

1) Let $f(z) = 2z^2 - 1$. Then the maximum value of $|f(z)|$ on the unit disc $D = \{z \in \mathbb{C} : |z| \leq 1\}$ equals

- a) 1 b) 2 c) 3 d) 4

2) Let

$$f(z) = \frac{1}{z^2 - 3z + 2}$$

Then the coefficient of $\frac{1}{z^3}$ in the Laurent series expansion of $f(z)$ for $|z| > 2$ is

- a) 1 b) 2 c) 3 d) 4

3) Let $f : \mathbb{C} \rightarrow \mathbb{C}$ be an arbitrary analytic function satisfying $f(0) = 0$ and $f(1) = 2$. Then

- a) there exists a sequence $\{z_n\}$ such that $|z_n| > n$ and $|f(z_n)| > n$
 b) there exists a sequence $\{z_n\}$ such that $|z_n| > n$ and $|f(z_n)| < n$
 c) there exists a bounded sequence $\{z_n\}$ such that $|f(z_n)| > n$
 d) there exists a sequence $\{z_n\}$ such that $z_n \rightarrow 0$ and $f(z_n) \rightarrow 2$

4) Define $f : \mathbb{C} \rightarrow \mathbb{C}$ by

$$f(x) = \begin{cases} 0 & \text{if } \operatorname{Re}(z) \text{ or } \operatorname{Im}(z) = 0 \\ z & \text{otherwise.} \end{cases}$$

Then the set of points where f is analytic is

- a) $\{z : \operatorname{Re}(z) \neq 0 \text{ and } \operatorname{Im}(z) \neq 0\}$
 b) $\{z : \operatorname{Re}(z) \neq 0\}$
 c) $\{z : \operatorname{Re}(z) \neq 0 \text{ or } \operatorname{Im}(z) \neq 0\}$
 d) $\{z : \operatorname{Im}(z) \neq 0\}$

5) Let $U(n)$ be the set of all positive integers less than n and relatively prime to n . Then $U(n)$ is a group under multiplication modulo n . For $n = 248$, the number of elements in $U(n)$ is

- a) 60 b) 120 c) 180 d) 240

6) Let $R[x]$ be the polynomial ring in x with real coefficients and let $I = (x^2 + 1)$ be the ideal generated by the polynomial $x^2 + 1$ in $R[x]$. Then

- a) I is a maximal ideal
 b) I is a prime ideal but NOT a maximal ideal

- c) I is NOT a prime ideal
- d) $R[x]/I$ has zero divisors

7) Consider Z_5 and Z_{20} as rings modulo 5 and 20, respectively. Then the number of homomorphisms $\phi : Z_5 \rightarrow Z_{20}$

- a) 1
- b) 2
- c) 4
- d) 5

8) Let Q be the field of rational numbers and consider Z_2 as a field modulo 2. Let

$$f(x) = x^3 - 9x^2 + 9x + 3$$

Then $f(x)$ is

- a) irreducible over Q but reducible over Z_2
- b) irreducible over both Q and Z_2
- c) reducible over Q but irreducible over Z_2
- d) reducible over both Q and Z_2

9) Consider Z_5 as a field modulo 5 and let

$$f(x) = x^5 + 4x^4 + 4x^3 + 4x^2 + x + 1$$

Then the zeros of $f(x)$ over Z_5 are 1 and 3 with respective multiplicity

- a) 1 and 4
- b) 2 and 3
- c) 2 and 2
- d) 1 and 2

10) Consider the Hilbert space $l^2 = \{x = \{x_n\} : x_n \in R, \sum_{n=1}^{\infty} x_n^2 < \infty\}$. Let

$$E = \left\{ \{x_n\} : |x_n| \leq \frac{1}{n} \text{ for all } n \right\}$$

be a subset of l^2 . Then

- a) $E^\circ = \{x : |x_n| < \frac{1}{n} \text{ for all } n\}$
- b) $E^\circ = E$
- c) $E^\circ = \{x : |x_n| < \frac{1}{n} \text{ for all but finitely many } n\}$
- d) $E^\circ = \phi$

11) Let X and Y be normed linear spaces and let $T : X \rightarrow Y$ be a linear map. Then T is continuous if

- a) Y is finite dimensional
- b) X is finite dimensional
- c) T is one to one
- d) T is onto

12) Let X be a normed linear space and let $E_1, E_2 \subseteq X$. Define

$$E_1 + E_2 = \{x + y : x \in E_1, y \in E_2\}$$

Then $E_1 + E_2$ is

- a) open if E_1 or E_2 is open

- b) NOT open unless both E_1 and E_2 are open
- c) closed if E_1 or E_2 is closed
- d) closed if both E_1 and E_2 are closed

13) For each $a \in R$, consider the linear programming problem Max. $z = x_1 + 2x_2 + 3x_3 + 4x_4$ subject to

$$\begin{aligned} ax_1 + 2x_3 &\leq 1 \\ x_1 + ax_2 + 3x_4 &\leq 2 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

Let $S = \{a \in R : \text{the given LP problem has a basic feasible solution}\}$. Then

- a) $S = \emptyset$
- b) $S = R$
- c) $S = (0, \infty)$
- d) $S = (-\infty, 0)$

14) Consider the linear programming problem

$$\text{Max. } z = x_1 + 5x_2 + 3x_3$$

subject to

$$\begin{aligned} 2x_1 - 3x_2 + 5x_3 &\leq 3 \\ 3x_1 + 2x_3 &\leq 5 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

Then the dual of this LP problem

- a) has a feasible solution but does NOT have a basic feasible solution
- b) has a basic feasible solution
- c) has infinite number of feasible solutions
- d) has no feasible solution

15) Consider a transportation problem with two warehouses and two markets. The warehouse capacities are $a_1 = 2$ and $a_2 = 4$ and the market demands are $b_1 = 3$ and $b_2 = 3$. Let x_{ij} be the quantity shipped from warehouse i to market j and c_{ij} be the corresponding unit cost. Suppose that $c_{11} = 1, c_{21} = 1$ and $c_{22} = 2$. Then $(x_{11}, x_{12}, x_{21}, x_{22}) = (2, 0, 1, 3)$ is optimal for every

- a) $c_{12} \in [1, 2]$
- b) $c_{12} \in [0, 3]$
- c) $c_{12} \in [1, 3]$
- d) $c_{12} \in [2, 4]$

16) The smallest degree of the polynomial that interpolates the data

x	-2	-1	0	1	2	3
$f(x)$	-58	-21	-12	-13	-6	27

is

a) 3

b) 4

c) 5

d) 6

17) Suppose that x_0 is sufficiently close to 3 . Which of the following iterations $x_{n+1} = g(x_n)$ will converge to the fixed point $x = 3$?

a) $x_{n+1} = -16 + 6x_n + \frac{3}{x_n}$

b) $x_{n+1} = \sqrt{3 + 2x_n}$

c) $x_{n+1} = \frac{3}{x_n - 2}$

d) $x_{n+1} = \frac{x_n - 3}{2}$