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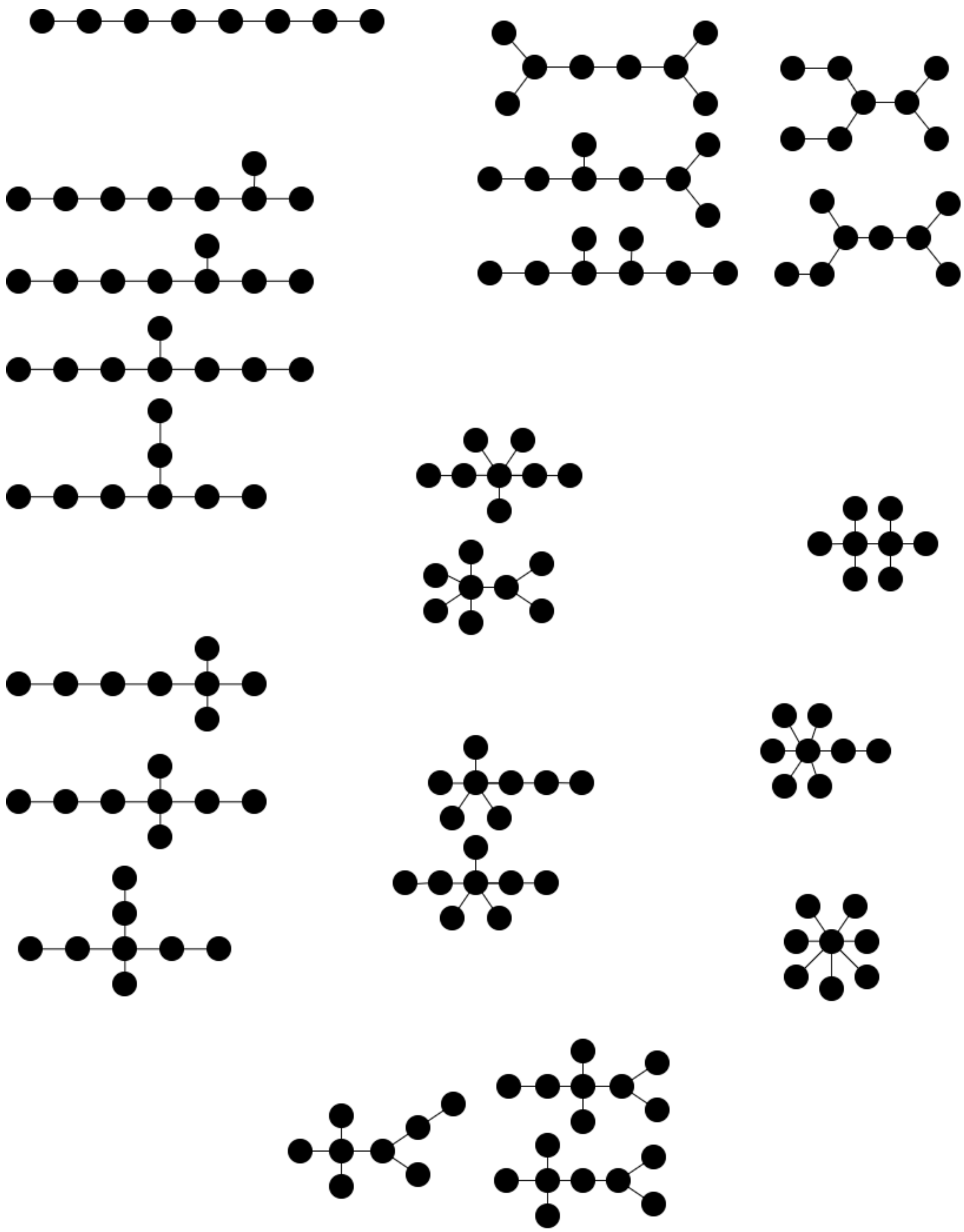
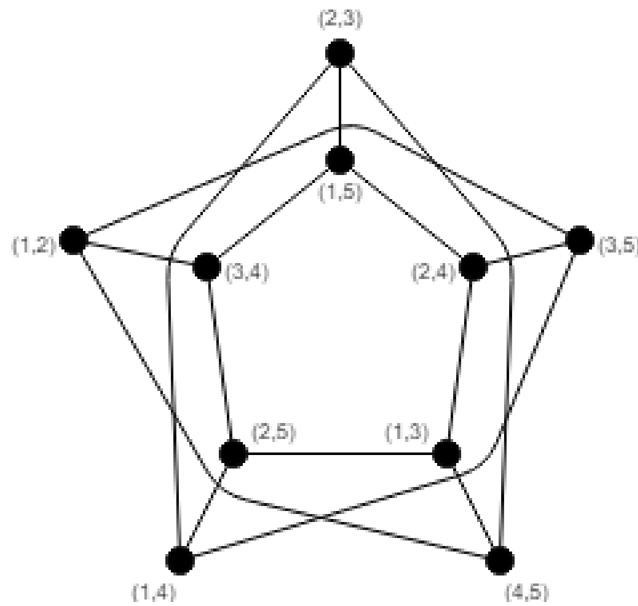
Homework 7

Problem #1
Please sort all trees on 8 vertices into homeomorphism classes.

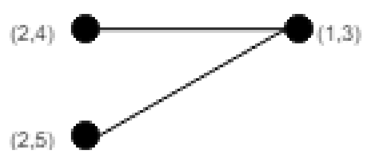
Here, I drew out all different trees on 8 vertices and grouped them up based on their homeomorphic classes:



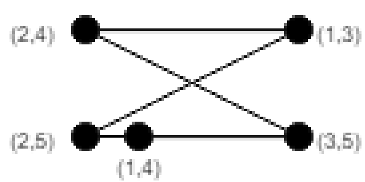
Problem #2
Show that the graph G (defined later) is not planar in two ways: (1) Use Kuratowski's Theorem, and (2) use the Euler identity $n - e + f = 2$.
Define $G = (V, E)$ as follows. Let $V = \{$2-sets of $[5]\}$, with vertices x and y adjacent if and only if $x \cap y = \emptyset$. First, we can use Kuratowski's Theorem, which states that a graph is planar if and only if it does not contain a subgraph that is a subdivision of K_5 or of $K_{3,3}$. First, let us being by showing our graph G:



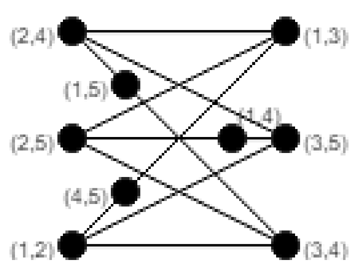
We can rearrange the different edges and vertices in this graph to show a subgraph of it does contain a subdivision of $K_(3,3)$. We can pick three vertices that are adjacent to one another, and from there determine how other vertices link up to the graph. So, let's first pick three adjacent vertices:



These vertices link directly to one another quite nicely. Now, let's look for another vertex we can connect onto. For this one, we will allow our links to pass through other vertices in the graph. This allows us to reorganize it into this:



Finally, let's look for other vertices and how they link up to one another following a similar approach:



As we can tell, we have been able to find a subgraph of G that is a subdivision of $K_(3,3)$. This means that the graph G is not planar.

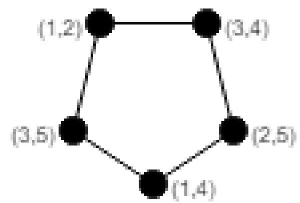
Now, using the Euler identity, we can start by determining how many faces our graph would need to have in order to be planar. By simply counting nodes and edges, we come to the conclusion:

$$n - e + f = 2$$
$$(10) - (15) + f = 2$$
$$f = 7$$

Our graph would need to have 7 faces for Euler's identity to hold up. Let's assume it to be a planar graph, if it is, then it must meet the property outlined in **Lemma 11.3.1**: Suppose G is a planar graph, $e(G)$ is the number of edges of G, and $R(G)$ is the set of regions created in some planar embedding of G; then

$$\sum_{r \in R(G)} d(r) = 2e(G).$$

Let's see if this property holds up. First, let's determine the minimum number of vertices needed to form a cycle in G. This number turns out to be 5, this is shown below:



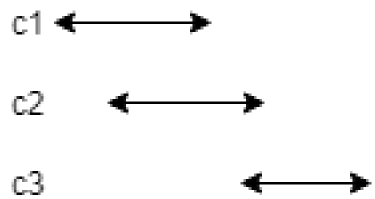
So, since the minimum number of nodes needed to form a face is 5, and the number of faces we require is 7, we can use the equality outlined earlier and determine if it holds up:

$$\sum_{r \in R(G)} d(r) = 2e(G)$$
$$35 = 7 * 5 \leq \sum_{r \in R(G)} d(r) = 2(15)$$
$$35 \leq \sum_{r \in R(G)} d(r) = 30$$

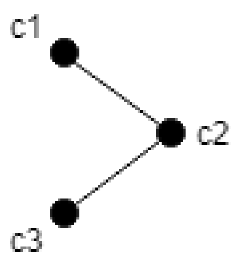
Since 35 is not lesser than, or equal to 30, then we have determined that G is indeed not planar using Euler's identity.

Problem #3
Preferably using Graph Theory, please model the following problem and describe a solution. Suppose $C = \{c_1, c_2, \ldots, c_n\}$ is a collection of chemicals which must be stored very carefully at very specific temperatures. For each $c_i \in C$, you know the lowest temperature at which it can be stored, call it ℓ_i, and highest temperature at which it can be stored, call it h_i. Here's the problem: Determine the smallest number of temperature-controlled storage units into which the chemicals can be stored.

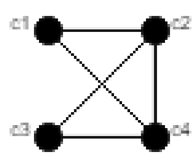
All chemicals can be stored within certain temperature ranges. Some chemicals may be stored with one another in the same container as long as the container is set to a temperature both chemicals share within their range. This previous fact applies to larger amounts of chemicals. If we assume each chemical to be a node, and an edge between them to be the relation in which both chemicals are capable of being stored together, we can end up with ranges like these:



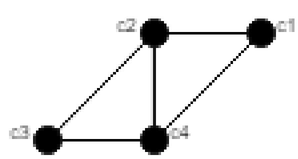
Where the lines represent temperature ranges each element can be stored in. Notice some of them overlap with one another. This representation can be visualized with the graph:



This graph shows that both c1 and c3 can be stored with c2. It also shows that c1 and c3 cannot be stored with each other. When storing these chemicals, we could start by looking at how c1 can be stored with c2, store them in the same container at the temperature both can be stored within, and finally store c3 in a separate container.
Another graph we could make for a different set of temperatures could be:



Which shows all chemicals are able to be stored with one another except c3 and c1. Slightly shifting the nodes gives us:



Which shows something interesting. All nodes that can be stored with one another can be represented as a fully connected graph. So, by organizing our graph into ways that display these fully connected graphs within it, we can find all the various ways to store our chemicals together. This also happens to be the method that allows us to store them in the least amount of containers possible. The reason this is the case is that, by following this method, we will utilize each chemical's relation with one another in the most efficient way possible.