2023 February 1 Shift 2

AI24BTECH11004-Bheri Sai Likith Reddy

1 SECTION-A

1) The sum

$$\sum_{n=1}^{\infty} \frac{2n^2 + 3n + 4}{(2n)!}$$

is equal to :

a) $\frac{11e}{2} + \frac{7}{2e}$
b) $\frac{13e}{4} + \frac{5}{4e} - 4$
c) $\frac{11e}{2} + \frac{7}{2e} - 4$
d) $\frac{13e}{4} + \frac{5}{4e}$

2) Let

$$S = \left\{ x \in R : 0 < x < 1 and 2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) \right\}.$$

If $n(S)$ denotes the number of elements in S then :

a) $n(S) = 2$ and only one element in S is less than $\frac{1}{2}$
b) $n(S) = 2$ and only one element in S is less than $\frac{1}{2}$
c) $n(S) = 2$ and only one element in S is less than $\frac{1}{2}$
d) $n(S) = 0$

3) Let $\vec{a} = 2\hat{i} - 7\hat{j} + 5\hat{k}$, $\vec{b} = \hat{i} + \hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} - 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r} \cdot \vec{b} = 0$, then $\left|\vec{r}\right|$ is equal to :

a) $\frac{11}{7}\sqrt{2}$
b) $\frac{11}{7}$
c) $\frac{11}{5}\sqrt{2}$
d) $\frac{\sqrt{914}}{7}$

4) If $A = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}$, then :

a) $A^{30} - A^{25} = 2I$
b) $A^{30} + A^{25} + A = I$
c) $A^{30} + A^{25} - A = I$
d) $A^{30} = A^{25}$

5) Two sice are thrown independently. Let A be the event that the number appeared on the 1^{st} die is less than the number appeared on the 2^{nd} die, B be the event that the number appeared on the number appeared on the 1^{st} die is even and that on the second die is odd, and C be the event that the number appeared on i^{st} die is odd and that on the 2^{nd} is even. Then

a) the number of favourable cases of the event $(A \cup B) \cap C$ is 6
b) A and B are mutually exchusive
c) The number of favourabel cases of the events A, B and C are $15, 6$ and 6 respectively
d) B and C are independent

6) Which of the following statements is a tautology ?
a) $p \rightarrow (p \wedge (p \rightarrow q))$
b) $(p \wedge q) \rightarrow (\neg (p) \rightarrow q)$
c) $(p \wedge (p \rightarrow q)) \rightarrow \neg q$
d) $p \vee (p \wedge q)$

7) The number of integral values of k, for which one root of the equation

$$x^2 - 8x + k = 0$$

lies in the interval $(2, 3)$, is:
a) 2
b) 0
c) 1
d) 3

8) Let $f : R - 0, 1 \rightarrow R$ be a function such that

$$f(x) + f\left(\frac{1}{1-x}\right) = 1 + x.$$

Then $f(2)$ is equal to :
a) $\frac{9}{2}$
b) $\frac{9}{4}$
c) $\frac{7}{4}$
d) $\frac{7}{3}$

9) Let the plane P pass through the intersection of the planes $2x + 3y - z = 2$ and $x + 2y + 3z = 6$, and be perpendicular to the plan $2x + y - z + 1 = 0$. If d is the distance of P from the point $(-7, 1, 1)$, then d_2 is equal to:
a) $\frac{250}{83}$
b) $\frac{15}{53}$
c) $\frac{25}{83}$
d) $\frac{250}{82}$

10) Let a, b be two real numbers such that $ab < 0$. If the complex number $\frac{1+ai}{b+i}$ is of unit modulus and $a + ib$ lies on the circle $|z - 1| = |2z|$, then a possible value of $\frac{1+|a|}{4b}$, where $[t]$ is greatest inter function, is :
a) $\frac{-1}{2}$
b) -1
c) 1
d) $\frac{1}{2}$

11) The sum of the abosolute maximum and minimum values of the function

$$f(x) = \left|x^2 - 5x + 6\right| - 3x + 2$$

in the interval $[-1, 3]$ is equal to:

a) 10
b) 12
c) 13
d) 24

12) Let $P(S)$ denote the power set of $S = \{1, 2, 3, \ldots, 10\}$. Define the relations R_1 and R_2 on $P(S)$ as AR_1B if $(A \cap B^c) \cup (B \cap A^c) = \phi$ adn AR_2B if $A \cup B^c = B \cup A^c$, $\forall$ A,B $\in$ P(S). Then :

a) both R_1 and R_2 are equivalence relations
b) only R_1 is an equivalence realtion
c) only R_2 is an euevalence realtaion
d) both R_1 and R_2 are not equivalence relations

13) The area of the region given by $\left\{(x, y) : xy \leq 8, 1 \leq y \leq x^2\right\}$ *is* :

a) $8\log_e^2 - \frac{13}{3}$
b) $16\log_e^2 - \frac{14}{3}$
c) $8\log_e^2 + \frac{7}{6}$
d) $16\log_e^2 + \frac{7}{3}$

14) Let $\alpha x = exp\left(x^\beta y^\gamma\right)$ be the solution of the differential equation

$$2x^2 y dy - \left(1 - xy^2\right) dx = 0,$$

$x > 0$, $y(2) = \sqrt{\log_e^2}$. Then $\alpha + \beta - \gamma$ equals :

a) 1
b) -1
c) 0
d) 3

15) The value of the integral

$$\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \frac{x + \frac{\pi}{4}}{2 - \cos 2x} dx$$

is :

a) $\frac{\pi^2}{6}$
b) $\frac{\pi^2}{12\sqrt{3}}$
c) $\frac{\pi^2}{3\sqrt{3}}$
d) $\frac{\pi^2}{6\sqrt{3}}$