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1) If the characteristic equation of a 3x3 matrix is $\lambda^3 - \lambda^2 + \lambda - 1 = 0$, then the matrix should be

   a) Hermitian b) unitary c) skew symmetric d) identity

2) $2 \lim_{(x,y)\to(0,0)} \frac{x^4+xy}{x^3-y^3}$ is

   a) 0 b) 1 c) -1 d) does not exist

3) If $f(z) = u + iv$ is an analytic function and $u - v = (x-y)^3 + kxy(x-y)$, then k is
   a) 2
   b) -4
   c) 6
   d) -8
4) The directional derivative at the point $P(1,2,3)$ to the surface $x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 3$ in the direction of the vector $\overrightarrow{OP}$, where O denotes the origin, is
   a) 0
   b) $\frac{2}{\sqrt{14}}$
   c) $\frac{3}{\sqrt{14}}$
   d) $\frac{6}{\sqrt{14}}$
5) If the solution of the differential equation

$$\frac{dy}{dx} + P(x)y = xy^3$$

   is $y^2\left(1 + ce^{x^2}\right) = 1$, c being an arbitrary constant, then $P(x)$ is

   a) $-x$ b) $\frac{x}{2}$ c) x d) $2x$

6) The system of equations

$$ax + by + a^2 = 0$$
$$bx + ay - b^2 = 0$$
$$x + y + a - b = 0$$

   a) admits unique solution if $a = b \neq 0$
   b) admits unique solution if $a = -b \neq 0$
   c) admits unique solution if $a = b = 0$
   d) does not admit unique solution

7) The matrix

$$\begin{pmatrix} l & 0 & \sin\theta \\ 0 & 1 & m \\ n & 0 & \cos\theta \end{pmatrix}$$

is orthogonal, if

a) $l = -\sin\theta, m = -\cos\theta, n = 0$
b) $l = -\sin\theta, m = 0, n = \cos\theta$
c) $l = \cos\theta, m = \sin\theta, n = 0$
d) $l = -\cos\theta, m = 0, n = \sin\theta$

8) The radius of convergence of the real power series

$$\sum_{m=0}^{\infty} \frac{(m!)^2}{(2m+1)!} x^m$$

is

a) 4
b) 3
c) 2
d) 1

9) The value of

$$\left(\int_0^{\frac{\pi}{2}} (\sin\theta)^{3/4}\, d\theta\right) x \left(\int_0^{\frac{\pi}{2}} (\sin\theta)^{-3/4} d\theta\right)$$

is

a) $\frac{2\pi}{3}(\sqrt{2}+1)$
b) $\frac{2\pi}{3}(\sqrt{2}-1)$
c) $\frac{\pi}{2}\sqrt{3}$
d) $-\frac{\pi}{2}\sqrt{3}$

10) If $f(z) = y\left(1+x^2\right) + x^2 + i\left(y^2+2y\right)x$ is differentiable at a point $z = z_0$, then $f'(z_0)$ is

a) 0
b) 1
c) i
d) $-i$

11) The value of the integral

$$\oint_{|z|=2} \frac{e^{1/z}}{(z-1)^2} dz$$

is

a) 0
b) $(2e\pi)i$
c) $(4e\pi)i$
d) $(4\pi)i$

12) The absolute value of the integral

$$\oint_c (-z dx + x dy + y dz),$$

where c is the curve obtained by the intersection of $x^2 + y^2 = a^2, a > 0$ and $y = z$, is

a) $\frac{\pi a^2}{\sqrt{2}}$
b) $\frac{\pi a^2}{\sqrt{3}}$
c) $\pi a^2 \sqrt{2}$
d) $2\pi a^2$

13) One of the values of

$$\frac{1}{(4x^2 D^2 + 8xD + 1)}(\ln x) \text{ where } D \equiv \frac{d}{dx},$$

is

a) $\ln x + 4$
b) $\ln x - 4$
c) $4 \ln x - 4$
d) $4 \ln x + 4$

14) A particular integral of the differential equation

$$\frac{d^2y}{dx^2} - y = \sec hx$$

is

a) $-(\cosh x)(\ln \cosh x) + x \sinh x$
b) $-(\sinh x)(\ln \cosh x) + x \cosh x$
c) $(\cosh x)(\ln \sinh x) + x \sinh x$
d) $(\sinh x)(\ln \cosh x) - x \cosh x$

15) If $u = u(x,t)$ is such that $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}, 0 \le x \le \pi, t \ge 0$, $u(0,t) = u(\pi,t) = 0$, $u(x,0) = 0$, $\frac{\partial u}{\partial t}(x,0) = \sin x$, then $u\left(\frac{\pi}{3}, \frac{\pi}{6}\right)$ is

a) $\frac{3}{4}$
b) $\frac{3}{8}$
c) $\frac{\sqrt{3}}{4}$
d) $\frac{\sqrt{3}}{8}$

16) The two lines of regression of the variables x and y are $4x+2.4y = 20$ and $1.6x+4y = 12$. The coefficient of correlation between x and y is

a) 0.49 b) −0.49 c) 0.35 d) −0.35

17) While solving the initial value problem $\frac{dy}{dx} + ky = 0, y(0) = 1$ at $x = h$ by fourth order Runge-Kutta method, the expression for k_3 is

a) $-kh + \frac{(kh)^2}{2!} - \frac{(kh)^3}{3!}$
b) $-kh + \frac{(kh)^2}{2} - \frac{(kh)^3}{3}$
c) $-kh + \frac{(kh)^2}{2} - \frac{(kh)^3}{4}$

d) $-k\left(1+\frac{h}{2}-\frac{h^2}{3}\right)$