Matrix Analysis Homework

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4.Determine the general solution for each of the following nonhomogeneous systems.

$$(a)\quad \begin{array}{r} x_1+2x_2+x_3+2x_4=3 \\ 2x_1+4x_2+x_3+3x_4=4 \\ 3x_1+6x_2+x_3+4x_4=5 \end{array}$$

Solution: Reducing the augmented matrix $[\mathbf{A}|\mathbf{b}]$ to $\mathbf{E}_{[\mathbf{A}|\mathbf{b}]}$ yields.

$$\mathbf{A} = \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 3 \\ 2 & 4 & 1 & 3 & 4 \\ 3 & 6 & 1 & 4 & 5 \end{array}\right) \rightarrow \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 3 \\ 0 & 0 & -1 & -1 & -2 \\ 0 & 0 & -2 & -2 & -4 \end{array}\right)$$
$$\rightarrow \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 3 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & -2 & -2 & -4 \end{array}\right) \rightarrow \left(\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) = \mathbf{E}_{[\mathbf{A}|\mathbf{b}]}$$

The system is consistent for the last column is nonbasic, Solve the reduced system for the basic variables x_1 and x_3 in terms of the free variables x_2 and x_4. So the general solution is

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1-2*x_2-x_4 \\ x_2 \\ 2-x_4 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \end{pmatrix}$$

$$(b)\quad \begin{array}{r} 2x+y+z=4 \\ 4x+2y+z=6 \\ 6x+3y+z=8 \\ 8x+4y+z=10 \end{array}$$

Solution: Reducing the augmented matrix $[\mathbf{A}|\mathbf{b}]$ to $\mathbf{E}_{[\mathbf{A}|\mathbf{b}]}$ yields.

$$
\mathbf{A} = \left(\begin{array}{ccc|c} 2 & 1 & 1 & 4 \\ 4 & 2 & 1 & 6 \\ 6 & 3 & 1 & 8 \\ 8 & 4 & 1 & 10 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & \frac{1}{2} & \frac{1}{2} & 2 \\ 4 & 2 & 1 & 6 \\ 6 & 3 & 1 & 8 \\ 8 & 4 & 1 & 10 \end{array}\right)
$$

$$
\rightarrow \left(\begin{array}{ccc|c} 1 & \frac{1}{2} & \frac{1}{2} & 2 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & -2 & -4 \\ 0 & 0 & -3 & -6 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & \frac{1}{2} & \frac{1}{2} & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -2 & -4 \\ 0 & 0 & -3 & -6 \end{array}\right)
$$

$$
\rightarrow \left(\begin{array}{ccc|c} 1 & \frac{1}{2} & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) = \mathbf{E}_{[\mathbf{A}|\mathbf{b}]}
$$

The system is consistent for the last column is nonbasic, Solve the reduced system for the basic variables x_1 and x_3 in terms of the free variables x_2 and x_4. So the general solution is

$$
\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 - \frac{1}{2}y \\ y \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + y \begin{pmatrix} -\frac{1}{2} \\ 1 \\ 0 \end{pmatrix}
$$