NUMBER PROPERTIES

Number Systems:

A Number is an abstract entity that represents a count or measurement.

All numbers fall in 2 categories Real Number, Complex Number

A Real Number can either be a Rational number or an irrational number.

Rational number can either be Natural numbers or Negative integers or Fractions

Natural Numbers

The natural numbers start off as follows: 1, 2, 3, 4, and 5 ... The "..." means that the list goes on forever. We give this set the name N.

If a number is in N, then its successor is also in N.

Thus, there is no greatest number, because we can always add one to get a larger one. N is an infinite set and hence can never be exhausted by removing its members one at a time.

Whole Numbers

If we add zero to our above list then we have the set of whole numbers.

i.e Whole numbers 0,1,2,3...

Negative numbers

Negative numbers are numbers which are less than zero. They are used to indicate a number that is opposite to the corresponding positive number (the absolute value), but equal in magnitude.

Example: -1, -2, -3, . . .

Remember -(n + 1)is always smaller than -n where n is a positive number.

Integer

Integers are the whole numbers, negative whole numbers, and zero.

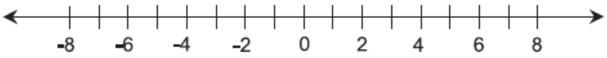
One of the numbers ..., -2, -1, 0, 1, 2, . . .

But numbers like 1/2, 4.00032, 2.5, Pi, and -9.90 are not integers.

Note that zero is neither positive nor negative.

It may help you to think of numbers as occurring along a line that stretches infinitely in both directions.

Numbers to the left of the 0 point are negative, numbers to the right are positive.



Along the number line there are a series of dots that correspond to whole numbers (integers).

The spaces between the whole numbers are occupied by the numbers that are not whole (they contain fractions, and are called real numbers ex.1/2, 4.00032, 2.5, Pi, and -9.90).

Even and Odd

The terms even and odd only apply to integers. A number is said to be an even number if it is divisible by 2 or else it is an odd number.

Even numbers are: 2, 4, 6, 8, 10. . . . .40, 42, 44,. . . 312, 314, ... 1008,1010, ...686860....

Odd numbers are: . . 5, 7, 9. . . . .41, 43, 45,. . . 311, 313, ... 1007,1009, ...686861....

Note:

- 2.5 is neither even nor odd.
- Zero, on the other hand, is even since it is 2 times some integer: it's 2 times 0.

To check whether a number is odd, see whether it's one more than some even number:

Example:

7 is odd since it's one more than 6, which is even.

Another way to say this is that zero is even since it can be written in the form 2*n, where n is an integer. Odd numbers can be written in the form 2*n + 1.

Again, this lets us talk about whether negative numbers are even and odd: -9 is odd since it's one more than -10, which is even.

Every positive integer can be factored into the product of prime numbers, and there's only one way to do it for every number.

Example:

280 = 2x2x2x5x7, and there's only one way to factor 280 into prime numbers

Rational Number

A rational number is a number that can be expressed as a fraction p/q where p and q are integers and $q \neq 0$.

i.e., Rational numbers are simply defined as ratios of integers. 1/2 is a rational number. 2/3 is also a rational number.

Note that all the integers are rational numbers, because you can think of them as the ratio of themselves to 1, as in 2 = 2/1 which is certainly the ratio of two integers, and so 2 is a rational number.
The decimal form of a rational number is either a terminating or repeating decimal.

Representation of rational numbers in decimal form
Any positive rational number p/q, after actual division, if necessary can be expressed as,
$p / q = m + r/q$
where m is non-negative integer and $0 \le r < q$
Example:
31/5 = 6 + 1/5 = 6.2

There are few fractions for which the right most digit (or set of right most digit) recurs endlessly.
Example:
1/3 =0.33333. . . . and 5/11 = 0.45454 . . . .
Note that the dots .... represent endless recurrence of digits.

The above examples are decimal numbers of the "non-terminating type".
In case of "non-terminating type" we have decimal fractions having an infinite number of digits. Some decimal fractions from this group have digits repeating infinitely. They are called "repeating or recurring " decimals.

In "endless recurring or infinite repeating" decimal fractions we can see that when p is actually divided by q the possible remainders are 1, 2, 3, ... , q -1. So one of them has to repeat itself in q steps. Thereafter the earlier numeral or group of numerals must repeat itself.
All the rational numbers thus can be represented as a finite decimal (terminating type) or as a recurring decimal.

Irrational Numbers
In mathematics, an irrational number is any real number that is not a rational number
i.e., one that cannot be written as a ratio of two integers
i.e., it is not of the form a/b where a and b are integers and b is not zero.
It can readily be shown that the irrational numbers are precisely those numbers whose expansion in any given base (decimal, binary, etc) never ends and never enters a periodic pattern.
The square root of 2 is a classic example of an irrational number: you cannot write it as the ratio of ANY two integers.

Prime Number
A prime number is a whole number that is not the product of two smaller numbers.

Note that the definition of a prime number doesn't allow 1 to be a prime number : 1 only has one factor, namely 1.

Prime numbers have exactly two factors, not "at most two" or anything like that. When a number has more than two factors it is called a composite number.
Here are the first few prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199, etc.

PRIME FACTORS
Suppose n is a natural number, then there exists a unique sequence of prime numbers p1, p2, p3, . . . , pm, such that both of the following statements are true:
$p1 \le p2 \le p3 \le . . . \le pm$
p1 x p2 x p3 . . . x pm
The numbers p1, p2, p3, . . . , pm are called the prime factors of the natural number.
Every natural number n has one, but only one, set of prime factors.
This is an important principle known as the Fundamental Theorem of Arithmetic.

Number of Prime Factors
A number N of the form am x bn x cp
where a, b, c are all prime factors of number N has (m + 1)(n + 1)(p + 1) no. of prime factors

What is the fastest way to determine if a number is Prime?
The easiest & simplest method is to divide the number up to the closet square root of that number.
Example:
Lets consider 53. Number close to 53 having a perfect square is 64 and its square root is 8. Now start dividing 53 from 2 to 8.
There is no such number between 2 to 8 which divides 53 so 53 is a prime number.

Composite Numbers

A composite number is a positive integer which is not prime (i.e., which has factors other than 1 and itself).

The first few composite numbers (sometimes called "composites" for short) are 4, 6, 8, 9, 10, 12, 14, 15, 16, . . .

Note that the number 1 is a special case which is considered to be neither composite nor prime.

Numeric Operations

1. A + 0 = A   A - 0= A
   A x 0 = 0   A/0= Value Does not exist
2. A x 1= 1   A + (-A) = 0
   A x (1/A) = 1
3. A + B = B + A
4. A x B = B x A
5. A (B + C) = AB + AC

Integer Roots

Suppose that ‘a’ is a positive real number. Also suppose that ‘n’ is a positive integer. Then n^{th} root of ‘a’ can also be expressed as the (1/ n) power of ‘a’.

Thus, the second root (or square root) is the same thing as the 1/2 power

the third root (or cube root) is the same thing as the 1/3 power

the fourth root is the same thing as the 1/4 power; and so on.

Irrational-Number Powers

Suppose that ‘a’ is a real number. Also suppose that ‘b’ is a rational number such that b = m / n,

where ‘m’ and ‘n’ are integers and ≠ 0.

Then the following formula holds true:

$a^b = a^{(m/n)} = a^{m(1/n)} = a^{(1/n)m}$

and $(1/a)^b = 1/(a^b)$

In case of a negative power $a^{-b} = (1/a^b)$

Important Formula

$A^{(b+c)} = A^bA^c$ and $A^{(b-c)} = A^b / A^c$

Let A be a real number. Let b and c be rational numbers.

Then the following formula holds good: $A^{(bc)} = (A^b)^c = (A^c)^b$

Dividing by 3

Add up the digits: if the sum is divisible by three, then the number is as well.

Examples:

111111: the digits add to 6 so the whole number is divisible by three. 87687687. The digits add up to 57, and 5 plus seven is 12, so the original number is divisible by three.

Dividing by 4

Look at the last two digits. If the number formed by its last two digits is divisible by 4, the original number is as well.

Examples:

100 is divisible by 4.

1732782989264864826421834612 is divisible by four also, because 12 is divisible by four.

Dividing by 5

If the last digit is a five or a zero, then the number is divisible by 5.

Dividing by 6

Check 3 and 2. If the number is divisible by both 3 and 2, it is divisible by 6 as well.

Dividing by 7

To find out if a number is divisible by seven, take the last digit, double it, and subtract it from the rest of the number.

Example:

If you had 203, you would double the last digit to get six, and subtract that from 20 to get 14. If you get an answer divisible by 7 (including zero), then the original number is divisible by seven. If you don't know the new number's divisibility, you can apply the rule again.

TEST - Take the number and multiply each digit beginning on the right hand side (ones) by 1, 3, 2, 6, 4, 5.

Repeat this sequence as necessary and Add the products.

If the sum is divisible by 7 - so is your number

Example:

Is 2016 divisible by 7?

6(1) +1(3) + 0(2) + 2(6) = 21 . 21 is divisible by 7 and we can now say that 2016 is also divisible by 7.

Dividing by 8

Check the last three digits. Since 1000 is divisible by 8, if the last three digits of a number are divisible by 8, then so is the whole number.

Example:

33333888 is divisible by 8; 33333886 isn't.

Dividing by 9
Add the digits. If that sum is divisible by nine, then the original number is as well.
Example:
12348 is divisible by 9; as the sum is 18
If the number ends in 0, it is divisible by 10.
Example: 20, 345ABCV80

PRIME FACTORISATION
A prime factorisation of a natural number can be expressed in the exponential form.
For example:
(i) 48 = 2 x 2 x 2x 2 x3 = 24 x 3
(ii) 420 = 2 x 2 x 3 x 5 x 7 = 22 x 3 x 5 x 7

LEAST COMMON MULTIPLE (L.C.M.)
A common multiple is a number that is a multiple of two or more numbers. The common multiples of 3 and 4 are 0, 12, 24, ...
The least common multiple (LCM) of two numbers is the smallest number (not zero) that is a multiple of both.

Method 1
Simply list the multiples of each number (multiply by 2, 3, 4, etc.) then look for the smallest number that appears in each list.
Example:
Find least common multiple for 5, 6, and 15.
Multiples of 5 are 10, 15, 20, 25, 30, 35, 40,...
Multiples of 6 are 12, 18, 24, 30, 36, 42, 48,...
Multiples of 15 are 30, 45, 60, 75, 90,....
Now, when you look at the list of multiples, you can see that 30 is the smallest number that appears in each list. Therefore, the least common multiple of 5, 6 and 15 is 30.

Method 2
To use this method factor each of the numbers into primes. Then for each different prime number in all of the factorizations, do the following...

1. Count the number of times each prime number appears in each of the factorizations.
2. For each prime number, take the largest of these counts.
3. Write down that prime number as many times as you counted for it in step 2.

The least common multiple is the product of all the prime numbers written down.
Example:
Find the least common multiple of 5, 6 and 15.

Factor into primes
Prime factorization of 5 is 5
Prime factorization of 6 is 2 x 3
Prime factorization of 15 is 3 x 5
* Notice the different primes are 2, 3 and 5.

Now, we do
Step #1 - Count number of times each prime number appears in each of the factorizations...
The count of primes in 5 is one 5
The count of primes in 6 is one 2 and one 3
count of primes in 15 is one 3 and one 5

Step #2 - For each prime number, take the largest of these counts. So we have...
The largest count of 2s is one
The largest count of 3s is one
The largest count of 5s is one

Step #3 - Since we now know the count of each prime number, you simply - write down that prime number as many times as you counted for it in step 2.
Here they are...2, 3, 5

Step #4 - The least common multiple is product of all prime numbers written down.
2 x 3 x 5 = 30
Therefore, the least common multiple of 5, 6 and 15 is 30.
So there you have it. *A quick and easy method for finding least common multiples*.

HIGHEST COMMON FACTOR (HCF)
H.C.F. of two natural numbers is the largest common factor (or divisor) of the given natural numbers.
In other words, H.C.F. is the greatest element of set of common factors of given numbers.
H.C.F. is also called Greatest Common Divisor (abbreviated G.C.D.)
Example:
Find the H.C.F. of 72, 126 and 270.
Solution:
Using Prime factorisation method
$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$
$126 = 2 \times 3 \times 3 \times 7 = 2^1 \times 3^2 \times 7^1$
$270 = 2 \times 3 \times 3 \times 3 \times 5 = 2^1 \times 3^3 \times 5^1$
H.C.F. of the given numbers
= product of common factors with least index
= 21 x 32

Using Division method find H.C.F. 72 & 126
72|126|1
72
54| 72|1
54
18| 54|3
54
0
H.C.F. of 72 and 126 = 18
Similarly calculate H.C.F. of 18 and 270 as 18
Hence H.C.F. of the given three numbers = 18

Guard Against the Probable Errors

1. 1 is **not** a prime prime number. 119 is also **not** a prime number.
2. Sum of two prime numbers can also be a prime number – this is possible only if one of the numbers is 2.
3. Sum of *k* integers is odd does **not** necessarily mean all the integers are odd – it is sufficient if an odd number of integers are odd.
4. If a number is divisible by *n* and *m*, it does **not** necessarily mean that the number is divisible by *nm* – it is so only if *n* and *m* are prime to each other.
5. HCF of a set of numbers **cannot** be greater than the smallest of the numbers.
6. LCM of a set of numbers **canno**t be less than the largest of the numbers.
7. The remainder is not always less than the dividend – it can also be equal to the dividend if the dividend is less than the divisor.
8. The conjugate of $(\sqrt{3} + 1)$ is **not** $(\sqrt{3} - 1)$ – it is $(1 - \sqrt{3})$.
9. The conjugate of $(i - 2)$ is **not** $(i + 2)$ – the correct conjugate is $(- 2 - i)$.

Example 1:
If *x* and *y* are odd integers which of the following cannot be odd?
a. x + y b. xy c. 2x + y d. x + 2y
Explanation:
Since sum of two odd integers is always even, $x + y$ cannot be odd.
Product of 2 odd integers is always odd
So *xy* is odd.
For any values,
2*x* and 2*y* are even and even plus odd is odd.
Hence $(2x + y)$ and $(2y + x)$ are odd.

Example 2:
If product of *k* integers is odd, which of the following must be true?
Solution:
I. *k* is odd.
II. All the *k* integers are odd.
III. At least one of the *k* integers is odd.
Product of an even number of odd integers is also odd. So, I is not necessary.
Product of an even integer with any number of odd or even integers is always even. So, if the product is odd all the integers must be odd.
=> II must be true.
II also implies III is not sufficient to get the product as odd. Hence III is not true. Therefore, only II is true.

Example 3:
If $pq = 60$, what is the maximum number of values *p* can take?
Solution:
Given condition implies that *p* and *q* are divisors of 60.
Now, 60 = 22.31.51 and hence the number of divisors of 60 = (2 + 1)(1 + 1)(1 + 1) = 12.
So, *p* can take any of these 12 values.

Example 4:
Find the HCF and LCM of 42, 91 and 154.
Solution:
$42 = 2 \times 3 \times 7$; $91 = 7 \times 13$; $154 = 2 \times 7 \times 11$.
The only common factor is 7 and hence the HCF = 7.

Example 5:
What is the value of *p* if *q* is 119, LCM and HCF of *p* and *q* are 595 and 17 respectively?
Solution:
$LCM \times HCF = 595 \times 17 = pq$
Given q = 119, $p = (595 \times 17)/119 = 85$.

Example 6:
A P. T. Master wants to make a formation with 299 students – 39 from Primary, 65 from Secondary and 195 from Higher Secondary sections, fulfilling all the following conditions:

1. Number of students in each row should be the same.
2. The front rows can have students from Primary section only.
3. The last rows can have students from Higher Secondary section only.

4. Students of Primary section cannot be accomodated in any row other than the front rows and the Higher Secondary students can be only in the last rows.
5. No student should be left behind.

If the P. T. Master wishes to have the formation with as few rows as possible, what should be the number of students per row?

Solution:

Conditions (2), (3) and (4) imply that each row can have students from one section only.

This in conjunction with (1) and (5) means that the number of students in each section should be a multiple of the number of students per row.

Or in other words, the number of students per row should be a factor of 39, 65 and 195.

More the number of students per row, less the number of rows. Thus the last condition requires the number of students per row to be as large as possible. Combining this with the earlier implication, the number of students per row should be the largest factor of 39, 65 and 195, which is nothing but the HCF.

Now, 39 = 3 × 13; 65 = 5 × 13; 195 = 15 × 13.

The HCF is 13.

Example 7:

When a number is divided by 9 the remainder is 3 and when the same number is divided by 5 the remainder is 3. What is the largest 3-digit number which satisfies this condition?

Solution:

Let n be the number.

Then by rule (2) above, (n – 3) is a multiple of 9 and also 5. So, the least possible value of (n – 3) is the LCM of 9 and 5, which is 45 and all other values (n – 3) can take must be multiples of 45.

Now, the largest 3-digit number is 999. On dividing 999 by 45, the remainder is 9 and again applying rule (2) above, (999 – 9) = 990 is a multiple of 45.

Thus, the largest possible 3-digit value for (n – 3) is 990 or the largest possible value for n is 993.

Example 8:

What is the largest 4-digit number that leaves a remainder of 6 when divided by 15?

Solution:

The largest 4-digit number is 9999. On dividing 9999 by 15, the quotient is 666with a remainder of 9. So, 9999 = (15 × 666) + 9, which in turn implies (9999 – 9) = 9990 is the largest 4-digit multiple of 15.

Hence (9990 + 6) = 9996 must be the largest 4-digit number yielding a remainder of 6 when divided by 15.

Example 9:

A number leaves a remainder 5 when divided by 7 and 7 when divided by 9. What is the least possible number that has this property?

Solution:

Noting (7 – 5) = 2 = (9 – 7), if n is the least possible number, then (n + 2) is a multiple of both 7 and 9. Hence least possible value for (n + 2) is the LCM of 7 and 9, viz. 63. Therefore, the least possible value for n is 61.

5 divided by 3 leaves a remainder of 2, 4 divided by 3 leaves a remainder of 1 and 7 divided by 3 leaves a remainder of 1.

Now, 11 divided by 3 leaves a remainder of 2 and 20 divided by 3 leaves a remainder of 2.

11 = 4 + 7

and remainder of 11/3 = (remainder of 4/3 + remainder of 7/3)

Also, 20 = 4 × 5 and the remainder of 20/3

= (remainder of 4/3 × remainder of 5/3)

Or in general, if R(n/k) denotes the remainder of n when divided by k,

R{(n + m)/k} = R(n/k) + R(m/k)

R{(n × m)/k} = R(n/k) × R(m/k)

And extending the second rule,

R{(nm)/k} = {R(n/k)}m

Example 10:

What is the remainder when 429 is divided by 63?

Solution:

43 is 64 which is just 1 more than 63 and hence R(43/63) = 1.

Now, 429 = (43)9 × 42 and so

R(429/63) = R[{(43)9 × 42}/63]

= R[{(43)9}/63] × R[(42)/63]

= [R(43/63)]9 × 16 = 19 × 16 = 16.

In general, to find the remainder when nm is divided by k, a step by step rule would be

Step 1: Identify a power t of n so that nt is very close to k.

Step 2: Divide m by t and get the quotient and remainder, say q and r respectively.

Step 3: Determine R(nt/k) and R(nr/k).

Step 4: Raise the first quantity of Step 3 to power q and multiply this by the second quantity to get the final answer.

Example 11:
If the remainder is 75 when a number is divided by 85, what is the remainder when the same number is divided by 17?
Solution:
Let the number be n. Then $n = 85k + 75$.
Now, $R(n/17) = R\{(85k + 75)/17\}$
$= R(85k/17) + R(75/17) = 0 + 7 = 7$.

Example 12:
What is the maximum power of 7 in 1024!?
Solution:
7)1024
7) 146
7) 20
2
Since 2 < 7, the process is stopped and the maximum power = 146 + 20 + 2 = 168.

Example 13:
Sum of $i^{101} + i^{102} + i^{103} + \ldots\ldots\ldots\ldots\ i^{200} = ?$
Solution:
$i^{101} = I;\ i^{102} = -1;\ i^{103} = -I;\ i^{104} = 1$.
So, $i^{101} + i^{102} + i^{103} + i^{104} = 0$.
Similarly,
$i^{105} + i^{106} + i^{107} + i^{108} = 0$
$i^{109} + i^{110} + i^{111} + i^{112} = 0$
$\ldots\ldots\ldots\ldots\ldots\ldots\ i^{197} + i^{198} + i^{199} + i^{200} = 0$.
Thus $i^{101} + i^{102} + i^{103} + \ldots\ldots\ldots\ldots\ i^{200} = 0$
To simplify a complex number with i in the denominator, multiply both the numerator and the denominator by the conjugate of the denominator.

Example 14:
Reduce $(14 – 5i)/(7 – 9i)$ to standard form.
Solution:
The conjugate of $(7 – 9i) = (7 + 9i)$.
Multiplying both the numerator and the denominator by the conjugate,
$(14 – 5i)/(7 – 9i)$
$= \{(14 – 5i) \times (7 + 9i)\}/\{(7 – 9i) \times (7 + 9i)\}$
$= (98 – 35i + 126i - 45i^2)/(49 - 81i^2)$
$= (98 + 91i + 45)/(49 + 81)$
$= (143 + 91i)/130$
$= (143/130) + (91/130)i$.
The principles in finding the square root of a complex number is the same as the one for irrational numbers.

Example 15:
What is the square root of $(32 – 126i)$?
Solution:
Let $\sqrt{(32 – 126i)} = (a + bi)$.
Squaring both sides,
$(32 – 126i) = a^2 – b^2 + 2abi$.
Or, $a^2 – b^2 = 32$ and $ab = – 63$.
Clearly, $a = 9$ and $b = – 7$.
So, $\sqrt{(32 – 126i)} = + (9 – 7i)$.

<u>Questionnaire for Practice</u>

1. $77^3 + 13^3 - 90^3$ is divisible by
   a. Both 13 and 17 b. Both 1 and 17
   c. Both 11 and 13 d. Both 3 and 19

2. If P is the set of prime numbers from 1 to 100, and all elements of P are multiplied, the product will be exactly divisible by
   a.10 b. 100 c. 1000 d. None of these

3. If n = 1 + x, where x is the product of 4 consecutive positive integers, then which of the following is/are true?
   I. n is odd II. n is prime
   III. n is a perfect square
   a. I and III only b. I and II only
   c. I only d. None of these

4. In the middle of a round pool lies a beautiful water-lily. The water-lily doubles in size every day. After exactly 20 days the complete pool will be covered by the lily. After how many days will half of the pool be covered by the water- lily?
   a. 10 b. 15 c. 17.5 d. 19

5. A fraction is first multiplied by itself and product so obtained is divided by the reciprocal of the original fraction. If the final result is the fraction $42\frac{7}{8}$, what is the original fraction?
   a. $\frac{2}{7}$ b. $\frac{3}{8}$ c. $\frac{11}{16}$ 4. None of these

6. If $(1.001)^{1259} = 3.52$ and $(1.001)^{2062} = 7.85$, then $(1.001)^{3321} = ?$
   a. 2.23 b. 4.33 c. 11.37 d. 27.64

7. There are 8 bags of rice looking alike, 7 of which have equal weight and one is slightly heavier. The weighing balance is of unlimited capacity. Using this balance, minimum number of weighings required to identify heavier bag is
   a. 2 b. 3 c. 4 d. 8

8. A number when divided by 187 leaves a reminder 62. What will be the reminder when 17 divide that number?
a. 8 b. 9 c. 10 d. 11

9. $n^3 + 2n$, for any natural number n. is always a multiple of
a. 3 b. 4 c.5 d. 6

10. If x =20^4 and y = 17 x 19 x 21 x 23 then
a. x > y b. x < y
c. x = y d. None of these

11. If 4 X 56 is divisible by 33 then X is
a. 3 b. 4 c. 5 d. 6

12. $2^{61} + 2^{62} + 2^{63} + 2^{64} + 2^{65}$ is divisible by
a. 3 b. 31 c. 11 d. 17

13. If x is a positive integer such that 3x+12 is perfectly divisible by x, then the number of possible values of x is
a. 2 b. 5 c. 6 d. 12

14. Two alarm clocks ring their alarms at regular intervals of 50 seconds and 48 seconds. If they first beep together at 12 noon, at what time will they beep again for the first time?
a. 2:10pm b.12:12pm c.12:11pm d.None

15. The remainder when 2^{31} is divided by 5
a. 2 b. 4 c. 8 d. 3

16. 434758X0 is divisible by 4, Find the number of possible values of X
a. 2 b. 4 c. 5 d. 3

17. A man earns Rs 20 on the first day and spends Rs 15 on the next day. He again earns Rs 20 on the third day spends Rs 15 on the fourth day. If he continues to save like this, how soon will he have Rs 60?
a. On 24th day b. On 19th day
c. On 17th day d. On 20th day

18. Unit digit in the sum $(264)^{102} + (264)^{103}$ is
a. 0 b. 4 c. 6 d.8

19. 100 oranges were distributed among friends equally. Had there been 5 more friends each would have received one orange less. How many friends were there?
a. 20 b. 25 c. 30 d. None

20. A number when divided by D leaves a remainder of 8 and if divided by 3D leaves 21. What is the remainder twice the number is divided by 3D?
a. 3 b. 8 c. 5 d. None

21. What is the highest power of 7 that divides 77!
a. 11 b. 12 c. 1 d. None

22. Which is greater among 4^{300} or 3^{400} ?
a. 4^{300} b. 3^{400}
c. Both are equal d. None

23. For a Natural Number n, $n^4 + n^2 + 1$ is
a. Odd b. Even
c. Either even or odd d. None

24. By what smallest number must 21600 be multiplied or divided in order to make it a perfect square?
a. 6 b. 5 c. 8 d. 10

25. Dividing by 3/8, then multiplying by 5/6 is equivalent to dividing by
a. 5/16 b. 16/40
c. 9/20 d. 40/18

26. If $32^{x-2} = 64/8^x$. Find the value of x
a. -2 b. 3 c. 2 d. -3

27. A call center agent has a list of 305 phone numbers of people in alphabetic order of names (but she does not have any of the names). She needs to quickly contact Deepak Sharma to convey a message to him. If each call takes 2 minutes to complete, and every call is answered, what is the minimum amount of time in which she can guarantee to deliver the message to Mr Sharma.
a. 610 minutes b. 18 minutes
c. 206 minutes d. 34 minutes

28. n and p are integers greater than 1, 5n is the square of a number, 75np is the cube of a number. The smallest value for n + p is
a. 14 b. 18 c. 20 d. 30 e. 50

29. For how many integer values of n will the value of the expression 4n + 7 be an integer greater than 1 and less than 200?
a. 48 b. 49 c. 50 d. 51 e. 52