$$(a(t) \cdot b(t))' = a'(t) \cdot b(t) + a(t) \cdot b'(t)$$

$$(a(t) \times b(t))' = a'(t) \times b(t) + a(t) \times b'(t)$$

$$(a(t), b(t), c(t))' = (a'(t), b(t), c(t)) + (a(t), b'(t), c(t)) + (a(t), b(t), c'(t))$$

混合积 $\uparrow$ $(a \times b) \cdot c = (b \times c) \cdot a = (c \times a) \cdot b$

Thm: 设 $a(t)$ 是处处非零的连续可微向量函数.

则 (1) $a(t)$ 长度不变 $\Longleftrightarrow a'(t) \cdot a(t) \equiv 0$

$(|a(t)|^2 = a(t) \cdot a(t)$ 定值 $\Rightarrow$ 求导$)$

(2) $a(t)$ 方向不变 $\Longleftrightarrow a'(t) \times a(t) \equiv \vec{0}$

"$\Rightarrow$" $a(t) = f(t) \cdot a \qquad a'(t) = f'(t) \cdot a \Rightarrow a'(t) \times a(t) \equiv \vec{0}$

"$\Leftarrow$" $b(t) = \dfrac{a(t)}{|a(t)|}$, $|b(t)| = 1 \Rightarrow b'(t) \cdot b(t) = 0$

$\Rightarrow b'(t) \cdot a(t) = 0.$

又 $b'(t) = \dfrac{a'(t)}{|a(t)|} + a(t) \left(\dfrac{1}{|a'(t)|}\right)'$,

$b'(t) \times a(t) = b'(t) \times \dfrac{a'(t)}{|a(t)|} = a(t) \times a'(t) = 0$

$\Rightarrow b'(t) \equiv 0$, $a(t)$ 方向不变.

$a \times (b \times c) = (a \cdot c) b - (a \cdot b) c$