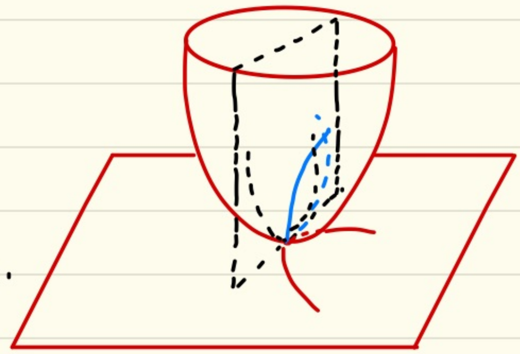
$$K=\frac{R_{1212}}{g}=-\frac{1}{\sqrt{EG}}\left[\left(\frac{(\sqrt{E})_v}{\sqrt{G}}\right)_v+\left(\frac{(\sqrt{G})_u}{\sqrt{E}}\right)_u\right]$$

$e_1=\alpha(s),\ e_3=n(s),\ e_2=e_3\times e_1$



$$\begin{pmatrix}e_1\\e_2\\e_3\end{pmatrix}'=\begin{pmatrix}0&k_g&k_n\\-k_g&0&\tau_g\\-k_n&-\tau_g&0\end{pmatrix}\begin{pmatrix}e_1\\e_2\\e_3\end{pmatrix},$$ k_n为曲线的法曲率.

$$k_g=\frac{d^2r(s)}{ds^2}\cdot e_2=r''(s)\cdot(n(s)\times r'(s))=(n(s),r'(s),r''(s))$$

$$\tau_g=\frac{de_2}{ds}\cdot n=(n'(s)\times r'(s))\cdot n=(n(s),n'(s),r'(s)).$$

测地曲率、法曲率、曲线曲率

$k_g^2+k_n^2=k^2$, $\quad k_n=k\cos\theta,\ \theta=\angle(\beta,n)$, $k_g=k\cos\tilde{\theta},\ \tilde{\theta}=\angle(\gamma,n)$.

$$k_g=e_1'\cdot e_2=\sqrt{g_{11}g_{22}-(g_{12})^2}\begin{vmatrix}\frac{du^1}{ds}&\frac{d^2u^1}{ds^2}+\Gamma^1_{\alpha\beta}\frac{du^\alpha}{ds}\frac{du^\beta}{ds}\\\frac{du^2}{ds}&\frac{d^2u^2}{ds^2}+\Gamma^2_{\alpha\beta}\frac{du^\alpha}{ds}\frac{du^\beta}{ds}\end{vmatrix}$$

Liouville 公式 (正交参数系)

$\mathrm{I}=E\,du^2+G\,dv^2$, $C:\begin{cases}u=u(s)\\v=v(s)\end{cases}$, $\frac{dr}{ds}=r_u\frac{du}{ds}+r_v\frac{dv}{ds}$,

$\alpha_1=\frac{1}{\sqrt{E}}du,\ \alpha_2=\frac{1}{\sqrt{G}}dv\Rightarrow r'(s)=\alpha_1\cos\theta+\alpha_2\sin\theta,\ \theta=\angle(\alpha_1,e_1)$

单位化.

$(\sqrt{f})_x=\frac{f'}{2\sqrt{f}}$, $(\ln f)_x=\frac{f'}{f}$