Do you like Texas hold 'em

STAT230 Real World Assignment 2024W

Eason Li, Johnson Ji

Falculty of Math
University of Waterloo
Canada
April 8, 2024

In this report, we consider the case a player uses the best five-card poker hand out of seven cards.

Starting Hands (with 2 players)

Out of the $\binom{52}{2}$ possible starting hands, there are 13×13 distinct types of starting hands since there are 13 ranks, there are 13 distinct pocket pairs (e.g. AA), $\binom{13}{2} = 78$ distinct suited 2-card combinations (e.g. AKs) and $\binom{13}{2} = 78$ distinct unsuited 2-card combinations (e.g. AKo). Notice that

$$13 \times 13 = 13 + \binom{13}{2} + \binom{13}{2}$$

It is intended to calculate the pre-flop (before any community cards are dealt) win rate for each distinct 2-card combination. For each distinct combination, there are

$$\left[\prod_{i=1}^{n-1} \binom{52-2i}{2}\right] \times \binom{52-2n}{5}$$

possible outcomes for n players. A simulation (Appendix) size of 100 000 is chosen for this section:

```
Pocket Pair:  5976
Suited:  23167
Unsuited:  70857
Contains Ace or King:  28643
Does Not Contain Ace or King:  71357
```

Recall that we consider in a game consisting two players. In theory, we know that the probability of getting pocket pair in a deck of 52 cards is

$$\frac{52}{52} \cdot \frac{3}{51} \approx \boxed{0.058823}$$

the probability for suited pair is

$$\frac{52}{52} \cdot \frac{12}{51} \approx \boxed{0.235294}$$

while the probability for unsuited pair is

$$\frac{52}{52} \cdot \frac{39}{51} - \frac{52}{52} \cdot \frac{3}{51} \approx \boxed{0.707014}$$

Lmao our simulations are pretty close to our theoretical values.

Poker Hands

The chance of making, winning and tying with each poker hand is investigated. The total number of possible outcomes for a hand of poker between n players is given by:

$$\left[\prod_{i=0}^{n-1} \binom{52-2i}{2}\right] \cdot \binom{52-2n}{2}$$

which is 2.781×10^{12} for just 2 players. Since it isn't realistic to go through all possible outcomes, a simulation (Appendix) size of 100 000 is chosen (with 2 players):

Player 1 :	*Player* 2 :
`Straight Flush: 4`	`Straight Flush: 5`
`Quad: 15`	`Quad: 11`
`Full House: 250`	`Full House: 259`
`Flush: 307`	`Flush: 274`
`Straight: 447`	`Straight: 439`
`Triple: 536`	`Triple: 461`
`Two Pairs: 2341`	`Two Pairs: 2368`
`Pair: 4355`	`Pair: 4395`
`High Card: 1745`	`High Card: 1788`

We can also calculate the theoretical expression of the absolute frequency of the occurences of each hands:

1. **Straight flush (including royal straight flush):**

$$\binom{10}{1}\binom{4}{1}\binom{46}{2} = 4,324$$

2. **Quad:**

$$\binom{13}{1}\binom{48}{3} = 224,848$$

3. **Full house:**

$$\left[\binom{13}{2}\binom{4}{3}^2\binom{44}{1}\right] + \left[\binom{13}{1}\binom{12}{2}\binom{4}{3}\binom{4}{2}^2\right] + \left[\binom{13}{1}\binom{12}{1}\binom{11}{2}\binom{4}{3}\binom{4}{2}\binom{4}{1}^2\right] \approx 3,473,184$$

4. **Flush:**

$$\left[\binom{4}{1} \times \left[\binom{13}{7} - 217\right]\right] + \left[\binom{4}{1} \times \left[\binom{13}{6} - 71\right] \times 39\right] + \left[\binom{4}{1} \times \left[\binom{13}{5} - 10\right] \times \binom{39}{2}\right] \approx 4,047,644$$

5. **Straight:**

$$[217 \times [4^7 - 756 - 4 - 84]] + [71 \times 36 \times 990] + \left[10 \times 5 \times 4 \times [256 - 3] + 10 \times \binom{5}{2} \times 2268\right] \approx 6,180,020$$

6. **Triple:**

$$\left[\binom{13}{5} - 10\right]\binom{5}{1}\binom{4}{1}\left[\binom{4}{1}^4 - 3\right] \approx 6,461,620$$

7. **Two pair:**

$$[1277 \times 10 \times [6 \times 62 + 24 \times 63 + 6 \times 64]] + \left[\binom{13}{3}\binom{4}{2}^3\binom{40}{1}\right] \approx 31,433,400$$

8. **One pair:**

$$\left[\binom{13}{6} - 71\right] \times 6 \times 6 \times 990 \approx 58,627,800$$

9. **High cards:**

$$1499 \times [4^7 - 756 - 4 - 84] \approx 23,294,460$$

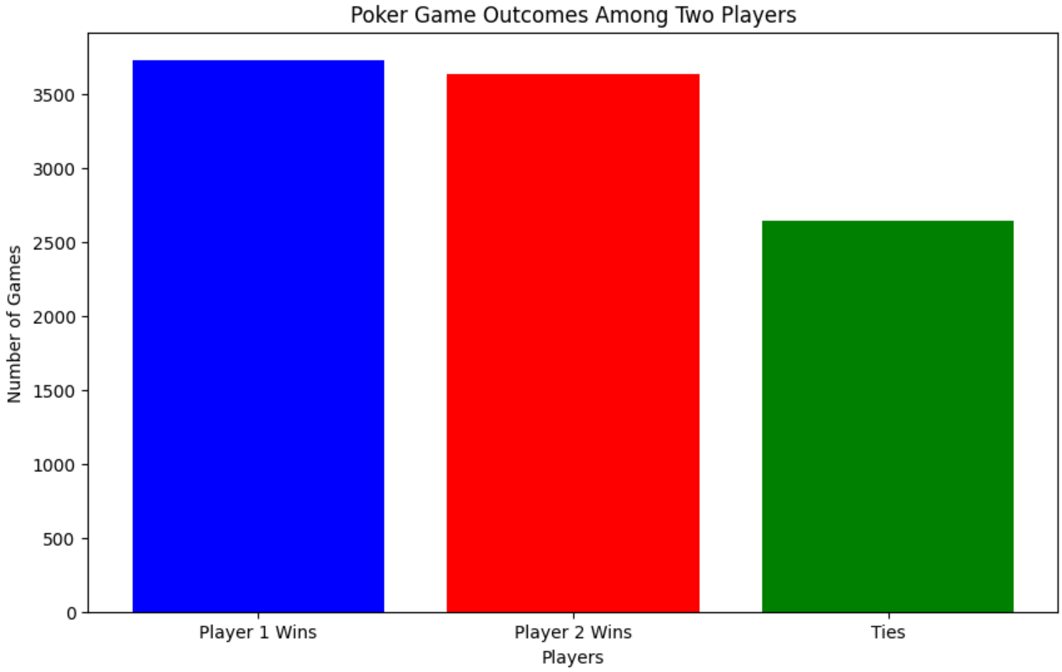
Therefore, since we know that the total number of possible combination of 7 cards is $\binom{52}{7} = 133,784,560$, hence we can find the probability of each hand:

Poker Hands	Probability
A$\heartsuit$ K$\heartsuit$ Q$\heartsuit$ J$\heartsuit$ 10$\heartsuit$ Strait Flush	$\frac{4,324}{133,784,560} \approx 0.0311\%$
A$\heartsuit$ A$\clubsuit$ A$\diamondsuit$ A$\spadesuit$ 6$\heartsuit$ Quad	$\frac{224,848}{133,784,560} \approx 0.168\%$
8$\heartsuit$ 8$\diamondsuit$ 8$\spadesuit$ K$\spadesuit$ K$\diamondsuit$ Full House	$\frac{3,473,184}{133,784,560} \approx 2.60\%$
8$\clubsuit$ 3$\clubsuit$ 6$\clubsuit$ J$\clubsuit$ A$\clubsuit$ Flush	$\frac{4,047,644}{133,784,560} \approx 3.03\%$
7$\heartsuit$ 8$\heartsuit$ 9$\diamondsuit$ 10$\spadesuit$ J$\diamondsuit$ Straight	$\frac{6,180,020}{133,784,560} \approx 4.62\%$
Q$\heartsuit$ Q$\spadesuit$ Q$\diamondsuit$ 5$\heartsuit$ A$\diamondsuit$ Triple	$\frac{6,461,620}{133,784,560} \approx 4.83\%$
3$\spadesuit$ 3$\clubsuit$ 6$\heartsuit$ 6$\heartsuit$ Q$\clubsuit$ Two pairs	$\frac{31,433,400}{133,784,560} \approx 23.5\%$
5$\heartsuit$ 5$\spadesuit$ 2$\clubsuit$ J$\clubsuit$ A$\diamondsuit$ One pair	$\frac{58,627,800}{133,784,560} \approx 43.8\%$
2$\diamondsuit$ 5$\spadesuit$ 6$\spadesuit$ J$\heartsuit$ A$\clubsuit$ High hands	$\frac{23,294,460}{133,784,560} \approx 17.4\%$

We can easily discover that all the probabilities sum up to be 1, thus we can that this is in fact a multinomial distribution.

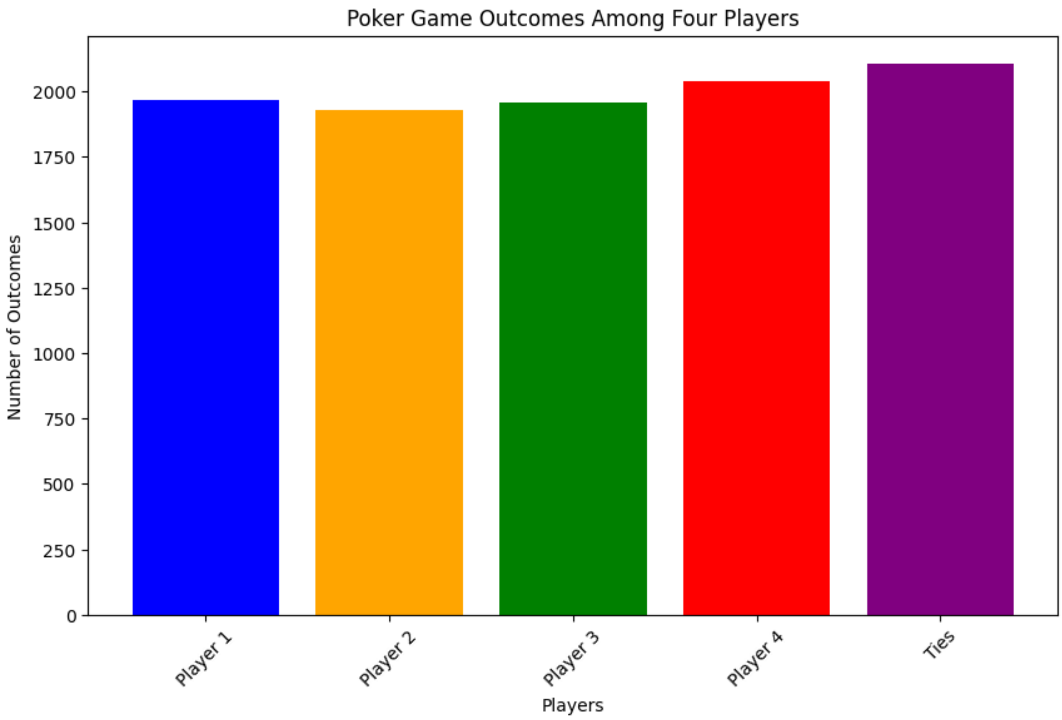
How many times does each player win in a 2-player poker game

We can also simulate how many times each player wins:



The bar chart presents the outcomes of poker games between two players. It shows the frequency of wins for Player 1, Player 2, and the number of games that ended in a tie. Statistically, we can observe that each no player has a significant advantage over the other. The small difference in the heights of the bars for Player 1 Wins and Player 2 Wins implies a roughly uniform distribution of outcomes between the two players.

Moreover, in a game of 4 player, the winning rates are still fairly close among all participants:



Appendices

Some code modelling poker hands:

```
        for _ in range(num_simulations):
            deck = create_deck()
            random.shuffle(deck)

            player1_cards = deck[:2]
            player2_cards = deck[2:4]
            community_cards = deck[4:9]

            player1_hand = find_best_hand(player1_cards + community_cards)
            player2_hand = find_best_hand(player2_cards + community_cards)

            hand_type_counters['Player 1'][player1_hand] += 1
            hand_type_counters['Player 2'][player2_hand] += 1

        for player, counters in hand_type_counters.items():
            print(f"{player}:")
            for hand_type, count in counters.items():
                print(f"{hand_type}: {count}")
            print()
```

This code simulates multiple rounds of a poker game, shuffling and distributing a deck of cards between two players and the community pool in each round. It eventually prints a summary of how often each type of hand occurred for each player, providing insights into the distribution of hand types over the simulated games.

Some code modelling winning frequencies in a two-player game:

```
        def simulate_games(num_games):
            player1_wins = 0
            player2_wins = 0
            ties = 0

            for _ in range(num_games):
                deck = create_deck()
                player1_hand, player2_hand, community_cards = deal_hands(deck)

                result = evaluate_and_compare_hands
                (player1_hand + community_cards, player2_hand + community_cards)

                if result == 'player1':
                    player1_wins += 1
                elif result == 'player2':
                    player2_wins += 1
                else:
                    ties += 1

            return player1_wins, player2_wins, ties
```

The "simulate_games" function runs a specified number of poker games between two players, counting wins for each player and ties. Wins for each player and ties are tallied and returned at the end, providing a summary of game results.