Assignment- 9-9.3-7

AI24BTECH11004-Bheri Sai Likith Reddy

Question: Find the area of the region enclosed by the curves $y^2 = x, x = \frac{1}{4}, y = 0$ and $x = 1$. (12,2022)

Solution: The parameters of the conic are

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = -\frac{1}{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix}, f = 0$$

for the line $x - \frac{1}{4} = 0$ paramenters are $h_2 = \begin{pmatrix} \frac{1}{4} \\ 0 \end{pmatrix}, m = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$:w

substituing in equation 9.1.1.3 we get $k_i = 1, -1$

that means points of intersection are $a_0 = \begin{pmatrix} \frac{1}{4} \\ \frac{1}{2} \end{pmatrix} and a_1 = \begin{pmatrix} \frac{1}{4} \\ \frac{-1}{2} \end{pmatrix}$

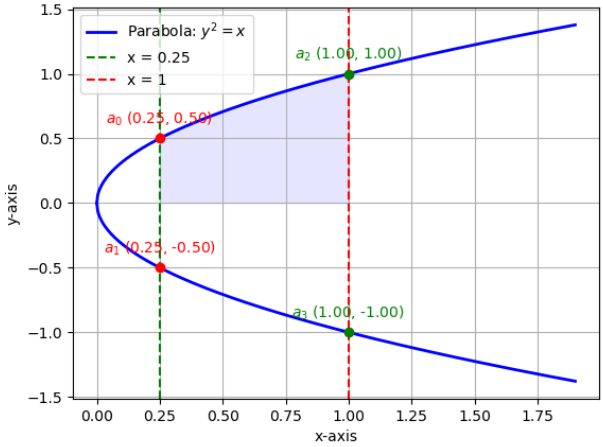
similarly point of intersentions with x=1 are $a_3 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, a_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

hence area of the region between the two lines and the parabola is

$$\int_{\frac{1}{4}}^{1} \sqrt{x}dx - \int_{\frac{1}{4}}^{1} 0dx = \frac{7}{6}$$

hence therefore required area is $\frac{7}{12}$ since the parabola is symmetric about x axis.

Point	Description
$a_0(0.25, 0.5), a_0(0.25, 0.5)$	Point of intersection of the line x=0.25 and the parabola $y^2 = x$
$a_2(1, 1), a_3(1, -1)$	Point of intersection of the line x=0.25 and the parabola $y^2 = x$ Point of intersection of the line x=1 and the parabola $y^2 = x$


Parabola: $y^2 = x$
x = 0.25
x = 1
a_0 (0.25, 0.50)
a_1 (0.25, -0.50)
a_2 (1.00, 1.00)
a_3 (1.00, -1.00)
y-axis
x-axis
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75