

$B1 = \{a^k \mid k \text{ is a multiple of } 1\} = \{a^0, a^1, a^2\}$

Use graph notation for proofs

Finite state transducer

M_k : for calculating modulo /of k

$\Sigma = \{0, 1, 2, \dots, 9\}$

If m_k ends at state q_1 , then the input of $\% k$ is i

Proof by induction

Prove base case

Induction step

Goal is to show if its true for $a = k$, then its true for $n = k+1$

Double statements need cartesian product FSM

Closure

A set of objects is closed under an operation if applying that operations to members of that set always results in a members of that set

Regular languages are closed under the three regular operations we just introduced (union, concatenation, star)

Can you look ahead to see why we care

We can build FA to recognize regular expression

Closure of union

Theorem 1.25 The class of regular languages is closed under the union operation.

$A1 \cup A2$

$M1$ recognize $A1$

$M2$ recognize $A2$

We build m that recognizes $A1xA2$

$= (Q1 \times Q2, \epsilon, \delta: ((q_i, q_j), a) = (\delta_1(q_i, a), \delta_2(q_j, a)))$

$M1$ recognize $A1$

$M2$ recognize $A2$

Use epsilon to branch to both