Do you like Texas hold 'em

STAT230 Real World Assignment 2024W

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In this report, we consider the case a player uses the best five-card poker hand out of seven cards.

The stages consist of a series of three cards ("the flop"), later an additional single card ("the turn"), and a final card ("the river"). Each player seeks the best five-card poker hand from any combination of the seven cards: the five community cards and their two hole cards.

For a 7-card hand to contain a Royal Flush, i.e.

A$\heartsuit$	K$\heartsuit$	Q$\heartsuit$	J$\heartsuit$	10$\heartsuit$

it must contain the specific set of 5 cards (Ace, King, Queen, Jack, 10 of the same suit), with the other 2 cards being any of the remaining 47 cards in the deck. Therefore, the probability of getting a Royal Flush in a group of seven cards can be evaluated as

$$P(\text{Royal Flush in 7-card poker}) = \frac{4 \times C(47, 2)}{C(52, 7)}$$

Out of the $\binom{52}{2}$ possible starting hands, there are 13×13 distinct types of starting hands since there are 13 ranks, there are 13 distinct pocket pairs (e.g. AA), $\binom{13}{2} = 78$ distinct suited 2-card combinations (e.g. AKs) and $\binom{13}{2} = 78$ distinct unsuited 2-card combinations (e.g. AKo). Notice that

$$13 \times 13 = 13 + \binom{13}{2} + \binom{13}{2}$$

It is intended to calculate the pre-flop (before any community cards are dealt) win rate for each distinct 2-card combination. For each distinct combination, there are

$$\left[\prod_{i=1}^{n-1} \binom{52-2i}{2} \right] \times \binom{52-2n}{5}$$

possible outcomes for n players. A simulation (Appendix) size of 100000 is chosen for this section (with $n = 2$ players):

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Pocket Pair:  5976
Suited:  23167
Unsuited:  76833
Contains Ace or King:  28643
Does Not Contain Ace or King:  71357
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The chance of making, winning and tying with each poker hand is investigated. The total number of possible outcomes for a hand of poker between n players is given by:

$$\left[\prod_{i=0}^{n-1} \binom{52-2i}{2} \right] \cdot \binom{52-2n}{2}$$

which is 2.781×10^{12} for just 2 players. Since it isn't realistic to go through all possible outcomes, a simulation size of 100 000 is chosen.