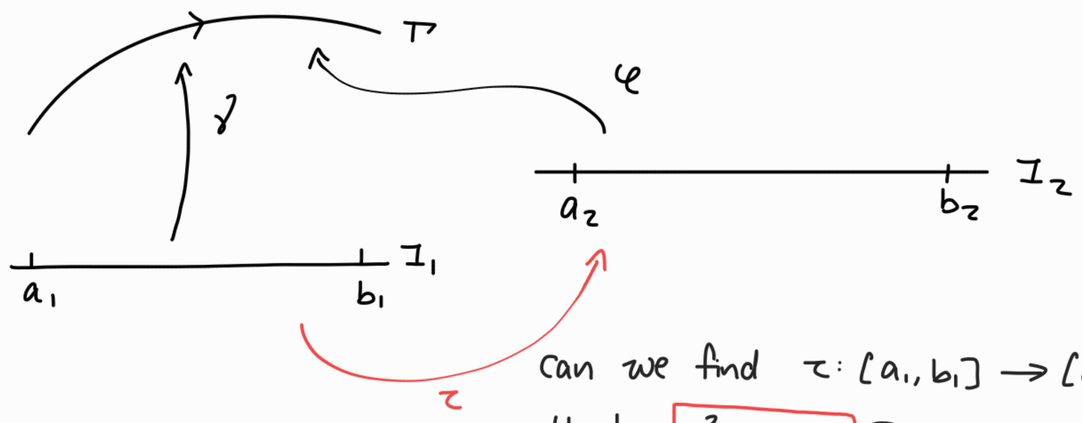
Proposition 2. *The integral of a 1-form over an oriented closed segment of a curve is independent of the parameter for parameters giving the same orientation. (When the orientation is reversed, the integral changes sign.)*



Can we find $\tau : [a_1, b_1] \to [a_2, b_2]$ such that $\boxed{\gamma = \varphi \circ \tau}$?

Yes, as long as we assume γ and φ are bijections between T and I_1, I_2.

$\boxed{\tau = \varphi^{-1} \circ \gamma}$

i.e. $\tau(a_1) = a_2$

$\tau(b_1) = b_2$

and $u = \tau(t)$

Need to show:

$$\int_{a_1}^{b_1} \omega(\gamma'(t))\,dt = \int_{a_2}^{b_2} \omega(\varphi'(u))\,du$$

$$\hookrightarrow \int_{a_1}^{b_1} \omega\left(\frac{d}{dt}\gamma(t)\right) dt$$

$$= \int_{a_1}^{b_1} \omega\left(\frac{d}{dt}(\varphi \circ \tau(t))\right) dt$$

$$= \int_{a_1}^{b_1} \omega\left(\varphi'(\tau(t)) \cdot \tau'(t)\right) dt$$

scalar

can move out because ω is linear

$$= \int_{a_1}^{b_1} \omega(\varphi'(\tau(t)))\,\tau'(t)\,dt$$

$$= \int_{\tau(a_1)}^{\tau(b_1)} \omega(\varphi'(\tau(t)))\,d\tau(t)$$

$$= \int_{a_2}^{b_2} \omega(\varphi'(u))\,du \quad \square$$