Physics 647 Homework 1

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(1)

(a) Magnitude: $\sqrt{a^2+g^2}$ direction, $\theta = \tan^{-1}(\frac{a}{g})$ with vertical.

(b) It tilts in the direction of the gravitational force from (a), it tilts to the right at an angle $\theta = \tan^{-1}(\frac{a}{g})$ with vertical.

(c) It tilts in the direction of the gravitational force from (a), it tilts to the right at an angle $\theta = \tan^{-1}(\frac{a}{g})$ with vertical. Pendulum frequency is Proportional to $\sqrt{\frac{g}{L}}$ so the second hand moves $\sqrt{\frac{\sqrt{a^2+g^2}}{g}}$ time faster and takes $60 \cdot \sqrt{\frac{g}{\sqrt{a^2+g^2}}}$ seconds to complete.

(2)

(a) The components of a lorentz tensor in a new frame are expressed as linear combinations of the components of the tensor in another frame, and a linear combination of 0s is 0.

(b) We assume $A_{\mu\nu} = -A_{\nu\mu}$.

$$\begin{aligned}
A'_{\mu\nu} =& \Lambda_\nu{}^{\sigma_1}\Lambda_\mu{}^{\sigma^2}A_{\sigma_1\sigma_2}\\
=& \Lambda_\nu{}^{\sigma_1}\Lambda_\mu{}^{\sigma^2}(-A_{\sigma_2\sigma_1})\\
=& -\Lambda_\nu{}^{\sigma_2}\Lambda_\mu{}^{\sigma^1}A_{\sigma_1\sigma_2}(\text{relable indices})\\
=& -\Lambda_\mu{}^{\sigma^1}\Lambda_\nu{}^{\sigma_2}A_{\sigma_1\sigma_2}\\
=& -A'_{\nu\mu}
\end{aligned}$$

We assume $S_{\mu\nu} = S_{\nu\mu}$.

$$\begin{aligned}
S'_{\mu\nu} =& \Lambda_\nu{}^{\sigma_1}\Lambda_\mu{}^{\sigma^2}S_{\sigma_1\sigma_2}\\
=& \Lambda_\nu{}^{\sigma_1}\Lambda_\mu{}^{\sigma^2}S_{\sigma_2\sigma_1}\\
=& \Lambda_\nu{}^{\sigma_2}\Lambda_\mu{}^{\sigma^1}S_{\sigma_1\sigma_2}(\text{relable indices})\\
=& \Lambda_\mu{}^{\sigma^1}\Lambda_\nu{}^{\sigma_2}S_{\sigma_1\sigma_2}\\
=& S'_{\nu\mu}
\end{aligned}$$

(c) $\frac{K^{\mu\nu}}{K^{\mu\sigma}} = \frac{K^{\rho\nu}}{K^{\rho\sigma}}$ for all choices of μ, ν, ρ, σ.

(d) We make use of the result (2.37), $\Lambda^\mu{}_\nu\Lambda_\mu{}^\rho = \delta^\rho_\nu$.

$$\begin{aligned}
\delta'^\mu_\nu =& \Lambda^\mu{}_\rho\Lambda_\nu{}^\sigma\delta^\rho_\sigma\\
=& \Lambda^\mu{}_\rho\Lambda_\nu{}^\sigma\left(\Lambda^\alpha{}_\sigma\Lambda_\alpha{}^\rho\right)\\
=& \left(\Lambda^\mu{}_\rho\Lambda_\alpha{}^\rho\right)\left(\Lambda_\nu{}^\sigma\Lambda^\alpha{}_\sigma\right)\\
=& \delta^\mu_\alpha\delta^\alpha_\nu\\
=& \delta^\mu_\nu
\end{aligned}$$

(3) $U^\mu = W^{\mu\nu}V_\nu$ where U^μ is a Lorentz vector holds in all Lorentz frams for arbitrary Lorentz vector V_μ. We have $U'^\mu = W'^{\mu\nu}V'_\nu$ where $U'^\mu = \Lambda^\mu{}_\nu U^\nu$ and $V'_\nu = \Lambda^\sigma_\nu V_\sigma$.

$$\begin{aligned}
U'^\mu &= W'^{\mu\nu}V'_\nu \\
\Lambda^\mu{}_\alpha U^\alpha &= W'^{\mu\nu}\Lambda^\sigma_\nu V_\sigma \\
\Lambda^\mu{}_\alpha W^{\alpha\rho}V_\rho &= W'^{\mu\nu}\Lambda^\sigma_\nu V_\sigma \\
\Lambda^\mu{}_\alpha W^{\alpha\rho}\left(\Lambda^\beta{}_\rho V_\beta\right) &= W'^{\mu\nu}\Lambda^\sigma_\nu\left(\Lambda^\beta{}_\sigma V_\beta\right) \text{(holds for arbitrary Lorentz vector)} \\
\left(\Lambda^\beta{}_\rho\Lambda^\mu{}_\alpha W^{\alpha\rho}\right)V_\beta &= W'^{\mu\nu}\left(\Lambda^\beta{}_\sigma\Lambda^\sigma_\nu\right)V_\beta \\
\left(\Lambda^\beta{}_\rho\Lambda^\mu{}_\alpha W^{\alpha\rho}\right)V_\beta &= W'^{\mu\nu}\delta^\nu_\beta V_\beta \\
\left(\Lambda^\beta{}_\rho\Lambda^\mu{}_\alpha W^{\alpha\rho}\right)V_\beta &= W'^{\mu\beta}V_\beta
\end{aligned}$$

Because this holds for arbitrary V_ν we conclude $W'^{\mu\beta} = \Lambda^\beta{}_\rho\Lambda^\mu{}_\alpha W^{\alpha\rho}$ is a Lorentz tensor.

(4) Wlog we will consider a coordinate system in which we have $k^2 = k^3 = 0$ we then have the condition $(k^0)^2 = (k^1)^2$ from k^μ being null, and we take these both to be nonzero, otherwise $k^\mu V_\mu = 0$ for all V. Then from the condition $k^\mu V_\mu = 0$ we have $k^0V^0 = k^1V^1 \Rightarrow V^0 = \frac{k^1V^1}{k^0} \Rightarrow (V^0)^2 = \frac{(k^1)^2(V^1)^2}{(k^0)^2} = (V^1)^2$. From the fact that $V^\mu V_\mu = -(V^0)^1 + (V^1)^2 + (V^2)^2 + (V^3)^2 \leq 0 \Rightarrow (V^2)^2 + (V^3)^2 \leq 0$ and since these are both nonegative numbers $(V^2)^2 = (V^3)^2 = 0 \Rightarrow V^2 = V^3 = 0$.

In the case $k^0 = k^1$ we have from $k^0V^0 = k^1V^1 \Rightarrow k^1V^0 = k^1V^1 \Rightarrow V^0 = V^1$ so clearly V^μ is a multiple of k^μ and $V^\mu = \frac{V^0}{k^0}k^\mu$. In the case $k^0 = -k^1$ we have from $k^0V^0 = k^1V^1 \Rightarrow -k^1V^0 = k^1V^1 \Rightarrow V^0 = -V^1$ and again V^μ is a multiple of k^μ and $V^\mu = -\frac{V^0}{k^0}k^\mu$ so we have the desired conclusion.