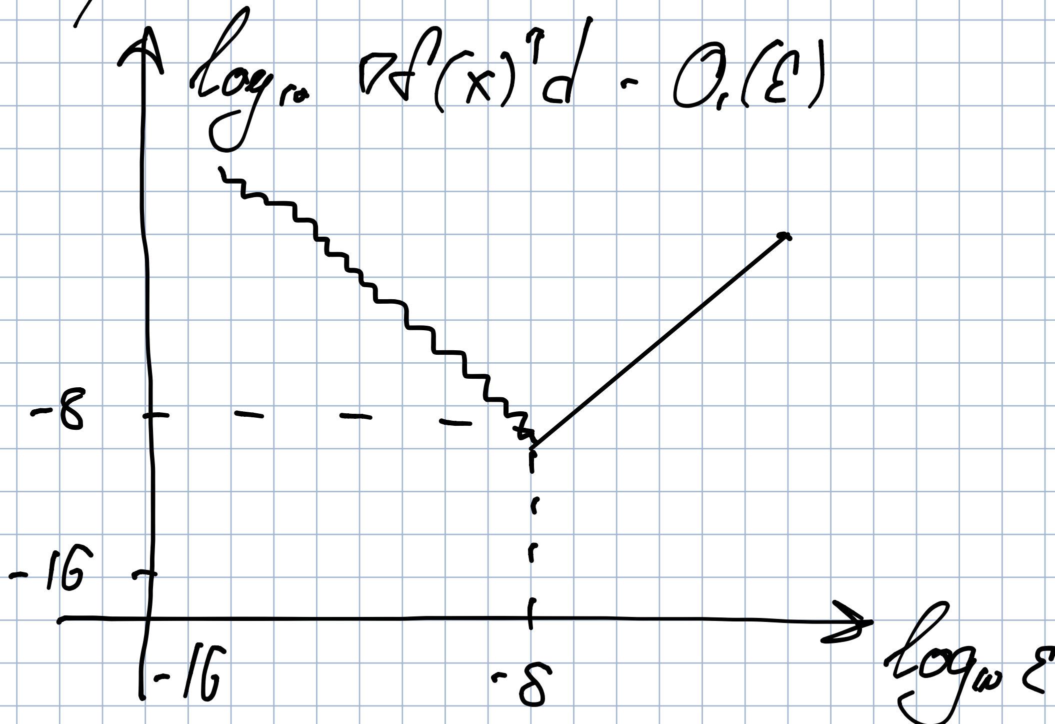
Разностное дифференц.

①

$$f(x+\varepsilon d) = f(x) + \nabla f(x)^T \varepsilon d + O(\varepsilon^2)$$

$$\nabla f(x)^T d = \underbrace{\frac{f(x+\varepsilon d) - f(x)}{\varepsilon}}_{O_1(\varepsilon)} + O(\varepsilon)$$

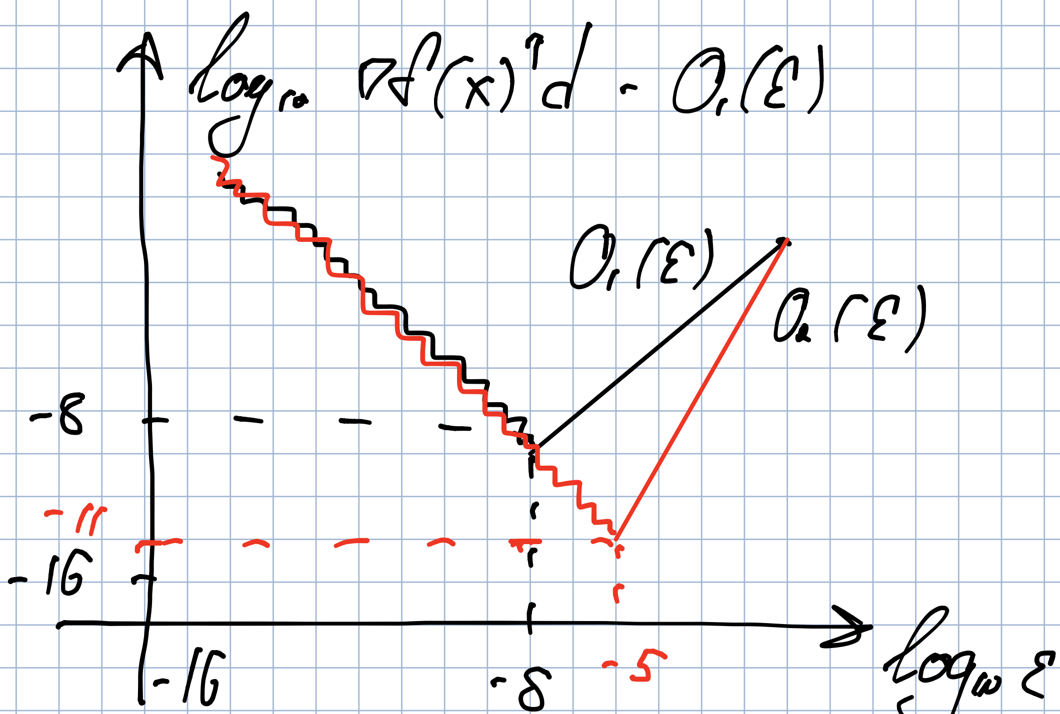
$$\varepsilon_{opt} = \sqrt{\frac{2L_0 \varepsilon_m}{L_2}} \approx \sqrt{\varepsilon_m}$$



② $f(x+\varepsilon d) = f(x) + \nabla f(x)^T d\varepsilon + \frac{1}{2}\varepsilon^2 d^T \nabla^2 f(x) d + O(\varepsilon^3)$

$f(x-\varepsilon d) = f(x) - \nabla f(x)^T \varepsilon d + \frac{1}{2} d^T \varepsilon^2 \nabla^2 f(x) d + O(\varepsilon^3)$

$$\nabla f(x)^T d = \underbrace{\frac{f(x+\varepsilon d) - f(x-\varepsilon d)}{2\varepsilon}}_{O_2(\varepsilon)} + O(\varepsilon^2)$$



$\varepsilon = \sqrt[3]{\varepsilon_m}$.

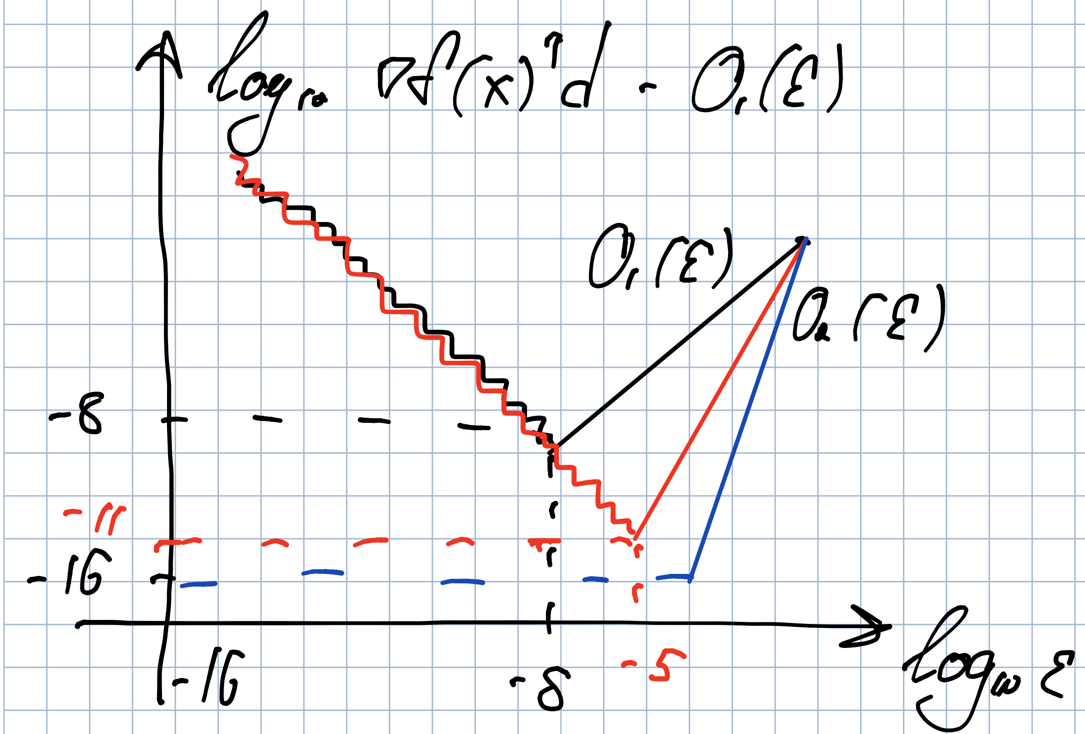
комп. ед.

③ $f(x + i\varepsilon d) =$

$$= f(x) + \nabla f(x)^T d\, i\varepsilon + \underline{O}(\varepsilon^2) + i\,\underline{O}(\varepsilon^3)$$

$$\nabla f(x)^T d = \underbrace{\frac{\operatorname{Im}(f(x+\varepsilon i d))}{\varepsilon}}_{O_3(\varepsilon)} + \underline{O}(\varepsilon^2)$$

Нет процедуры вычитания и нет потери точности.



Град. спуск.

Пример.

(1) $f(x,y) = \frac{1}{2}x^2 - \frac{1}{2}y^2 \to \min\limits_{x,y}$

$\nabla f(x,y) = (x, -y)$

$\nabla^2 f(x,y) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $\lambda_{max} = 1$

По норме можно ограничить сверху.

$\Rightarrow$ Функция с липш. град.

$(x_1, y_1) = (x_0, 0) - \alpha(x_0, 0) = (x_0 - \alpha x_0, 0) =$

$= (1-\alpha)(x_0, 0)$

$(x_k, y_k) = [(1-\alpha)^k x_0, 0] \to 0$ (седловая т.)

Для произвольных x_0, y_0.

$(x_k, y_k) = [(1-\alpha)^k x_0, (1+\alpha)^k y_0] \to -\infty$

② $f(x) = \frac{1}{3}|x|^3, \quad x \in \mathbb{R}.$

$$\nabla f(x) = x^2 \cdot \mathrm{sign}(x) = x \cdot |x|$$

$$x_{k+1} = x_k - \alpha x_k \cdot |x_k| = x_k (1 - \alpha |x_k|)$$

$$x_{k+1} = x_k - \alpha x_k^2$$

$$x_{k+1} - x_k = -\alpha x_k^2$$

$$\dot{x} = -\alpha x_k^2$$

$$dx = -\alpha x_k^2 \, dt$$

$$\frac{dx}{x_k^2} = -\alpha \, dt$$

$$\frac{1}{x_k} = \alpha t + C \Rightarrow x = \frac{1}{\alpha t + C}$$

$$x_{k+1} = x_k - \alpha x_k^2 \quad \Big/ \cdot \frac{1}{x_{k+1} \cdot x_k}$$

$$\frac{1}{x_k} = \frac{1}{x_{k+1}} - \frac{\alpha x_k}{x_{k+1}} > 1$$

$$\frac{1}{x_{k+1}} = \frac{1}{x_k} + \alpha \frac{x_k}{x_{k+1}} > \frac{1}{x_k} + \alpha > \frac{1}{x_{k-1}} + 2\alpha > \dots$$

$$> \frac{1}{x_0} + (k+1)\alpha$$

$$\boxed{x_{k+1} < \frac{1}{\frac{1}{x_0} + (k+1)\alpha}}$$