Problem Set 8

Due Dec 26, before class

1. MLE in GLMs with a binary covariate

The MLEs for GLMs do not have explicit formulas in general. But when the covariate x_i containing 1 and a binary covariate $z_i \in \{0, 1\}$, the MLEs do have simple formulas. Find the MLEs in the Normal linear model, binary logistic regression model, and Poisson regression model with $x_i = (1, z_i)$ where z_i is binary. Then find the variance estimators of $\hat{\beta}$ in the three models.

2. MLE in GLMs with a binary covariate with possible misspecification

Following the last problem, find the variance estimators of $\hat{\beta}$ in the three models without assuming the models are correct.

3. Cluster-robust standard error in OLS with a cluster-specific binary regressor

Consider a special case in GEE with $x_{it} = (1, x_i)^T$ and $x_i \in \{0, 1\}$ for $i = 1, \ldots, n$ and view "1" as treatment and "0" as control. Show that the coefficient of x_i in the OLS fit of y_{it} on x_{it} (using independent working covariance) equals $\hat{\tau} = \bar{y}_1 - \bar{y}_0$, where

$$\bar{y}_1 \;=\; \sum_{i=1}^{n}\sum_{t=1}^{n_i} x_i y_{it}/N_1, \quad \bar{y}_0 \;=\; \sum_{i=1}^{n}\sum_{t=1}^{n_i}(1-x_i) y_{it}/N_0$$

with $N_1 = \sum_{i=1} n_i x_i$ and $N_0 = \sum_{i=1}^{n} n_i(1 - x_i)$ denoting the total number of observations under treatment and control, respectively. Further show that the cluster-robust standard error of $\hat{\tau}$ equals the square root of

$$\frac{\sum_{i=1}^{n} x_i R_i^2}{N_1^2} + \frac{\sum_{i=1}^{n}(1 - x_i)R_i^2}{N_0^2},$$

where

$$R_i \;=\; \begin{cases} \sum_{t=1}^{n_i}(y_{it} - \bar{y}_1), & \text{if } x_i = 1, \\ \sum_{t=1}^{n_i}(y_{it} - \bar{y}_0), & \text{if } x_i = 0. \end{cases}$$

4. Robust standard errors in the *gym* data

We analyzed the *gym* data for each week using the models for count outcomes in class. Report the robust standard errors in the analysis.

5. Pooled analysis of the *gym* data

Re-analyze the *gym* data using exchangeable working covariance matrix. You can read the original article to help you choose link functions and time periods.

"Royer, H., Stehr, M., and Sydnor, J. (2015). Incentives, commitments, and habit formation in exercise: evidence from a field experiment with workers at a fortune-500 company. *American Economic Journal: Applied Economics*, 7:51–84."