

# Discreteness in Neural Natural Language Processing

Lili Mou<sup>a</sup> Hao Zhou<sup>b</sup> Lei Li<sup>b</sup>

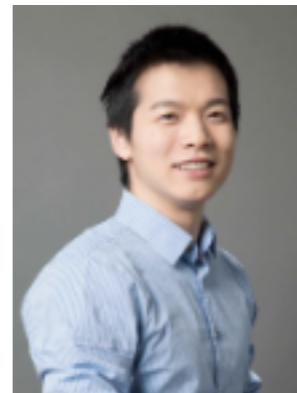
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EMNLP-IJCNLP 2019 Tutorial



# Part III: Discrete Latent Space



# Roadmap

- Definitions & Examples
- General techniques
  - Maximum likelihood estimation
  - Reinforcement learning
  - Gumbel-softmax
  - Step-by-step Attention
- Case studies
  - Weakly supervised semantic parsing
  - Unsupervised syntactic parsing

# Latent Variable

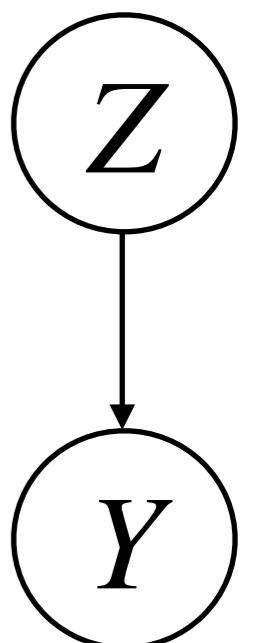
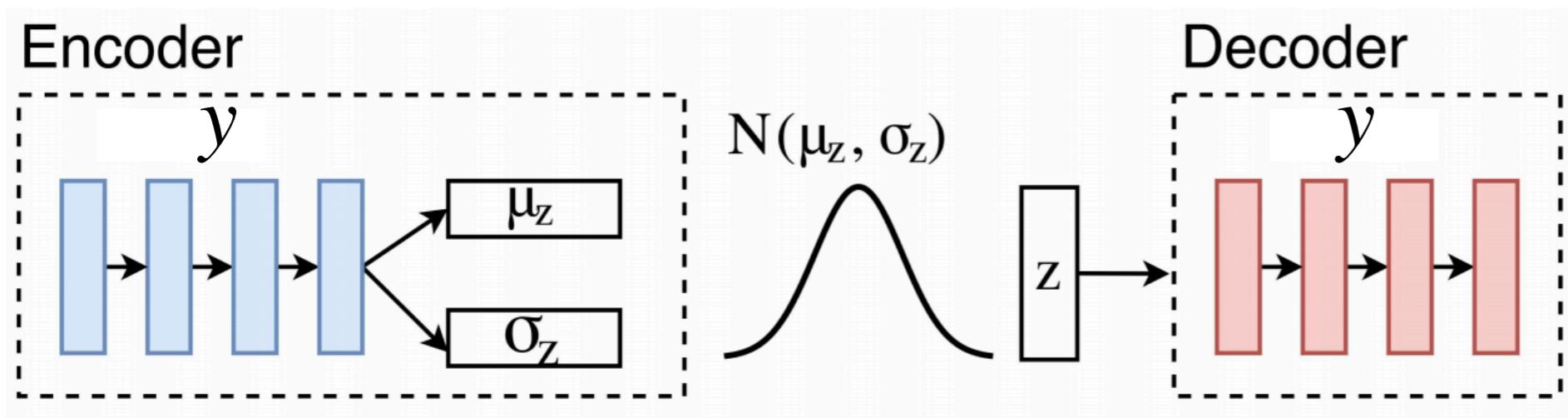
- Consider a probabilistic model on  $(x, y, z)$ 
  - $x$ : Discriminative (conditional)
  - $y$ : Generative (joint)
  - $z$ : Unknown during both training and prediction
- Their relations depend on applications.
- The definition here is based on the **model**  $p(z, y | x)$ , instead of the **task**

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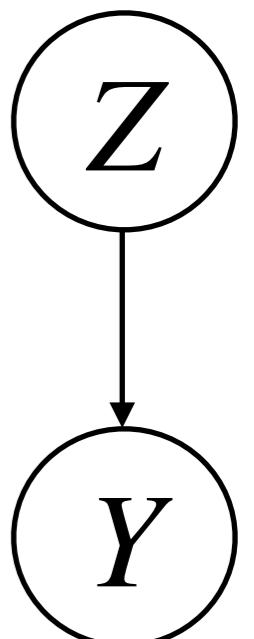
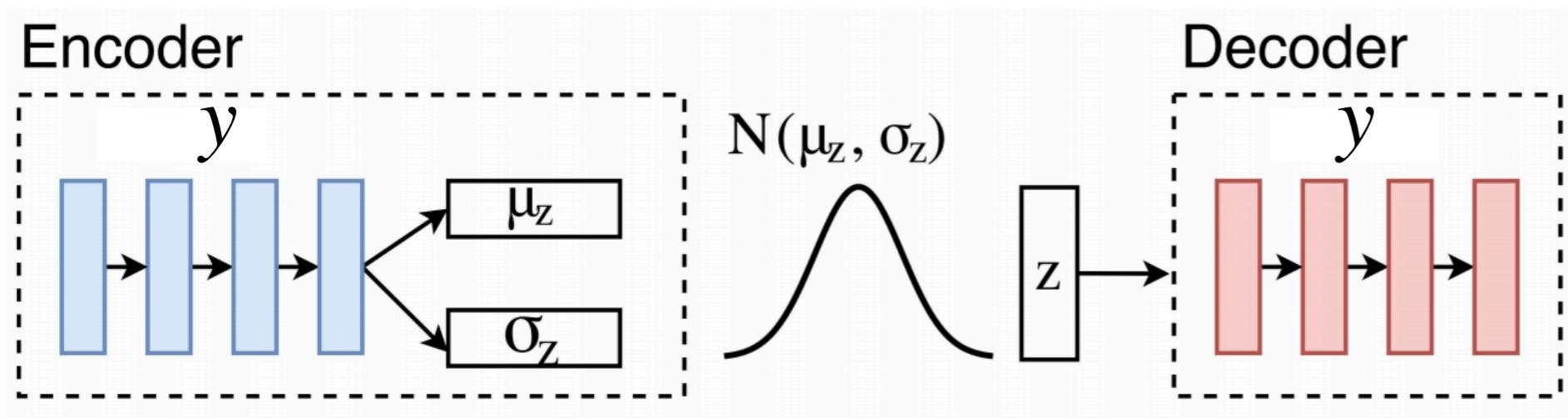
# Examples

- Continuous latent variable
  - **Variational autoencoder (VAE)**
  - A data point  $y$  is subject to some latent variable  $z$
  - Encoder: recognizing  $z$  from  $y$
  - Decoder: generating  $y$  from  $z$



# Examples: VAE

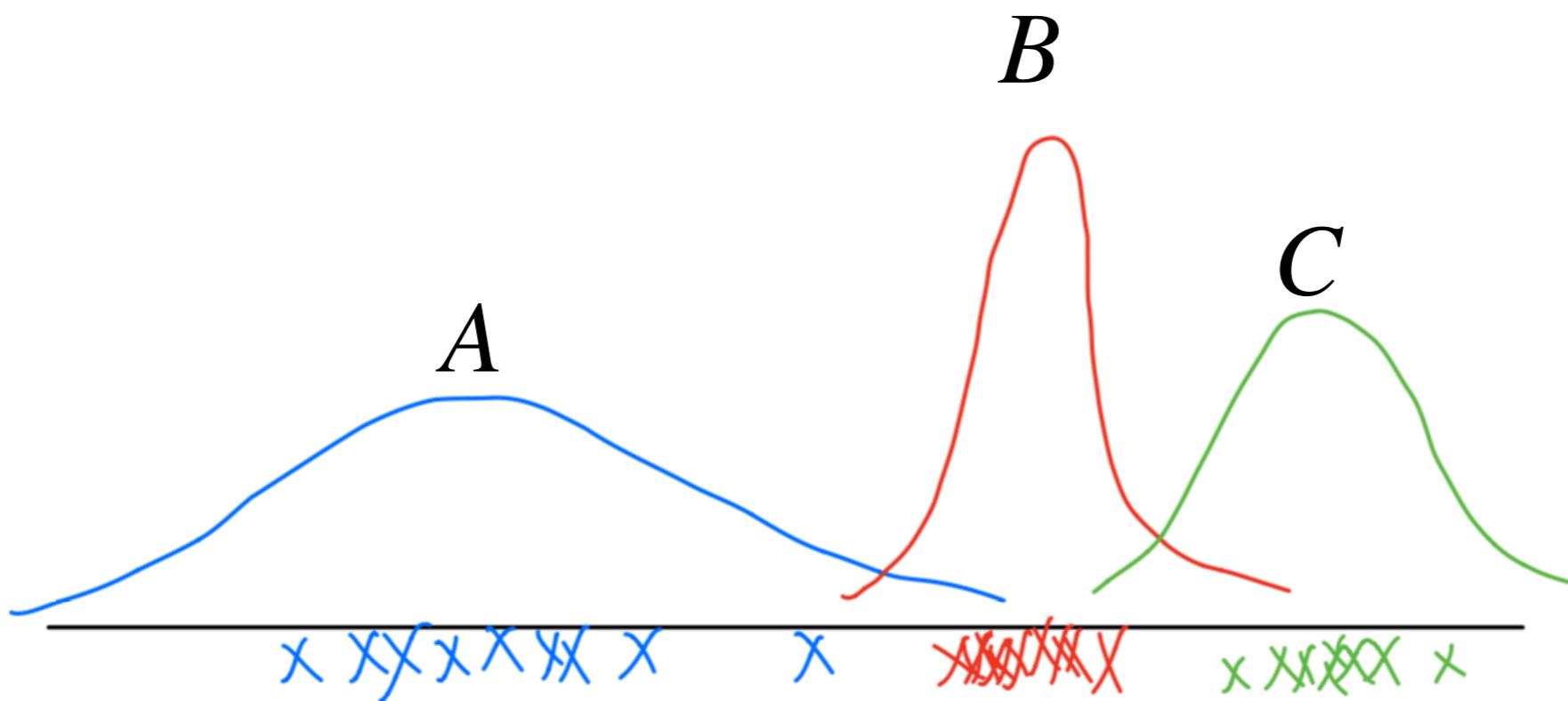
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# Examples: GMM

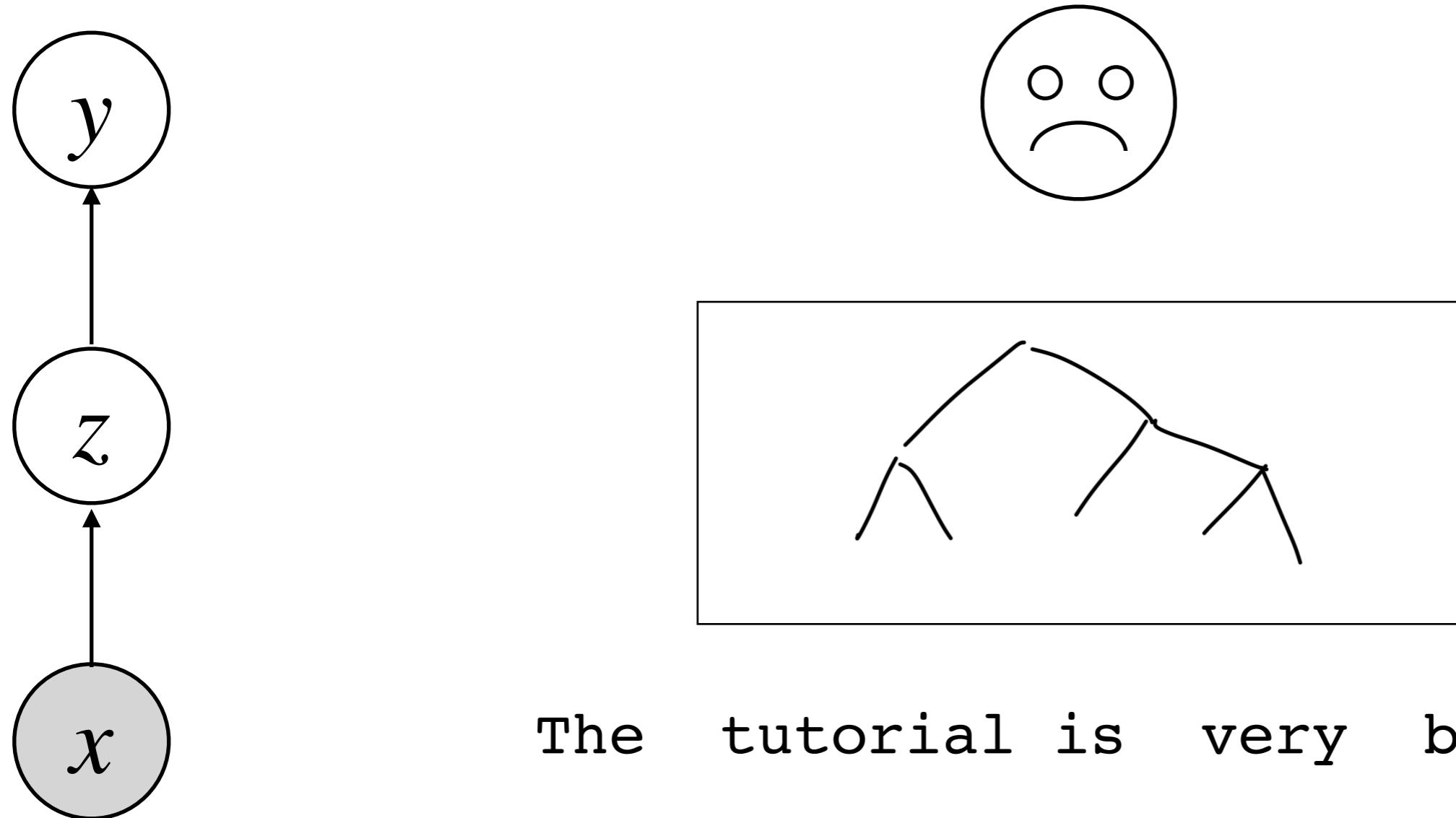
- Discrete latent variable: Clustering with Gaussian mixtures

$$z \in \{A, B, C\}$$



# Examples: Latent Tree Induction

- Discrete latent variable: Syntactic parse trees



The tutorial is very boring

Latent variables may play a role in discriminative models

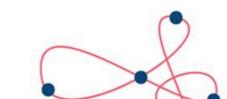
# General Criteria for Latent Variables

- Training
  - Marginalization
    - ▶ Something of  $\mathbb{E}$
    - ▶  $\mathbb{E}$  of something
    - ▶ All sorts of approx. for  $\mathbb{E}$
- Inference (depending on applications)
  - Target prediction: Predict  $y$  by marginalizing  $z$
  - Latent variable prediction: predict  $z$ 
    - ▶ *Max a posteriori*
    - ▶ Sampling

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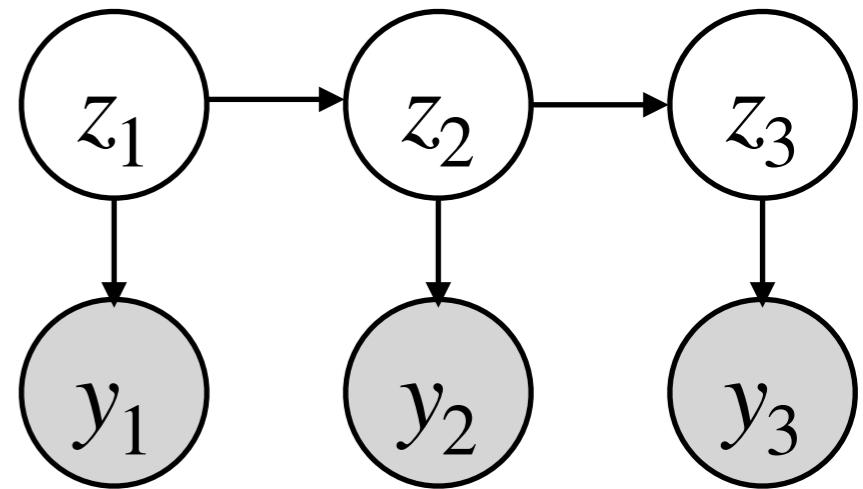
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# Maximum Likelihood Estimation



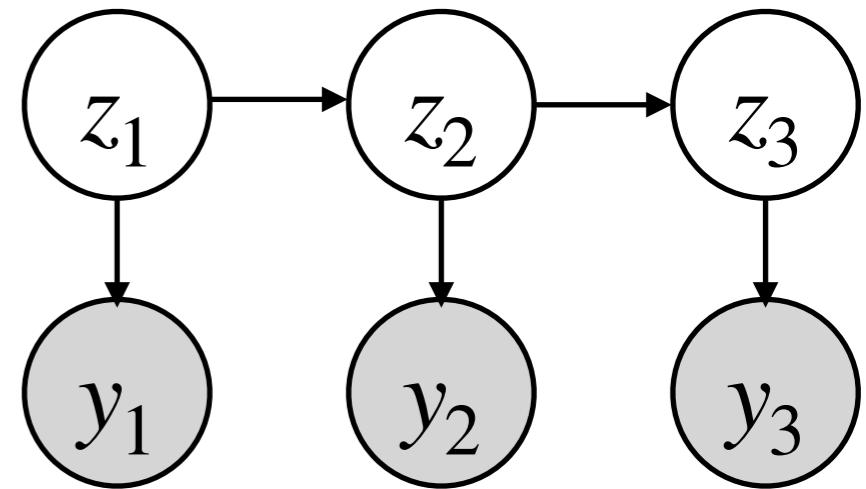
# Hidden Markov Models

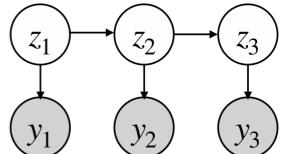
- Observed tokens:  $y_1, y_2, \dots, y_T$
- Latent states:  $z_1, \dots, z_T$
- Generative procedure
  - Choose  $z_1$  (omitted here)
  - For every step  $t$ :
    - ▶ Pick  $z_t \sim p(z_t | z_{t-1})$
    - ▶ Emit  $y_t \sim p(y_t | z_t)$
  - Suppose both parametrized by probability tables
- Example
  - $y_1, y_2, \dots, y_T$ : a sequence of words
  - $z_1, z_2, \dots, z_T$ : POS tags



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# Hidden Markov Models

- **E-step** (expectation for sufficient statistics)

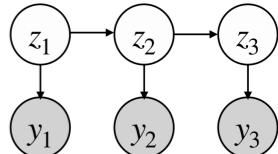
- Expectation of a state, that is,  $\gamma_t(i) \triangleq \mathbb{E}[z_t = i | \cdot]$
- Expectation of two consecutive states, that is,  $\xi_t(i, j) \triangleq \mathbb{E}[z_t = i, z_{t+1} = j | \cdot]$
- Computed by

$$\gamma_t(i) = \frac{\alpha_t(i)\beta_t(i)}{p(Y)} \quad \xi_t(i, j) = \frac{\alpha_t(i)p_{\theta}(x_t | z_n = i)p_{\theta}(z_t = j | z_{t-1} = i)\beta_t(j)}{p(Y)}$$

where

$$\alpha_t(i) \triangleq p(y_{1:t}, z_t = i) \quad \text{and} \quad \beta_t(i) \triangleq p(y_{t+1:T} | z_t = i)$$

are given by dynamic programming



# Hidden Markov Models

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  - Expectation of two consecutive states, that is,  $\xi_t(i, j) \triangleq \mathbb{E}[z_t = i, z_{t+1} = j | \cdot]$
- **M-step** (MLE by soft counting)

$$p(z_t = j | z_{t-1} = i) = \frac{\sum_{t=1}^{T-1} \xi_t(i, j)}{\sum_{t=1}^{T-1} \gamma_t(i)}$$

$$p(x | z_t = j) = \frac{\sum_{t=1}^T \gamma_t(j) \mathbb{1}\{X_t = x\}}{\sum_{t=1}^T \gamma_t(j)}$$

# EM as MLE

$$\ell(\theta_{t+1}) = \sum_i \log p(y_i; \theta_{t+1})$$

$$= \sum_i \log \left( \sum_z p(y_i, z; \theta_{t+1}) \right)$$

[Lower bound holds for any  $q_t$ ]

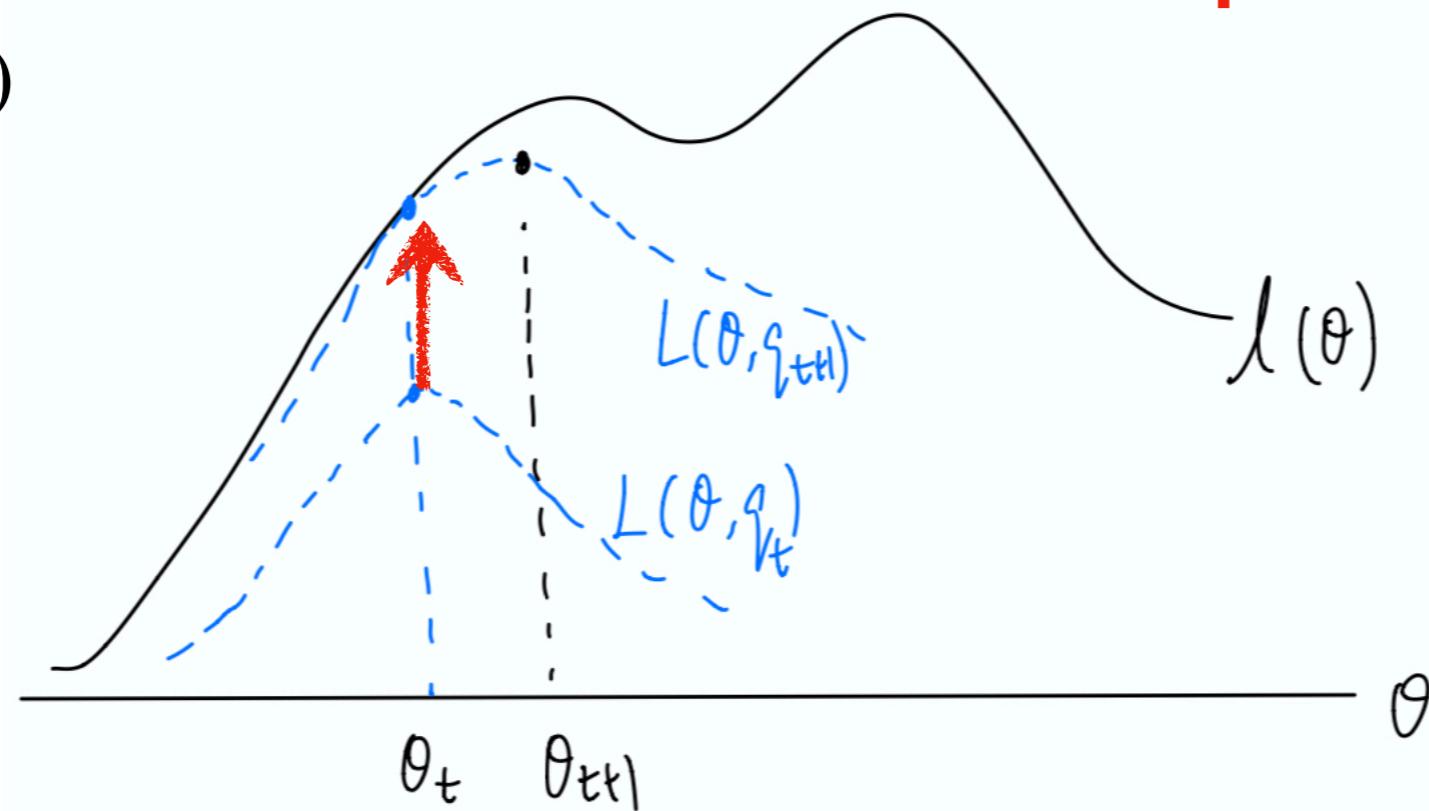
$$\geq \sum_i \sum_z q_t(z | y_i) \log \frac{p(y_i, z; \theta_{t+1})}{q_t(z | y_i)}$$

**M-step:**  $\theta_{t+1} = \arg \max \{ \cdot \}$

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**E-step:** make lower bound tight

$$= \ell(\theta_t)$$



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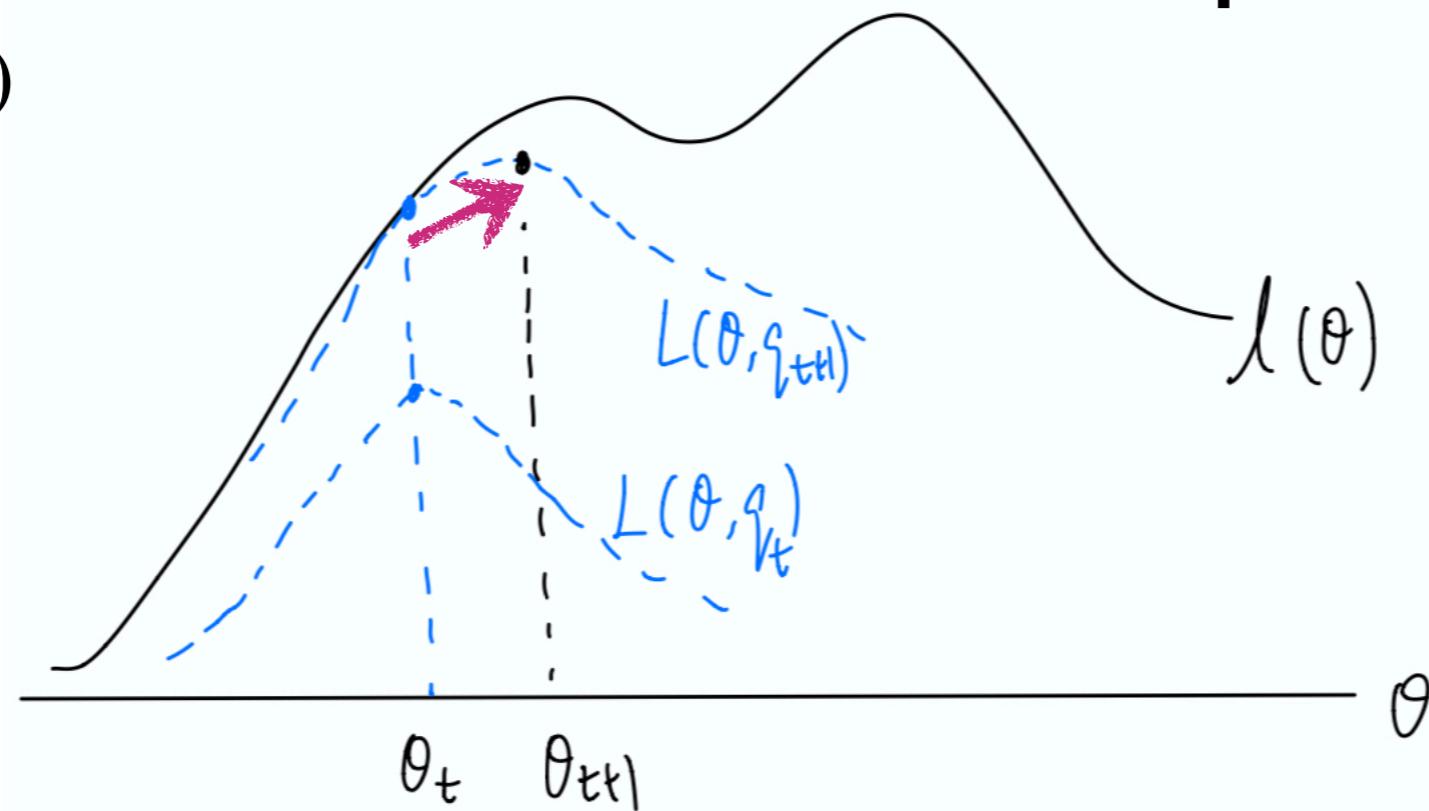
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# Back Propagation

$$\log p(Y|\theta) = \log \left( \sum_z p(Y, z|\theta) \right)$$

- Complexity of BP =  $\mathcal{O}$ (Complexity of FP)
- EM is BP

$$p(y, z|x) = \frac{1}{Z} \exp \left\{ \sum_i \theta_i f_i \right\}$$

$$\frac{\partial}{\partial \theta_i} \log p(y, z|x) = \mathbb{E}_{z \sim p(z|x,y)}[f_i] - \mathbb{E}_{y,z \sim p(y,z|x)}[f_i]$$

Eisner, Jason. Inside-outside and forward-backward algorithms are just backprop (tutorial paper). In *Proceedings of the Workshop on Structured Prediction for NLP*, 2016.

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# Other Treatments

$$\log p(Y|\theta) = \log \left( \sum_z p(Y, z|\theta) \right)$$

- Exact marginalization (enumeration as in GMM, DP as in HMM)
- Hard-EM: Choose the single best  $z$ 
  - E.g.,  $K$ -means clustering
- Choose top- $N$  latent variables
  - Beam search
- Sampling

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# Latent Variables in Discriminative Model

- In GMM and HMM
  - We model the joint probability  $p(z, y)$
- Sometimes we have discriminative variables
  - We predict  $y$  from  $x$  with  $z$  being a latent variable

$$\log p_{\theta}(y \mid \textcolor{red}{x}) = \log \left( \sum_z p_{\theta}(y, z \mid \textcolor{red}{x}) \right)$$

# Massage

maximize

$$\log \left( \sum_z p(z) p(Y|z, \theta) \right)$$


maximize

$$\sum_z p(z) \log(p(Y|z, \theta))$$

↓ generalize

maximize

$$\sum_z p(z) R(Y|z, \theta)$$

# Reinforcement Learning



# Markov Decision Process

- In a time series,  $t = 1, 2, \dots, T$ 
  - We are in some states,  $s_1, s_2, \dots, s_T$
  - We take action  $a_1, a_2, \dots, a_T$
  - We have reward  $r_1, r_2, \dots, r_T$

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- Formally, MDP:  $\langle S, A, P, R, \gamma \rangle$

$S$  : Set of states

$A$  : Set of actions

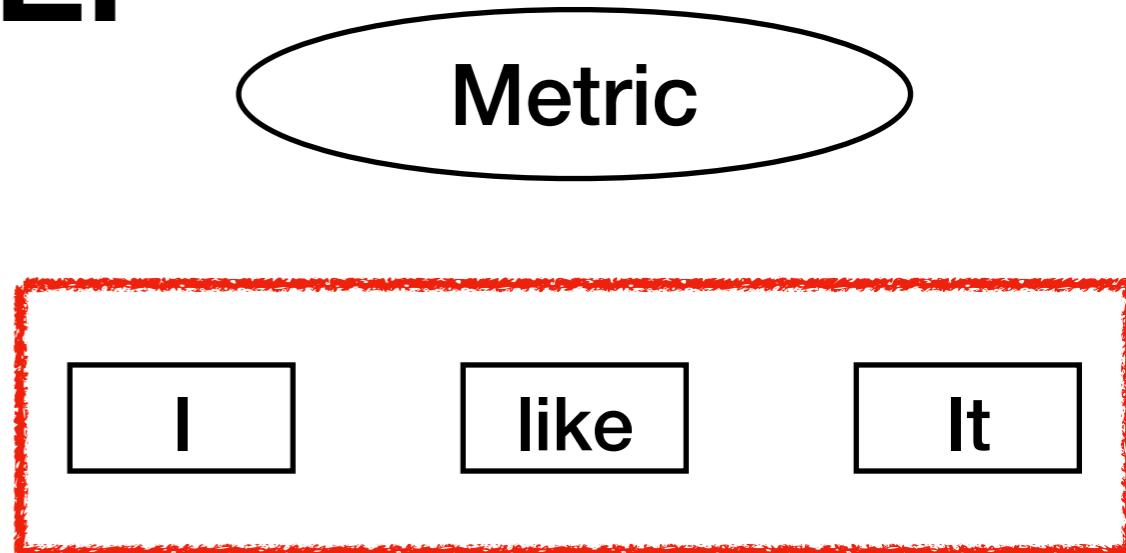
$$P_{ss'}^a = \mathbb{P}[S_{t+1} = s' | S_t = s, A_t = a]$$

$R_s^a$  : Reward at state  $s$  with action  $a$

$\gamma$  : Discount factor in  $[0,1]$

# MDP in NLP

- Consider a text generation task  
(we assume latent)



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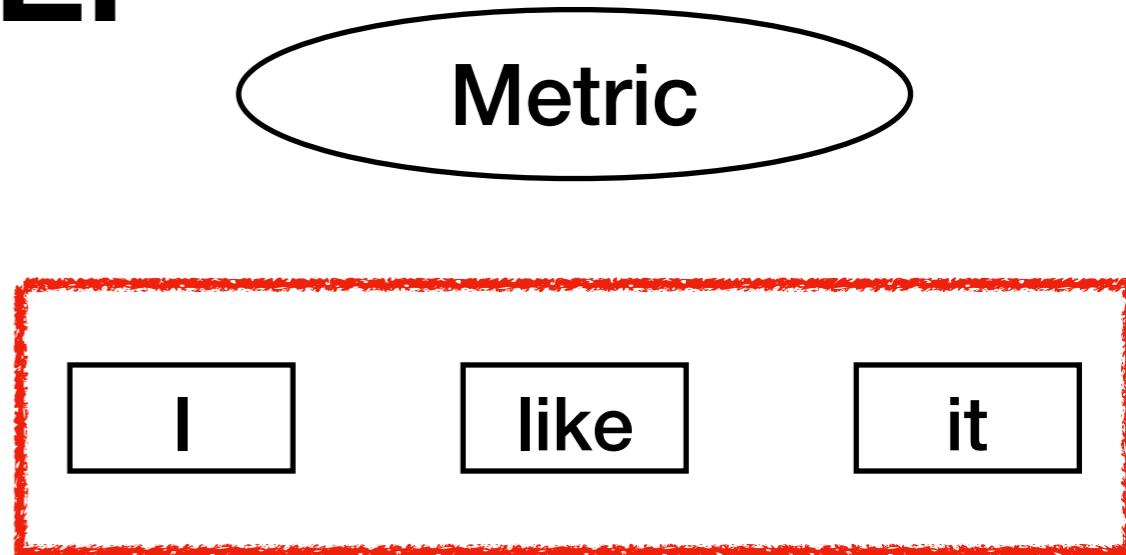
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Src info

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**States:** Src & generated words  
Usually approximated by NN

$a]$

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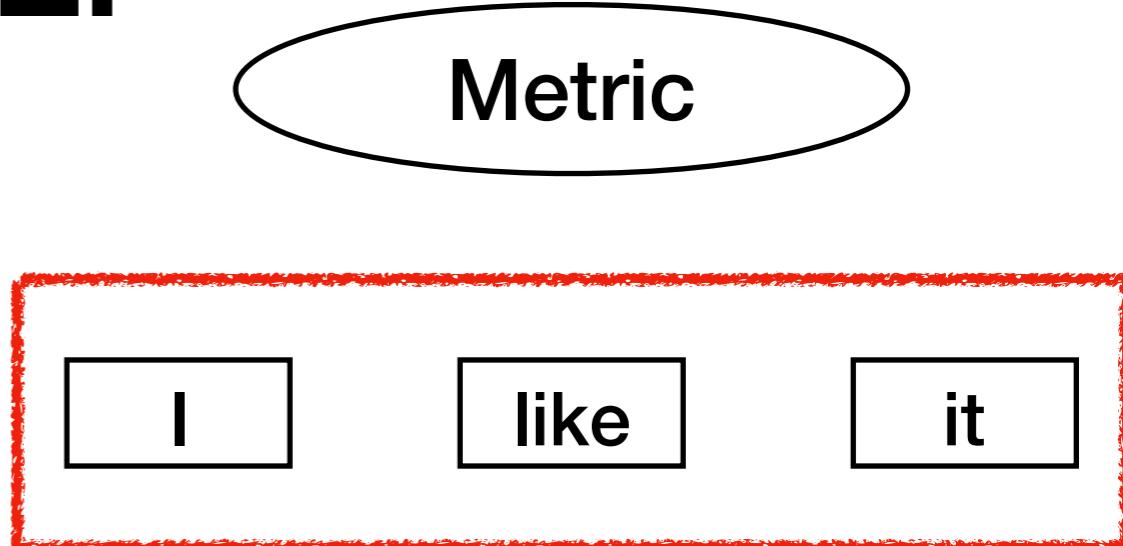
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**Actions:** all words in vocabulary,  
usually very large

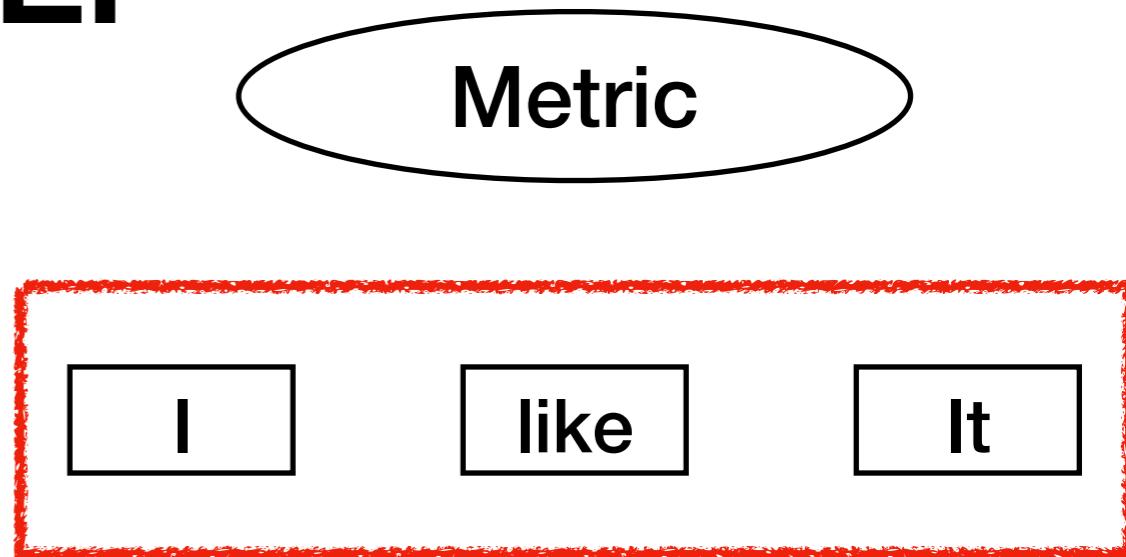
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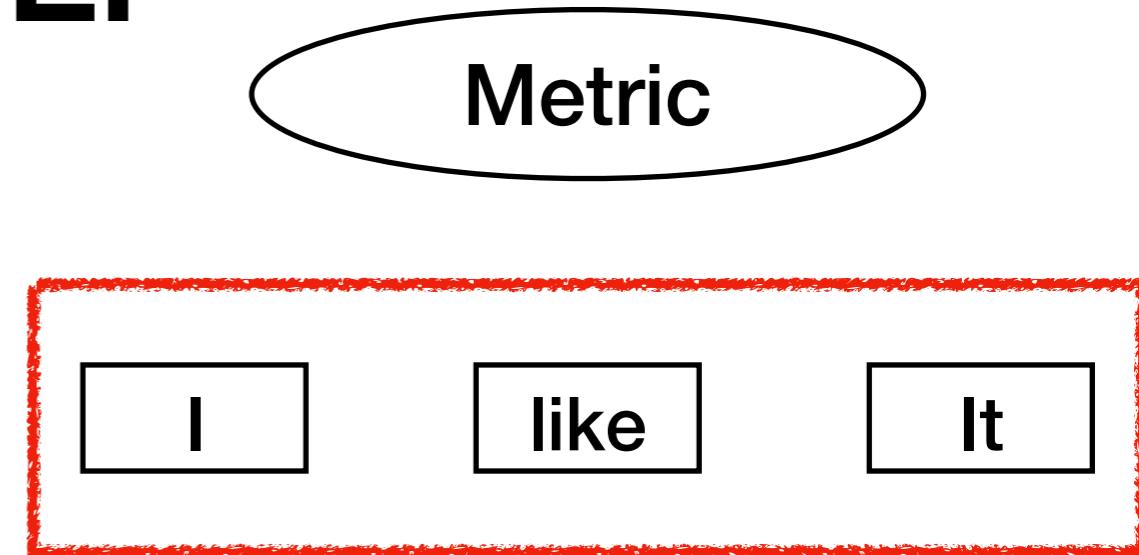
$$P_{ss'}^a = \mathbb{P}[S_{t+1} = s' | S_t = s, A_t = a]$$

**Transition:** deterministic

action  $a$

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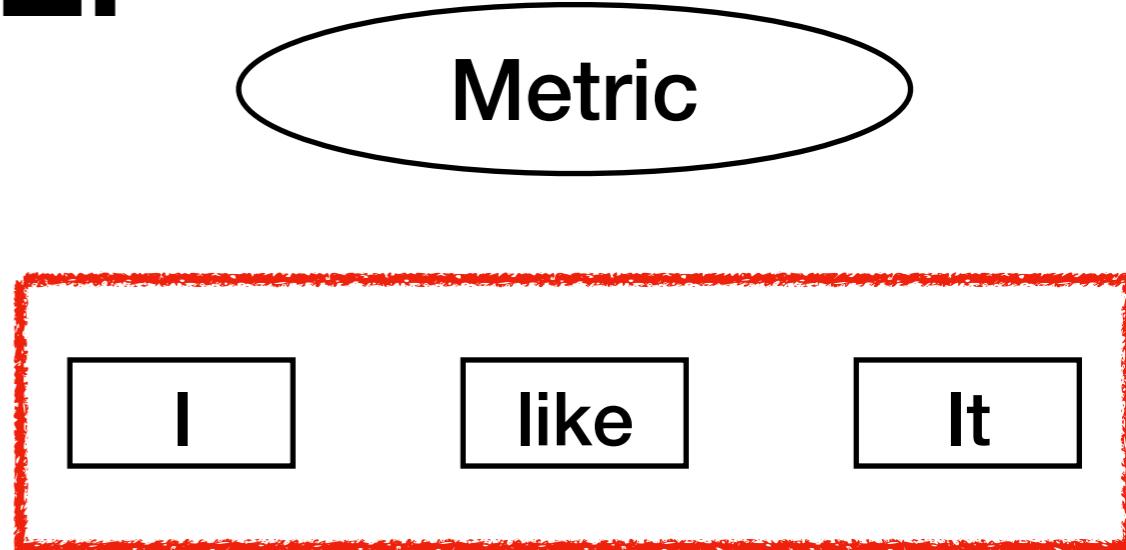
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**Reward:** measure of success,  
usually very sparse

Src info

# MDP in NLP

- Consider a text generation task  
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- Formally, MDP:  $\langle S, A, P, R, \gamma \rangle$

$S$  : Set of states

$A$  : Set of actions

**Discount:** doesn't  
matter too much

$s_t = s, A_t = a]$   
 $s$  with action  $a$

$\gamma$  : discount factor in  $[0,1]$

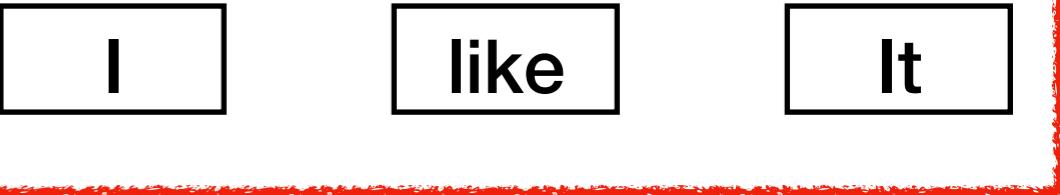
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# REINFORCE

- Stochastic policy
  - Action given state (called policy) modeled by probability
  - Model  $p(action | \cdot)$
  - Action is our latent variable, called  $z$
- Monte Carlo sampling
  - Sampling until the end of episode (data point)
  - No bootstrapping
- Goal is to maximize

$$\mathbb{E}_z R(Y|z; \theta)$$

Metric



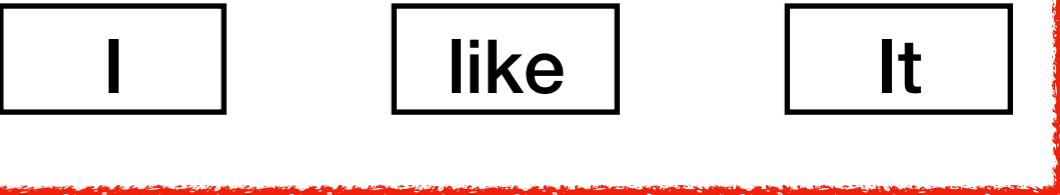
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For simplicity, we here only consider the reward at the end of the sequence

Src info

# REINFORCE: MC Policy Gradient

$$\underset{\theta}{\text{minimize}} \quad \mathbb{E}_{z_1, \dots, z_T \sim p_{\theta}} [ -R(y_1, \dots, y_n | z_1, \dots, z_T) ]$$

*Statisticians seem to be pessimistic creatures who think in terms of losses.*

*Decision theorists in economics and business talk instead in terms of gains (utility).*

James O. Berger (1985). *Statistical Decision Theory and Bayesian Analysis*.

# REINFORCE: MC Policy Gradient

$$\underset{\theta}{\text{minimize}} \quad \mathbb{E}_{z_1, \dots, z_T \sim p_{\theta}} [-R(y_1, \dots, y_n | z_1, \dots, z_T)]$$

Suppose we only have final reward

Otherwise,  $z_t$  is contributing to  $R_t, \dots, R_T$

$$\nabla_{\theta} \mathbb{E}_{z_1, \dots, z_T} [-R]$$

$$= \sum_{z_1, \dots, z_T} \frac{\nabla_{\theta} p_{\theta}(z_1, \dots, z_T) \cdot (-R)}{z_1, \dots, z_T}$$

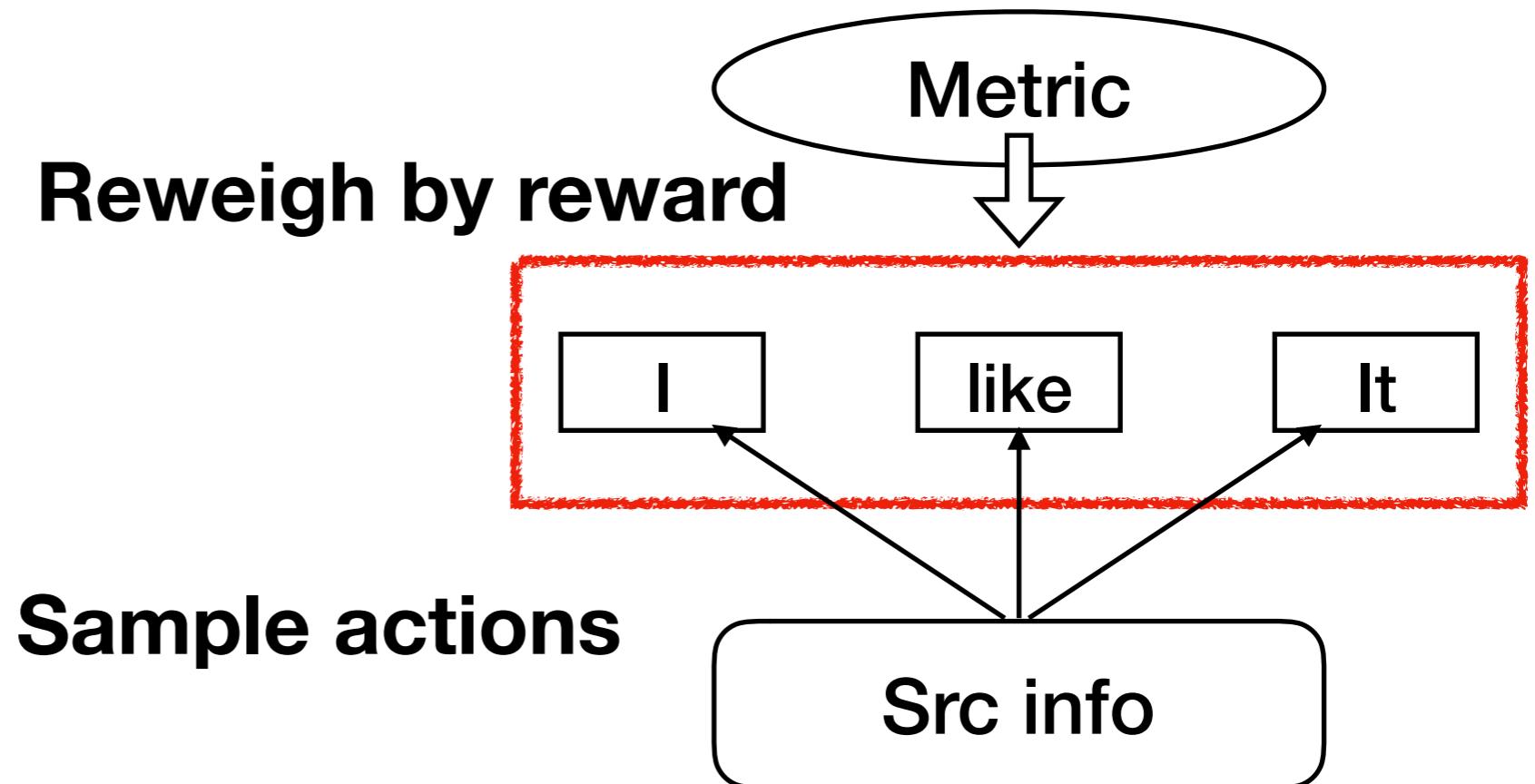
$$= \sum_{z_1, \dots, z_T} \frac{p_{\theta}(z_1, \dots, z_T) \nabla_{\theta} \log p_{\theta}(z_1, \dots, z_T) \cdot (-R)}{z_1, \dots, z_T}$$

$\mathbb{E}$

# REINFORCE vs Supervised

- Sample a few sequences of actions
- Pretend that they are groundtruth
- But reweigh it by (minus) reward

$$-\mathbb{E}_{z} [R \cdot \nabla_{\theta} \log p_{\theta}(z)]$$



# High Variance of REINFORCE

$$-\mathbb{E}_{z \sim R} [\nabla_{\theta} \log p_{\theta}(z)] (R - B)$$

## Baseline

- Mean
- Per-data mean
- $\hat{V}(s)$ 
  - Critic, which can be learned by  $(R - V(s))^2$

# RL vs MLE

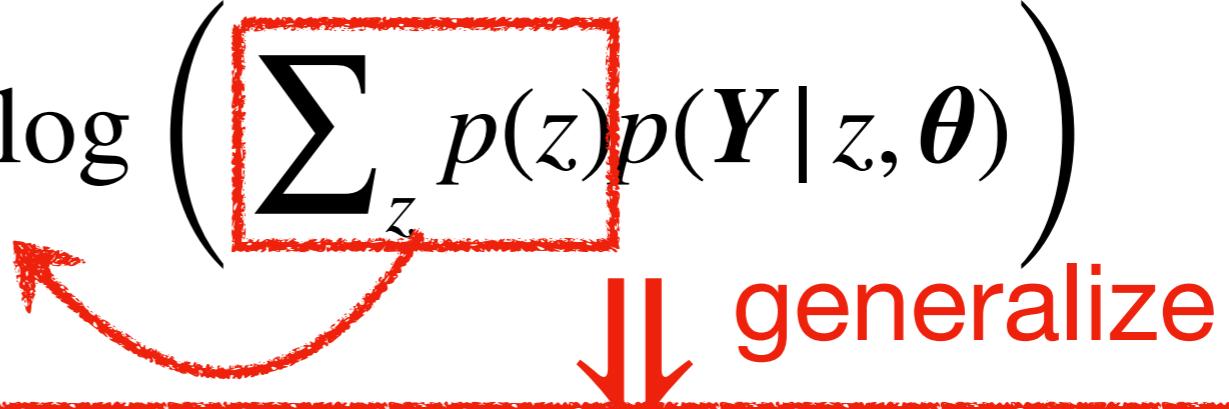
<b>Method</b>	<b>Approximation of <math>E_q[\cdot]</math></b>	<b>Exploration strategy</b>	<b>Gradient weight <math>q(\mathbf{z})</math></b>
REINFORCE	Monte Carlo integration	independent sampling	$p_\theta(\mathbf{z} \mid x)$
BS-MML	numerical integration	beam search	$p_\theta(\mathbf{z} \mid x, R(\mathbf{z}) \neq 0)$
RANDOMER	numerical integration	randomized beam search	$q_\beta(\mathbf{z})$

Guu K, Pasupat P, Liu EZ, Liang P. From language to programs: Bridging reinforcement learning and maximum marginal likelihood. In *ACL*, 2017.

# Massage

maximize

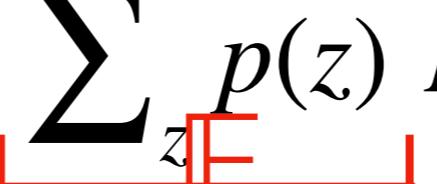
$$\log \left( \sum_z p(z) p(Y|z, \theta) \right)$$



generalize

maximize

$$\sum_z p(z) R(Y(z))$$



$$\mathbb{E} R(Y(z))$$

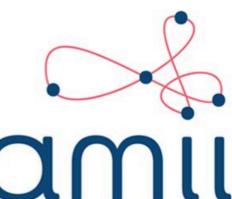
$$z \sim p_\theta(z)$$

reparametrize

maximize

$$\mathbb{E}_{\epsilon \in p(\epsilon)} J(Y(z_\theta(\epsilon)))$$

# Gumbel-softmax



# Reparametrization Trick

- **If**  $z \sim p_{\theta}(z) \iff \epsilon \sim p(\epsilon), z = f_{\theta}(\epsilon)$
- And **if**  $f$  is a differentiable function w.r.t  $\theta$
- **Then** life would be much easier

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- **Then** life would be much easier
- Gaussian distribution

$$z \sim \mathcal{N}(\mu, \sigma) \iff \epsilon \sim \mathcal{N}(0, 1), z = f_{\mu, \sigma}(\epsilon) = \mu + \sigma \cdot \epsilon$$

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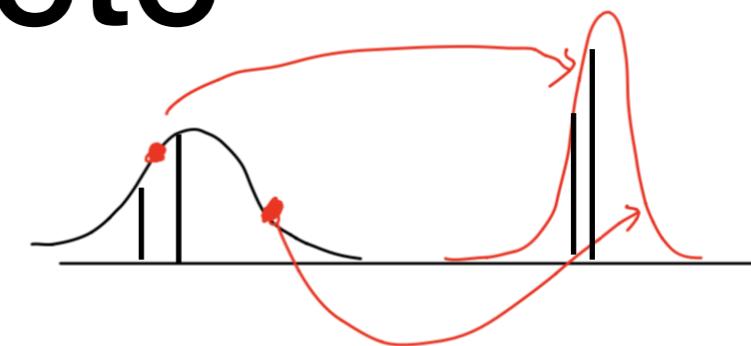
$$z \sim \mathcal{N}(\mu, \sigma) \iff \epsilon \sim \mathcal{N}(0, 1), z = f_{\mu, \sigma}(\epsilon) = \mu + \sigma \cdot \epsilon$$

- This doesn't happen in the **discrete** case

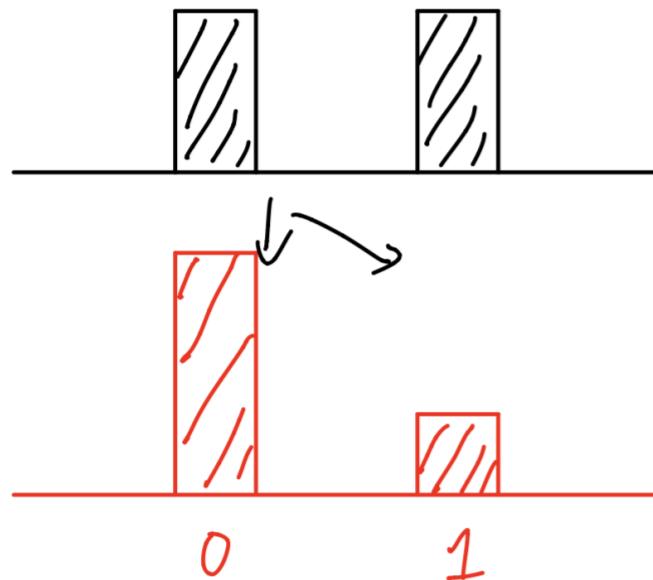
# Continuous vs Discrete

- Closer look at continuous reparametrization

$$z \sim \mathcal{N}(\mu, \sigma) \iff \epsilon \sim \mathcal{N}(0,1), z = f_{\mu, \sigma}(\epsilon) = \mu + \sigma \cdot \epsilon$$

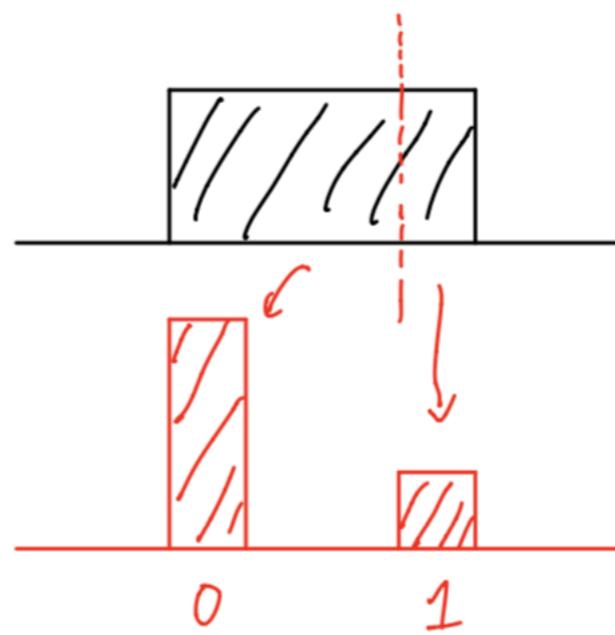


- Discrete  $\rightarrow$  Discrete



Infeasible in general

- Continuous  $\rightarrow$  Discrete



$f = \text{CDF}^{-1}$  not differentiable

# Reparametrization is still feasible

- Gumbel-max

$$z \sim \text{one\_hot}[\text{Cat}(\pi_1, \pi_2, \dots, \pi_n)]$$

$$\Updownarrow$$

$$z = \text{one\_hot} \left[ \arg \max_{i \in \{1, \dots, n\}} \{g_i + \log \pi_i\} \right]$$

$$g_i \sim \text{Gumbel}(0,1) \iff g = -\log(-\log(u)), u \sim U(0,1)$$

Gumbel EJ. Statistical theory of extreme values and some practical applications: a series of lectures. US Government Printing Office; 1948.

# Reparametrization is still feasible

- Gumbel-max

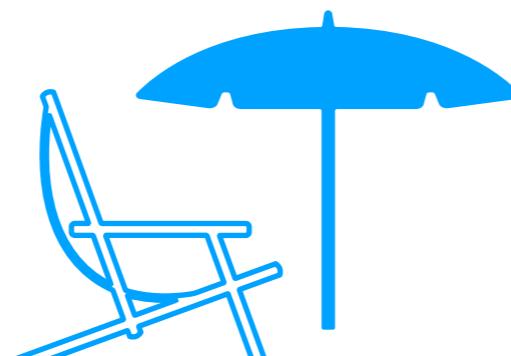
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$$g_i \sim \text{Gumbel}(0,1) \iff g = -\log(-\log(u)), u \sim U(0,1)$$

- Gumbel-max itself doesn't help much
- But we can **relax**



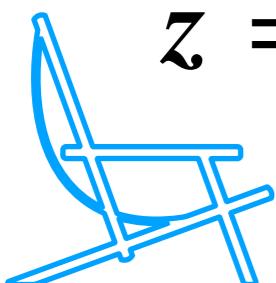
# Gumbel-Softmax

$$g = -\log(-\log(u)), u \sim U(0,1)$$

$$z = \text{one\_hot} \left[ \arg \max_{i \in \{1, \dots, n\}} \{g_i + \log \pi_i\} \right]$$



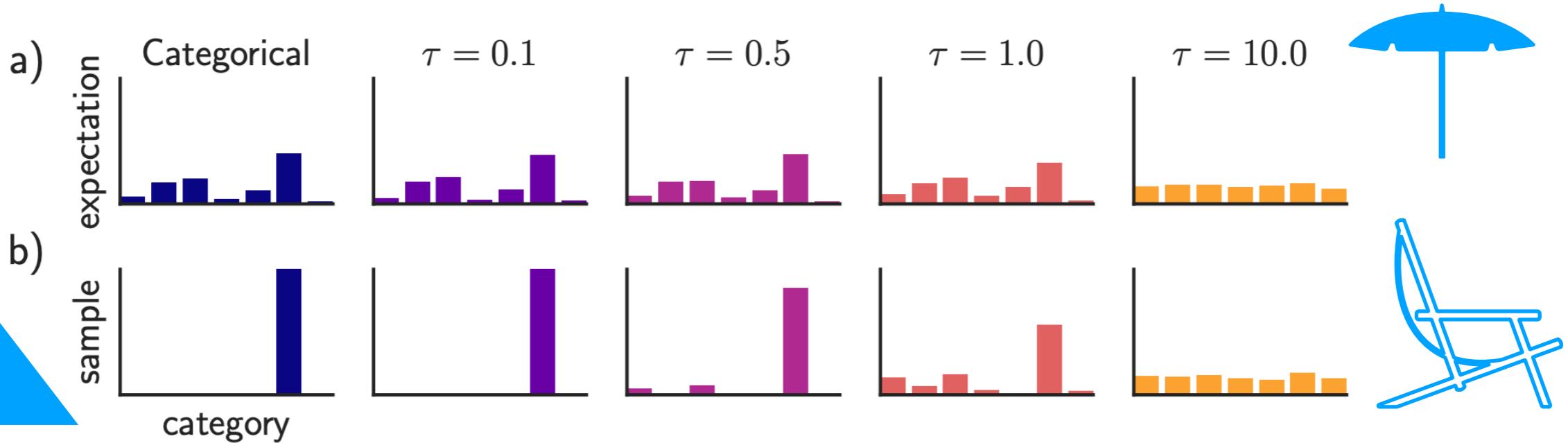
$$\tilde{z} = \text{softmax}_{i \in \{1, \dots, n\}} \{(g_i + \log \pi_i) / \tau\}$$



# Gumbel-Softmax

$$z = \text{one\_hot} \left[ \arg \max_{i \in \{1, \dots, n\}} \{g_i + \log \pi_i\} \right]$$

$$\tilde{z} = \text{softmax}_{i \in \{1, \dots, n\}} \{g_i + \log \pi_i\}$$



- Interpolation between one-hot **sample** and uniform
- Interpolation considers distribution info

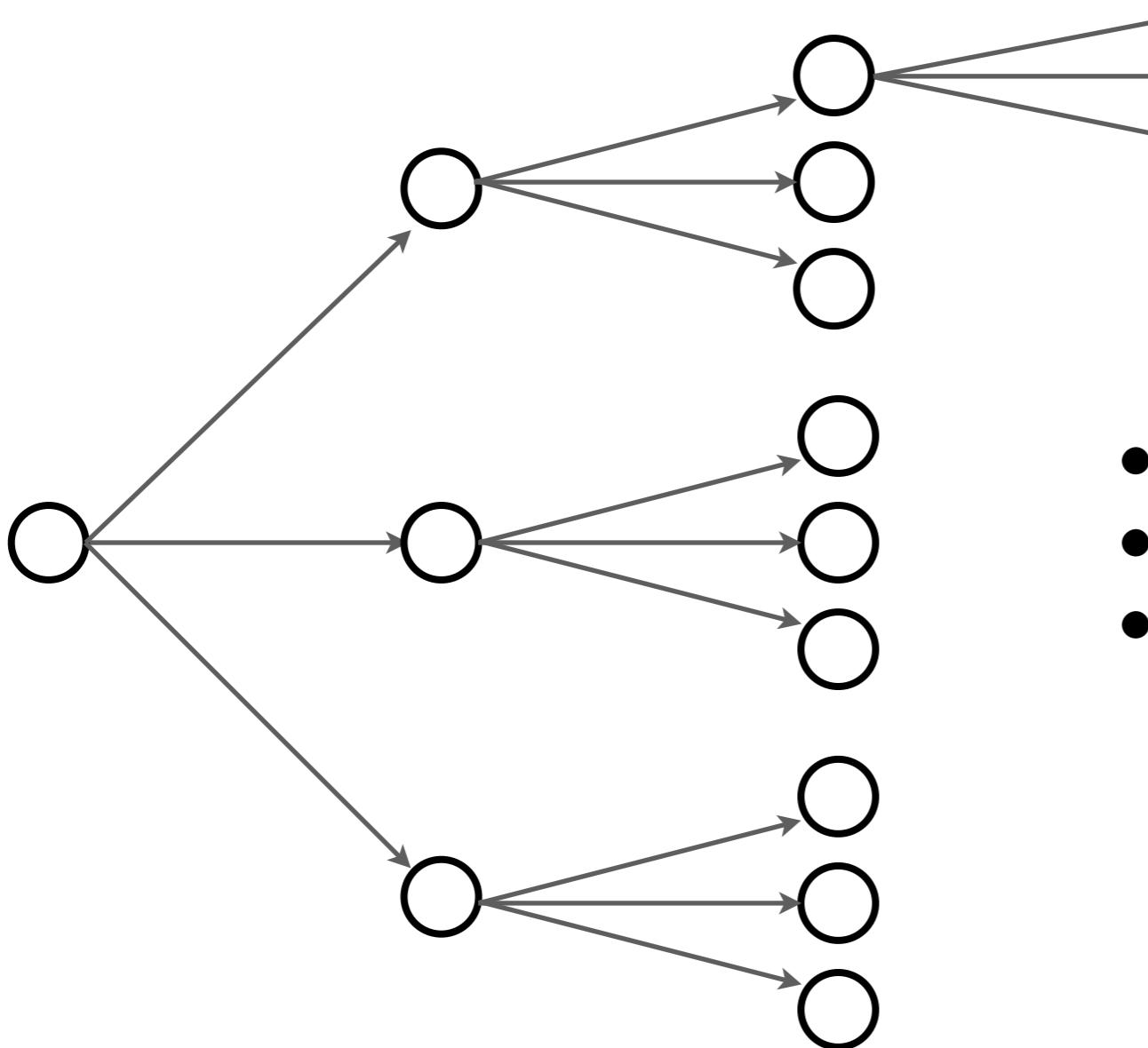
# Gumbel-Softmax in NN

$$z = \text{one\_hot} \left[ \arg \max_{i \in \{1, \dots, n\}} \{g_i + \log \pi_i\} \right]$$

$$\tilde{z} = \text{softmax}_{i \in \{1, \dots, n\}} \{g_i + \log \pi_i\}$$

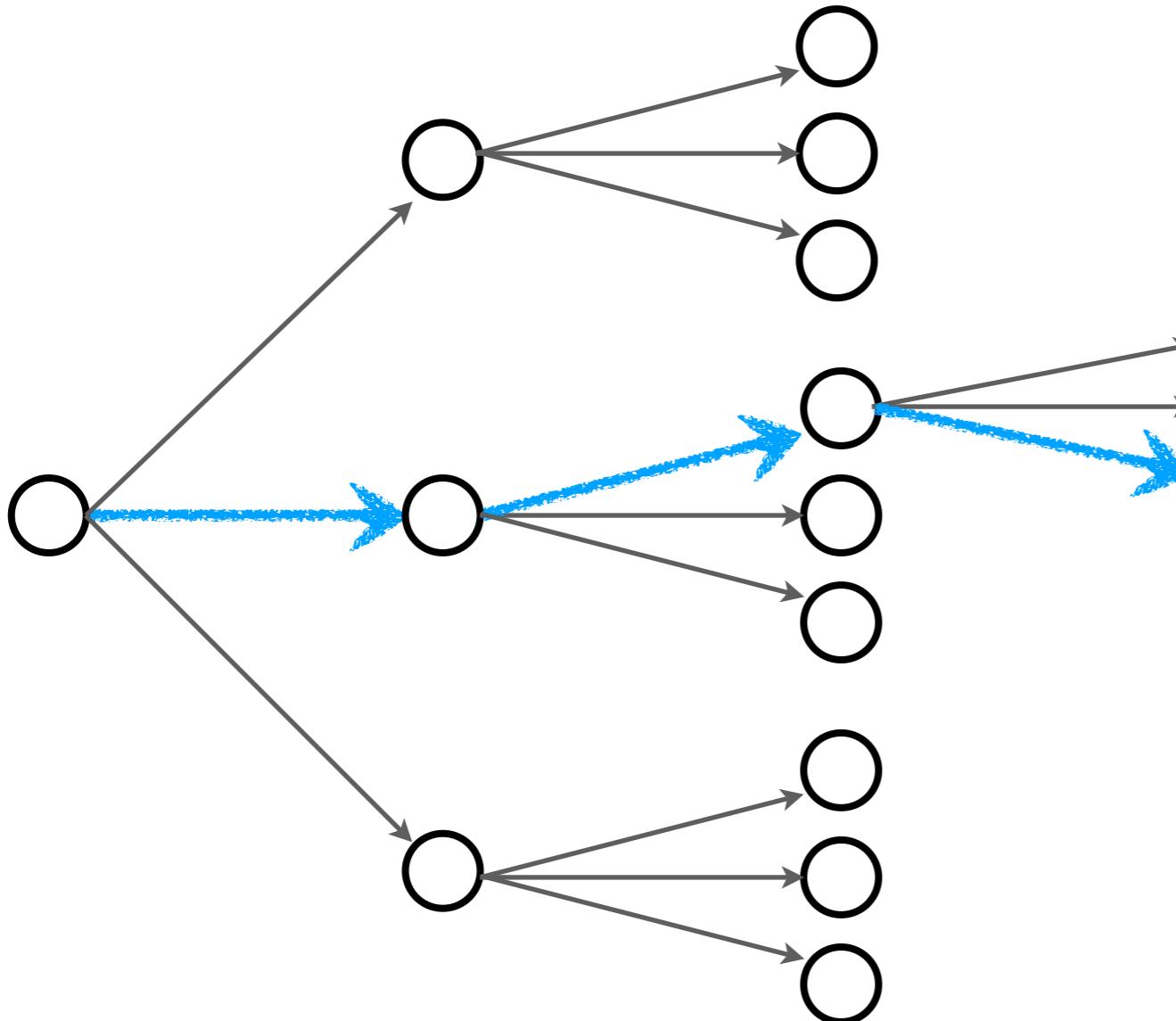
- **Straight-through Gumbel-softmax**
  - Forward prop: Sample **one** action
  - Backward prop: Relax by  $\tilde{z}$
- **Gumbel-softmax**
  - Both forward/backprop relaxed by  $\tilde{z}$

# Exponential Search Space



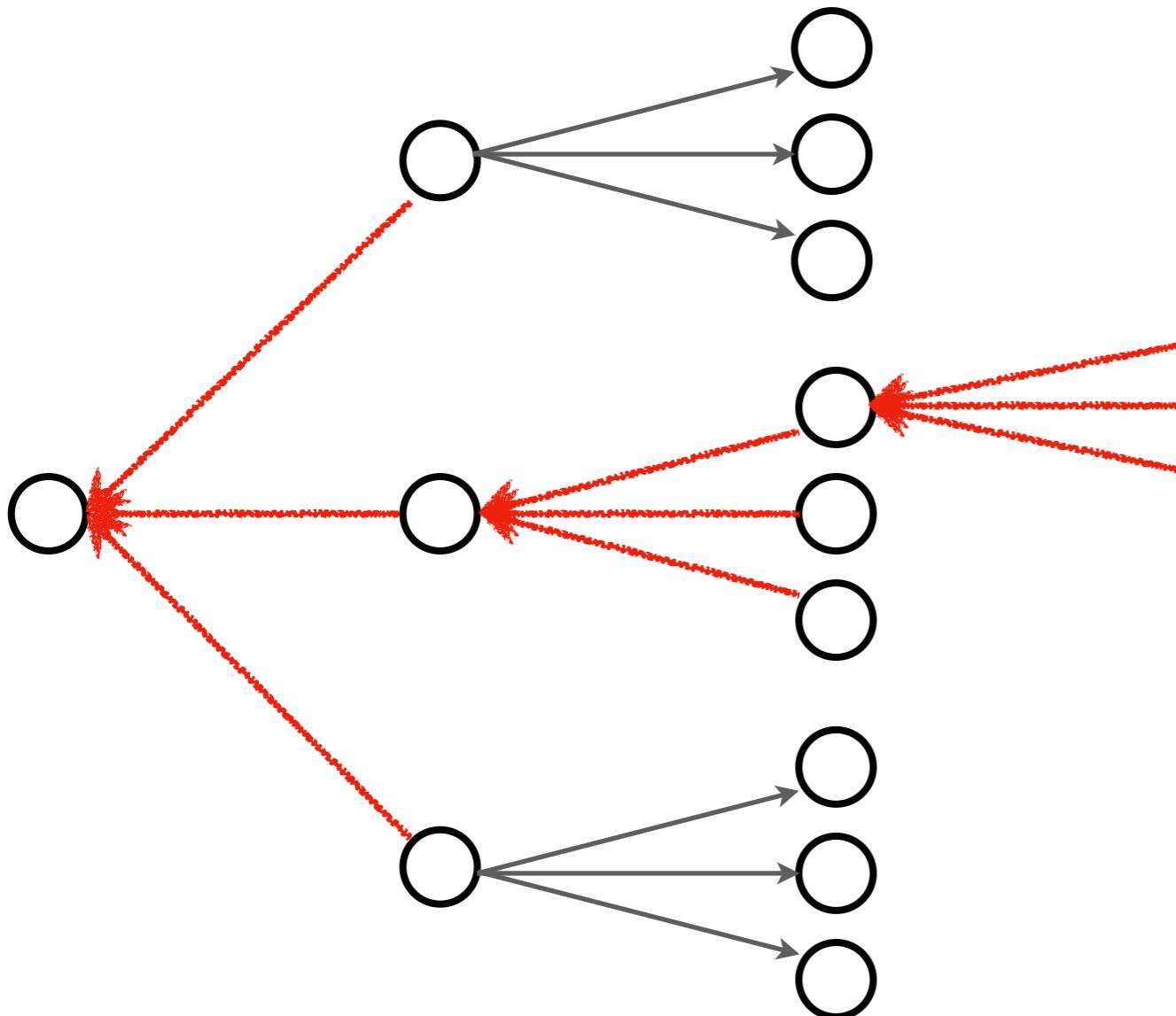
- Single discrete variable is not too bad
- But, space  $\propto \exp(\text{step})$

# Exponential Search Space



- Gumbel-softmax straight-through (ST)
  - Forward: sample one action
  - Backward: Relax by Gumbel-softmax

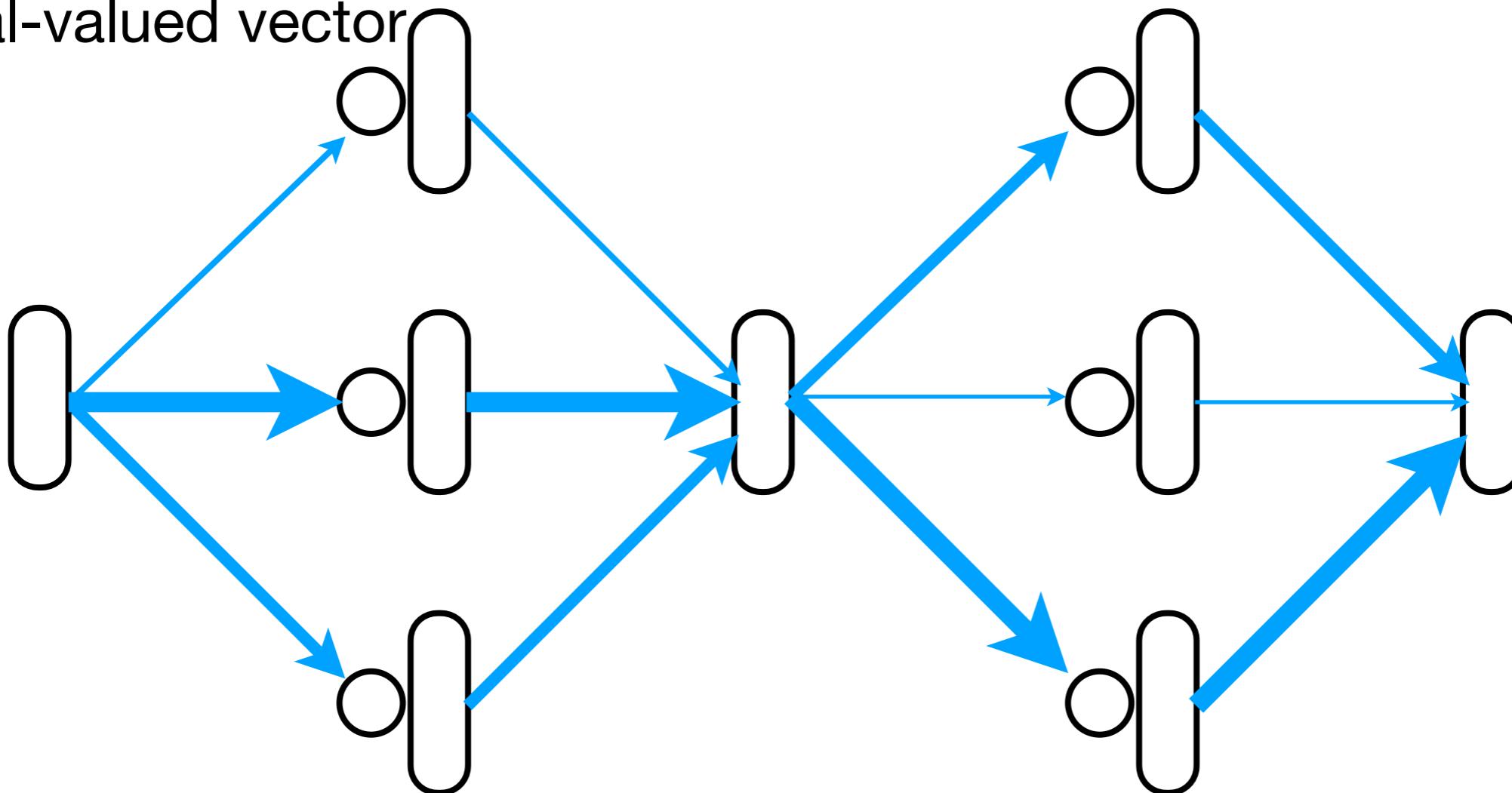
# Exponential Search Space



- Gumbel-softmax straight-through (ST)
  - Forward: Sample one action
  - Backward: Relax by Gumbel-softmax

# Exponential Search Space

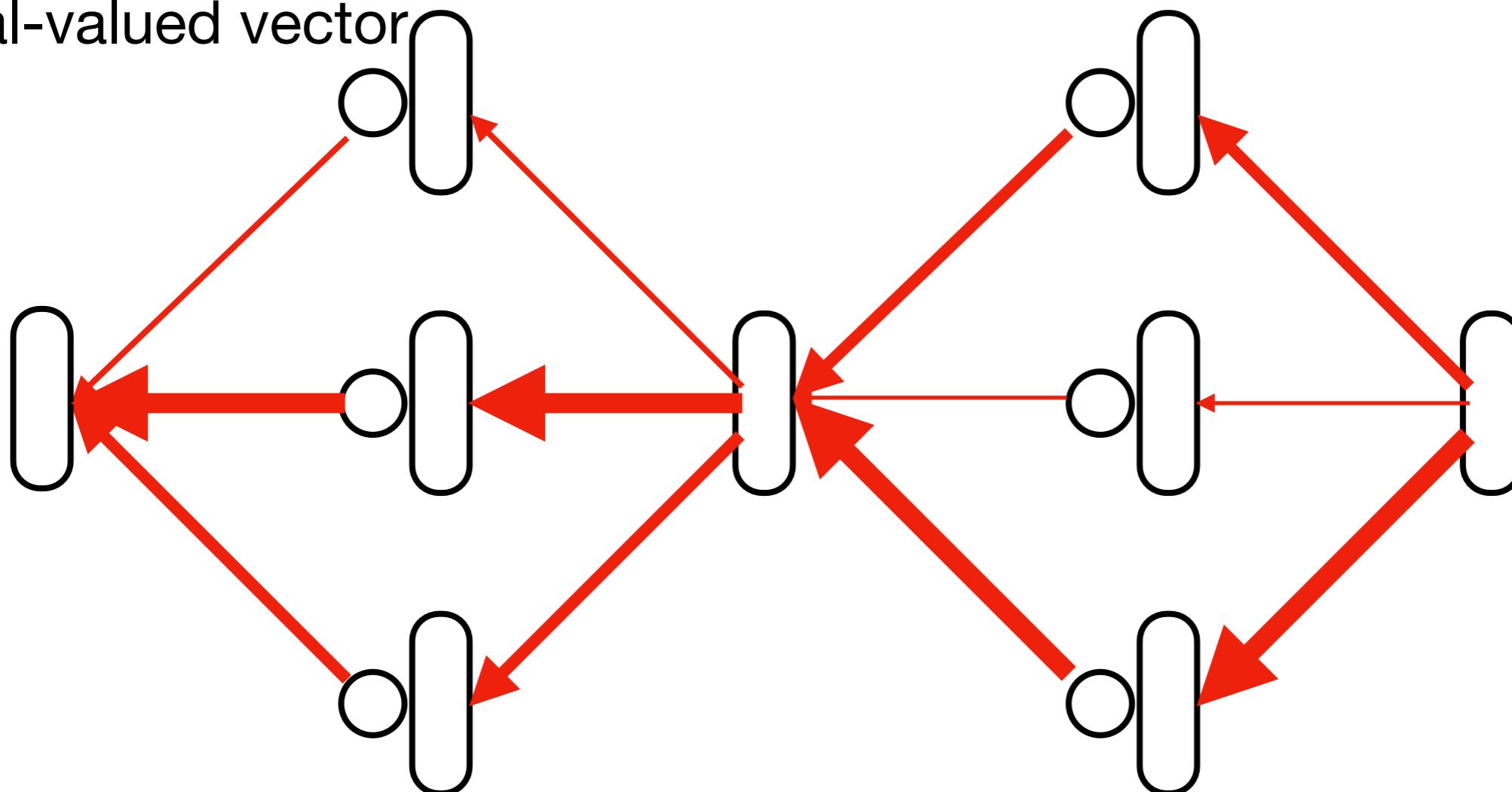
Discrete actions represented by  
real-valued vector



- Gumbel softmax (non-ST)
  - Forward: Relax
  - Backward: Relax

# Exponential Search Space

Discrete actions represented by  
real-valued vector



- Gumbel softmax (non-ST)
    - Forward: Relax
    - Backward: Relax

# Gumbel vs. RL

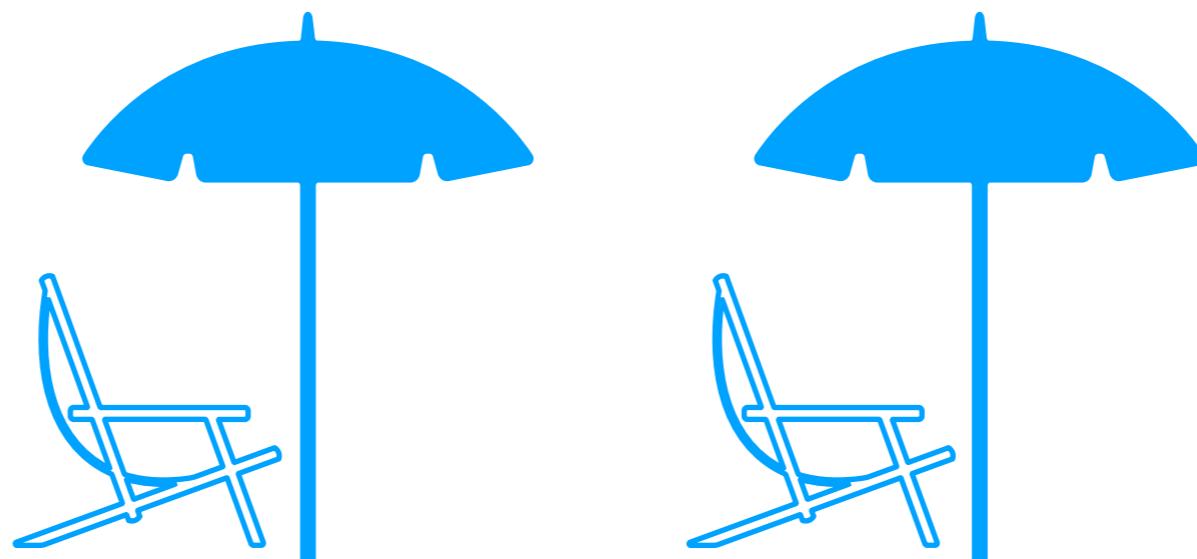
**Provable**      Mostly empirical

- **RL: unbiased**, high variance
  - Works with any reward (theoretically)
- **Gumbel: biased**, low variance (still involves sampling)
  - Works with differentiable loss

# Gumbel vs. RL

**Provable**      Mostly empirical

- **RL: unbiased, high variance**
  - Works with any reward (theoretically)
- **Gumbel: biased, low variance (still involves sampling)**
  - Works with differentiable loss
- **We may relax more**



# Massage

maximize

$$\log \left( \sum_z p(z) p(Y|z, \theta) \right)$$

↑  
generalize

maximize

$$\mathbb{E}_{z \sim p_\theta(z)} R(Y(z))$$

reparametrize  
↓

maximize

$$\mathbb{E}_{\epsilon \in p(\epsilon)} J(Y(z_\theta(\epsilon)))$$



# Massage

maximize

$$\log \left( \sum_z p(z) p(Y|z, \theta) \right)$$

↑  
generalize

maximize

$$\mathbb{E}_{z \sim p_\theta(z)} R(Y(z))$$

reparametrize

maximize

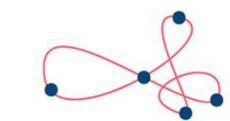
$$\mathbb{E}_{\epsilon \in p(\epsilon)} J(Y(z_\theta(\epsilon)))$$

relax

maximize

$$J(Y(\mathbb{E}_{z \sim p_\theta(z)}[z]))$$

# Step-by-step Attention



# Attention

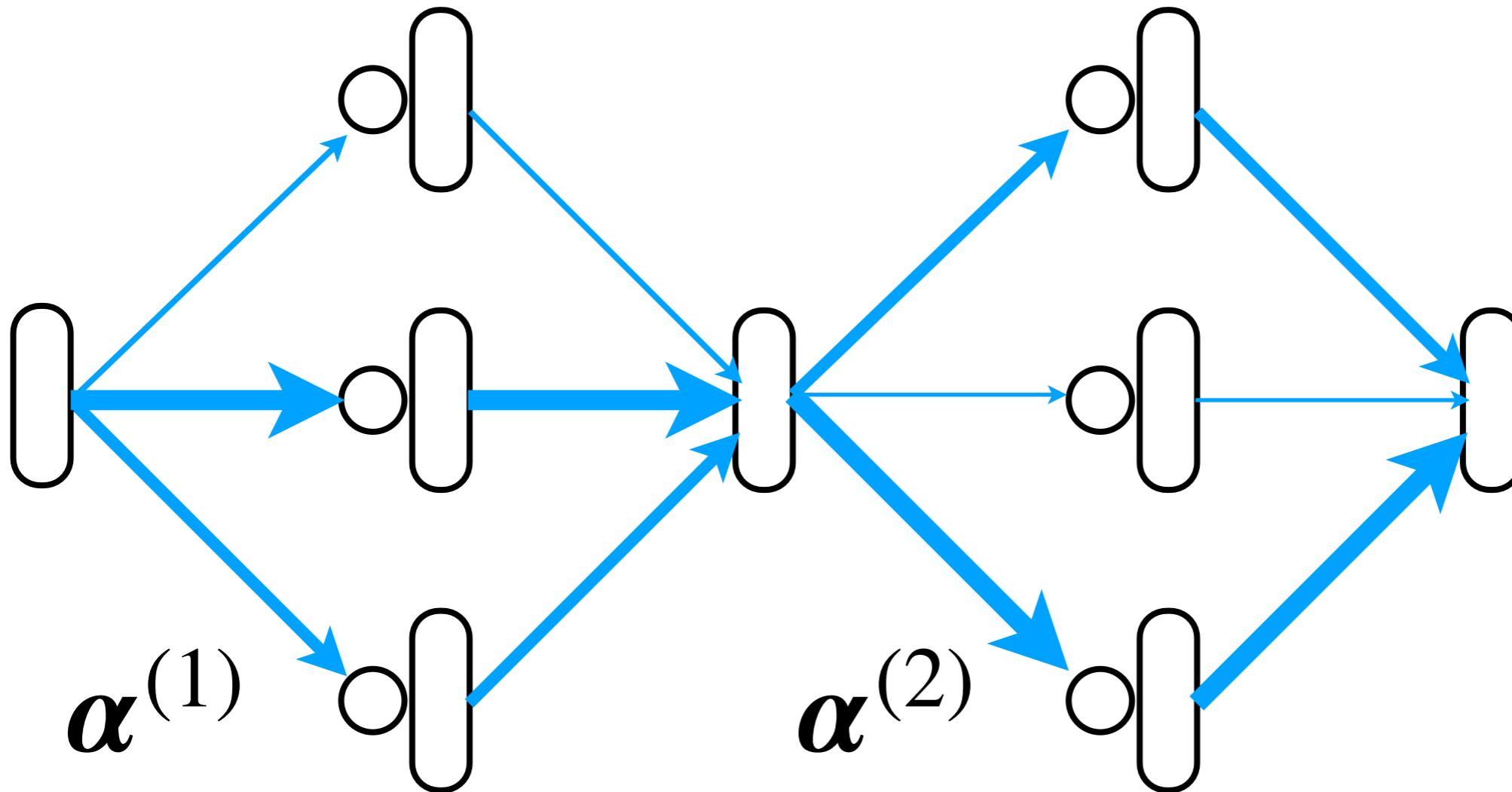
- Your current querying state  $q$
- $z \in \{1, \dots, n\} : n$  discrete actions
  - Each could be represented as a continuous vector  $z_i$
- Attention mechanism

**Unnormalized measure**  $\tilde{\alpha}_i = \exp\{s(q, z_i)\}$

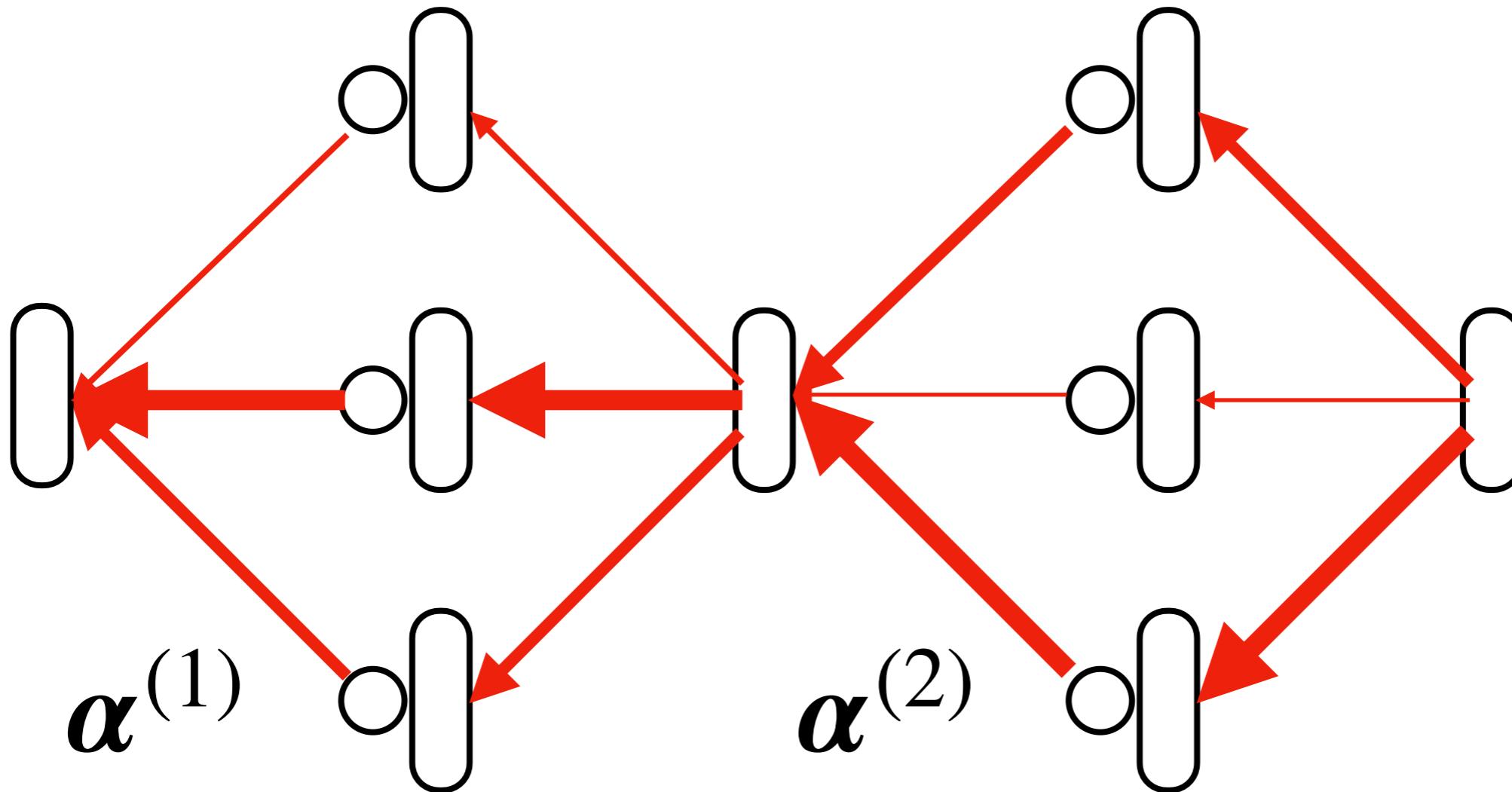
**Attention probability**  $\alpha_i = \frac{\tilde{\alpha}_i}{\sum_j \tilde{\alpha}_j}$

**Attention content**  $c = \sum_i \alpha_i z_i$

# Step-by-step Attention



# Step-by-step Attention



# Attention vs Gumbel softmax

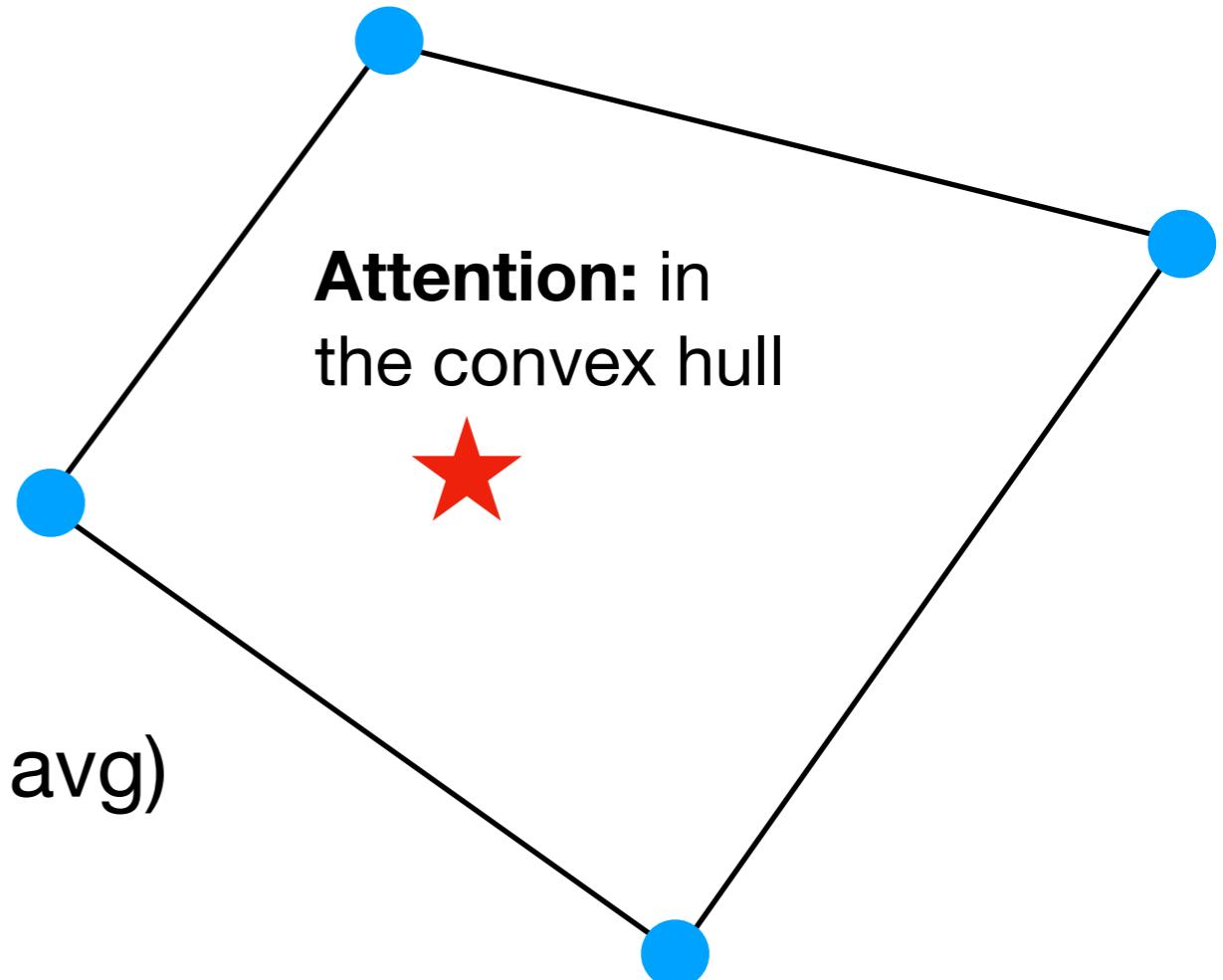
- Both relaxing **hard action** with **soft probability**
  - Attention: Directly using **predicted probability**
  - Gumbel: Using Gumbel-softmax distribution
    - ▶ Interpolation between **one-hot sample** and **uniform**
    - ▶ during which **predicted probability** is considered

# Pros & Cons of Attention

- Pros
  - Easy to use and understand
  - No sampling is involved

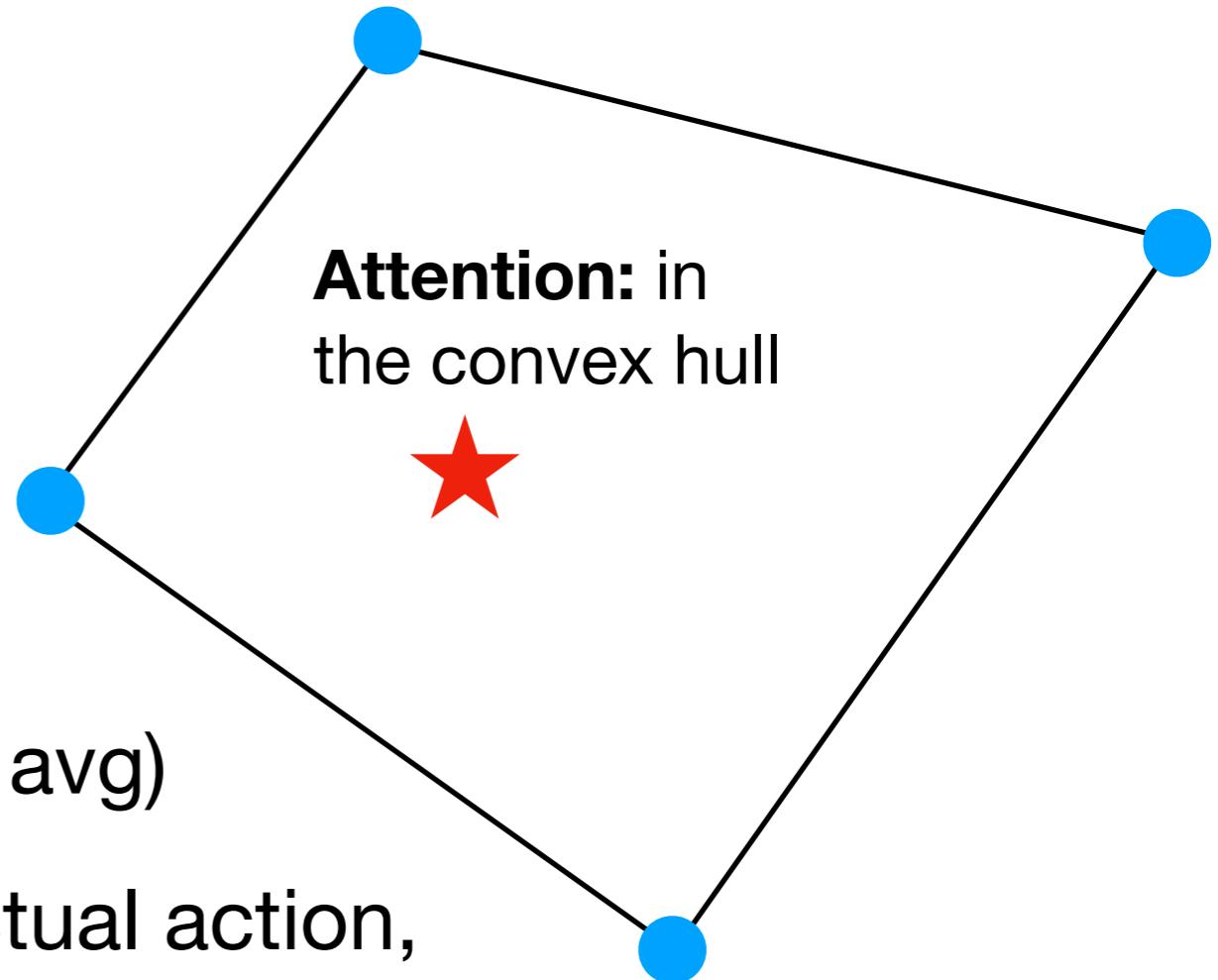
# Pros & Cons of Attention

- Pros
  - Easy to use and understand
  - No sampling is involved
- Cons
  - Landed in no-man's land (mode avg)



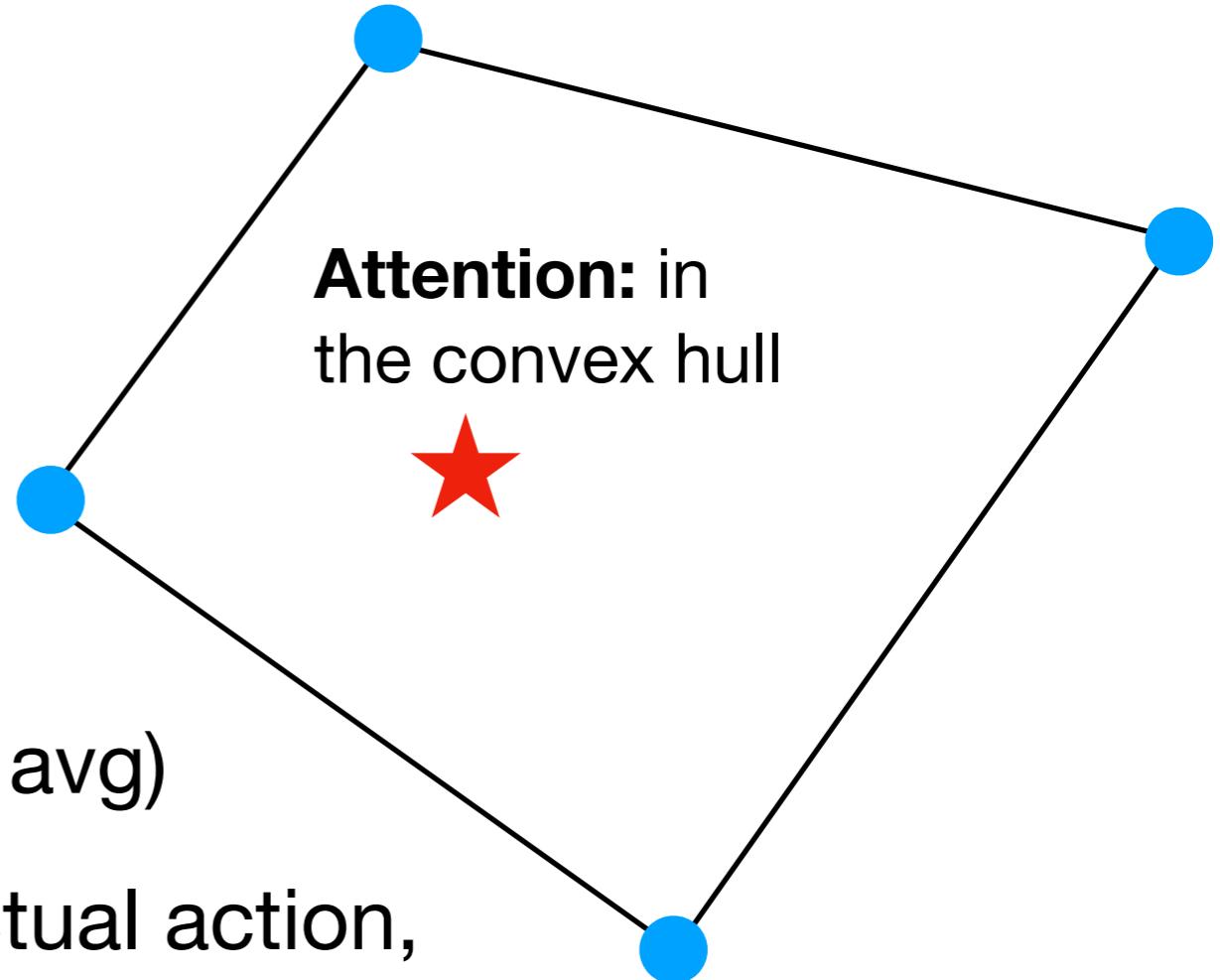
# Pros & Cons of Attention

- Pros
  - Easy to use and understand
  - No sampling is involved
- Cons
  - Landed in no-man's land (mode avg)
    - If you **don't** care about the actual action,  
It's fine 😊
  - E.g., attentions in Transformer are all soft



# Pros & Cons of Attention

- Pros
  - Easy to use and understand
  - No sampling is involved



- Cons
  - Landed in no-man's land (mode avg)
    - If you **don't** care about the actual action,

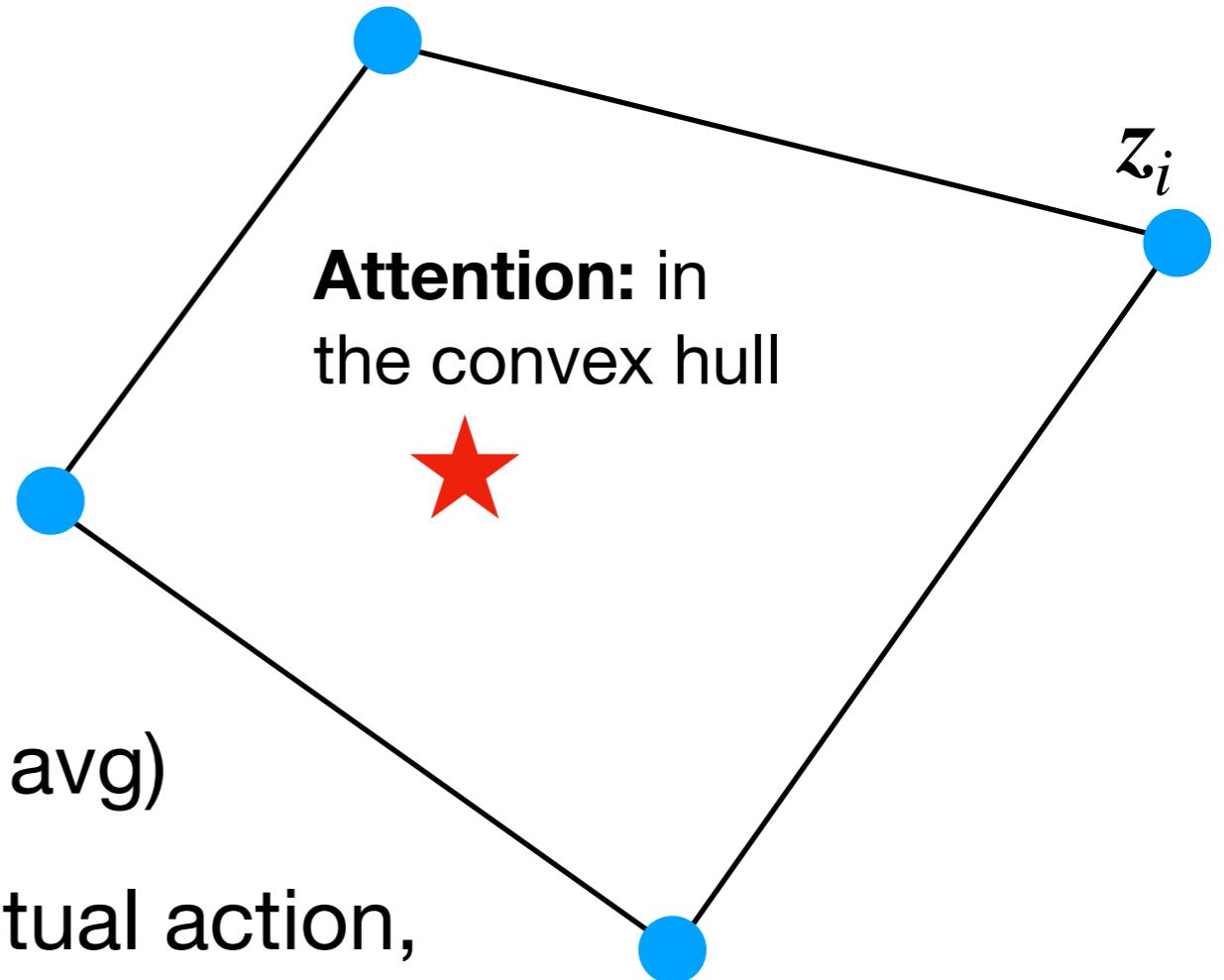
It's fine 😊

This is not too wrong.  
“Meaning is use” – Wittgenstein

In machine learning,  
how you train is how you predict

# Pros & Cons of Attention

- Pros
  - Easy to use and understand
  - No sampling is involved



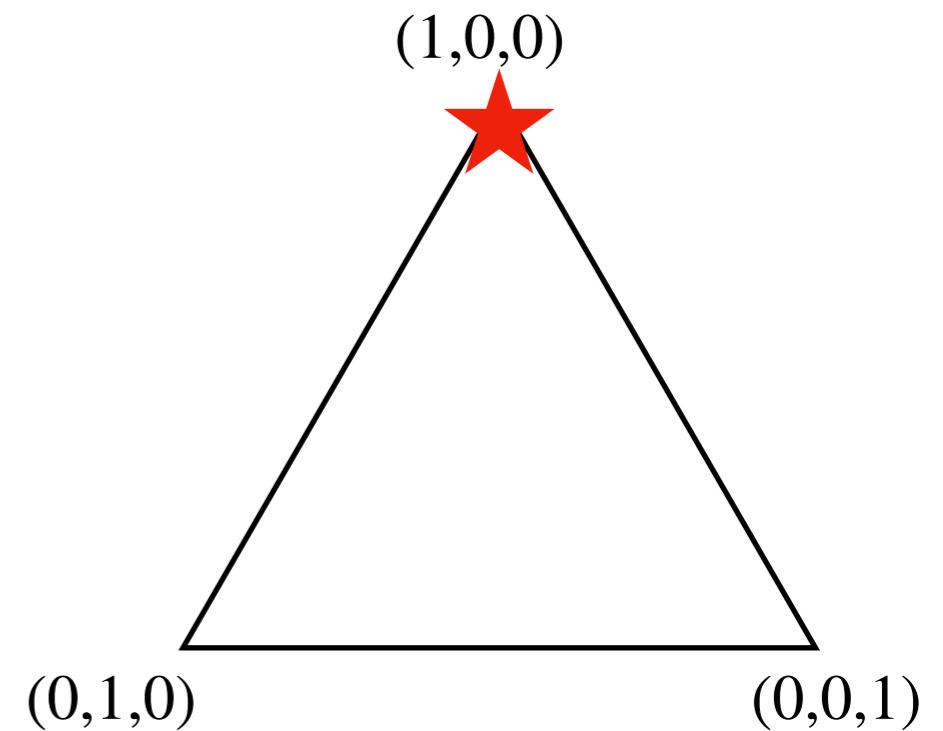
- Cons
  - Landed in no-man's land (mode avg)
    - ▶ If you **don't** care about the actual action,  
It's fine 😊
    - ▶ If you **do** care about the actual action,  
Discrepancy between training and prediction

# More Treatments of the Simplex

- Argmax

$$\boldsymbol{\alpha} = \operatorname{argmax}_{\boldsymbol{\alpha} \in \Delta} \mathbf{s}^T \boldsymbol{\alpha}$$

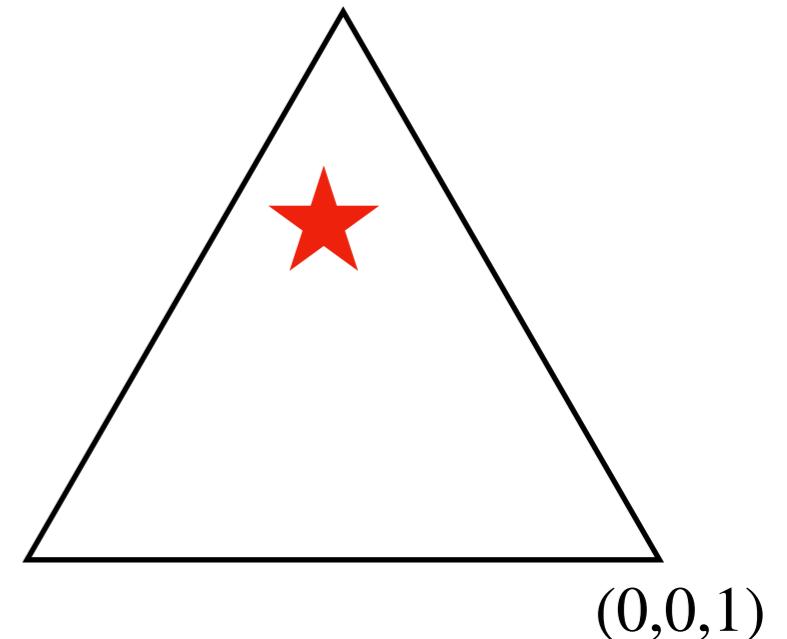
- Choose the largest element of  $\mathbf{s}$
- Result in one-hot  $\boldsymbol{\alpha}$  (assuming no ties)



# More Treatments of the Simplex

- Softmax

$$\alpha = \frac{\exp\{s\}}{\sum_i \exp\{s_i\}}$$
$$= \operatorname{argmax}_{\alpha \in \Delta} s^\top \alpha + \mathcal{H}(\alpha)$$

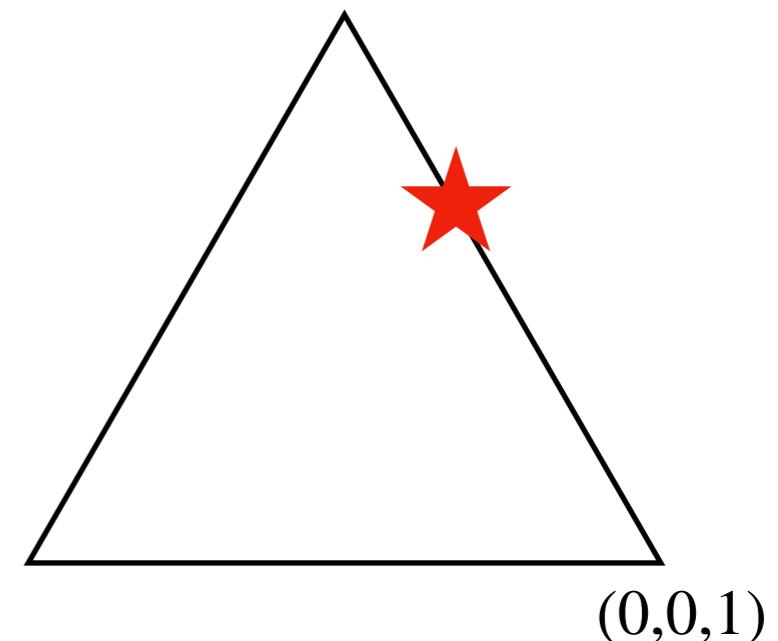


- Always dense

# More Treatments of the Simplex

- Sparsemax

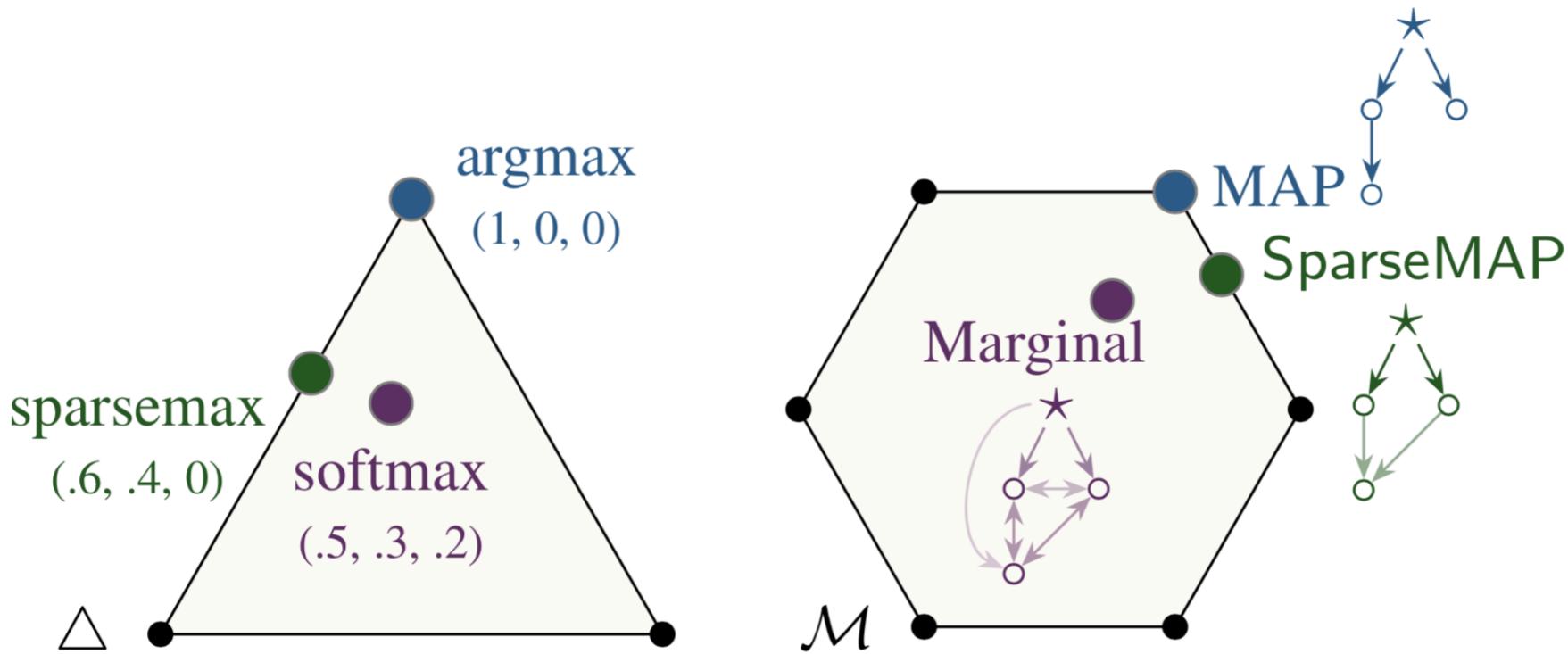
$$\alpha = \operatorname{argmax}_{\alpha \in \Delta} s^T \alpha - \frac{1}{2} \|\alpha\|^2$$



- Denser than argmax
- Sparser than softmax

Martins, A. and Astudillo, R., June. From softmax to sparsemax: A sparse model of attention and multi-label classification. In *ICML*, 2016.

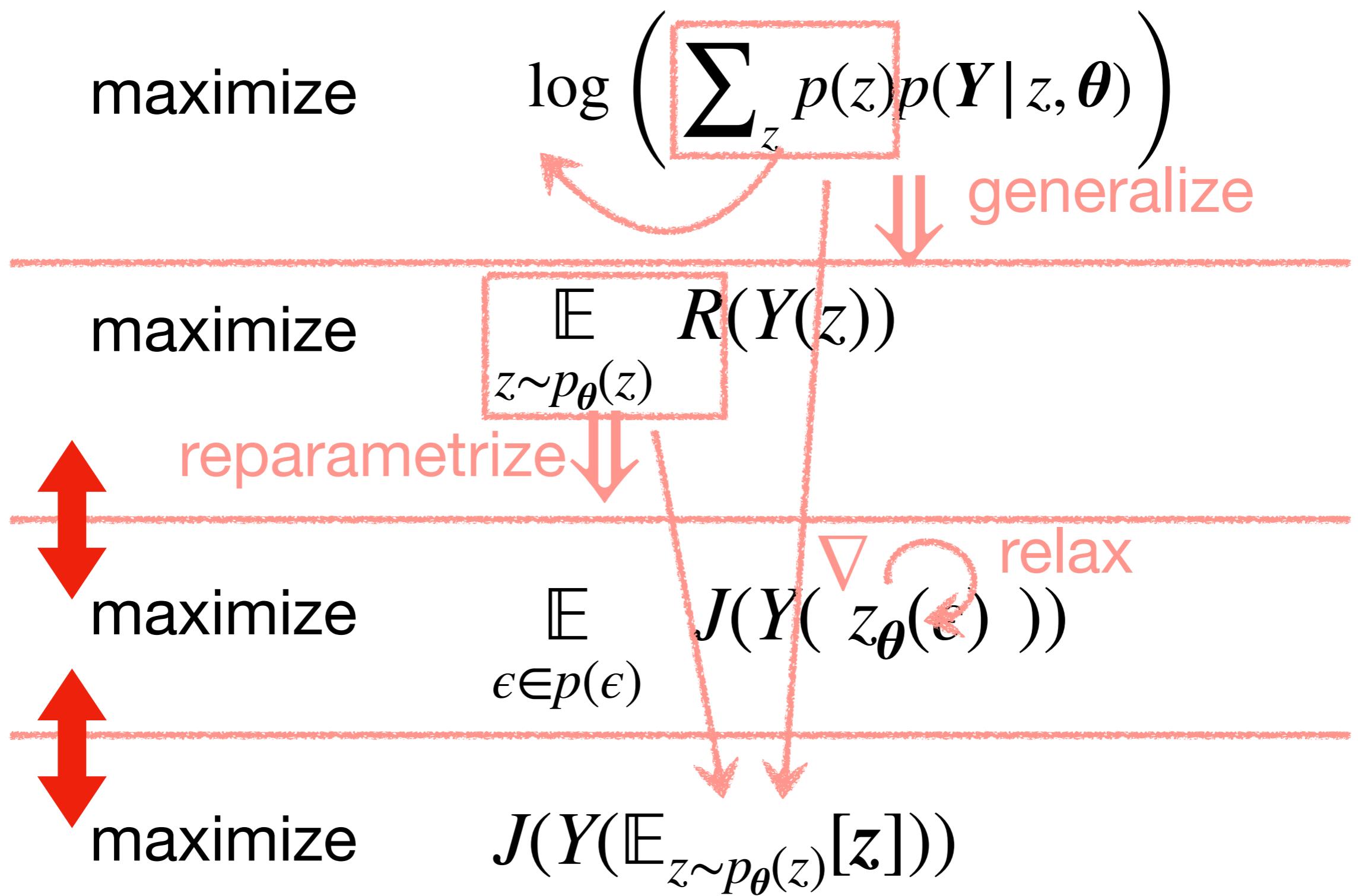
# Extending Simplex to Polytope



- Structured prediction
  - A set of latent variables
  - Log-linear model on the set of (latent) variables

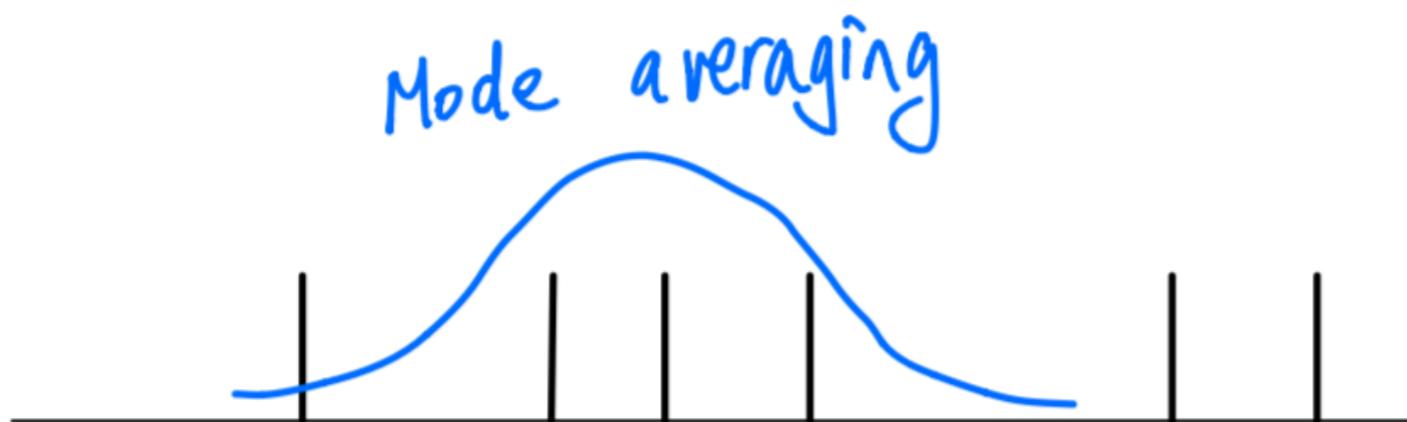
Niculae, V., Martins, A.F., Blondel, M. and Cardie, C. SparseMAP:  
Differentiable sparse structured inference. In *ICML*, 2018.

# Massage



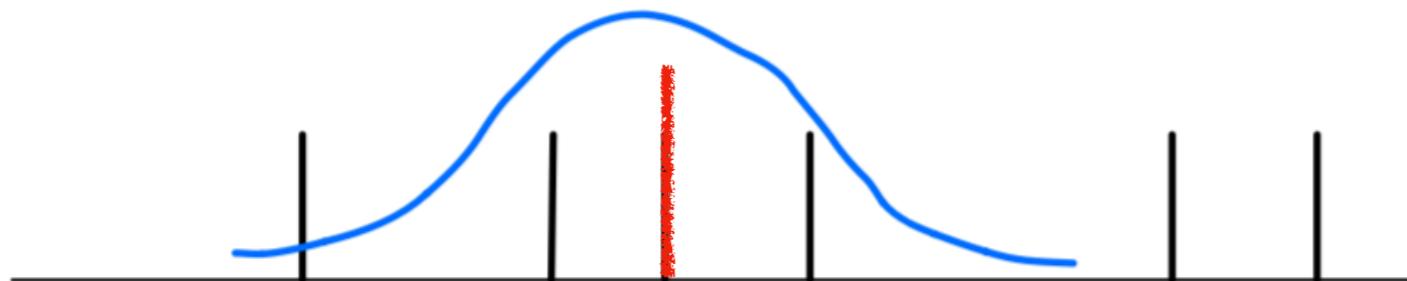
# Combining Mode Avg/Sampling

- First, do mode averaging
  - Exploring all modes simultaneously
  - Having a general sense of the search space
- Then, do mode sampling
  - To learn more accurate actions



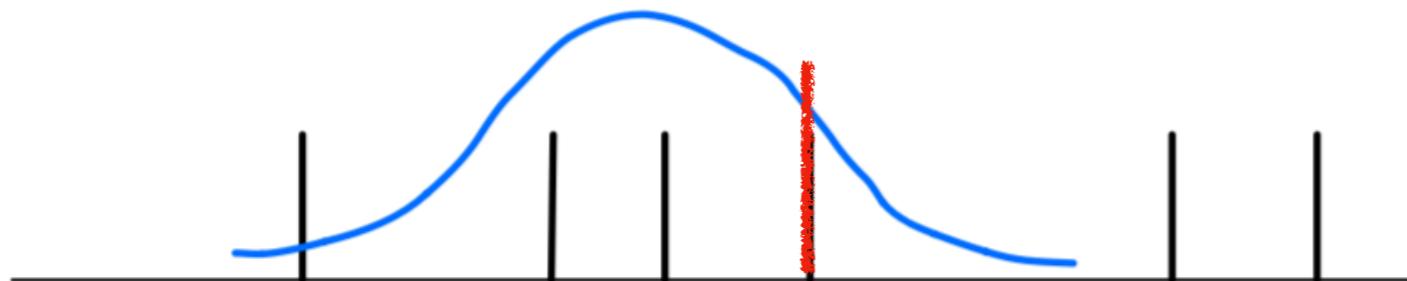
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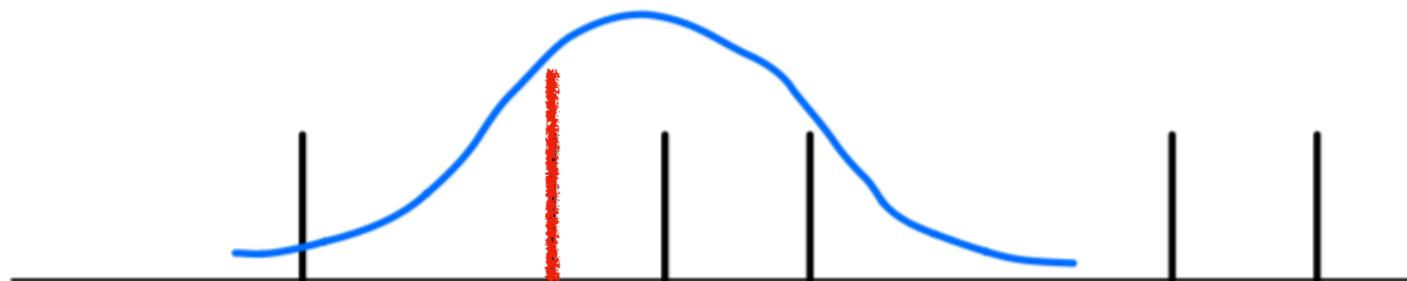
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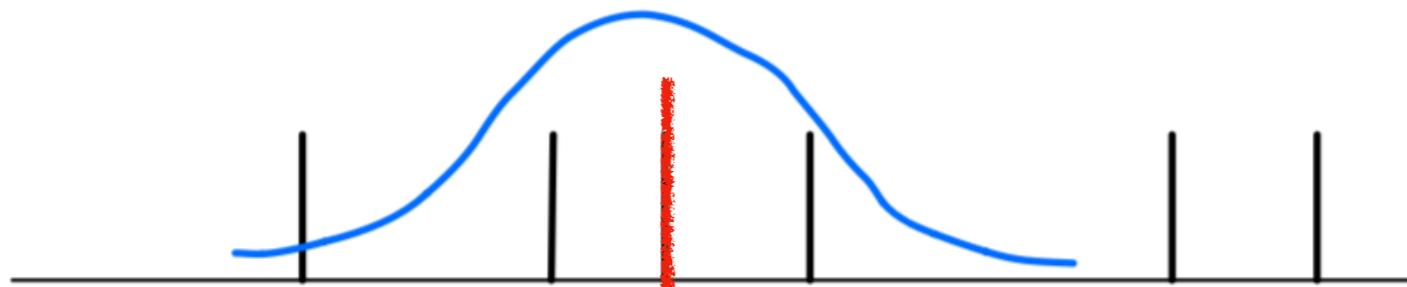
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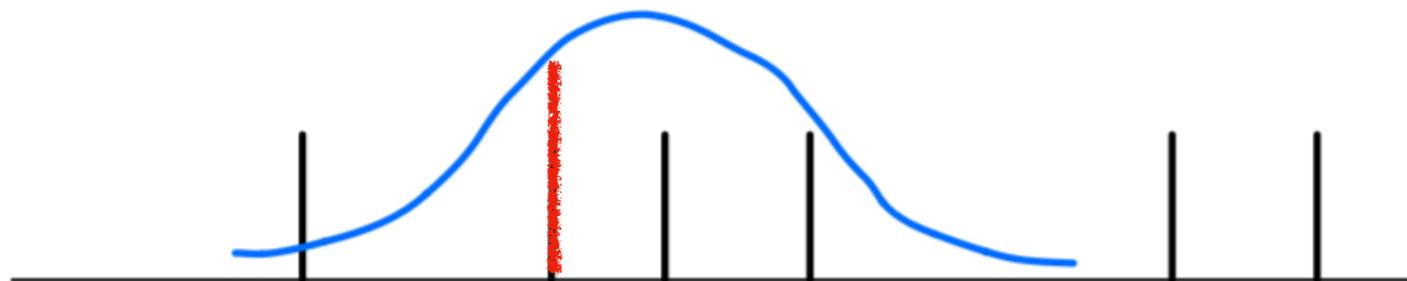
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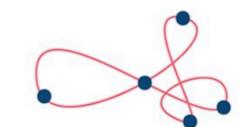


# Combining Mode Avg/Sampling

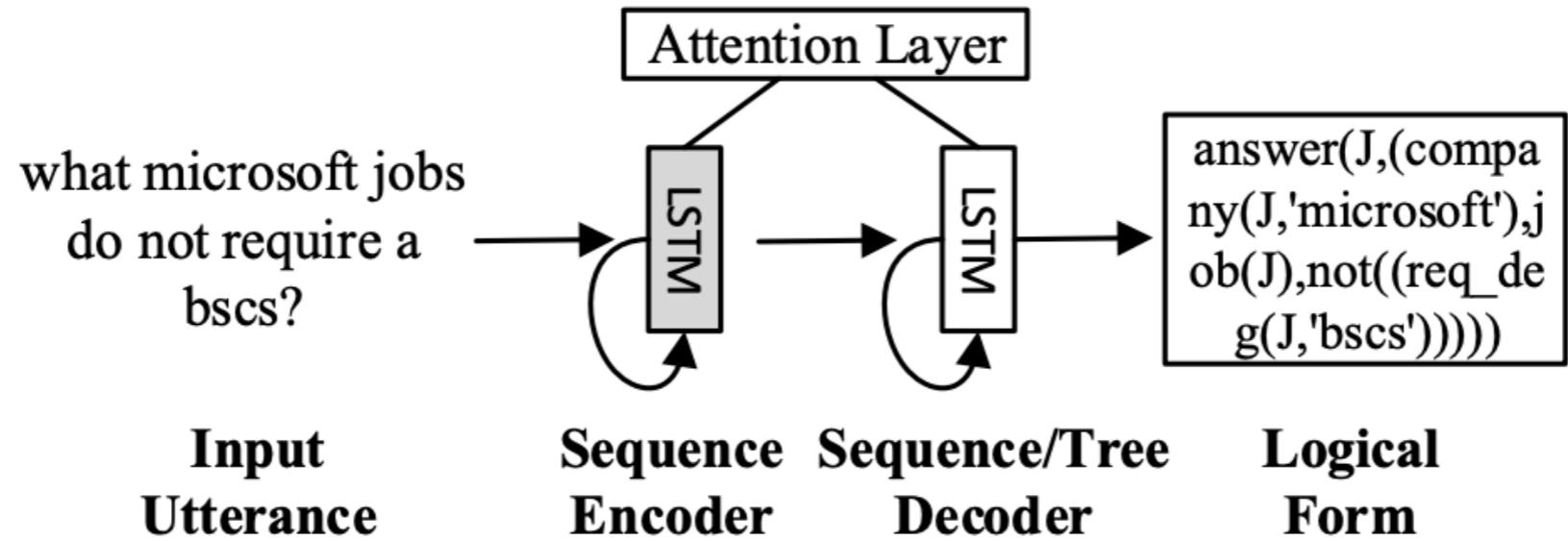
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# Application: Semantic Parsing



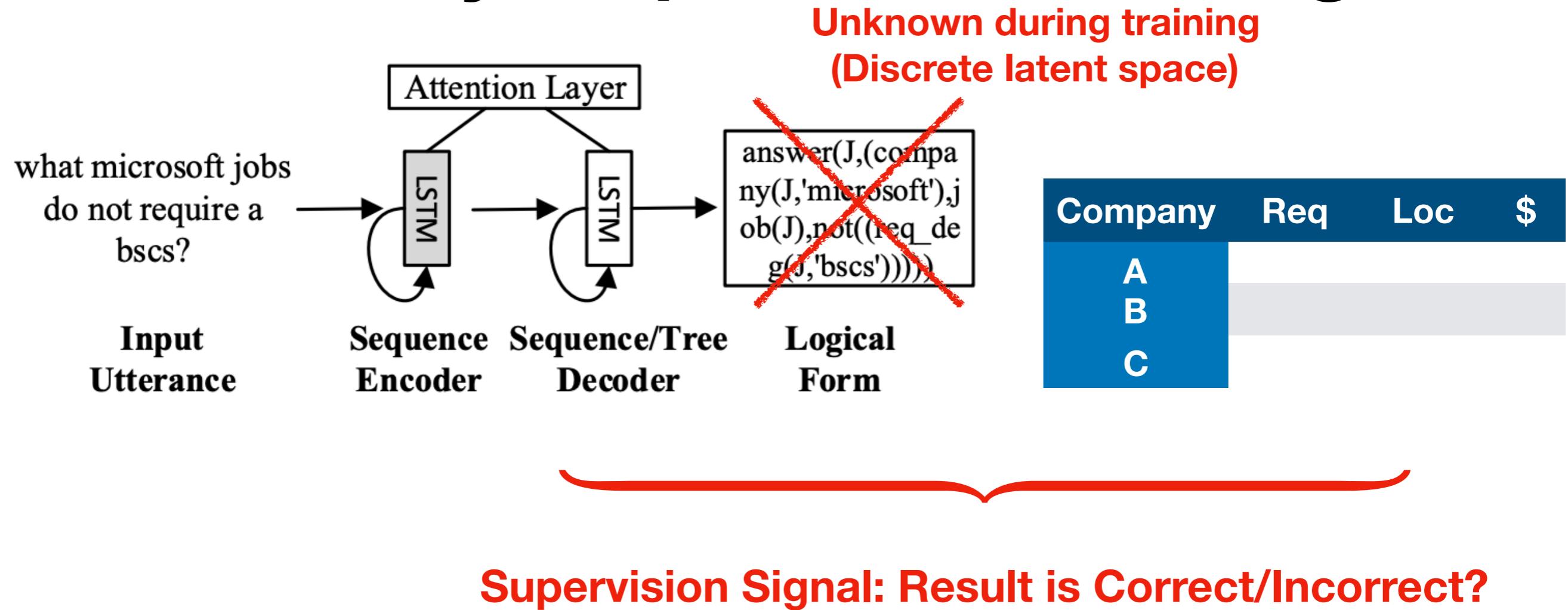
# Semantic Parsing



- Fully supervised setting:
  - Input natural language, and
  - Output logical forms
- Both are known during training

Dong, Li, and Mirella Lapata. Language to logical form with neural attention. In *ACL*, 2016.

# Weakly Supervised setting



# RL Approach

## Predefined primitive operators

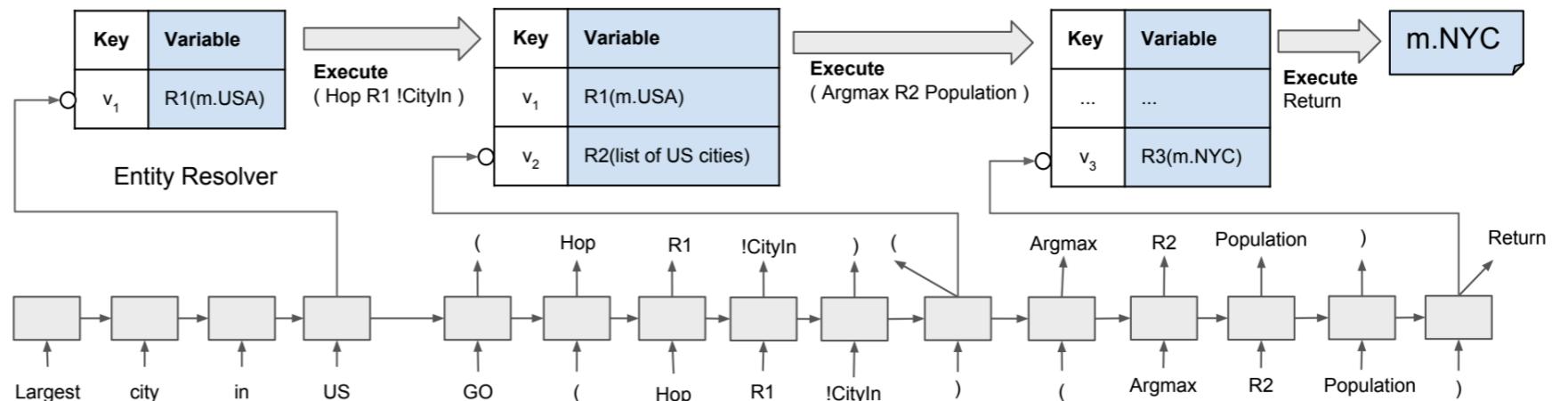
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$$\begin{aligned}
 (\text{Hop } r \ p) &\Rightarrow \{e_2 | e_1 \in r, (e_1, p, e_2) \in \mathbb{K}\} \\
 (\text{ArgMax } r \ p) &\Rightarrow \{e_1 | e_1 \in r, \exists e_2 \in \mathcal{E} : (e_1, p, e_2) \in \mathbb{K}, \forall e : (e_1, p, e) \in \mathbb{K}, e_2 \geq e\} \\
 (\text{ArgMin } r \ p) &\Rightarrow \{e_1 | e_1 \in r, \exists e_2 \in \mathcal{E} : (e_1, p, e_2) \in \mathbb{K}, \forall e : (e_1, p, e) \in \mathbb{K}, e_2 \leq e\} \\
 (\text{Filter } r_1 \ r_2 \ p) &\Rightarrow \{e_1 | e_1 \in r_1, \exists e_2 \in r_2 : (e_1, p, e_2) \in \mathbb{K}\}
 \end{aligned}$$


---

Table 1: Interpreter functions.  $r$  represents a variable,  $p$  a property in Freebase.  $\geq$  and  $\leq$  are defined on numbers and dates.

## Seq2Seq-like model



RL training  
(BS better than sampling)

$$\begin{aligned}
 J^{RL}(\theta) &= \sum_x \mathbb{E}_{P_\theta(a_{0:T}|x)}[R(x, a_{0:T})], \\
 \nabla_\theta J^{RL}(\theta) &= \sum_x \sum_{a_{0:T}} P_\theta(a_{0:T} | x) \cdot [R(x, a_{0:T}) - \\
 &\quad B(x)] \cdot \nabla_\theta \log P_\theta(a_{0:T} | x),
 \end{aligned}$$

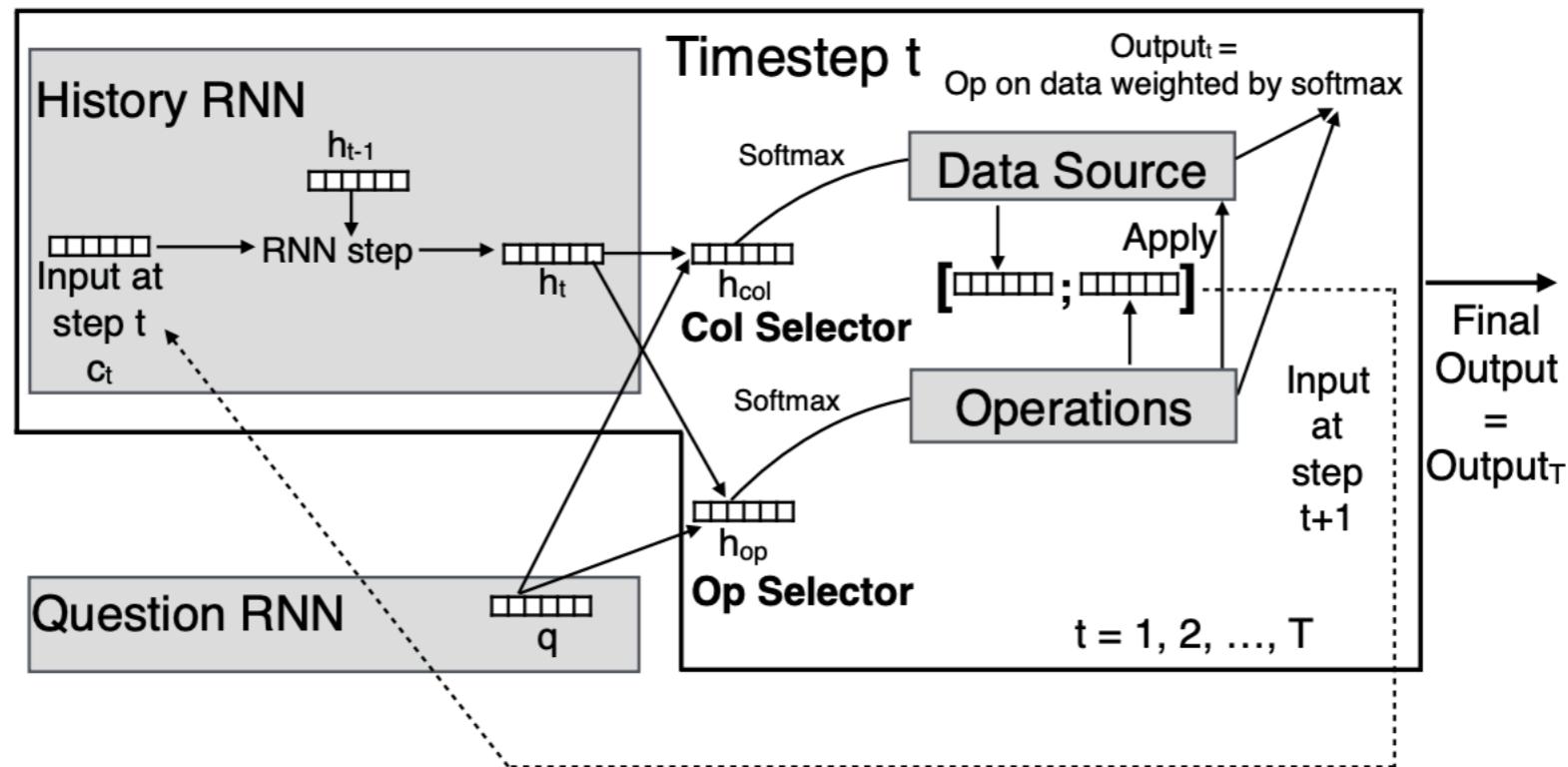
# MLE

<b>Method</b>	<b>Approximation of <math>E_q[\cdot]</math></b>	<b>Exploration strategy</b>	<b>Gradient weight <math>q(\mathbf{z})</math></b>
REINFORCE	Monte Carlo integration	independent sampling	$p_\theta(\mathbf{z} \mid x)$
BS-MML	numerical integration	beam search	$p_\theta(\mathbf{z} \mid x, R(\mathbf{z}) \neq 0)$
RANDOMMER	numerical integration	randomized beam search	$q_\beta(\mathbf{z})$

- Show close relationship between RL and MLE

Guu, K., Pasupat, P., Liu, E.Z. and Liang, P. From language to programs: Bridging reinforcement learning and maximum marginal likelihood. In ACL, 2017.

# Attention on Execution Results

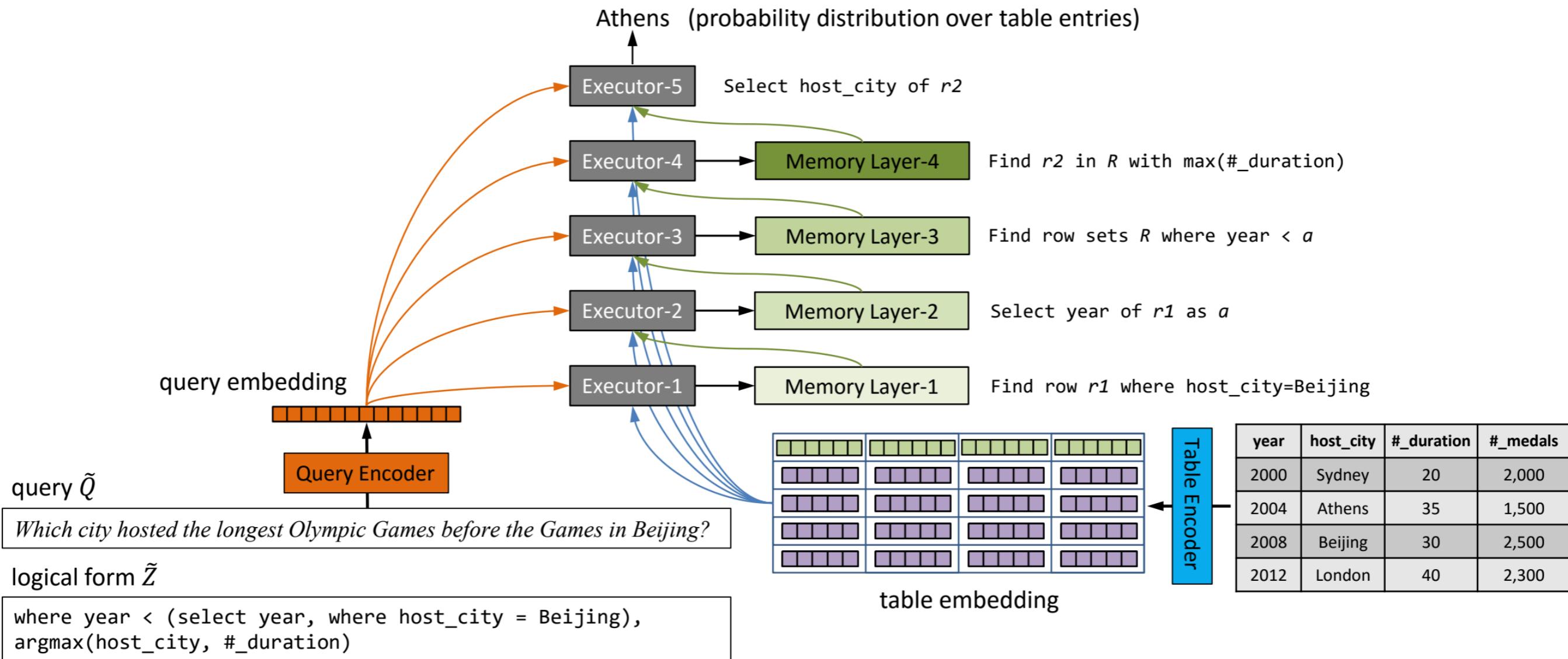


$$\text{scalar\_answer}_t = \alpha_t^{op}(\text{count}) \text{count}_t + \alpha_t^{op}(\text{difference}) \text{diff}_t + \sum_{j=1}^C \alpha_t^{col}(j) \alpha_t^{op}(\text{sum}) \text{sum}_t[j],$$

$$\text{lookup\_answer}_t[i][j] = \alpha_t^{col}(j) \alpha_t^{op}(\text{assign}) \text{assign}_t[i][j], \forall (i, j) i = 1, 2, \dots, M, j = 1, 2, \dots, C$$

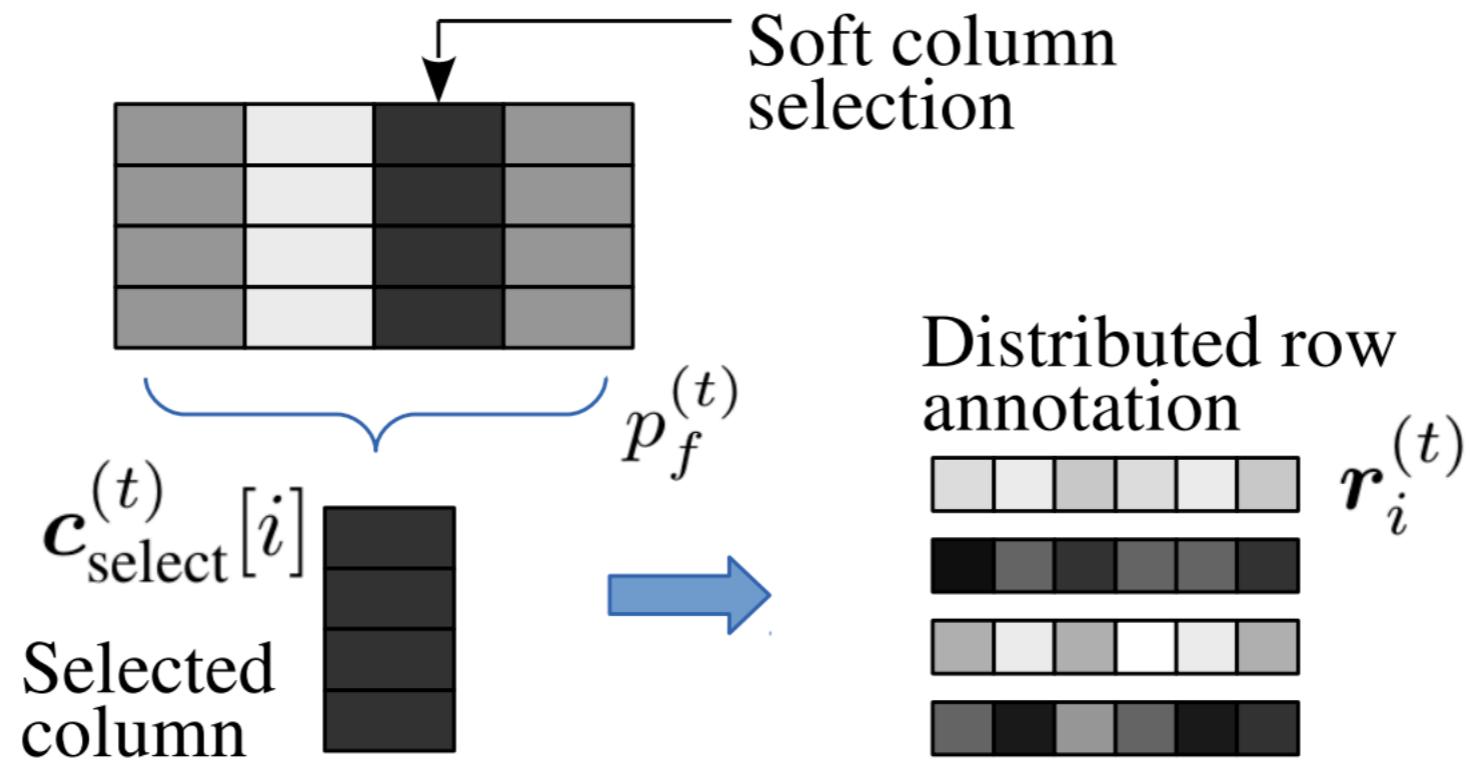
**Primitive operator + Step-by-step attn on results**

# Attention as Execution Itself



Yin, P., Lu, Z., Li, H. and Kao, B., 2015. Neural enquirer:  
Learning to query tables with natural language. In *IJCAI*, 2016.

# Neural Executor

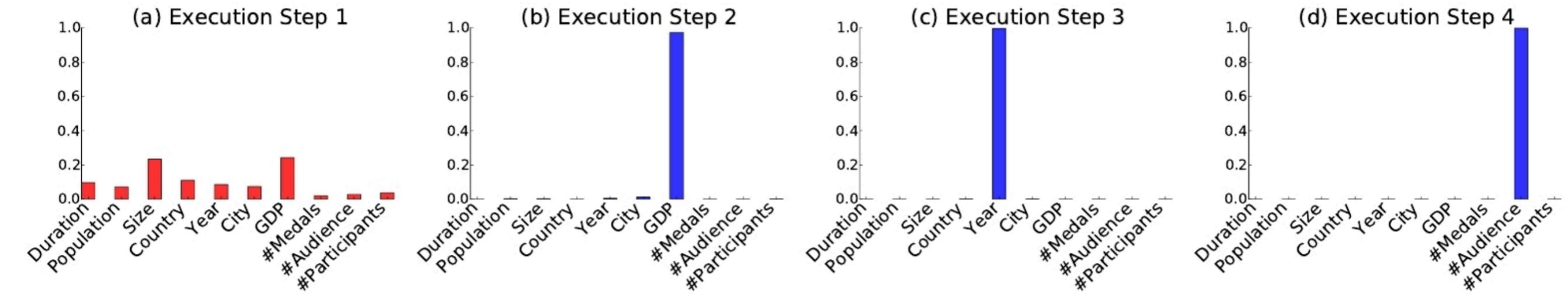


- Attention-based column selection
- Distributed representation for row selection
  - Not subject to primitive operators
  - Not fully explainable either

Yin, P., Lu, Z., Li, H. and Kao, B., 2015. Neural enquirer:  
Learning to query tables with natural language. In *IJCAI*, 2016.

# Attention as Execution Itself

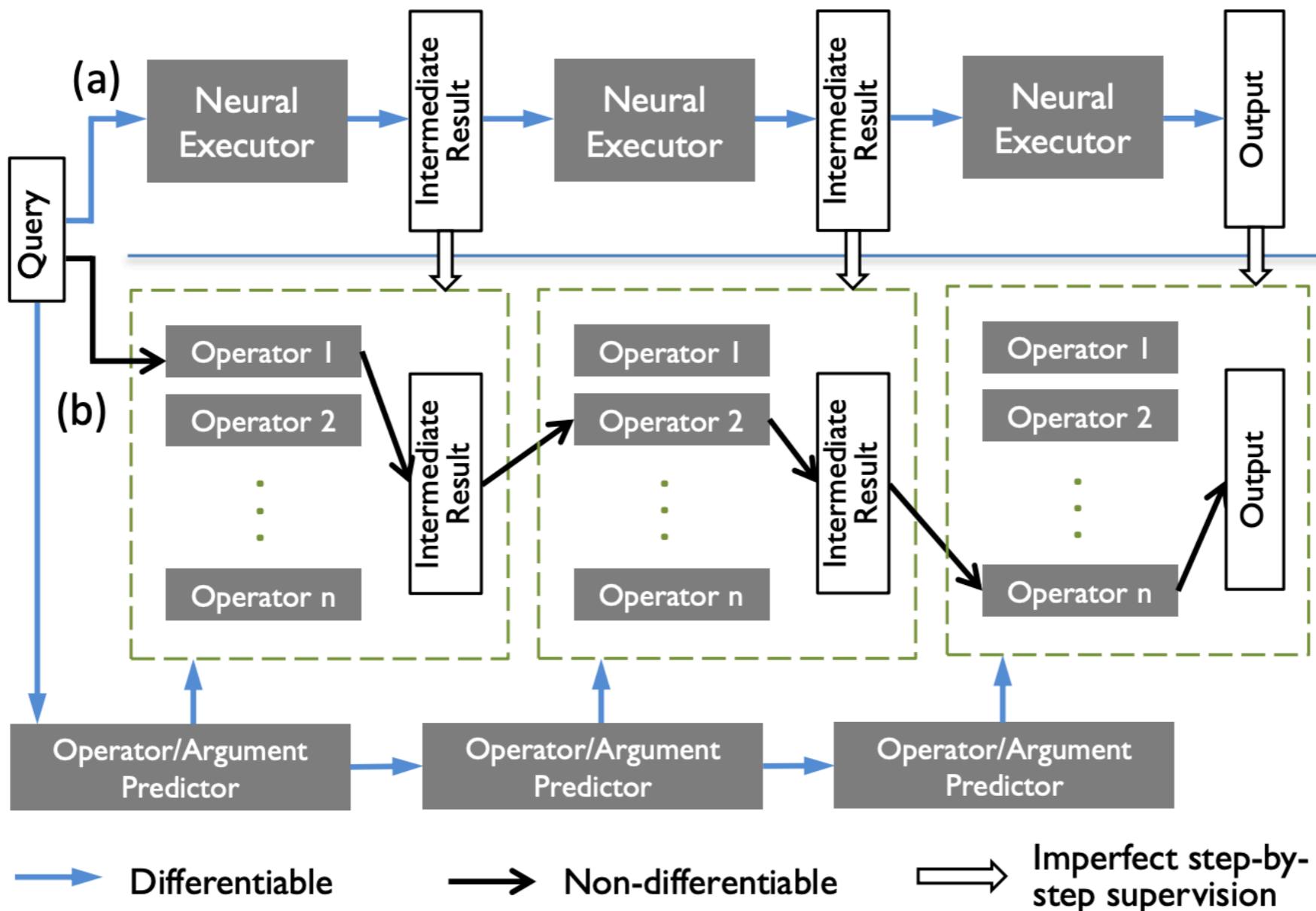
Query: How many people watched the earliest game whose host country GDP is larger than the game in Cape Town?



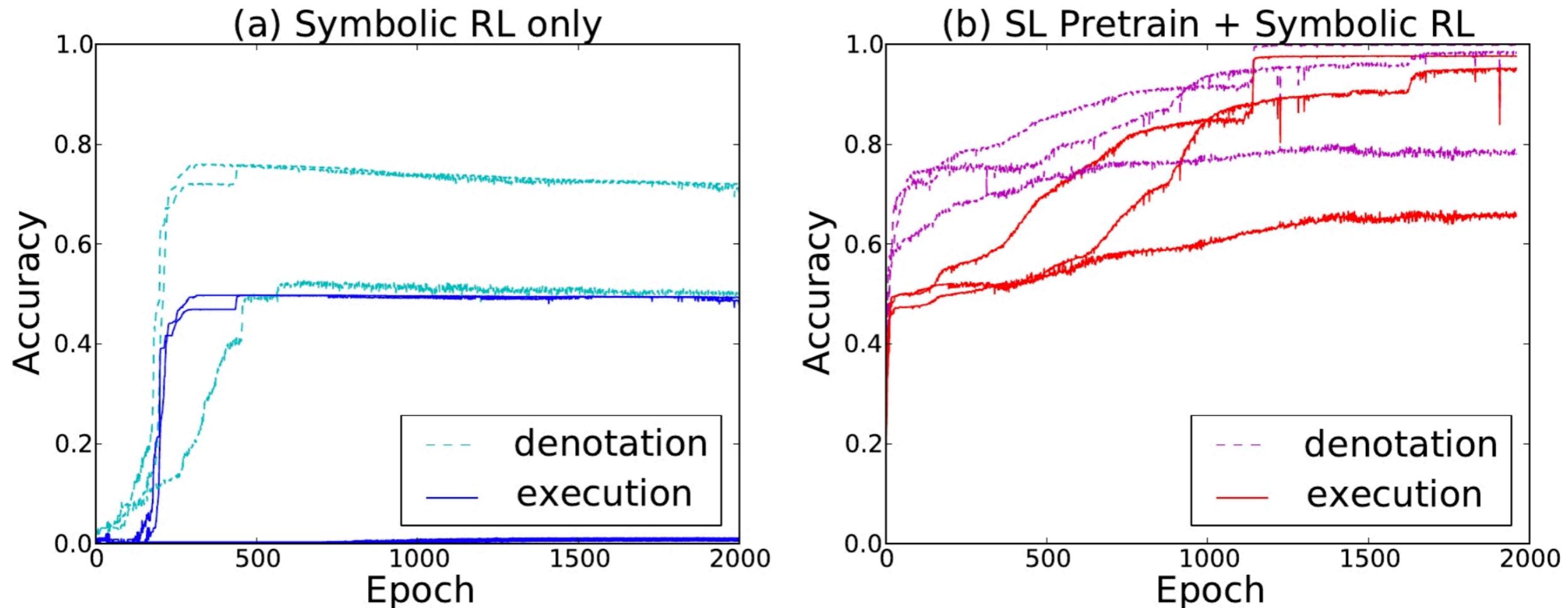
Step-by-step attention does learn meaningful things

Yin, P., Lu, Z., Li, H. and Kao, B., 2015. Neural enquirer:  
Learning to query tables with natural language. In *IJCAI*, 2016.

# Attention + RL

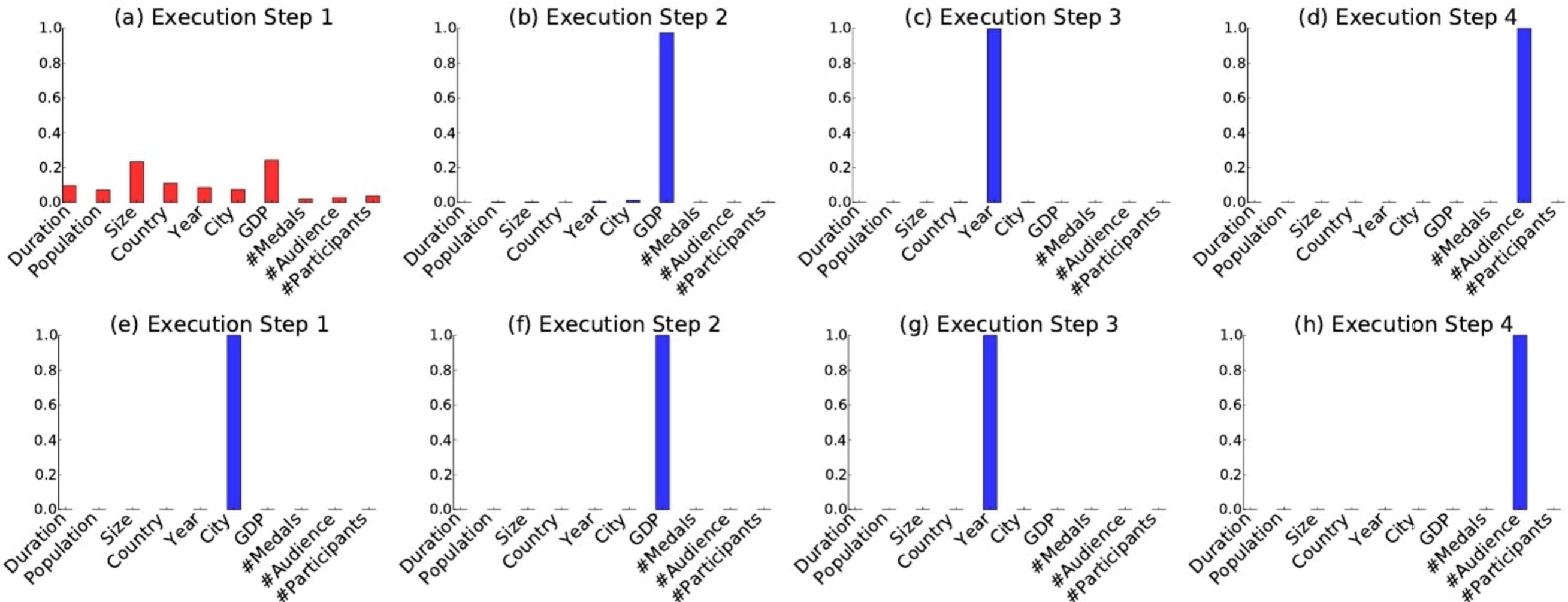


# Attention-based initialization is important



# Attention-based initialization is important

**Query:** How many people watched the earliest game whose host country GDP is larger than the game in Cape Town?



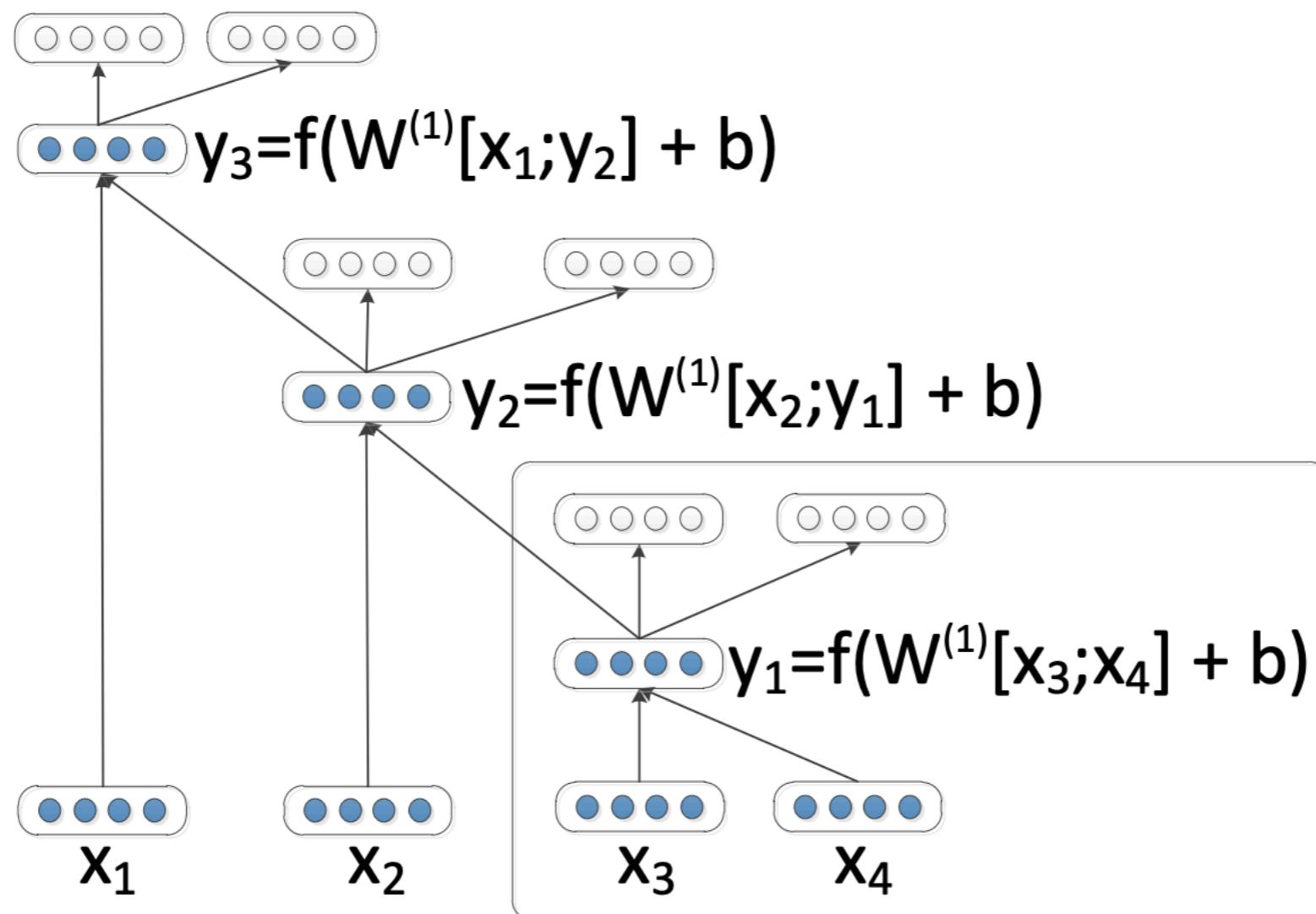
# Application: Syntactic Parsing (Unsupervised)



amii



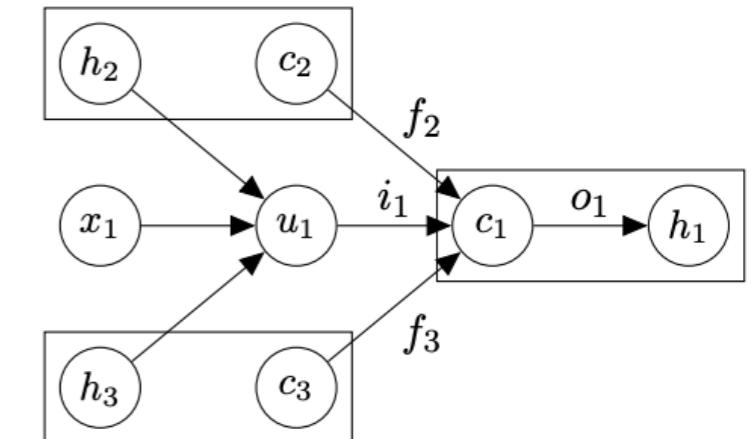
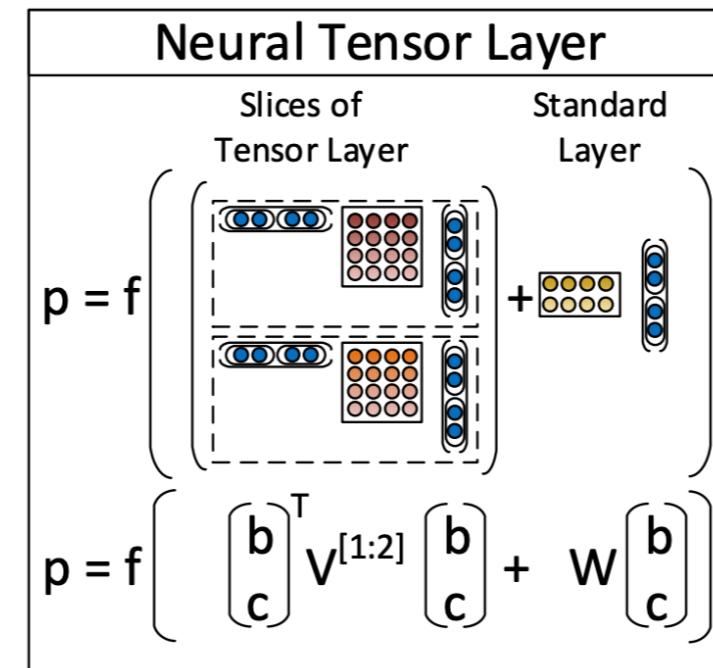
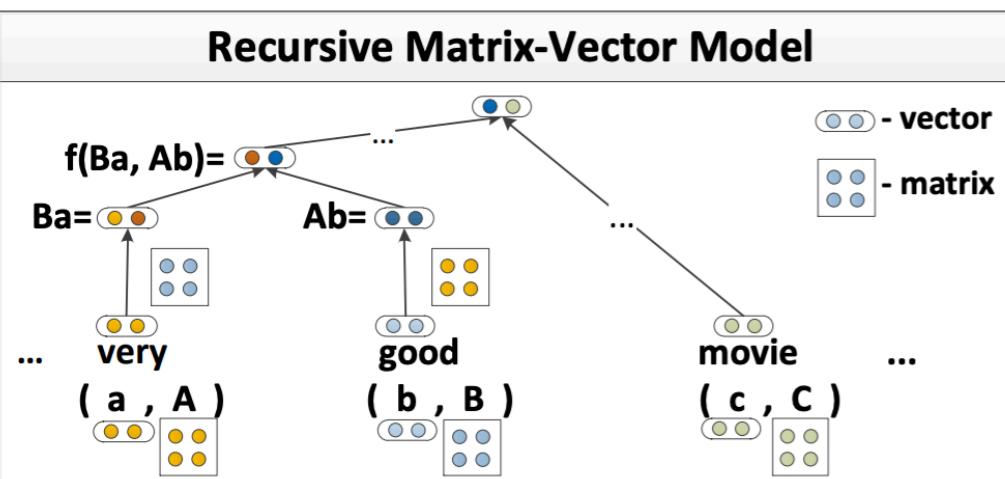
# Recursive Autoencoder



Induce tree structures by minimizing reconstruction on an AE

Socher, Richard, Jeffrey Pennington, Eric H. Huang, Andrew Y. Ng, and Christopher D. Manning. Semi-supervised recursive autoencoders for predicting sentiment distributions. In *EMNLP*, 2011.

# Recursive Neural Network



- Parsing by auto-encoding never worked
- Standard RecursiveNN is based on external parse trees  
I.e., Tree structures are constant

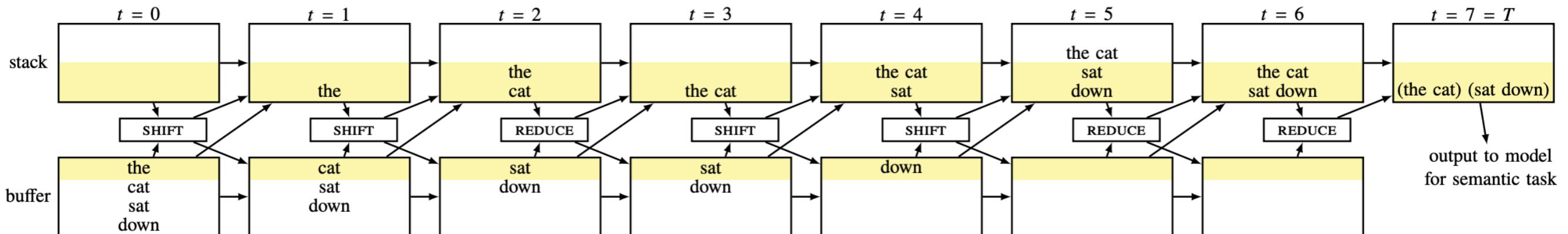
Sheng, Socher, et al. Improved semantic representations from tree-structured long short-term memory networks. In *ACL*, 2015.

Socher, R., et al. Recursive deep models for semantic compositionality over a sentiment treebank. In *EMNLP*, 2013.

Socher R., et al. Semantic compositionality through recursive matrix-vector spaces. In *EMNLP*, 2012.

# SPINN

## Stack-augmented Parser-Interpreter Neural Network

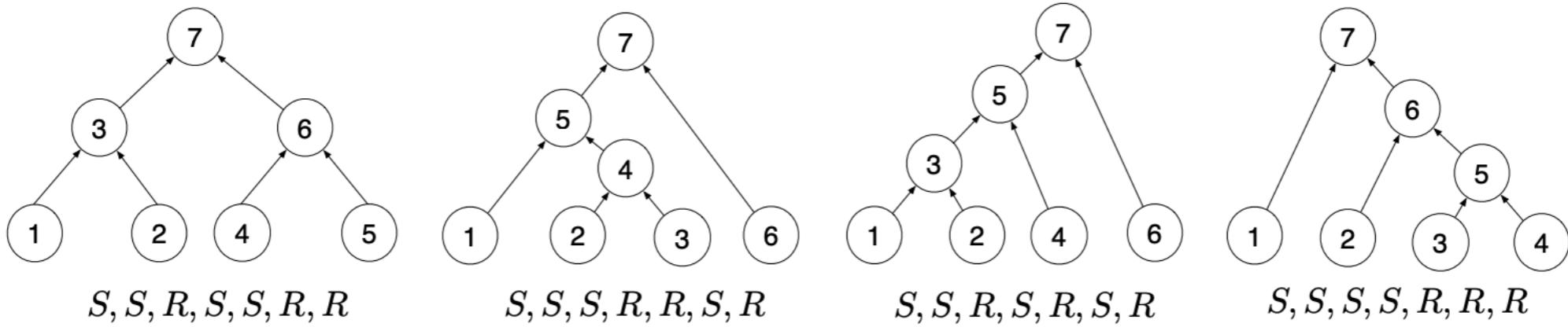


(b) The fully unrolled SPINN for *the cat sat down*, with neural network layers omitted for clarity.

- Shift-reduce parser jointly trained with downstream task
- Supervision provided by Standford Parser

Bowman, S.R., Gauthier, J., Rastogi, A., Gupta, R., Manning, C.D. and Potts, C., 2016. A fast unified model for parsing and sentence understanding. In *ACL*, 2016.

# RL-SPINN



- Still shift-reduce parser
- Semi-supervised or unsupervised
- Trained by RL

$$\mathcal{R}(\mathbf{W}) = \mathbb{E}_{\pi(\mathbf{a}, \mathbf{s}; \mathbf{W}_R)} \left[ \sum_{t=1}^T r_t a_t \right]$$

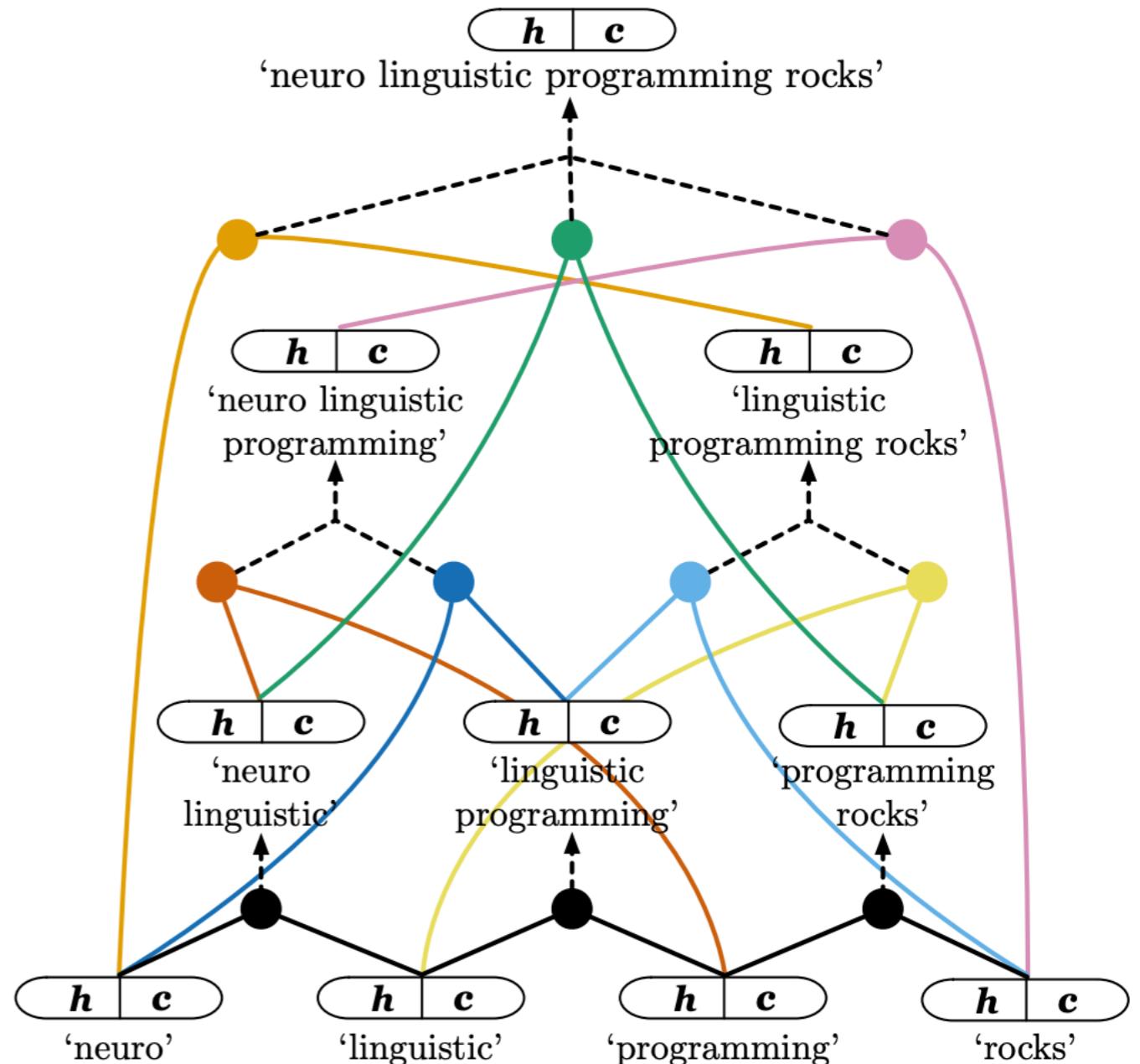
Yogatama, D., Blunsom, P., Dyer, C., Grefenstette, E. and Ling, W.. Learning to compose words into sentences with reinforcement learning. In *ICLR*, 2017.

# Chart-style Parser

- Implicitly considering all possible trees
- Not exact marginalization
- Step-by-step fusion/attention

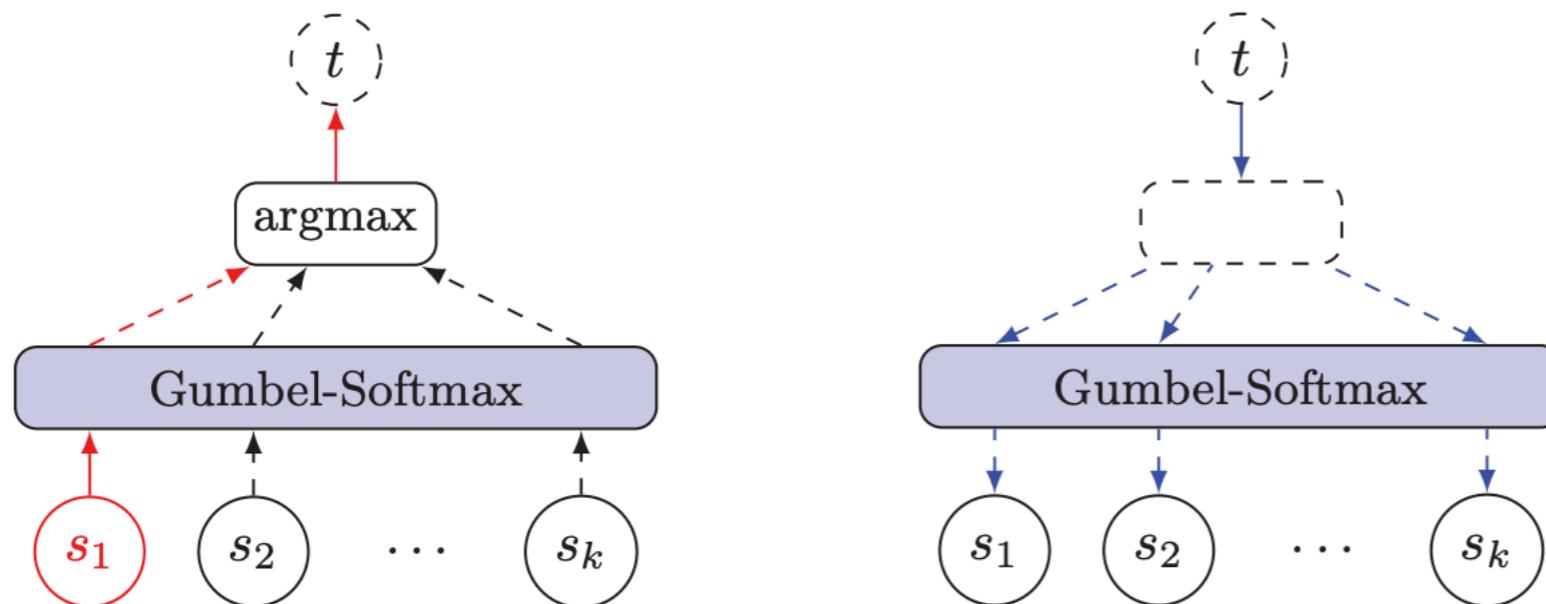
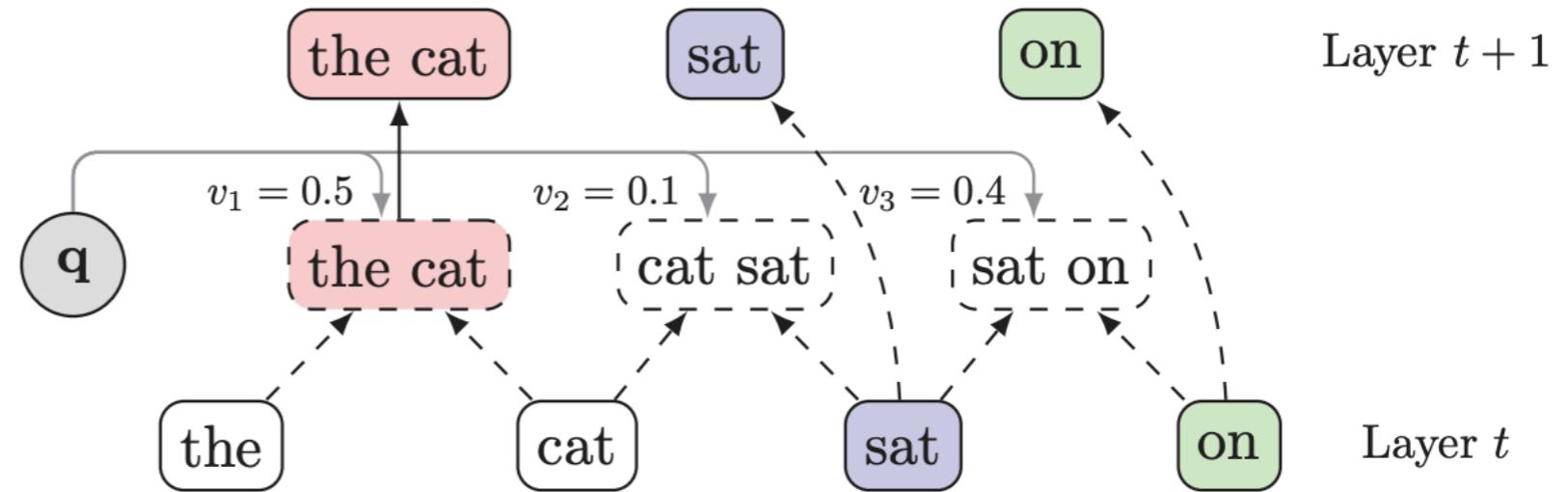
$$s_i = \text{softmax}(e_i/t),$$

$$c = \sum_{i=1}^n s_i c_i, \quad h = \sum_{i=1}^n s_i h_i$$



# Pyramid

- ST-Gumbel



# Main issues with these models

[William et al., TACL'18]

- Trees are not consistent across random init.
- Do not resemble real trees

[Shi et al., EMNLP'18]

- All trees are similar to downstream performance
- Balanced trees are slightly better

Williams, A., Drozdov, A. and Bowman, S.R. Do latent tree learning models identify meaningful structure in sentences? *TACL*, 2018.

Shi, H., Zhou, H., Chen, J. and Li, L., 2018. On tree-based neural sentence modeling. In *EMNLP*, 2018.

# Proximal Policy Optimization

- Train the policy  $K$  steps

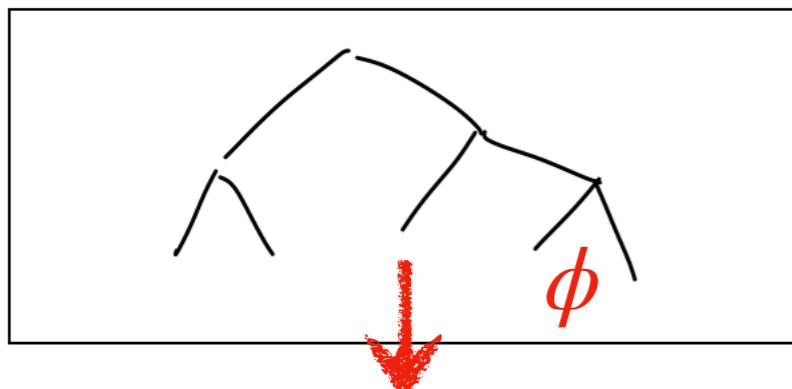
$$\hat{\mathbb{E}}_t [r_\phi(t) \ell(f_\theta(x, t), y)] \quad r_\phi(t) = \frac{p_\phi(t|x)}{p_{\phi_{\text{old}}}(t|x)}$$

- Clip gradient

$$\hat{\mathbb{E}}_t [\max \{r_\phi(t) \ell(f_\theta(x, t), y), r_\phi^c(t) \ell(f_\theta(x, t), y)\}]$$

 $\theta$ 

**Exact gradient, easy to learn**



**RL, difficult to learn**

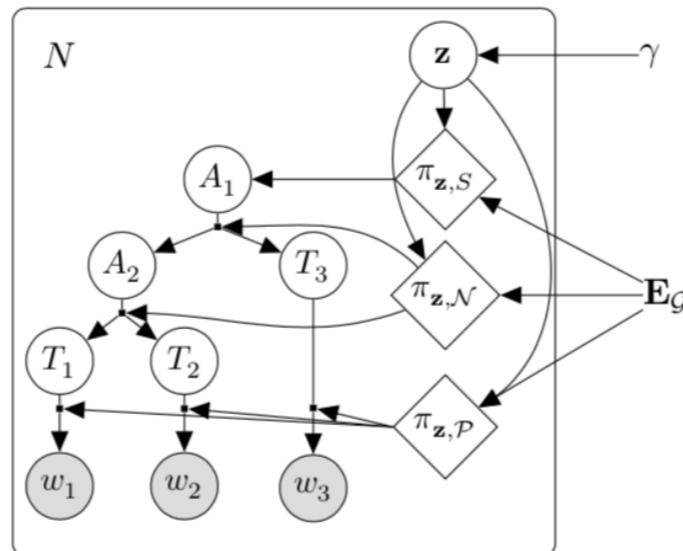
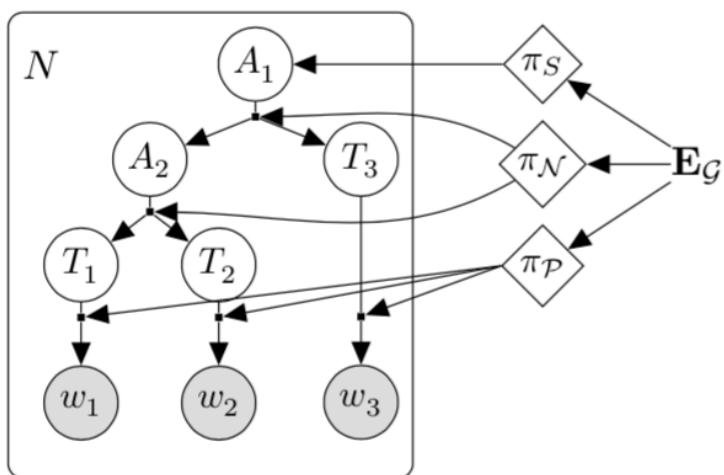
The tutorial is very boring

# Compound PCFG

- Over-parametrize PCFG into a Gaussian continuous space
  - Shown to be easier to train and more linguistically plausible

$$\begin{aligned} \log p_{\theta}(\mathbf{x}) &= \log \left( \int p_{\theta}(\mathbf{x} | \mathbf{z}) p_{\gamma}(\mathbf{z}) d\mathbf{z} \right) \\ &= \log \left( \int_{t \in \mathcal{T}_{\mathcal{G}}(\mathbf{x})} \sum p_{\theta}(t | \mathbf{z}) p_{\gamma}(\mathbf{z}) d\mathbf{z} \right) \end{aligned}$$

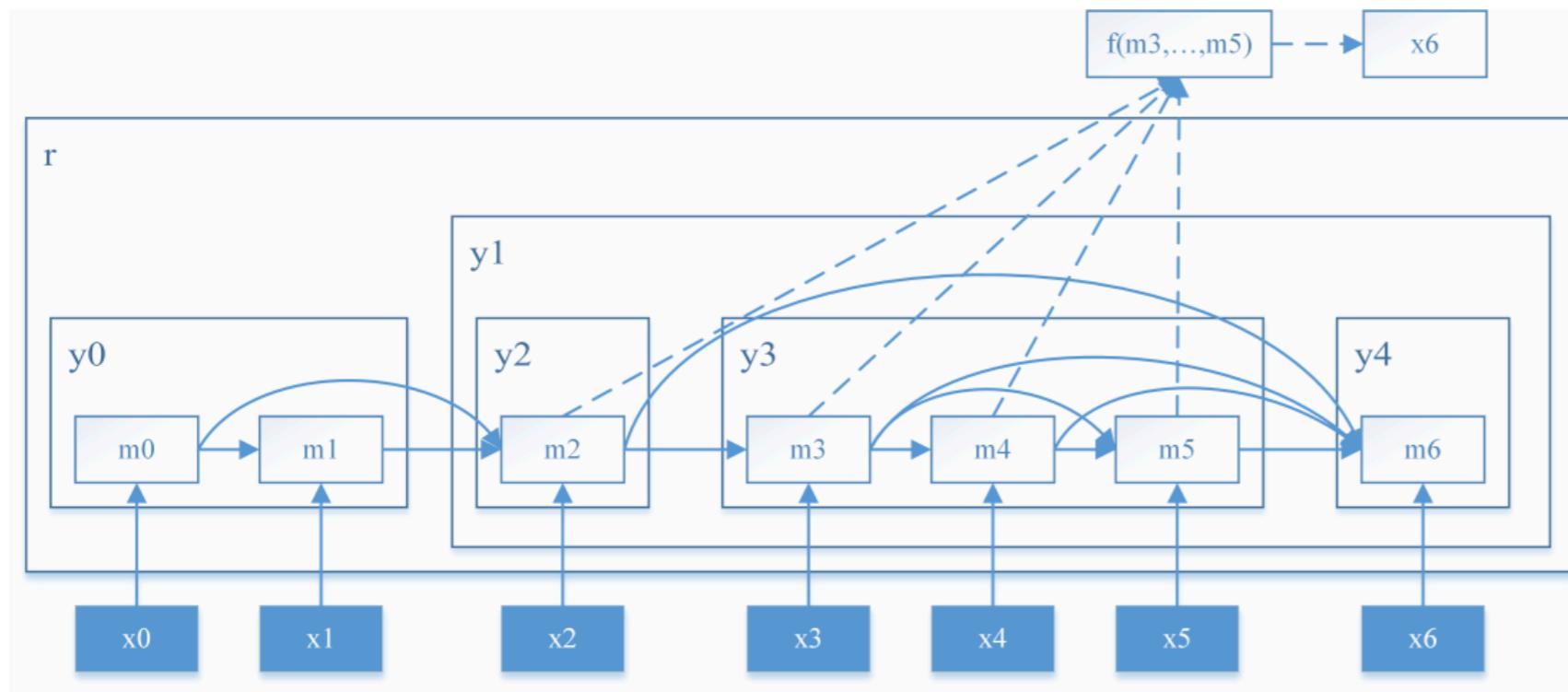
**VAE**      **Exact by inside all**



# PRPN

Parsing-Reading-Predict Networks

- Language modeling is important
- Structured attention, based on “syntactic distance”

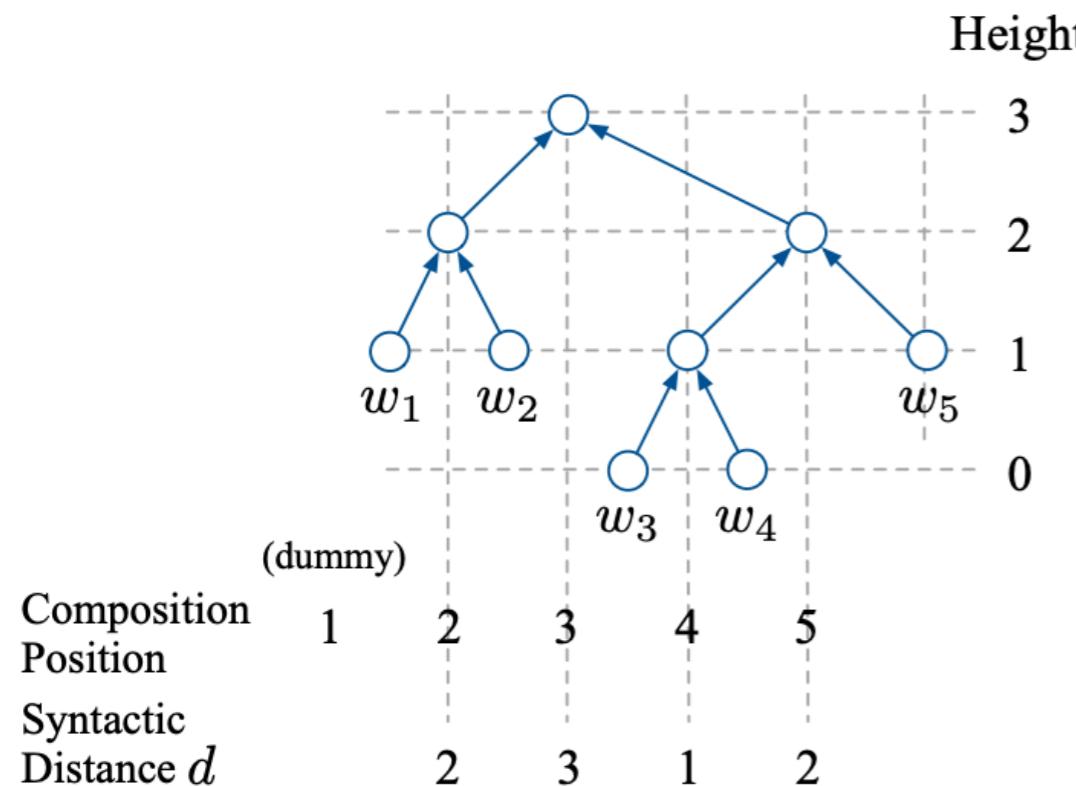


# PRPN

Parsing-Reading-Predict Networks

- Syntactic distance  $d$  (learned in an unsupervised way)

Difference of  $d$ :  $\alpha_j^t = \frac{\text{hardtanh}(\tau(\hat{d}_t - \hat{d}_j)) + 1}{2} \in [0,1]$



Multiplicative accumulation

$$g_i^t = \prod_{j=i+1}^{t-1} \alpha_j^t$$

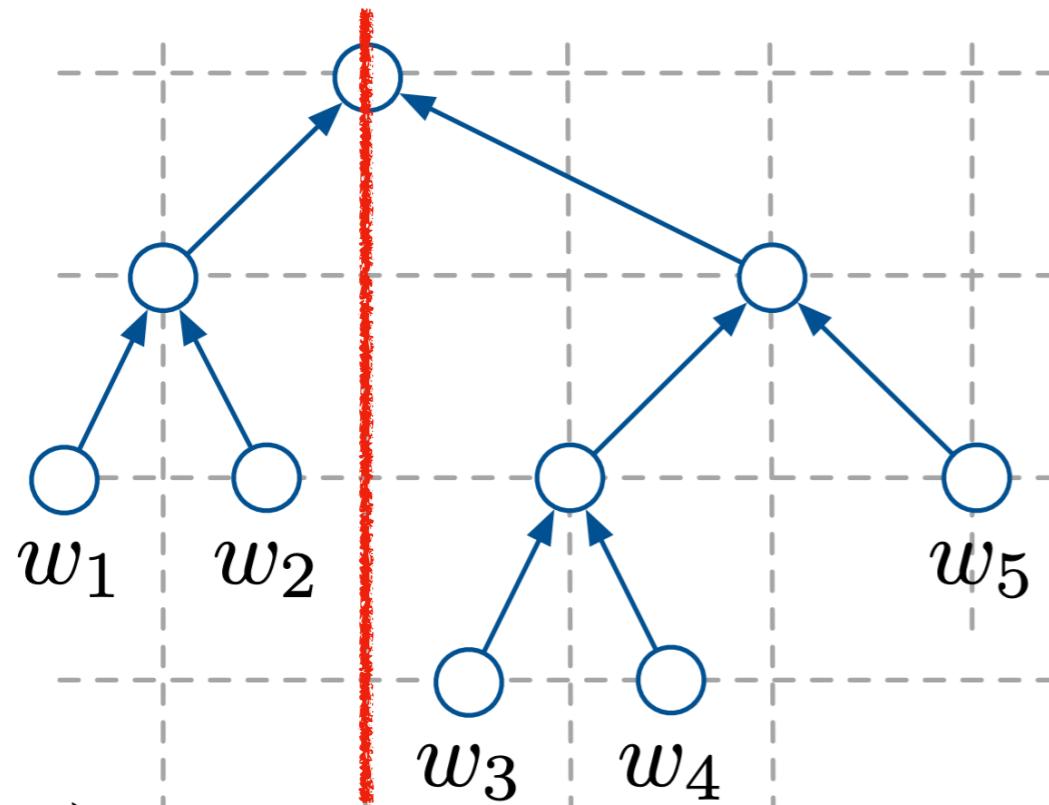
Reweigh self-attn.

$$s_i^t = \frac{g_i^t}{\sum_{i=1}^{t-1} g_i^t} \tilde{s}_i^t$$

# PRPN

Parsing-Reading-Predict Networks

- Prediction



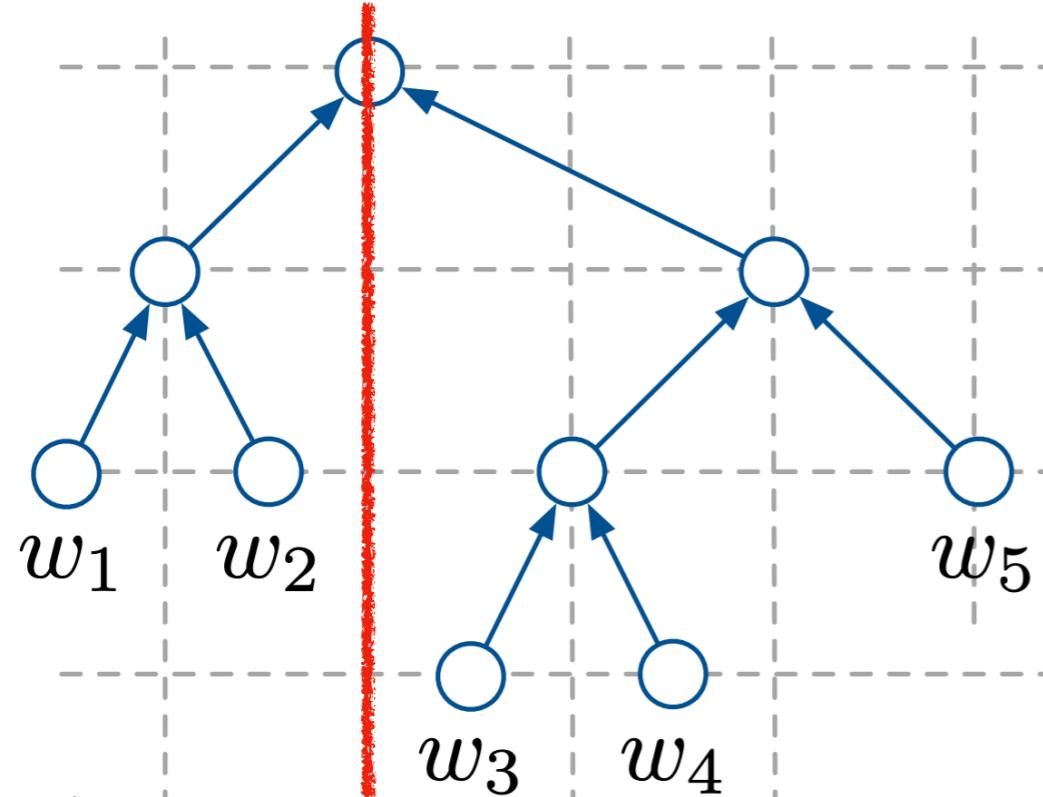
$$(w_1, w_2) | (w_3 w_4 w_5)$$

Intuitive way/In paper

# PRPN

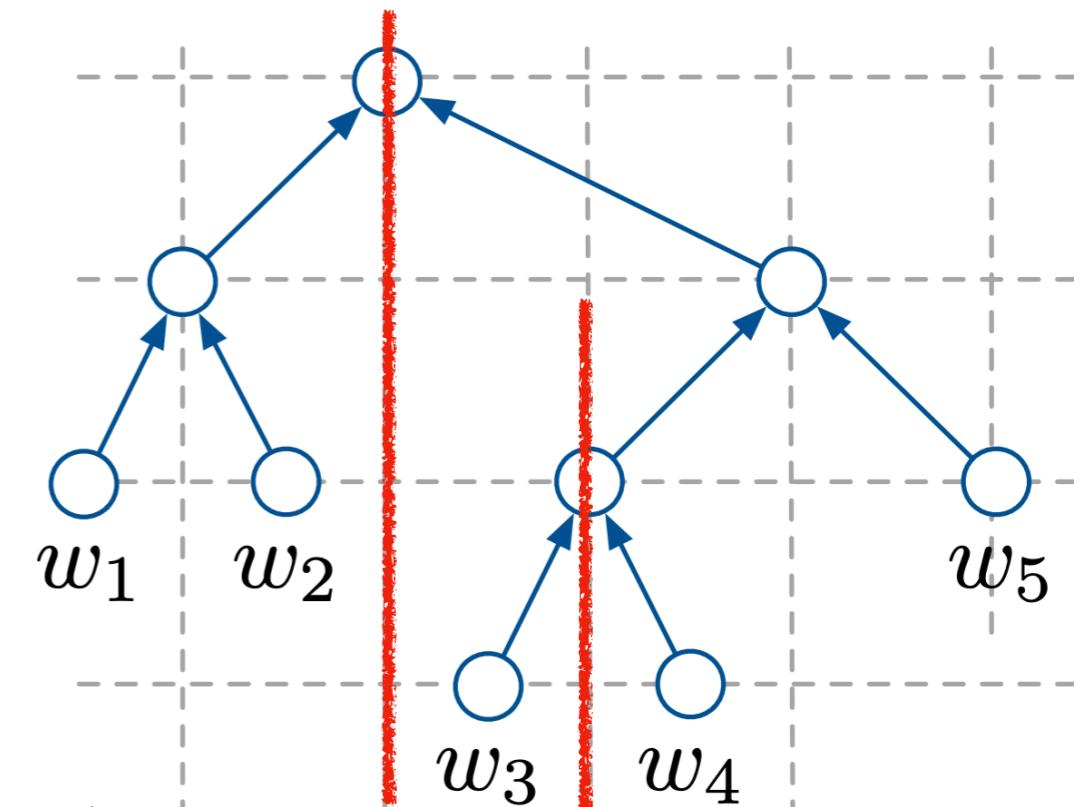
Parsing-Reading-Predict Networks

- Prediction



$(w_1, w_2) | (w_3 w_4 w_5)$

Intuitive way/In paper



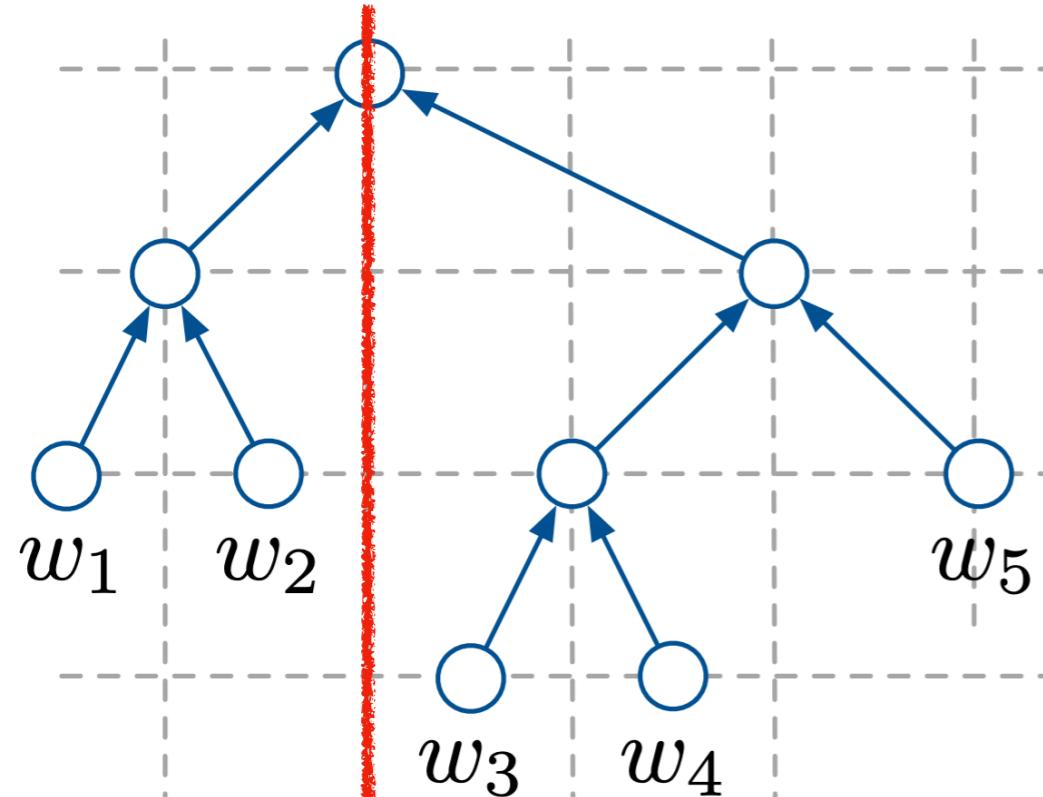
$(w_1, w_2) | (w_3 | (w_4 w_5))$

In Appendix/Code

# PRPN

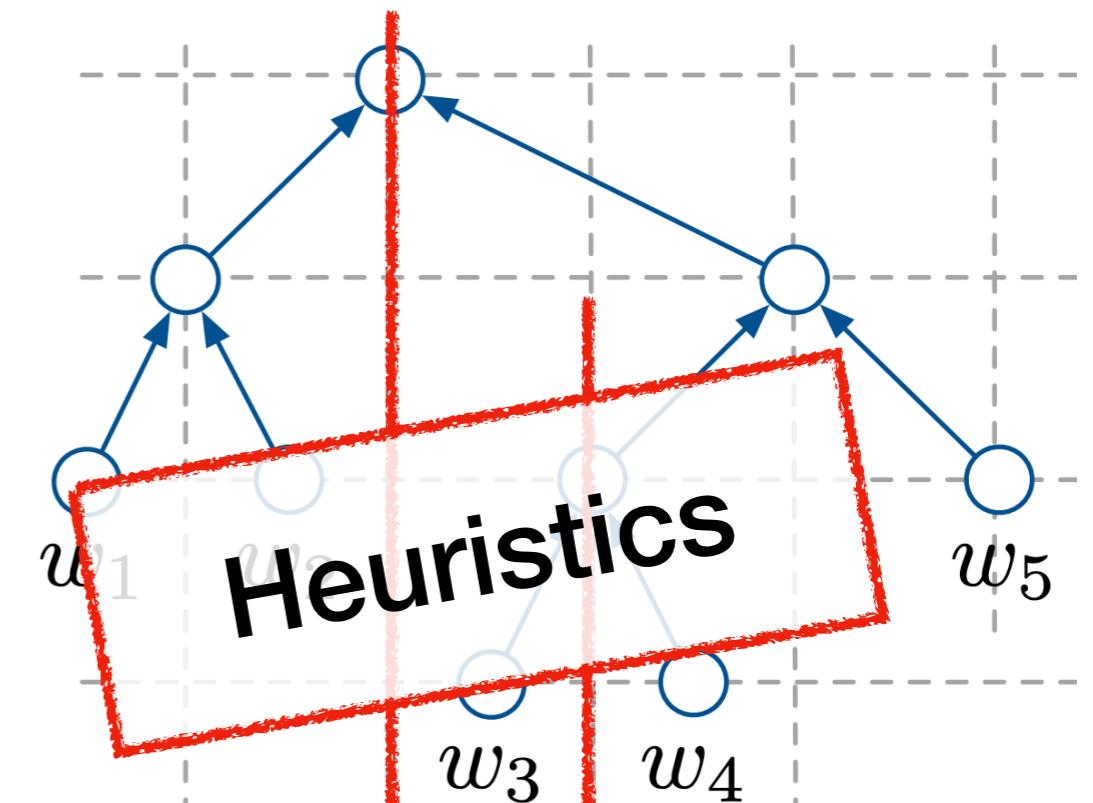
Parsing-Reading-Predict Networks

- Prediction



$(w_1, w_2) | (w_3 w_4 w_5)$

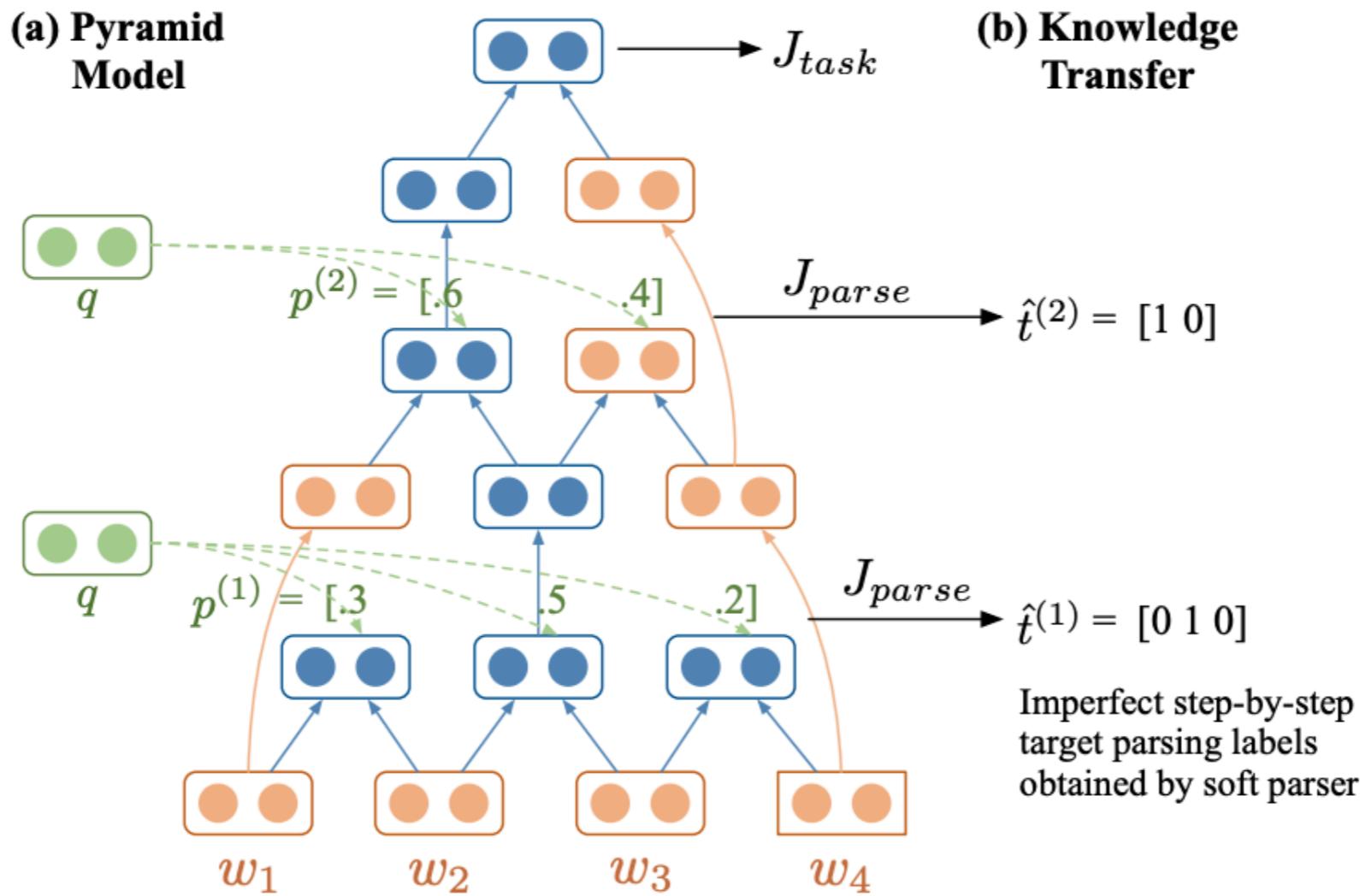
Intuitive way/In paper



$(w_1, w_2) | (w_3 | (w_4 w_5))$

In Appendix/Code

# Combining Both Worlds



- Step1: Step-by-step learning from PRPN
- Step2: Policy improvement by ST-Gumbel

# Results

Model	Mean $F$	w/o Punctuation		w/ Punctuation		
		Self-agreement	RB-agreement	Mean $F$	Self-agreement	RB-agreement
Left-Branching	20.7	-	-	18.9	-	-
Right-Branching	<b>58.5</b>	-	-	18.5	-	-
Balanced-Tree	39.5	-	-	22.0	-	-
ST-Gumbel	36.4	57.0	33.8	21.9	56.8	<b>38.1</b>
PRPN	46.0	48.9	51.2	51.6	65.0	27.4
Imitation (SbS only)	45.9	49.5	62.2	52.0	<b>70.8</b>	20.6
Imitation (SbS + refine)	53.3 <sup>†</sup>	<b>58.2</b>	<b>64.9</b>	<b>53.7<sup>†</sup></b>	67.4	21.1

- Our results show
  - Language modeling is good, but semantic oriented tasks also help
  - ST-Gumbel works if meaningful initialized

# Summary

<b>MLE</b>	maximize	$\log \left( \sum_z p(z) p(Y z, \theta) \right)$
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<b>RL</b>	maximize	$\mathbb{E}_{z \sim p_\theta(z)} R(Y(z))$
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<b>Gumbel softmax</b>	maximize	$\mathbb{E}_{\epsilon \in p(\epsilon)} J(Y(z_\theta(\epsilon)))$
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<b>Attention</b>	maximize	$J(Y(\mathbb{E}_{z \sim p_\theta(z)}[z]))$
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- Case studies
  - Weakly supervised semantic parsing
  - Unsupervised syntactic parsing

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