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1 SECTION-A

1) Let $f(z) = 2z^2 - 1$. Then the maximum value of $|f(z)|$ on the unit disc $D = \{z \in C : |z| \leq 1\}$ equals

a) 1 b) 2 c) 3 d) 4

2) Let

$$f(z) = \frac{1}{z^2 - 3z + 2}$$

Then the coefficient of $\frac{1}{z^3}$ in the Laurent series expansion of $f(z)$ for $|z| > 2$ is

a) 1 b) 2 c) 3 d) 4

3) Let $f : C \to C$ be an arbitrary analytic function satisfying $f(0) = 0$ and $f(1) = 2$. Then
   a) there exists a sequence $\{z_n\}$ such that $|z_n| > n$ and $|f(z_n)| > n$
   b) there exists a sequence $\{z_n\}$ such that $|z_n| > n$ and $|f(z_n)| < n$
   c) there exists a bounded sequence $\{z_n\}$ such that $|f z_n| > n$
   d) there exists a sequence $\{z_n\}$ such that $z_n \to 0$ and $f(z_n) \to 2$

4) Define $f : C \to C$ by

$$f(x) = \begin{cases} 0 & \text{if Re } (z) \text{ or Im}(z)\text{=0} \\ z & \textit{otherwise.} \end{cases}$$

Then the set of points where f is analytic is
   a) $\{z : Re(z) \neq 0 \text{and} Im(z) \neq 0\}$
   b) $\{z : Re(z) \neq 0\}$
   c) $\{z : Re(z) \neq 0 or Im(z) \neq 0\}$
   d) $\{z : Im(z) \neq 0\}$

5) Let $U(n)$ be the set of all positive integers less than n and relatively prime to n. Then $U(n)$ is a group under multiplication modulo n. For $n = 248$, the number of elements in $U(n)$ is

a) 60 b) 120 c) 180 d) 240

6) Let $R[x]$ be the polynomial ring in x with real coefficients and let $I = \left(x^2 + 1\right)$ be the ideal generated by the polynomial $x^2 + 1$ in $R[x]$. Then

a) I is a maximal ideal
b) I is a prime ideal but NOT a maximal ideal
c) I is NOT a prime ideal
d) $R[x]/I$ has zero divisors

7) Consider Z_5 and Z_{20} as rings modulo 5 and 20, respectively. Then the number of homomorphisms $\phi : Z_5 \to Z_{20}$

a) 1 b) 2 c) 4 d) 5

8) Let Q be the field of rational numbers and consider Z_2 as a field modulo 2. Let

$$f(x) = x^3 - 9x^2 + 9x + 3$$

Then $f(x)$ is
a) irreducible over Q but reducible over Z_2
b) irreducible over both Q and Z_2
c) reducible over Q but irreducible over Z_2
d) reducible over both Q and Z_2

9) Consider Z_5 as a field modulo 5 and let

$$f(x) = x^5 + 4x^4 + 4x^3 + 4x^2 + x + 1$$

Then the zeros of $f(x)$ over Z_5 are 1 and 3 with respective multiplicity

a) 1 and 4
b) 2 and 3
c) 2 and 2
d) 1 and 2

10) Consider the Hilbert space $l^2 = \left\{x = \{x_n\} : x_n \in R, \sum_{n=1}^{\infty} x_n^2 < \infty\right\}$. Let

$$E = \left\{\{x_n\} : |x_n| \leq \frac{1}{n} \text{ for all } n\right\}$$

be a subset of l^2. Then
a) $E^\circ = \left\{x : |x_n| < \frac{1}{n} \text{for all n}\right\}$
b) $E^\circ = E$
c) $E^\circ = \left\{x : |x_n| < \frac{1}{n} \text{for all but finitely many n}\right\}$
d) $E^\circ = \phi$

11) Let X and Y be normed linear spaces and let $T : X \to Y$ be a linear map. Then T is continuous if
a) Y is finite dimensional
b) X is finite dimensional
c) T is one to one
d) T is onto

12) Let X be a normed linear space and let $E_1, E_2 \subseteq X$. Define

$$E_1 + E_2 = \{x + y : x \in E_1, y \in E_2\}$$

Then $E_1 + E_2$ is

a) open if E_1 or E_2 is open
b) NOT open unless both E_1 and E_2 are open
c) closed if E_1 or E_2 is closed
d) closed if both E_1 and E_2 are closed

13) For each $a \in R$, consider the linear programming problem Max. $z = x_1+2x_2+3x_3+4x_4$ subject to

$$ax_1 + 2x_3 \leq 1$$
$$x_1 + ax_2 + 3x_4 \leq 2$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Let $S = \{a \in R :$ the given LP problem has a basic feasible solution$\}$. Then

a) $S = \phi$
b) $S = R$
c) $S = (0, \infty)$
d) $S = (-\infty, 0)$

14) Consider the linear programming problem
Max. $z = x_1 + 5x_2 + 3x_3$ subject to

$$2x_1 - 3x_2 + 5x_3 \leq 3$$
$$3x_1 + 2x_3 \leq 5$$
$$x_1, x_2, x_3 \geq 0$$

Then the dual of this LP problem

a) has a feasible solution but does NOT have a basic feasible solution
b) has a basic feasible solution
c) has infinite number of feasible solutions
d) has no feasible solution

15) Consider a transportation problem with two warehouses and two markets. The warehouse capacities are $a_1 = 2$ and $a_2 = 4$ and the market demands are $b_1 = 3$ and $b_2 = 3$. Let x_{ij} be the quantity shipped from warehouse i to market j and c_{ij} be the corresponding unit cost. Suppose that $c_{11} = 1, c_{21} = 1$ and $c_{22} = 2$. Then $(x_{11}, x_{12}, x_{21}, x_{22}) = (2, 0, 1, 3)$ is optimal for every

a) $c_{12} \in [1, 2]$
b) $c_{12} \in [0, 3]$
c) $c_{12} \in [1, 3]$
d) $c_{12} \in [2, 4]$

16) The smallest degree of the polynomial that interpolates the data

x	-2	-1	0	1	2	3
$f(x)$	-58	-21	-12	-13	-6	27

is

a) 3 b) 4 c) 5 d) 6

17) Suppose that x_0 is sufficiently close to 3 . Which of the following iterations $x_{n+1} = g(x_n)$ will converge to the fixed point $x = 3$?

a) $x_{n+1} = -16 + 6x_n + \frac{3}{x_n}$
b) $x_{n+1} = \sqrt{3 + 2x_n}$
c) $x_{n+1} = \frac{3}{x_n - 2}$
d) $x_{n+1} = \frac{x_n^2 - 3}{2}$