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Perishable Inventory Routing Problem

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ABSTRACT

This study addresses the Perishable Inventory Routing Problem (PIRP) by comparing a deterministic Co-Solver approach and the NSGA-II genetic algorithm. PIRP integrates inventory and vehicle routing under perishability constraints, seeking to maximize profit, improve service, and reduce emissions. The Co-Solver, using exact optimization, provides high-quality solutions rapidly, while NSGA-II's evolutionary principles explore a broader range of trade-offs but require longer runtimes. Results show that Co-Solver solutions are on par with NSGA-II for profit and emissions, making it suitable for large-scale or time-sensitive problems. Ultimately, these approaches complement each other, offering valuable insights for decision-makers in complex perishable supply chains.

Keywords: *Perishable Inventory Routing problem, Co-Solver, Profit Maximization, Service Level Optimization.*

1. INTRODUCTION

Despite advancements in technology, managing perishable products continues to pose significant challenges across various industries, including the food industry (Kumar et al., 2020), the agri-food sector (Weerabahu et al., 2022), pharmaceuticals (Singh et al.), and blood transfusion centers (Beliën & Forcé, 2012). The perishability of these products adds a layer of complexity to supply chain management, where efficient coordination of inventory and transportation is critical. These elements are recognized as essential logistical drivers of the supply chain (Chopra & Meindl, 2015) and form the foundation of the Perishable Inventory Routing Problem (PIRP). In addressing PIRP, the objective is to optimize inventory levels and routing decisions to reduce waste, preserve product quality, and meet time-sensitive demands.

The Perishable Inventory Routing Problem (PIRP) represents a complex optimization challenge that integrates inventory management and vehicle routing under perishability constraints. With growing consumer demand for timely deliveries, companies face the dual challenge of maintaining high service levels while minimizing costs and environmental impacts. This multi-objective problem is particularly relevant in industries such as food, pharmaceuticals, and fresh produce logistics.

To address these challenges, this research draws inspiration from two established frameworks: a deterministic optimization approach rooted in the Traveling Thief Problem (TTP)(Bonyadi et.al.) and a multi-objective genetic algorithm (NSGA-II)(Rahimi et.al.). While TTP emphasizes deterministic co-solver strategies for dynamic optimization, NSGA-II leverages evolutionary principles to identify trade-offs among conflicting objectives. We tried to find the best of the worlds by comparing them.

The primary goals of this project are to:

1. Formulate a mathematical model that integrates inventory and routing decisions for perishable products.
2. Implement both Co-Solver and NSGA-II approaches to explore the trade-offs between profit, service levels, and greenhouse gas emissions.
3. Evaluate the efficacy of these methods across diverse and large-scale test cases.

2. PROBLEM FORMULATION

The PIRP is modeled as a multi-objective optimization problem, incorporating inventory and routing decisions, penalties for unmet demand, and constraints for perishability, capacity, and timing. Below is a detailed explanation of its elements:

PARAMETERS AND VARIABLES

Decision Variables:—

x_{ijkut} : Binary variable: 1 if vehicle u travels from retailer i to retailer j at time t.
$q^{dd'}_{ifkutt}$: Quantity delivered from facility f to retailer i for product k at time t.
B_{ift} : Backordered quantity at retailer i at time t.
I_{ift} : Inventory held at retailer i at time t.
EX_{ift} : Expired products at retailer i at time t.
z_{ikut} : Binary variable: 1 if retailer i is visited by vehicle u at time t.
C_{ift} : Capacity utilized by products at retailer i.
T_{ikut} : Travel time of vehicle u at time t visiting retailer i.
r_{it} : Retailer service delay at time t.

Parameters:—

R_{ft} : Revenue per unit of product from facility f at time t.
h_{if} : Holding cost per unit of inventory at retailer i.
γ_{if} : Backordering cost per unit at retailer i.
θ_f : Recycling cost per unit at facility f.
f_{cku} : Fixed transportation cost for vehicle u.
v_{ku} : Variable transportation cost per kilometer for vehicle u.
c_{ij} : Distance between retailers i and j.
LC, UC : Loading and unloading costs per unit, respectively.
GT_{ku} : Greenhouse gas emission cost per kilometer for vehicle u.
GEE : Emission cost of expired products.
MA_f, GU_{id} : Emission costs related to recycling and handling of goods.
d_{ift} : Demand at retailer i at time t.
IC_{if} : Inventory capacity at retailer i.
SP_{ku} : Speed of vehicle u.
S_{ij} : Travel time between retailers i and j.
G : A large constant for subtour elimination.

OBJECTIVE FUNCTIONS

1. Profit Maximization (f_1):—

$$\begin{aligned}\max f_1 = &\sum_{i,f,k,dd',t} R_{ft} \cdot (q_{ifkutt}^{dd'} - EX_{ift}) - \sum_{i,f,t} h_{if} \cdot I_{ift} - \sum_{i,f,t} \gamma_{if} \cdot B_{ift} \\ &- \sum_{i,f,t} \theta_f \cdot EX_{ift} - \sum_{(i,j),k,u,t} (f_{cku} \cdot z_{ikut} + v_{ku} \cdot c_{ij} \cdot x_{ijkut}) - \sum_{i,f,k,dd',t} (LC \cdot q_{ifkutt}^{dd'} + UC \cdot q_{ifkutt}^{dd'}).\end{aligned} \tag{1}$$

2. Service Level Optimization (f_2):—

$$\min f_2 = \beta_d \cdot \left(\frac{\sum_{i,t} n_{it}}{\sum_{i,k,u,t} z_{ikut}} \right) + \beta_B \cdot \left(\frac{\sum_{i,f,t} B_{ift}}{\sum_{i,f,t} d_{ift}} \right) + \beta_r \cdot \left(\frac{\sum_{i,t} r_{it}}{\sum_{i,k,u,t} z_{ikut}} \right). \tag{2}$$

3. GHG Emission Minimization (f_3):—

$$\min f_3 = \sum_{(i,j),k,u,t} GT_{ku} \cdot v_{ku} \cdot c_{ij} \cdot x_{ijkut} + \sum_{i,f,t} GEE \cdot EX_{ift} + \sum_{i,f,k,dd',t} MA_f \cdot GU_{id} \cdot q_{ifkutt}^{dd'}. \tag{3}$$

CONSTRAINTS

1. Inventory Flow Balance:—

$$I_{ift} - B_{ift} = I_{if(t-1)} - d_{ift} + \sum_{k,u,dd',t'} q_{ifkutt}^{dd'} - EX_{ift} - B_{if(t-1)} \quad \forall i, f, t. \tag{4}$$

2. Backorder Satisfaction:—

$$B_{ift} \geq \sum_{k,u,dd',t'} q_{ifkutt}^{dd'} \quad \forall i, f, t. \tag{5}$$

3. Holding vs Backorder Constraint:—

$$I_{ift} \cdot B_{ift} = 0 \quad \forall i, f, t. \tag{6}$$

4. Capacity Constraint:—

$$\sum_{k,u,dd',t'} q_{ifkutt}^{dd'} \leq IC_{if} - I_{if(t-1)} \quad \forall i, f, t. \tag{7}$$

5. Expiry and Recycling:—

$$EX_{ift} \leq I_{ift} \quad \forall i, f, t. \tag{8}$$

6. Time Constraints:—

$$T_{ikut} + \sum_{f,d,dd',t'} q_{ifkutt}^{dd'} \cdot SP_{ku} + c_{ij} \cdot S_{ij} \leq T_{jkut} + G(1 - x_{ijkut}) \quad \forall (i,j), k, u, t. \tag{9}$$

7. Routing Constraints:—

$$\sum_{j \in M'} x_{ijkut} = \sum_{j \in M'} x_{jikur} = z_{ikut} \quad \forall i \in M', k, u, t. \tag{10}$$

$$\sum_{k,u,t} z_{ikut} \leq 1 \quad \forall i, t. \tag{11}$$

$$\sum_{k,u,t} z_{ikut} \geq 1 \quad \forall i \in M. \tag{12}$$

8. Binary and Non-Negativity Constraints:—

$$x_{ijkut}, z_{ikut} \in \{0, 1\}, \quad B_{ift}, I_{ift}, EX_{ift}, q_{ifkutt}^{dd'} \geq 0 \quad \forall i, j, k, u, t. \tag{13}$$

3. METHODOLOGY

3.1. *NSGA-II Approach*

The NSGA-II is a genetic algorithm designed for multi-objective optimization. It generates a diverse set of Pareto-optimal solutions, balancing trade-offs among conflicting objectives.

Steps:—

1. **Initialization:** Randomly generate a population of solutions, each encoding inventory allocations and vehicle routes.
2. **Fitness Evaluation:** Compute the objective functions (f_1, f_2, and f_3) for each solution.
3. **Selection:** Apply Pareto dominance to identify solutions that are not dominated by others.
4. **Crossover and Mutation:** Perform genetic operations to create new solutions and explore the search space.
5. **Sorting and Crowding Distance:** Rank solutions based on dominance and maintain diversity using crowding distance.
6. **Iteration:** Repeat steps 2-5 until convergence to an optimal Pareto front.

3.2. *Co-Solver Approach*

The Co-Solver employs deterministic optimization to solve PIRP by alternating between inventory allocation and routing subproblems. It ensures strict constraint satisfaction and dynamic weight adjustments.

Steps:—

1. **Initial Route Generation:** Use heuristics (e.g., nearest neighbor) to create a feasible initial route.
2. **Inventory Allocation Subproblem:** Solve for I_{ift}, B_{ift}, and EX_{ift} using Gurobi, maximizing profit while considering capacity and perishability.
3. **Routing Subproblem:** Solve a Traveling Salesman Problem (TSP) using Gurobi to minimize transportation costs and emissions within time windows.
4. **Dynamic Weight Adjustment:** Adjust the weights for profit, service level, and emissions dynamically over iterations to explore trade-offs.
5. **Iterative Optimization:** Alternate between solving the inventory allocation and routing subproblems until convergence to a high-quality solution.

3.3. *Summary of Co-Solver Objectives*

The Co-Solver approach integrates the following objectives within its optimization framework:

- **Profit (f_1):** This objective is directly maximized by incorporating revenue from product sales and subtracting costs associated with holding inventory, backordering, recycling expired products, and transportation.
- **Service Level (f_2):** The service level is indirectly optimized by minimizing backorders and routing delays. Additional penalties for unmet demand and delays are included to enhance the service level.
- **Emissions (f_3):** Greenhouse gas emissions are minimized by accounting for transportation emissions, recycling emissions, and loading/unloading emissions in the objective function.

3.4. *Randomized Data Generation*

- Distances: Symmetric matrix with random values in a defined range.
- Profits, Weights, and Demands: Random values within realistic ranges to simulate varied product and retailer characteristics.
- Perishability: Randomly assigned shelf lives to represent product constraints.
- Vehicle Capacity: Calculated as a percentage of total product weights to introduce capacity restrictions.

3.5. *Scalability Testing*

- Varying the number of retailers from 10 to 50 to evaluate scalability.
- Testing under tight and loose time windows to observe the impact on routing feasibility.

3.6. *Performance Metrics*

- Profit, travel cost, and emissions were used as primary metrics to compare NSGA-II and Co-Solver approaches.

- Runtime analysis highlighted computational efficiency differences.

4. RESULTS

In the cases where the number of retailers are low, the nsga2 is providing better solutions than the Co-solver. The reasons might be the following:

Nature of Co-Solver:

The Co-Solver uses exact mathematical programming (via Gurobi), which is highly effective for well-structured problems, such as those defined by linear constraints and objectives. For large test cases, the problem's structure (e.g., well-defined capacity, routing, and time constraints) aligns with the Co-Solver's capabilities, allowing it to find optimal solutions efficiently.

Impact of Diversity:

A more diverse test set introduces variability in constraints (e.g., perishability, capacity, demand). The Co-Solver systematically evaluates feasible solutions within these constraints, providing robust results.

Deterministic Optimization:

Unlike NSGA-II, which is stochastic, the Co-Solver deterministically solves the problem. This deterministic approach avoids randomness and ensures consistent performance across runs.

Impact on Large Test Sets:

In large, complex problems, stochastic algorithms like NSGA-II may require more iterations to converge or struggle with local optima. The Co-Solver's deterministic nature ensures it systematically explores and resolves complexities.

Explicit Constraint Integration:

The Co-Solver explicitly incorporates constraints (e.g., inventory balance, capacity limits, perishability, backorders) into its formulation, ensuring feasible solutions. For large and diverse datasets with many constraints, this explicit handling is a significant advantage.

NSGA-II Limitation:

NSGA-II handles constraints indirectly, often using penalties in the fitness function. For complex constraints, this indirect handling can lead to infeasible or suboptimal solutions.

Performance on Diverse Inputs:

The Co-Solver adapts well to variability in demand, perishability, and routing due to its flexible constraint-based formulation. It efficiently identifies feasible solutions even for extreme test cases (e.g., high variability in retailer demands or distant routes).

Dynamic Weight Adjustments:

The Co-Solver adjusts objective weights (e.g., profit, service level, emissions) dynamically across iterations. This ensures balanced optimization even for diverse datasets where trade-offs vary significantly.

Test Case Impact:

For large datasets, the importance of profit, emissions, and service levels may differ for each instance. The Co-Solver adapts its focus accordingly, ensuring robust performance.

Below are the Images for the various test cases that we have tested for both the methods (Co-solver and nsga2)

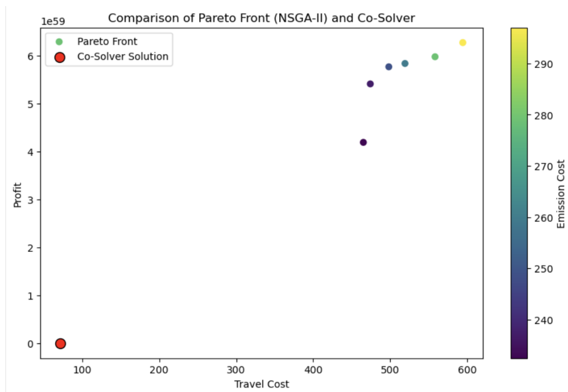

Figure 1. Test case small

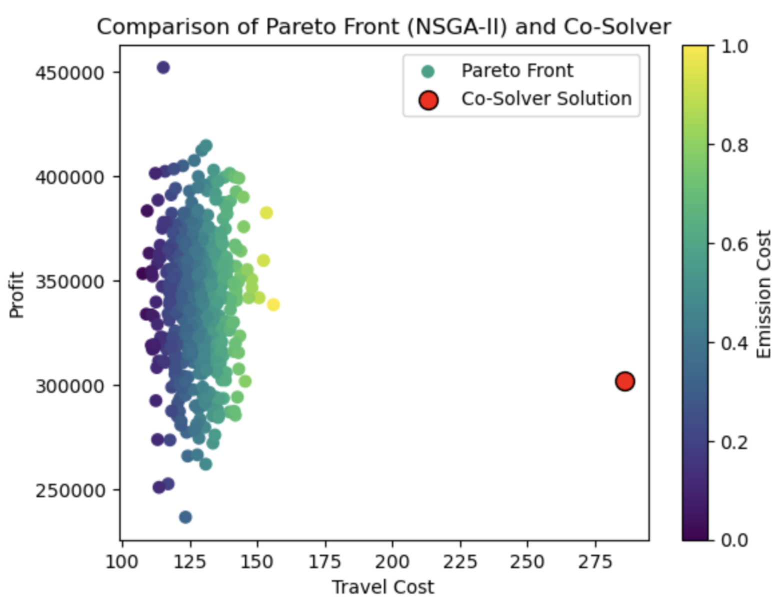

Figure 2. Test case large 1 (30 retailers)

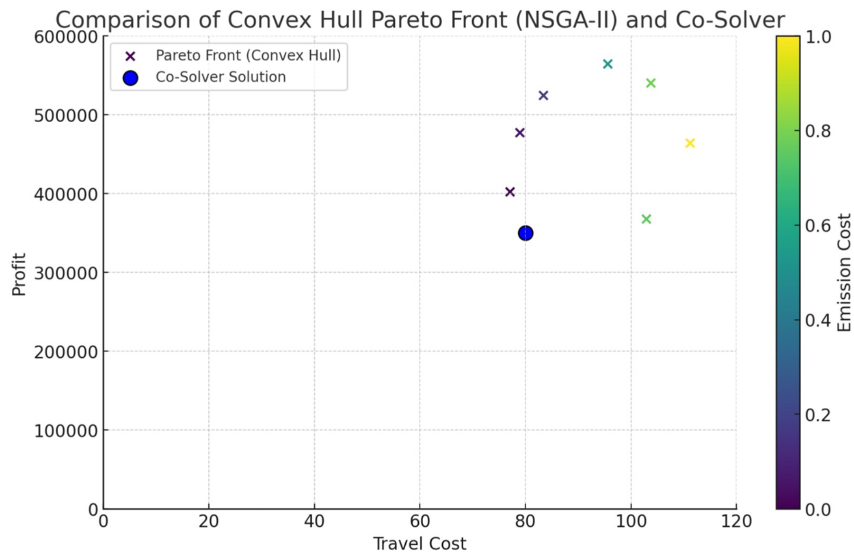

Figure 3. Large test case 2 (20 retailers)

From the above images, we can observe that the solution provided by the co-solver is on par with the pareto provided by the nsga2. The main usage of the co-solver is the time with which it provides the solution. The average runtime for the Co-solver is 1.7s compared to 37s for the nsga2. The above runtimes mentioned are for the smaller test case. For the larger test cases, the Co-solver takes around 5-10 minutes and the nsga2 consumes around 2-3hrs of time.

5. CONCLUSION AND FUTURE DIRECTIONS

5.1. *Conclusion*

This study addressed the Perishable Inventory Routing Problem (PIRP) using two optimization methodologies: the Co-Solver and NSGA-II. Both approaches effectively tackled the multi-objective nature of PIRP, including profit maximization, service level optimization, and emission minimization.

The Co-Solver, leveraging exact optimization via Gurobi, excelled with its deterministic nature, explicitly handling constraints like inventory balance and perishability. Its dynamic weight adjustment ensured robust performance across diverse test cases. Importantly, the Co-Solver achieved faster runtimes compared to NSGA-II by systematically exploring feasible solutions within a deterministic framework, providing solutions comparable to NSGA-II in terms of profit and emissions.

NSGA-II, with its evolutionary approach, generated a diverse Pareto front of trade-off solutions. This made it effective for exploring the solution space and balancing multiple objectives. However, its stochastic nature resulted in longer runtimes for large and diverse problem instances, where the Co-Solver often achieved equivalent solutions more efficiently.

In summary, the Co-Solver is well-suited for time-sensitive, large-scale problems requiring deterministic results, while NSGA-II excels in generating diverse trade-offs. Together, these approaches complement each other, providing a robust framework for solving complex PIRP scenarios.

5.2. *Future Directions*

As PIRP (Perishable and Inventory Routing Problem) continues to present new challenges and opportunities, we propose several directions for advancing research in this field. These ideas serve as an initial framework, and we encourage researchers to explore beyond them and contribute fresh perspectives.

1. **Hybrid Algorithms:** Combining evolutionary methods with exact solvers could enhance the efficiency and accuracy of PIRP solutions, addressing complex problem instances more effectively.

2. **Pareto Front Insights:** Extending research on analyzing and visualizing Pareto fronts in multi-objective decision-making could lead to better insights and trade-off analyses in PIRP solutions.

3. **Real-World Challenges:** Adapting PIRP models to handle stochastic demand, perishability, deterioration, and disruptions would improve their applicability in dynamic and uncertain environments.

4. **Multi-Echelon Supply Chains:** Expanding PIRP to include multi-echelon logistics networks could optimize supply chain performance, contributing to broader operational efficiency.

These ideas are just the seeds of what's possible. We encourage you to take these as inspiration but don't feel bound by them. The field of PIRP offers immense innovation potential, and new ideas and creative approaches are encouraged to push the boundaries further.

6. REFERENCES

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