{1} \times \left[\binom{13}{5} - 10\right] \times \binom{39}{2}\right] \approx 4,047,644$$

5. **Straight:**

$$[217 \times [4^7 - 756 - 4 - 84]] + [71 \times 36 \times 990] + \left[10 \times 5 \times 4 \times [256 - 3] + 10 \times \binom{5}{2} \times 2268\right] \approx 6,180,020$$

6. **Triple:**

$$\left[\binom{13}{5} - 10\right]\binom{5}{1}\binom{4}{1}\left[\binom{4}{1}^4 - 3\right] \approx 6,461,620$$

7. **Two pair:**

$$[1277 \times 10 \times [6 \times 62 + 24 \times 63 + 6 \times 64]] + \left[\binom{13}{3}\binom{4}{2}^3\binom{40}{1}\right] \approx 31,433,400$$

8. **One pair:**

$$\left[\binom{13}{6} - 71\right] \times 6 \times 6 \times 990 \approx 58,627,800$$

9. **High cards:**

$$1499 \times [4^7 - 756 - 4 - 84] \approx 23,294,460$$

Therefore, since we know that the total number of possible combination of 7 cards is $\binom{52}{7} = 133,784,560$, hence we can find the probability of each hand:

Poker Hands	Probability
A♡ K♡ Q♡ J♡ 10♡ Strait Flush	$\dfrac{4,324}{133,784,560} \approx 0.0311\%$
A♡ A♣ A♢ A♠ 6♡ Quad	$\dfrac{224,848}{133,784,560} \approx 0.168\%$
8♡ 8♢ 8♠ K♠ K♢ Full House	$\dfrac{3,473,184}{133,784,560} \approx 2.60\%$
8♣ 3♣ 6♣ J♣ A♣ Flush	$\dfrac{4,047,644}{133,784,560} \approx 3.03\%$
7♡ 8♡ 9♢ 10♠ J♢ Straight	$\dfrac{6,180,020}{133,784,560} \approx 4.62\%$
Q♡ Q♠ Q♢ 5♡ A♢ Triple	$\dfrac{6,461,620}{133,784,560} \approx 4.83\%$
3♠ 3♣ 6♡ 6♡ Q♣ Two pairs	$\dfrac{31,433,400}{133,784,560} \approx 23.5\%$
5♡ 5♠ 2♣ J♣ A♢ One pair	$\dfrac{58,627,800}{133,784,560} \approx 43.8\%$
2♢ 5♠ 6♠ J♡ A♣ High hands	$\dfrac{23,294,460}{133,784,560} \approx 17.4\%$

We can easily discover that all the probabilities sum up to be 1, thus we can that this is in fact a multinomial distribution.

How many times does each player win in a 2-player poker game

We can also simulate how many times each player wins:
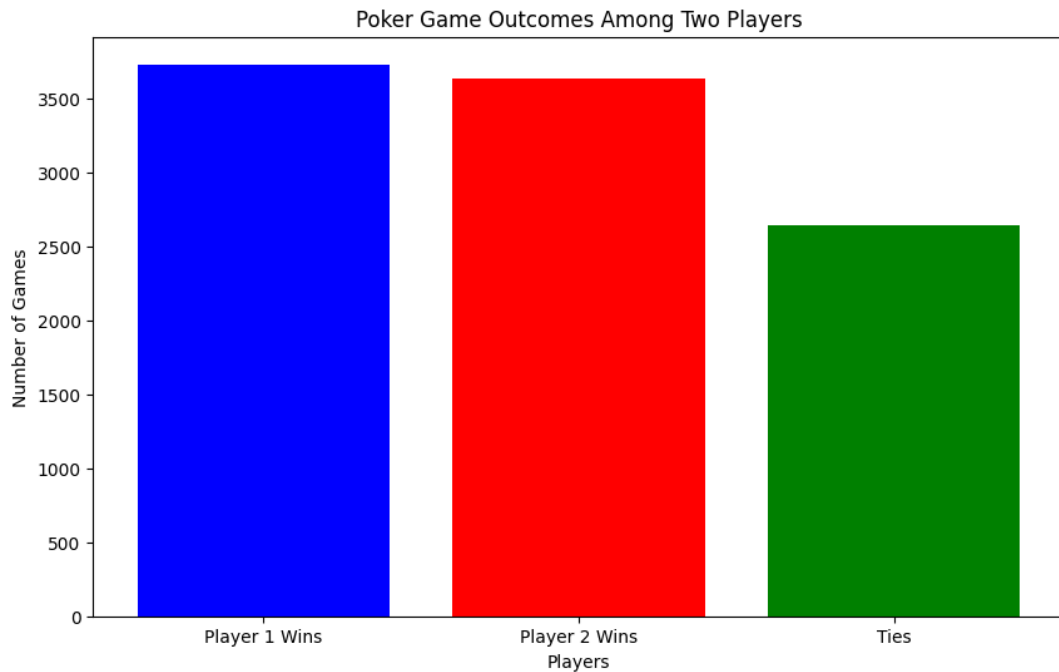
The bar chart presents the outcomes of poker games between two players. It shows the frequency of wins for Player 1, Player 2, and the number of games that ended in a tie. Statistically, we can observe that each no player has a significant advantage over the other. The small difference in the heights of the bars for Player 1 Wins and Player 2 Wins implies a roughly uniform distribution of outcomes between the two players.

Moreover, in a game of 4 player, the winning rates are still fairly close among all participants:
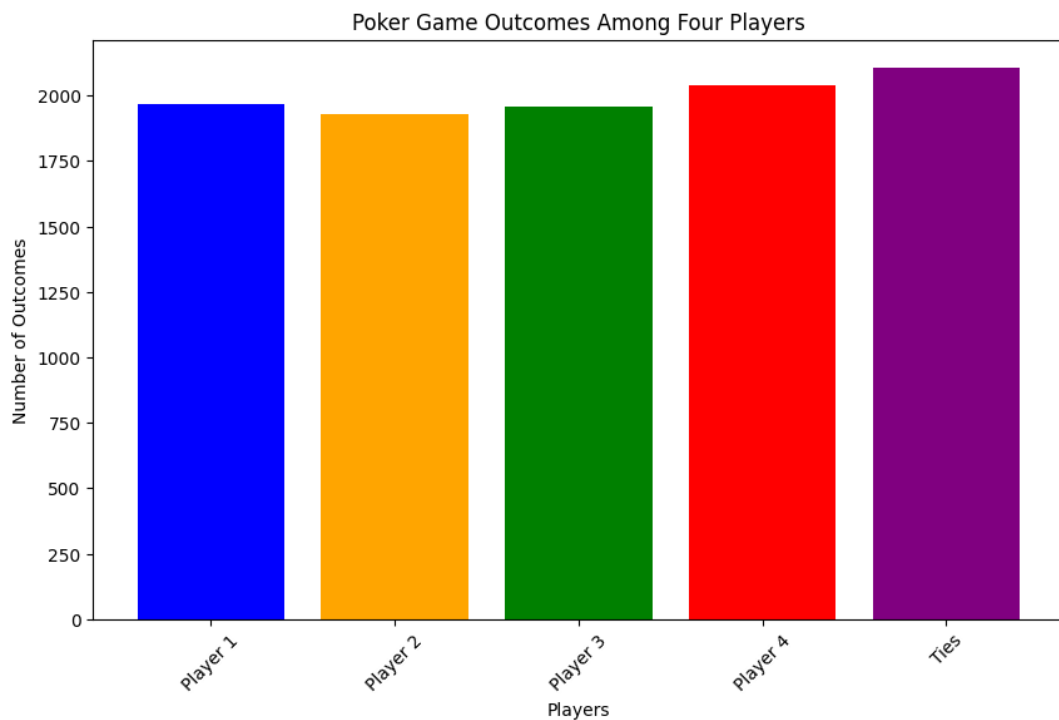
Out Odds

An out in a poker hand can be loosely defined as any card in the deck that will improve your current hand more than your opponents. Probability regarding outs is especially important in actual poker play, as they are a large influential factor regarding your chances of winning the pot. For instance, suppose that you had the following hand:

A♡ K♡

And the turn has revealed the following cards:

6♡ 3♡ 7♣ 10♢

In this situation, your outs on the river are if any heart is revealed ($13 - 4 = 9$ possibilities) to form a flush, or if any ace or king is revealed ($3 + 3 = 6$ possibilities) to form top pair. Summing these together, we obtain that our total number of outs is 15, and our out odds are $15/(52 - 6) \approx 32.6\%$.

Expected Value of Calling

Continuing this example, suppose that the current pot has \$50, and our opponent has just raised by \$30. We can calculate the expected value of calling, and use this to determine whether a call is profitable or not. By using our out odds as a rough substitute for our equity, or the chance that we win the pot, we can use the formula for expected value:

$$E[X] = P(\text{winning}) \cdot \text{gain} - P(\text{losing}) \cdot \text{loss}$$

So for instance, in this example, our probability of winning would be 32.6%, and our probability of losing would be $100\% - 32.6\% = 67.4\%$. Our gain would be equal to \$80, and our loss would be \$30 because we are essentially putting in \$30 to have a chance of winning the pot after our opponent has raised. Plugging these values into the formula, we obtain:

$$E[X] = 0.326 \cdot \$80 - 0.674 \cdot \$30 = \$5.83$$

Hence, a call is theoretically profitable.

However, this model skips out on many other details associated with the game of poker. For one, this assumes that an out is equivalent to winning the round, which isn't really reasonable. The opponent could have any range of cards that beat your hand, and there really isn't any way to know for certain. Furthermore, there's also the knowledge of knowing how likely your opponent is to bluff. Depending on this factor, it may be ideal to bet on unprofitable calls. But above all, these formulae are ultimately guidelines for you to follow. Statistics can and will help guide you along in times of confusion, but its your intuition that is your strongest weapon, and what will win you the most games.

Citations

[1] A. Bărbulescu. Texas Hold'em Poker probabilities. Available at `http://probability.infarom.ro/holdempoker.html`

[2] PokerStrategy.com. Texas Hold'em probabilities. Available at `https://www.pokerstrategy.com/strategy/various-poker/texas-holdem-probabilities/`

[3] Wikipedia contributors. Poker probability. Wikipedia, The Free Encyclopedia. Available at `https://en.wikipedia.org/wiki/Poker_probability`

Appendices

Some code modelling poker hands:

```python
    for _ in range(num_simulations):
        deck = create_deck()
        random.shuffle(deck)

        player1_cards = deck[:2]
        player2_cards = deck[2:4]
        community_cards = deck[4:9]

        player1_hand = find_best_hand(player1_cards + community_cards)
        player2_hand = find_best_hand(player2_cards + community_cards)

        hand_type_counters['Player 1'][player1_hand] += 1
        hand_type_counters['Player 2'][player2_hand] += 1

    for player, counters in hand_type_counters.items():
        print(f"{player}:")
        for hand_type, count in counters.items():
            print(f"{hand_type}: {count}")
        print()
```

This code simulates multiple rounds of a poker game, shuffling and distributing a deck of cards between two players and the community pool in each round. It eventually prints a summary of how often each type of hand occurred for each player, providing insights into the distribution of hand types over the simulated games.

Some code modelling winning frequencies in a two-player game:

```python
    def simulate_games(num_games):
        player1_wins = 0
        player2_wins = 0
        ties = 0

        for _ in range(num_games):
            deck = create_deck()
            player1_hand, player2_hand, community_cards = deal_hands(deck)

            result = evaluate_and_compare_hands
            (player1_hand + community_cards, player2_hand + community_cards)

            if result == 'player1':
                player1_wins += 1
            elif result == 'player2':
                player2_wins += 1
            else:
                ties += 1

        return player1_wins, player2_wins, ties
```

The "simulate_games" function runs a specified number of poker games between two players, counting wins for each player and ties. Wins for each player and ties are tallied and returned at the end, providing a summary of game results.