2021-st

AI24BTECH11004

1) Let Y follow $N_8\left(0, I_8\right)$ distribution, where I_8 is the $8x8$ identity matrix. let $Y^T \sum_1 Y$ and $Y^T \sum_2 Y$ be independent and follow central chi-square distributions with 3 and 4 degrees of freedom, respectively, where $\sum_1$ and $\sum_2$ are $8x8$ matrices and Y^T denotes transpose of Y. Then which of the following statements is/are true ?
P : $\sum_1$ and $\sum_2$ are idempotent.
Q : $\sum_1 \sum_2 = 0$, where 0 is the $8x8$ zero matrix.
   a) P only
   b) Q only
   c) Both P and Q
   d) Neither P nor Q

The next 12 questions are of numerical answer type(NAT) and carry TWO mark each (no negative marks).

2) Let (X, Y) have a bivariate normal distribution with the joint probability density function

$$f_{X,Y}(x, y) = \frac{1}{\pi} e^{\left(\frac{3}{2}xy - \frac{25}{32}x^2 - 2y^2\right)}, -\infty < x, y < \infty$$

Then $8E(XY)$ equals ______

3) Let $f : RxR \to R$ be defined by $f(x, y) = 8x^2 - 2y$, where R denotes the set of all real numbers. If M and m denotes the maximum and minimum values of f, respectively, on the set $\left\{(x, y) \in RxR : x^2 + y^2 = 1\right\}$, then $M - m$ equals ______ (round off to 2 decimal places).

4) Let $A = \begin{pmatrix} a & u_1 & u_2 & u_3 \end{pmatrix}$, $B = \begin{pmatrix} b & u_1 & u_2 & u_3 \end{pmatrix}$ and $= \begin{pmatrix} u_2 & u_3 & u_1 & a+b \end{pmatrix}$ be three $4x4$ real matrices, where $a, b, u_1, u_2$and u_3 are$4x1$ real column vectors. Let det(A),det(B) and det(C) denote the determinants of the matrices $A, B1andC$, respectively. If det(A) = 6 and det(B) = 2, then det($A + B$)-det(C) equals ______

5) Let X be a random variable having the moment generating function

$$M(t) = \frac{e^t - 1}{t(1-t)}, t < 1$$

Then $P(X > 1)$ equals ______ (round off to 2 decimal places.)

6) Let $\{X_n\}_n \geq 1$ be a sequence of independent and identically distributed random variables each having uniform distribution on $[0, 3]$. Let Y be a random variable, independent of $\{X_n\}_n \geq 1$, having probability mass function

$$P(Y = k) = \begin{cases} \frac{1}{(e-1)k!}, k = 1, 2, ..., \\ 0, \text{otherwise} \end{cases}$$

7) Let $\{X_n\}_n \geq 1$ be a sequence of independent and identically distributed random variables each having probability density function

$$f(x) = \begin{cases} e^{-x}, x > 0 \\ 0, otherwise. \end{cases}$$

Let $X_{(n)} = \max\{X_1, X_2, \ldots, X_n\}$ for $n \geq 1$. If Z is the random variable to which $\{X_{(n)} - \log_e n\}_{n\geq 1}$ converges in distribution, as $n \to \infty$, then the median of Z equals ______ (round off to 2 decimal places.)

8) Consider an amusement park where visitors are arriving according to Poisson process with rate 1. Upon arrival, a visitor spends a random amount of time in the park and then departs. The time spent by the visitors are independent of one another, as well as f the arrival process, and have common probability density function

$$f(x) = \begin{cases} e^{-x}, x > 0 \\ 0, otherwise. \end{cases}$$

If at a given time point, there are 10 visitors in the park and p is the probability that there will be exactly two more arrivals before the next departure, $\frac{1}{p}$ equals ______

9) let $\{0.90, 0.50, 0.01, 0.95\}$ be a realization of a random sample of size 4 from the probability density function

$$f(x) = \begin{cases} \frac{\theta}{1-\theta} x^{(2\theta-1)/(1-\theta)}, 0 < x < 1, \\ 0, otherwise, \end{cases}$$

where $0.5 \leq \theta < 1$. Then the maximum likelihood estimate of θ based on the observed sample equals ______ (**round off to** 2 **decimal places**.)

10) Let a random sample of size 100 from a normal population with unknown mean μ and variance 9 give the sample mean 5.608. Let $\Phi(.)$ denote the distribution function of the standard normal random variable. If $\Phi(1.96) = 0.975, \Phi(1.64) = 0.95$ and the uniformly most powerful unbiased test based on sample mean is used to test $H_0 : \mu = 5.02$ against $H_1 : \mu \neq 5.02$, then the p-value equals ______ (round off to 3 decimal places).

11) Let X be a discrete random variable with probability mass function $p \in \{p_0, p_1\}$, where

x	7	8	9	10
$p_1(x)$	0.69	0.10	0.16	0.05
$p_0(x)$	0.90	0.05	0.04	0.01

To test $H_0 : p = p_0$ against $H_1 : p = p_1$, th power of the most powerful test of size 0.05, based on X, equals ______ (round off to 2 decimal places).

12) Let $X_1, X_2, \ldots, X_{10}$ be a random sample from a probability density function

$$f_\theta(x) = f(x - \theta), -\infty < x < \infty,$$

where $-\infty < \theta < \infty$ and $f(-x) = f(x)$ for $-\infty < x < \infty$. For testing $H_0 : \theta = 1.2$ against $H_1 : \theta \neq 1.2$, let T^+ denote the Wilcoxson Signed-rank est statistic. If η denotes the probability of the event $\{T^+ < 50\}$ under H_0, then 32η equals ______ (round off to 2 decimal places).

13) Consider the multiple linear regression model

$$Y_i = \beta_0 + \beta_1 x_{1,i} + \beta_2 x_{2,i} + \ldots \beta_{22} x_{22,i} + \epsilon_i, i = 1, 2, \ldots, 123,$$

where, for $j = 0, 1, 2, \ldots, 22$, $\beta_j' s$ are unknown parameters and $\epsilon_i' s$ are independent and identically distributed $N\left(0, \sigma^2\right), \sigma > 0$, random variables.
If the sum of squares due to regression is 338.92, the total sum of squares is 522.30 and R^2_{adj} denotes the value of adjusted R^2, then $100R^2 adj$ equals ______ (round off to 2 decimal places).