Do you like Texas hold 'em

STAT230 Real World Assignment 2024W

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In this report, we consider the case a player uses the best five-card poker hand out of seven cards.

The stages consist of a series of three cards ("the flop"), later an additional single card ("the turn"), and a final card ("the river"). Each player seeks the best five-card poker hand from any combination of the seven cards: the five community cards and their two hole cards.

For a 7-card hand to contain a Royal Flush, i.e.

A♡	K♡	Q♡	J♡	10♡

it must contain the specific set of 5 cards (Ace, King, Queen, Jack, 10 of the same suit), with the other 2 cards being any of the remaining 47 cards in the deck. Therefore, the probability of getting a Royal Flush in a group of seven cards can be evaluated as

$$P(\text{Royal Flush in 7-card poker}) = \frac{4 \times C(47, 2)}{C(52, 7)}$$

Starting Hands (with 2 players)

Out of the $\binom{52}{2}$ possible starting hands, there are 13×13 distinct types of starting hands since there are 13 ranks, there are 13 distinct pocket pairs (e.g. AA), $\binom{13}{2} = 78$ distinct suited 2-card combinations (e.g. AKs) and $\binom{13}{2} = 78$ distinct unsuited 2-card combinations (e.g. AKo). Notice that

$$13 \times 13 = 13 + \binom{13}{2} + \binom{13}{2}$$

It is intended to calculate the pre-flop (before any community cards are dealt) win rate for each distinct 2-card combination. For each distinct combination, there are

$$\left[\prod_{i=1}^{n-1} \binom{52-2i}{2}\right] \times \binom{52-2n}{5}$$

possible outcomes for n players. A simulation (Appendix) size of 100 000 is chosen for this section:

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Pocket Pair:  5976
Suited:  23167
Unsuited:  70857
Contains Ace or King:  28643
Does Not Contain Ace or King:  71357
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Recall that we consider in a game consisting two players. In theory, we know that the probability of getting pocket pair in a deck of 52 cards is

$$\frac{52}{52} \cdot \frac{3}{51} \approx \boxed{0.058823}$$

the probability for suited pair is

$$\frac{52}{52} \cdot \frac{12}{51} \approx \boxed{0.235294}$$

while the probability for unsuited pair is

$$\frac{52}{52} \cdot \frac{39}{51} - \frac{52}{52} \cdot \frac{3}{51} \approx \boxed{0.707014}$$

Lmao our simulations are pretty close to our theoretical values.

Poker Hands

The chance of making, winning and tying with each poker hand is investigated. The total number of possible outcomes for a hand of poker between n players is given by:

$$\left[\prod_{i=0}^{n-1} \binom{52-2i}{2}\right] \cdot \binom{52-2n}{2}$$

which is 2.781×10^{12} for just 2 players. Since it isn't realistic to go through all possible outcomes, a simulation (Appendix) size of 100 000 is chosen (with 2 players):

Player 1 :	*Player* 2 :
`Straight Flush: 4`	`Straight Flush: 5`
`Quad: 15`	`Quad: 11`
`Full House: 250`	`Full House: 259`
`Flush: 307`	`Flush: 274`
`Straight: 447`	`Straight: 439`
`Triple: 536`	`Triple: 461`
`Two Pairs: 2341`	`Two Pairs: 2368`
`Pair: 4355`	`Pair: 4395`
`High Card: 1745`	`High Card: 1788`

We can also calculate the theoretical expression of the absolute frequency of the occurences of each hands:

1. **Straight flush (including straight flush):**

$$\binom{10}{1}\binom{4}{1}\binom{46}{2} = 4,324$$

2. **Quad:**

$$\binom{13}{1}\binom{48}{3} = 224,848$$

3. **Full house:**

$$\left[\binom{13}{2}\binom{4}{3}^2\binom{44}{1}\right] + \left[\binom{13}{1}\binom{12}{2}\binom{4}{3}\binom{4}{2}^2\right] + \left[\binom{13}{1}\binom{12}{1}\binom{11}{2}\binom{4}{3}\binom{4}{2}\binom{4}{1}^2\right] \approx 3,473,184$$

4. **Flush:**

$$\left[\binom{4}{1} \times \left[\binom{13}{7} - 217\right]\right] + \left[\binom{4}{1} \times \left[\binom{13}{6} - 71\right] \times 39\right] + \left[\binom{4}{1} \times \left[\binom{13}{5} - 10\right] \times \binom{39}{2}\right] \approx 4,047,644$$

5. **Straight:**

$$[217 \times [4^7 - 756 - 4 - 84]] + [71 \times 36 \times 990] + \left[10 \times 5 \times 4 \times [256 - 3] + 10 \times \binom{5}{2} \times 2268\right] \approx 6,180,020$$

6. **Triple:**

$$\left[\binom{13}{5} - 10\right]\binom{5}{1}\binom{4}{1}\left[\binom{4}{1}^4 - 3\right] \approx 6,461,620$$

7. **Two pair:**

$$[1277 \times 10 \times [6 \times 62 + 24 \times 63 + 6 \times 64]] + \left[\binom{13}{3}\binom{4}{2}^3\binom{40}{1}\right] \approx 31,433,400$$

8. **One pair:**

$$\left[\binom{13}{6} - 71\right] \times 6 \times 6 \times 990 \approx 58,627,800$$

9. **High cards:**

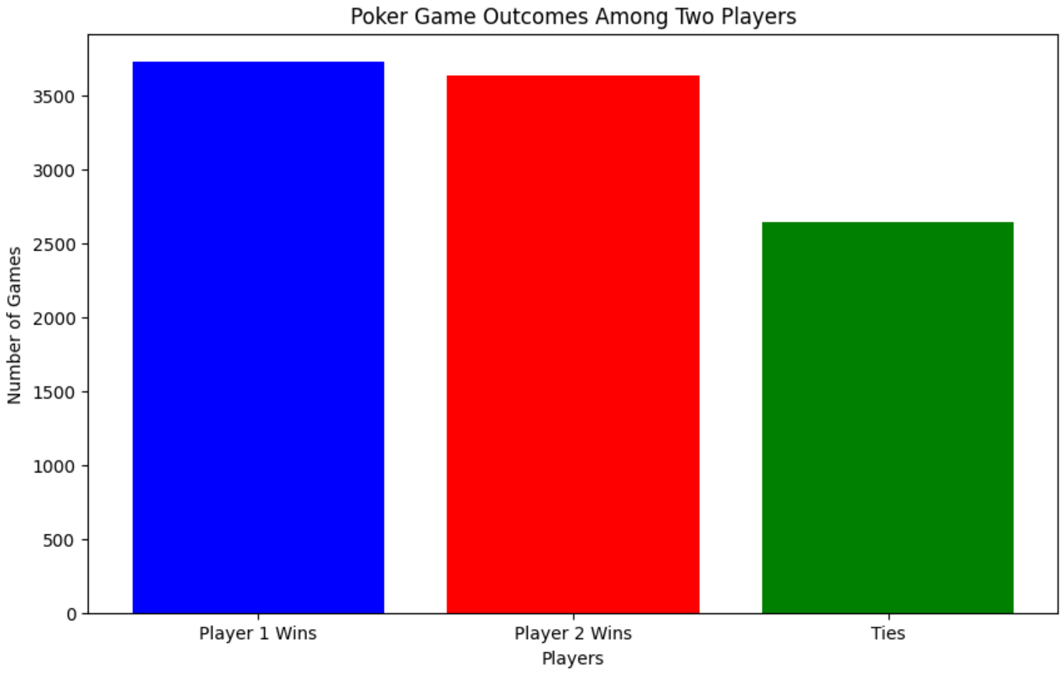
$$1499 \times [4^7 - 756 - 4 - 84] \approx 23,294,460$$

Therefore, since we know that the total number of possible combination of 7 cards is $\binom{52}{7} = 133,784,560$, hence we can find the probability of each hand:

Poker Hands	Probability
A♡ K♡ Q♡ J♡ 10♡ Strait Flush	$\frac{4,324}{133,784,560} \approx 0.0311\%$
A♡ A♣ A♢ A♠ 6♡ Quad	$\frac{224,848}{133,784,560} \approx 0.168\%$
8♡ 8♢ 8♠ K♠ K♢ Full House	$\frac{3,473,184}{133,784,560} \approx 2.60\%$
8♣ 3♣ 6♣ J♣ A♣ Flush	$\frac{4,047,644}{133,784,560} \approx 3.03\%$
7♡ 8♡ 9♢ 10♠ J♢ Straight	$\frac{6,180,020}{133,784,560} \approx 4.62\%$
Q♡ Q♠ Q♢ 5♡ A♢ Triple	$\frac{6,461,620}{133,784,560} \approx 4.83\%$
3♠ 3♣ 6♡ 6♡ Q♣ Two pairs	$\frac{31,433,400}{133,784,560} \approx 23.5\%$
5♡ 5♠ 2♣ J♣ A♢ One pair	cell 2
2♢ 5♠ 6♠ J♡ A♣ High hands	cell 2

How many times does each player win in a 2-player poker game

We can also simulate how many times each player wins:



where we can find that the number of wins fairly close among the two players.
Moreover, in a game of 4 player, the winning rates are still fairly close among all paarticipants: