Dupin 标形：$r(0,0)$为坐标原点，找u,v使得$(r_u(0,0), r_v(0,0), n(0,0))$

单位正交，有$r_{uu}\cdot n = k_1 E = k_1$，$r_{uv}\cdot n = 0$，$r_{vv}\cdot n = k_2$.

即 $r(u,v) = (u+o(1))e_1 + (v+o(1))e_2 + \frac{1}{2}(k_1u^2+k_2v^2)n + o(1)$

$\Rightarrow$ 近似曲面：$\begin{cases} x = u \\ y = v \\ z = \frac{1}{2}(k_1u^2+k_2v^2) \end{cases}$ P172.

k_1, k_2 一个为0：抛物柱
二个为0：平面
同号：椭圆抛物面
异号：双曲抛物面

$r = r(u^1, u^2)$，$r_1 = \frac{\partial r}{\partial u^1}$，$r_2 = \frac{\partial r}{\partial u^2}$ 为两个切向量.

$dr = r_1du^1 + r_2du^2 = \sum_{\alpha=1}^{2} r_\alpha du^\alpha := r_\alpha du^\alpha$ （指标上下相同表求和）

$S_{\alpha\beta}T^{\alpha\beta\gamma} = \sum_{\alpha=1}^{2}\sum_{\beta=1}^{2} S_{\alpha\beta}T^{\alpha\beta\gamma}$. $P_\alpha^\alpha = \sum_{\alpha=1}^{2} P_\alpha^\alpha = P_1^1 + P_2^2$.

$g_{\alpha\beta} = r_\alpha r_\beta$，即 $E = g_{11}$, $F = g_{12} = g_{21}$, $G = g_{22}$. $\mathrm{I} = g_{\alpha\beta}du^\alpha du^\beta$.

$b_{\alpha\beta} = r_{\alpha\beta}\cdot n$，即 $L = b_{11}$, $M = b_{12} = b_{21}$, $N = b_{22}$. $\mathrm{II} = b_{\alpha\beta}du^\alpha du^\beta$.

$g = \det(g_{\alpha\beta}) = g_{11}g_{22} - (g_{12})^2$，$b = \det(b_{ab}) = b_{11}b_{12} - (b_{12})^2$.

$g_{\alpha\beta}$的逆矩阵记为$g^{\alpha\beta}$，有 $g_{\alpha\beta}g^{\beta\gamma} = \delta_\gamma^\alpha = \begin{cases} 1, & \alpha = \gamma \\ 0, & \alpha \neq \gamma \end{cases}$.

$\begin{pmatrix} g^{11} & g^{12} \\ g^{21} & g^{22} \end{pmatrix} = \frac{1}{g}\begin{pmatrix} g_{22} & -g_{12} \\ -g_{21} & g_{11} \end{pmatrix}$.

S的自然标架为$\{r; r_1, r_2, n\}$，$\frac{\partial r_\alpha}{\partial u^\beta} = \Gamma_{\alpha\beta}^\gamma r_\gamma + C_{\alpha\beta}n$. $r_{\alpha\beta}$

$n_\beta \leftarrow \frac{\partial n}{\partial u^\beta} = D_\beta^\gamma r_\gamma + D_\beta n$.