IIITB COMET FOUNDATION

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Exercises: 6.3 and 6.4

EXERCISE 6.3

1. Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at $x = 4$.

2. Find the slope of the tangent to the curve $y = \dfrac{x-1}{x-2}$, $x \neq 2$ at $x = 10$.

3. Find the slope of the tangent to curve $y = x^3 - x + 1$ at the point whose x-coordinate is 2.

4. Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose x-coordinate is 3.

5. Find the slope of the normal to the curve $x = a\cos^3\theta$, $y = a\sin^3\theta$ at $\theta = \dfrac{\pi}{4}$.

6. Find the slope of the normal to the curve $x = 1 - a\sin\theta$, $y = b\cos^2\theta$ at $\theta = \dfrac{\pi}{2}$.

7. Find points at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$ is parallel to the x-axis.

8. Find a point on the curve $y = (x-2)^2$ at which the tangent is parallel to the chord joining the points $(2, 0)$ and $(4, 4)$.

9. Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$.

10. Find the equation of all lines having slope -1 that are tangents to the curve $y = \dfrac{1}{x-1}$, $x \neq 1$.

11. Find the equation of all lines having slope 2 which are tangents to the curve $y = \dfrac{1}{x-3}$, $x \neq 3$.

12. Find the equations of all lines having slope 0 which are tangent to the curve $y = \dfrac{1}{x^2 - 2x + 3}$.

13. Find points on the curve $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$ at which the tangents are

(i) parallel to x-axis
(ii) parallel to y-axis.

14. Find the equations of the tangent and normal to the given curves at the indicated points:

(i) $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at $(0, 5)$
(ii) $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at $(1, 3)$
(iii) $y = x^3$ at $(1, 1)$
(iv) $y = x^2$ at $(0, 0)$
(v) $x = \cos t$, $y = \sin t$ at $t = \dfrac{\pi}{4}$

15. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is

(a) parallel to the line $2x - y + 9 = 0$

(b) perpendicular to the line $5y - 15x = 13$.

16. Show that the tangents to the curve $y = 7x^3 + 11$ at the points where $x = 2$ and $x = -2$ are parallel.

17. Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to the y-coordinate of the point.

18. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

19. Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which the tangents are parallel to the x-axis.

20. Find the equation of the normal at the point (am^2, am^3) for the curve $ay^2 = x^3$.

item Find the equation of the normals to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$.

21. Find the equations of the tangent and normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$.

22. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$.

23. Find the equations of the tangent and normal to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ at the point (x_0, y_0).

24. Find the equation of the tangent to the curve $y = \sqrt{3x - 2}$ which is parallel to the line $4x - 2y + 5 = 0$.

Choose the correct answer in Exercises 26 and 27.

26. The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is

(A) 3

(B) $\dfrac{1}{3}$

(C) -3

(D) $-\dfrac{1}{3}$

27. The line $y = x + 1$ is a tangent to the curve $y^2 = 4x$ at the point

(A) $(1, 2)$

(B) $(2, 1)$

(C) $(1, -2)$

(D) $(-1, 2)$

EXERCISE 6.4

1. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

(i) $\sqrt{25.3}$

(ii) $\sqrt{49.5}$

(iii) $\sqrt{0.6}$

(iv) $(0.009)^{\frac{1}{3}}$

(v) $(0.999)^{\frac{1}{10}}$

(vi) $(15)^{\frac{1}{4}}$

(vii) $(26)^{\frac{1}{3}}$

(viii) $(255)^{\frac{1}{4}}$

(ix) $(82)^{\frac{1}{4}}$

(x) $(401)^{\frac{1}{2}}$

(xi) $(0.0037)^{\frac{1}{2}}$

(xii) $(26.57)^{\frac{1}{3}}$

(xiii) $(81.5)^{\frac{1}{4}}$

(xiv) $(3.968)^{\frac{3}{2}}$

(xv) $(32.15)^{\frac{1}{5}}$

2. Find the approximate value of $f(2.01)$, where $f(x) = 4x^2 + 5x + 2$.

3. Find the approximate value of $f(5.001)$, where $f(x) = x^3 - 7x^2 + 15$.

4. Find the approximate change in the volume V of a cube of side x metres caused by increasing the side by 1%.

5. Find the approximate change in the surface area of a cube of side x metres caused by decreasing the side by 1%.

6. If the radius of a sphere is measured as 7 m with an error of 0.02 m, then find the approximate error in calculating its volume.

7. If the radius of a sphere is measured as 9 m with an error of 0.03 m, then find the approximate error in calculating its surface area.

8. If $f(x) = 3x^2 + 15x + 5$, then the approximate value of $f(3.02)$ is

(A) 47.66

(B) 57.66

(C) 67.66

(D) 77.66

9. The approximate change in the volume of a cube of side x metres caused by increasing the side by 3% is

(A) $0.06x^3$ m^3

(B) $0.6x^3$ m^3

(C) $0.09x^3$ m^3

(D) $0.9x^3$ m^3