

# Word Embeddings & Language Modeling

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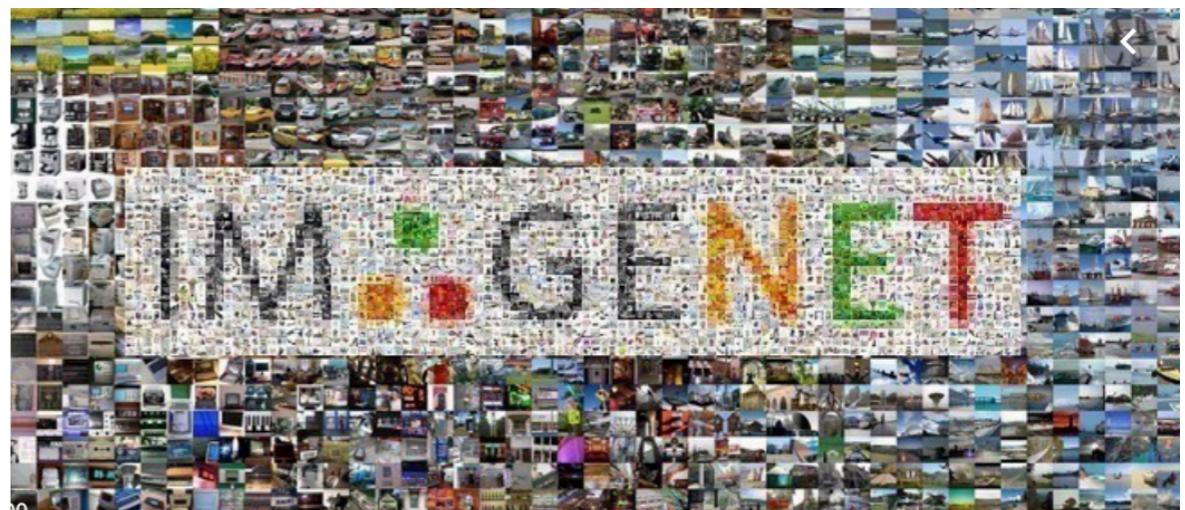
# Last Lecture

- Logistic regression/Softmax: Linear classification
- Non-linear classification
  - Non-linear feature engineering
  - Non-linear kernel
  - Non-linear function composition
- Neural networks
  - Forward propagation: Compute activation
  - Backward propagation: Compute derivative  
(greedy dynamic programming)

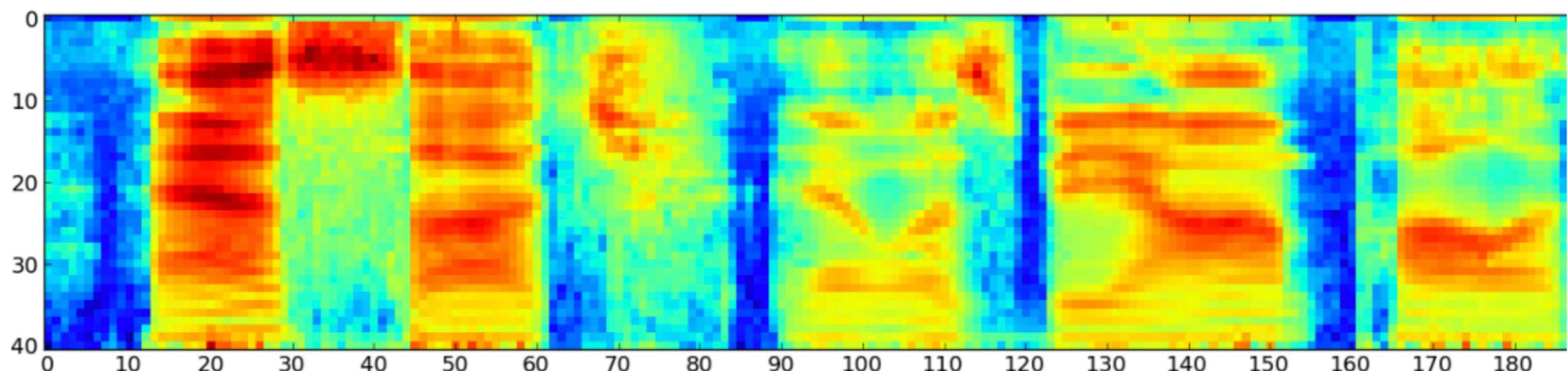


# Advantages of DL

- Work with raw data
  - Images processing: pixels
  - Speech processing: frequency



ImageNet



[Graves+, ICASSP'13]



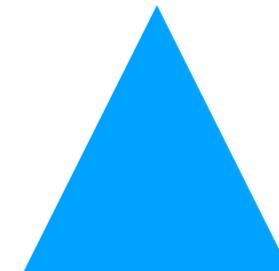
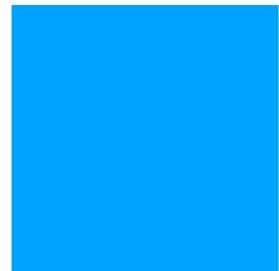
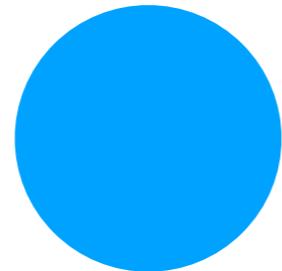
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# How about Language?

- The raw input of language

*I like the course*

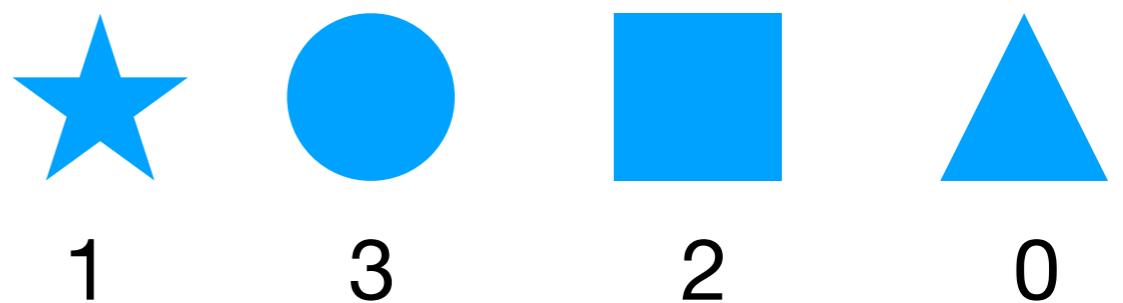
- Problem: **Words are discrete tokens!**



# Representing Words

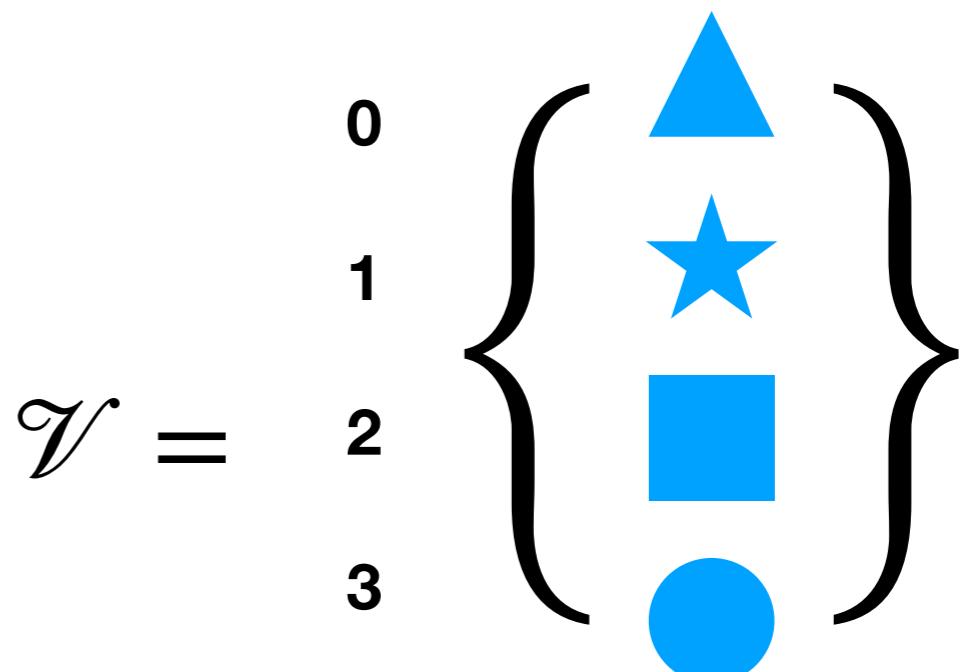
- **Attempt#1:**

- By index in the vocabulary



- Problem

- Introducing artefacts
    - Order, metric, inner-product
    - Extreme non-linearity

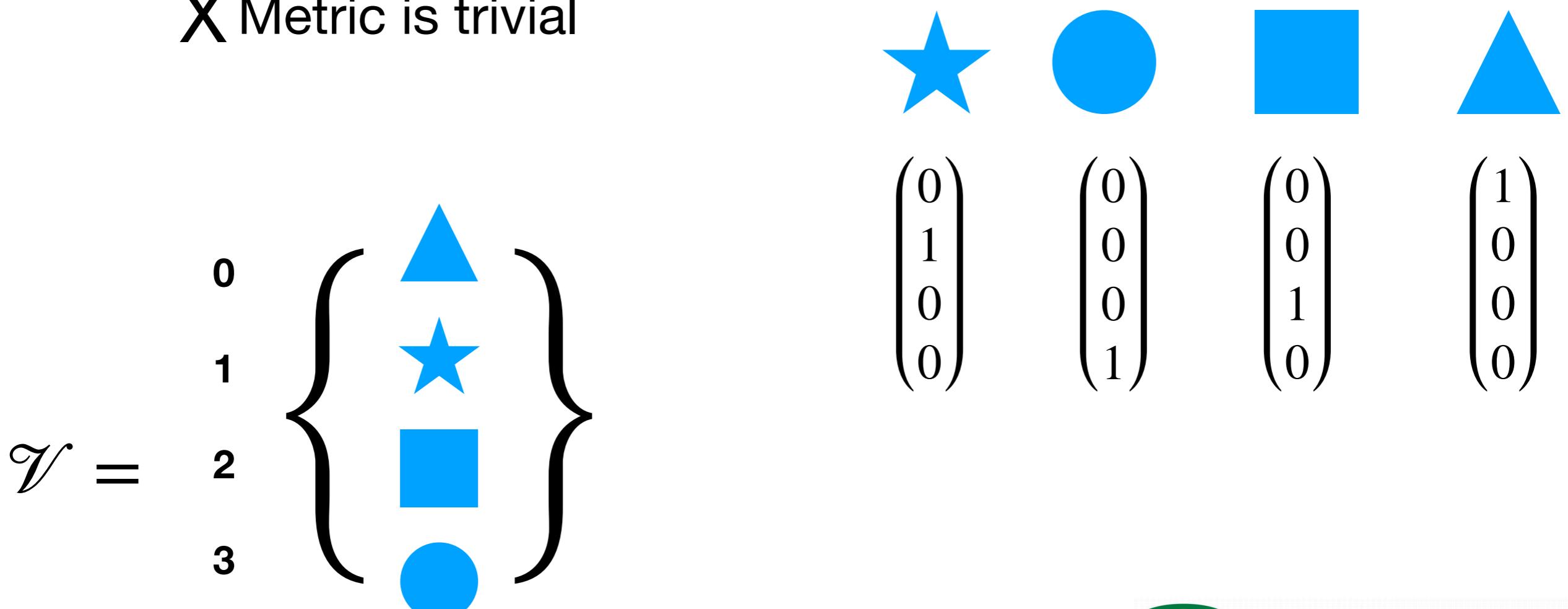


# Representing Words

- **Attempt#2:** One-hot representation

✗ Separability doesn't generalize

✗ Metric is trivial



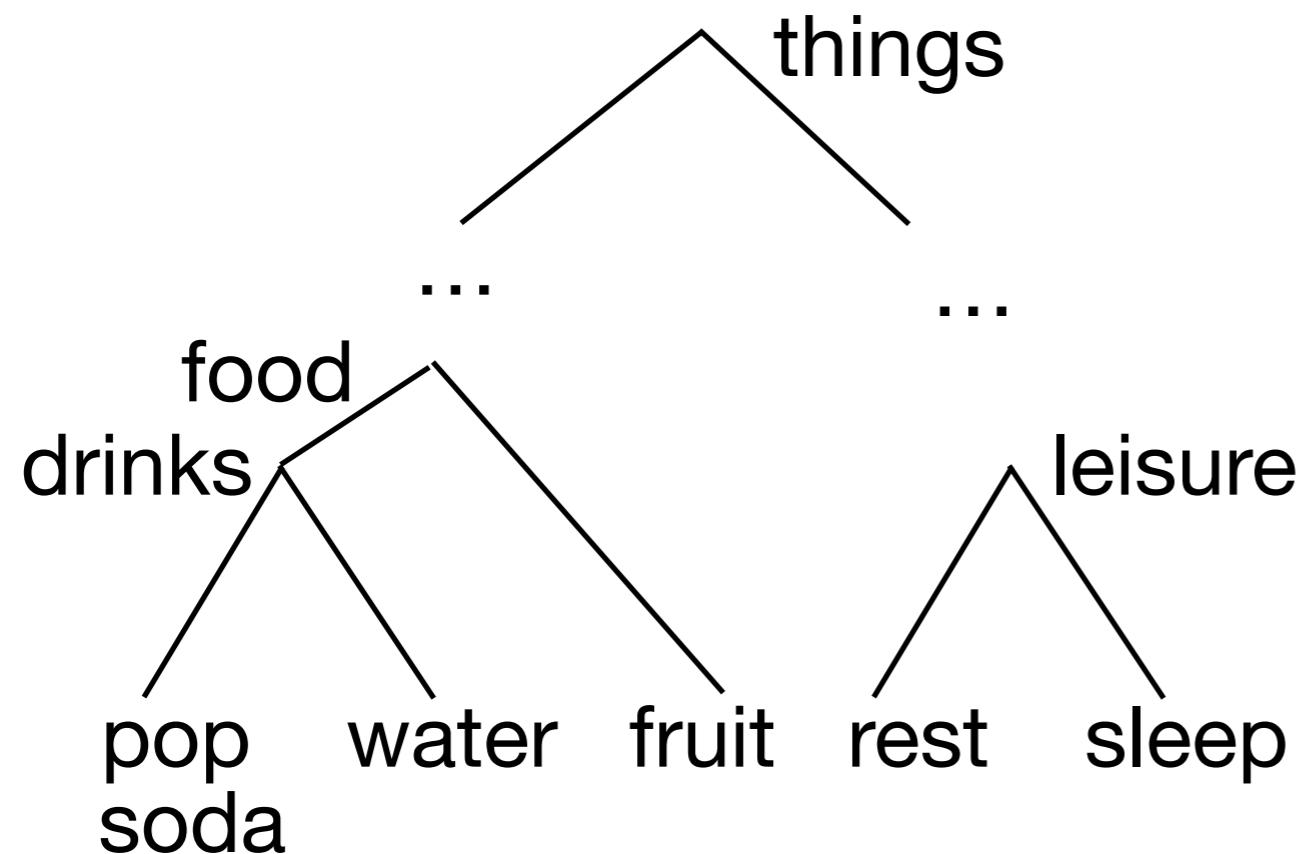


# Metric in the Word Space

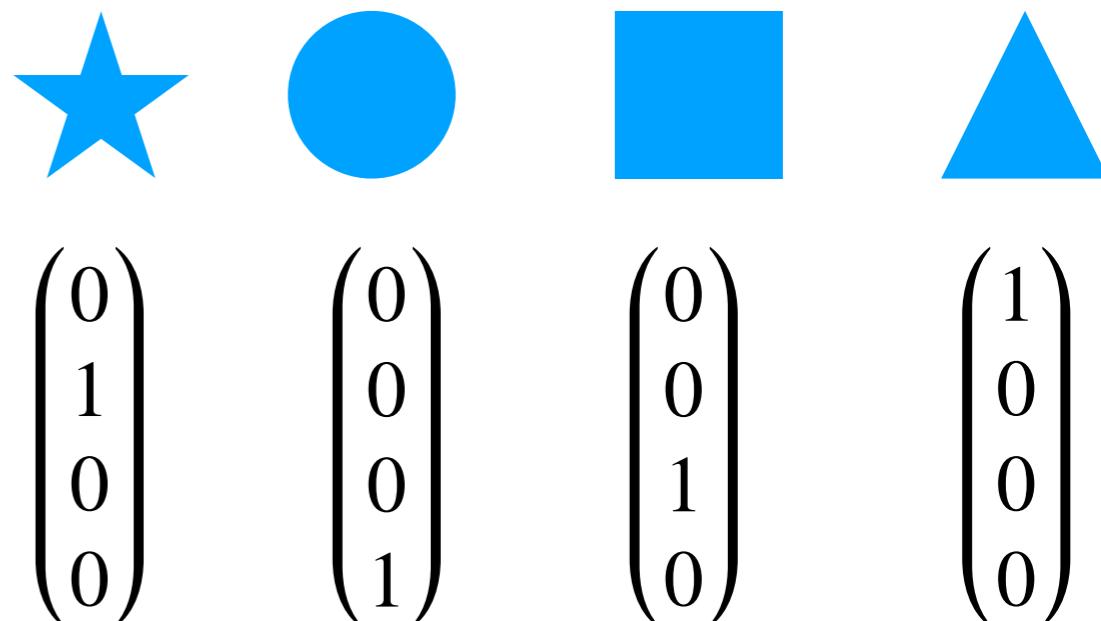
- Design a **metric**  $d(\cdot, \cdot)$  to evaluate the “distance” of two words in terms of some aspect
  - E.g., semantic similarity

*I'd like to have some pop/soda/water/fruit/rest*

- Traditional method: WordNet distance (if it's a metric).



If not, doesn't matter.





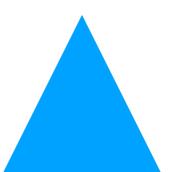
# Metric in the Word Space

- Design a **metric**  $d(\cdot, \cdot)$  to evaluate the “distance” of two words in terms of some aspect
  - E.g., semantic similarity

*I'd like to have some pop/soda/water/fruit/rest*

- A straightforward metric on one-hot vector:
  - Discrete metric

$$d(x_i, x_j) = 1 \text{ if } x_i = x_j, 0 \text{ otherwise}$$



**Non-informative**

$$\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

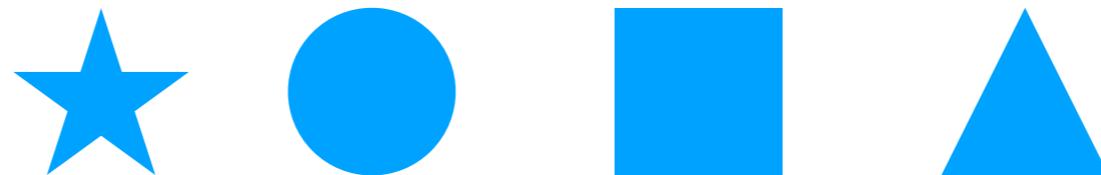
$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$



# ID and One-Hot



**ID representation**

1    3    2    0

**One-hot representation**

$$\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

**Dimension**

One-dimensional

$|\mathcal{V}|$ -dimensional

**Metric**

**Euclidean**

Artefact

Non-informative

**Discrete**

Non-informative

Non-informative

**Learnable**

Difficult

Possible but may not generalize  
Need to explore more



# Something in Between

- Map a word to a low-dimensional space
  - Not as low as one-dimensional ID representation
  - Not as high as  $|\mathcal{V}|$ -dimensional one-hot representation
- **Attempt#3:** Word vector representation (a.k.a., word embeddings)
  - Mapping a word to a vector
  - Equivalent to linear transformation of one-hot vector

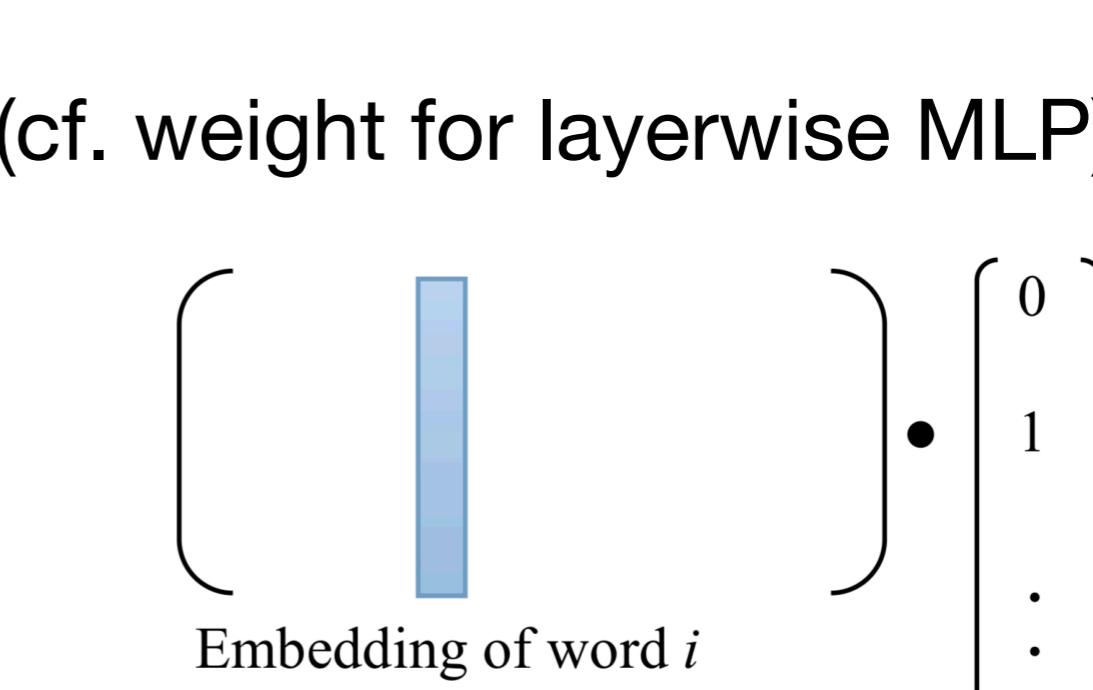
$$\left( \begin{array}{c} \text{Blue vertical bar} \\ \vdots \end{array} \right) \cdot \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}$$

Embedding of word  $i$   
retrieved by matrix-vector  
multiplication

One-hot representation of  
word  $i$  (sparse)



# Obtaining the Embedding Matrix

- Attemp#1: Treat as neural weights as usual
    - Random initialization & gradient descent
  - Properties of the embedding matrix
    - Huge,  $|\mathcal{V}| \times d_{NN}$  parameters (cf. weight for layerwise MLP)
    - Sparsely updated
  - Nature of language
    - Power law distribution
  - Good if corpus is large

The diagram illustrates the retrieval of a word embedding. It shows a vertical blue bar representing the "Embedding of word  $i$ " positioned between two curly braces. The left brace spans the width of the blue bar, and the right brace spans the width of the vector on the right. A dot product symbol ( $\cdot$ ) is placed between the blue bar and the vector. To the right of the vector is a vertical ellipsis, indicating it continues downwards. The vector itself is a column with entries 0, 1,  $\vdots$ , and 0, representing a sparse vector where only the  $i$ -th entry is non-zero.

$$\begin{pmatrix} & \text{---} \\ & \text{---} \\ & \text{---} \\ & \text{---} \\ & \text{---} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ \vdots \\ \vdots \\ 0 \end{pmatrix}$$

Embedding of word  $i$   
retrieved by matrix-vector  
multiplication

One-hot representation of  
word  $i$  (sparse)

# Embedding Learning

- **Attempt #2:**
  - Manually specifying the distance metric/inner-product, etc.
  - Humans are not rational
- **Attempt #3:**
  - Pre-training on a massive corpus with a different (pre-training) objective
  - Then, we can fine-tune those pre-trained embeddings in almost any specific task.

# Pretraining Criterion

- Language Modeling
  - Given a corpus  $\mathbf{x} = x_1 x_2 \cdots x_t$
  - Goal: Maximize  $p(\mathbf{x})$
- Is it meaningful to view language sentences as a random variable?
  - Frequentist: Sentences are repetitions of i.i.d. experiments
  - Bayesian: Everything unknown is a random variable

# Factorization

- $p(\mathbf{x}) = p(x_1, \dots, x_t)$  cannot be parametrized
- Factorizing a giant probability

$$\begin{aligned} p(\mathbf{x}) &= p(x_1, \dots, x_t) \\ &= p(x_1)p(x_2 | x_1)\cdots p(x_t | x_1, \dots, x_{t-1}) \end{aligned}$$

- Still unable to parametrize, especially  $p(x_n | x_1, \dots, x_{n-1})$
- **Questions:**
  - Can we decompose any probabilistic distribution defined on  $\mathbf{x}$  into this form? Yes.
  - Is it necessary to decompose the distribution a probabilistic distribution in this form? No.



# Markov Assumptions

$$\begin{aligned} p(\mathbf{x}) &= p(x_1, \dots, x_t) \\ &= p(x_1)p(x_2 | x_1) \cdots p(x_t | x_1, \dots, x_{t-1}) \end{aligned}$$

- Independence
  - Given the current “state,” independent with previous ones
  - State at step  $t$ :  $(x_{t-n+1}, x_{t-n+2}, \dots, x_{t-1})$
  - $x_t \perp x_{\leq t-n} | x_{t-n+1}, x_{t-n+2}, \dots, x_{t-1}$
- Stationary property
  - $p(x_{\textcolor{red}{t}} | x_{t-1}, \dots, x_{t-n+1}) = p(x_{\textcolor{red}{s}} | x_{s-n+1}, \dots, x_{s-1})$  for all  $\textcolor{red}{t}, \textcolor{red}{s}$

# Parametrizing $p(\mathbf{w})$

$$\begin{aligned} p(\mathbf{x}) &= p(x_1, \dots, x_t) \\ &= p(x_1)p(x_2 | x_1) \cdots p(x_t | x_1, \dots, x_{n-1}) \\ &\approx p(x_1)p(x_2 | x_1) \cdots p(x_n | x_1, \dots, x_{t-n+1}) \end{aligned}$$

**Direct parametrization:**

Each multinomial distribution is directly parametrized

$$p(w_n | w_1, \dots, w_{n-1}) \quad (\text{notation abuse})$$

# N-gram Model

$$\begin{aligned} p(\mathbf{x}) &= p(x_1, \dots, x_n) \\ &= p(x_1)p(x_2 | x_1) \cdots p(x_n | x_1, \dots, x_{n-1}) \\ &\approx p(x_1)p(x_2 | x_1) \cdots p(x_n | x_1, \dots, x_{t-n+1}) \end{aligned}$$

$$\hat{p}(w_n | w_1, \dots, w_{n-1}) = \frac{\#w_1 \cdots w_n}{\#w_1 \cdots w_{n-1}}$$

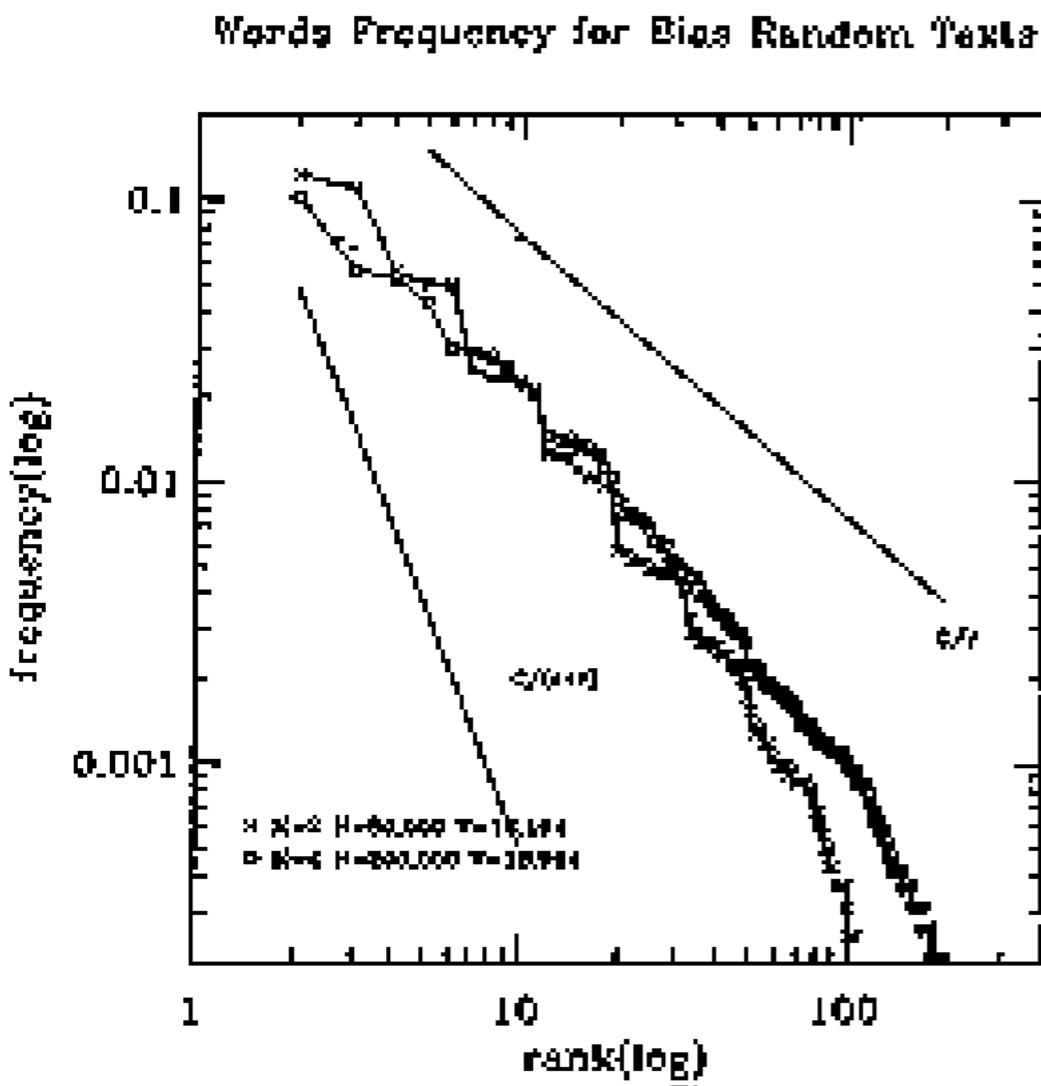
## Questions:

- How many multinomial distributions?
- How many parameters in total?



# Problems of n-gram models

- $\#\text{para} \propto \exp(n)$
- Power-law distribution
  - Severe data sparsity even if  $n$  is small



- Normal distribution
$$p(x) \propto \exp(-\tau x^2)$$
- Power-law distribution
$$p(x) \propto x^{-k}$$

# Smoothing Techniques

- Add-one smoothing
- Interpolation smoothing
- Backoff smoothing

Useful link: <https://nlp.stanford.edu/~wcmac/papers/20050421-smoothing-tutorial.pdf>

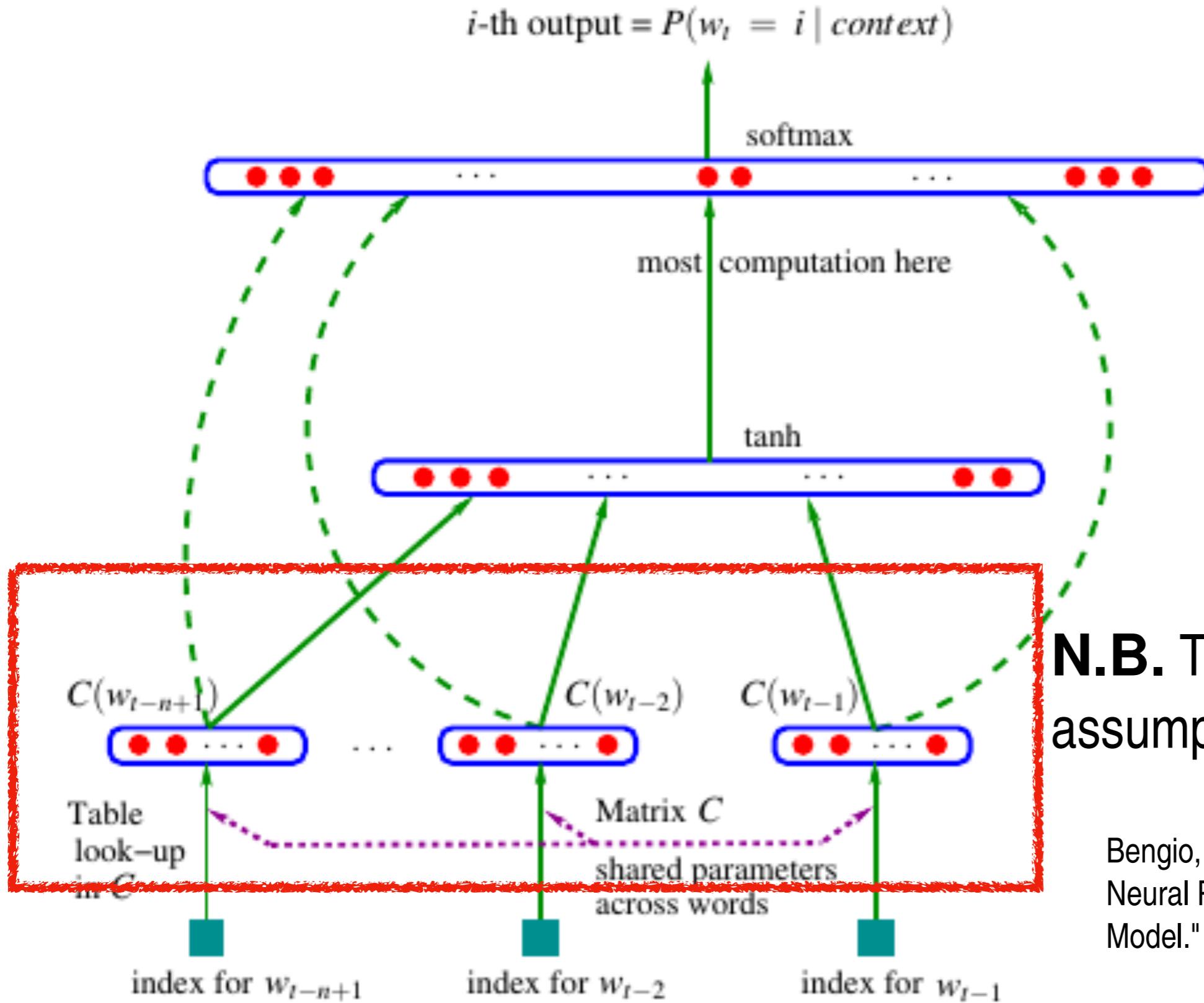


# Parametrizing LM by NN

- Is it possible to parametrize LM by NN?
- Yes
  - $p(w_n | w_1, \dots, w_{n-1})$  is a classification problem
  - NNs are good at (esp. non-linear) classification



# Feed-Forward Language Model



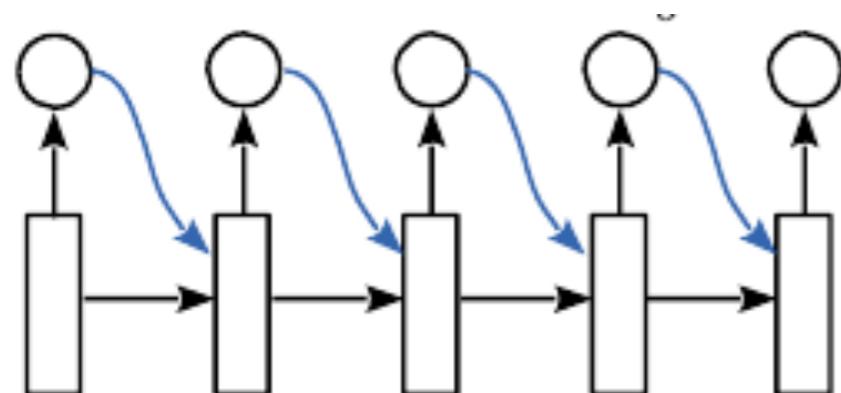
**N.B.** The Markov assumption also holds.

Bengio, Yoshua, et al. "A Neural Probabilistic Language Model." JMLR. 2003.

By product: Embeddings are pre-trained in a meaningful way

# Recurrent Neural Language Model

- RNN keeps one or a few hidden states
- The hidden states change at each time step according to the input



$$\begin{aligned} \mathbf{h}_t &= \text{RNN}(\mathbf{x}_t, \mathbf{h}_{t-1}) \\ &= f(W_{\text{in}} \mathbf{x}_t + W_{\text{hid}} \mathbf{h}_{t-1}) \\ p(w_t | \mathbf{w}_0^{t-1}) &\approx \text{softmax}(W_{\text{out}} \mathbf{h}_t) \end{aligned}$$

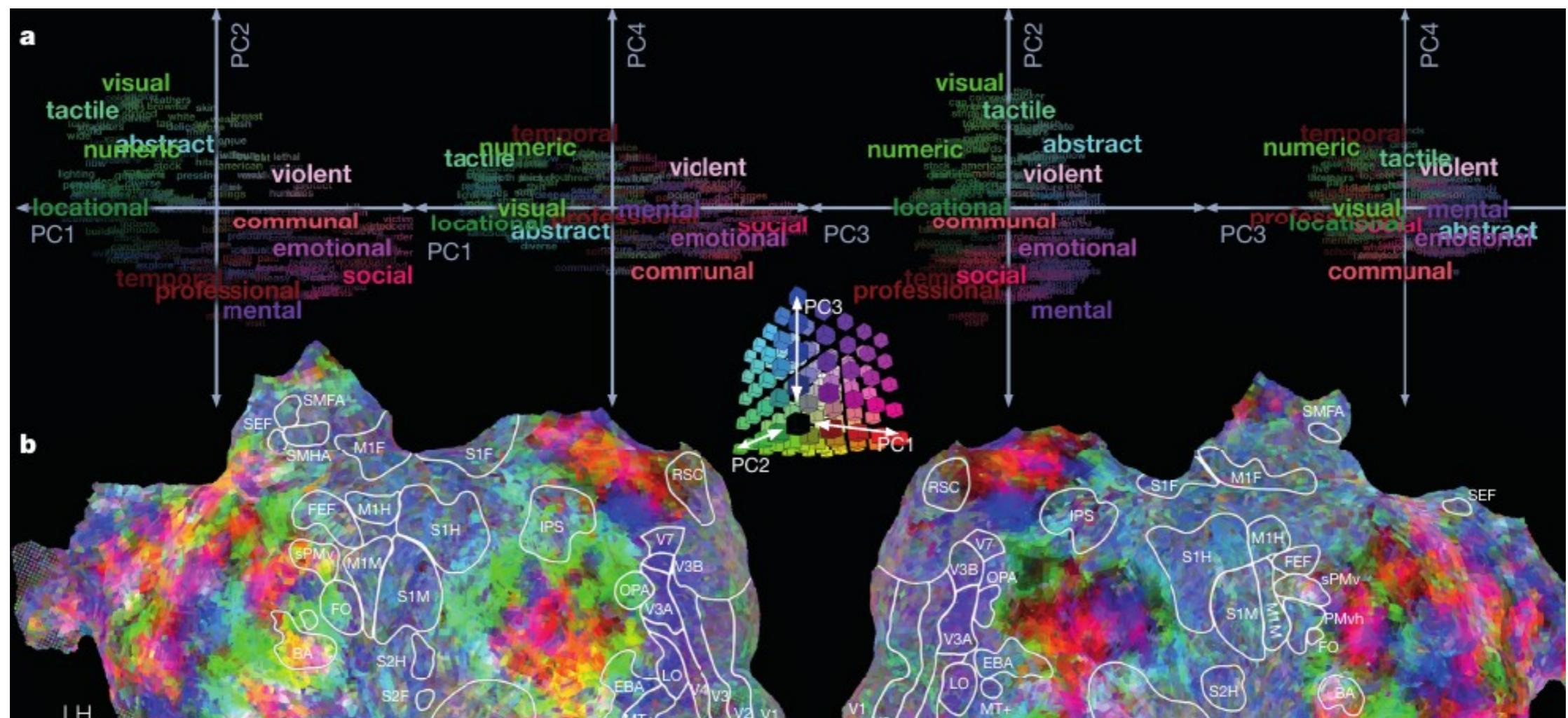
- RNN directly parametrizes  $p(\mathbf{w}) = \prod_{t=1}^m p(w_t | \mathbf{w}_1^{t-1})$   
rather than  $p(\mathbf{w}) \approx \prod_{t=1}^m p(w_t | \mathbf{w}_{t-n+1}^{t-1})$

Mikolov T, Karafiat M, Burget L, Cernocky J, Khudanpur S. Recurrent neural network based language model. In INTERSPEECH, 2010.

# How can we use word embeddings?

- Embeddings demonstrate the internal structures of words
  - Relation represented by vector offset
$$\text{"man"} - \text{"woman"} = \text{"king"} - \text{"queen"} \quad [\text{Mikolov+NAACL13}]$$
  - Word similarity
- Embeddings can serve as the initialization of almost every supervised task
  - A way of pretraining
  - **N.B.:** may not be useful when the training set is large enough

# Word Embeddings in our Brain

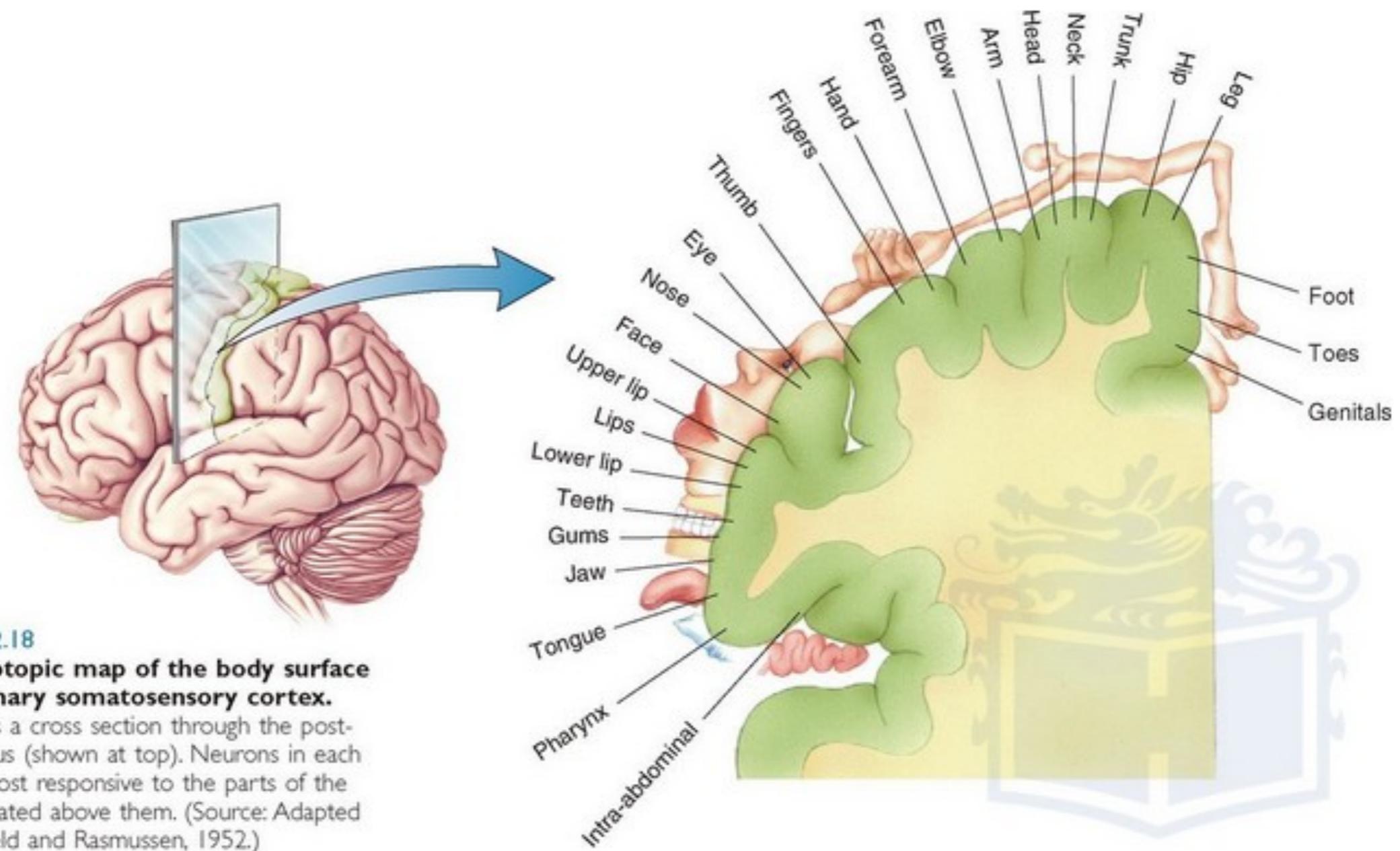


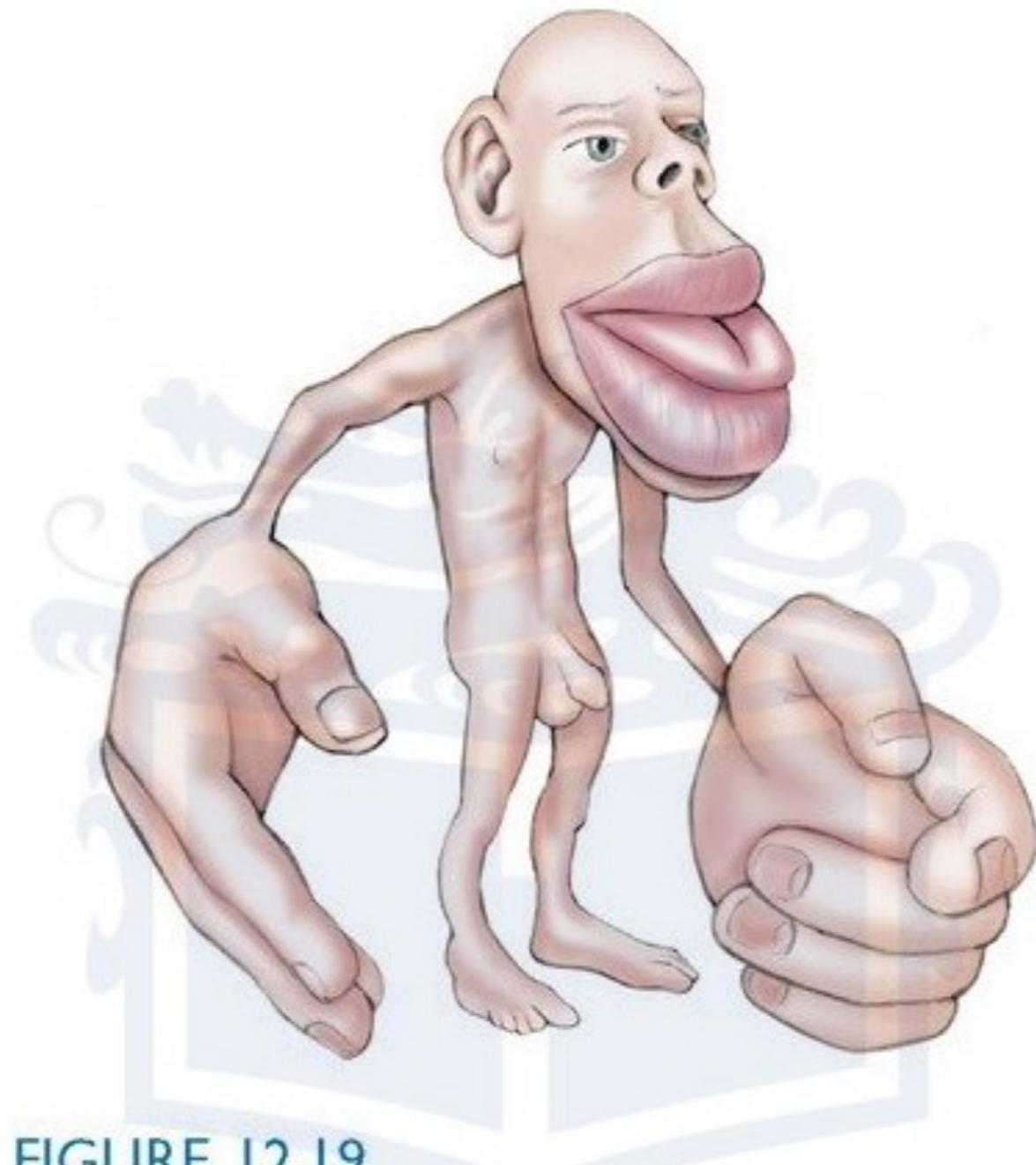
Huth, Alexander G., et al. "Natural speech reveals the semantic maps that tile human cerebral cortex." *Nature* 532.7600 (2016): 453-458.





# “Somatotopic Embeddings” in our Brain





**FIGURE 12.19**  
**The homunculus.**



# Complexity Concerns

- Time complexity
  - Hierarchical softmax [1]
  - Negative sampling: Hinge loss [2], Noisy contrastive estimation [3]
- Memory complexity
  - Compressing LM [4]
- Model complexity
  - Shallow neural networks are still too “deep.”
  - CBOW, SkipGram [3]

[1] Mnih A, Hinton GE. A scalable hierarchical distributed language model. NIPS, 2009.

[2] Collobert R, Weston J, Bottou L, Karlen M, Kavukcuoglu K, Kuksa P. Natural language processing (almost) from scratch. JMLR, 2011.

[3] Mikolov T, Chen K, Corrado G, Dean J. Efficient estimation of word representations in vector space. arXiv preprint arXiv:1301.3781. 2013

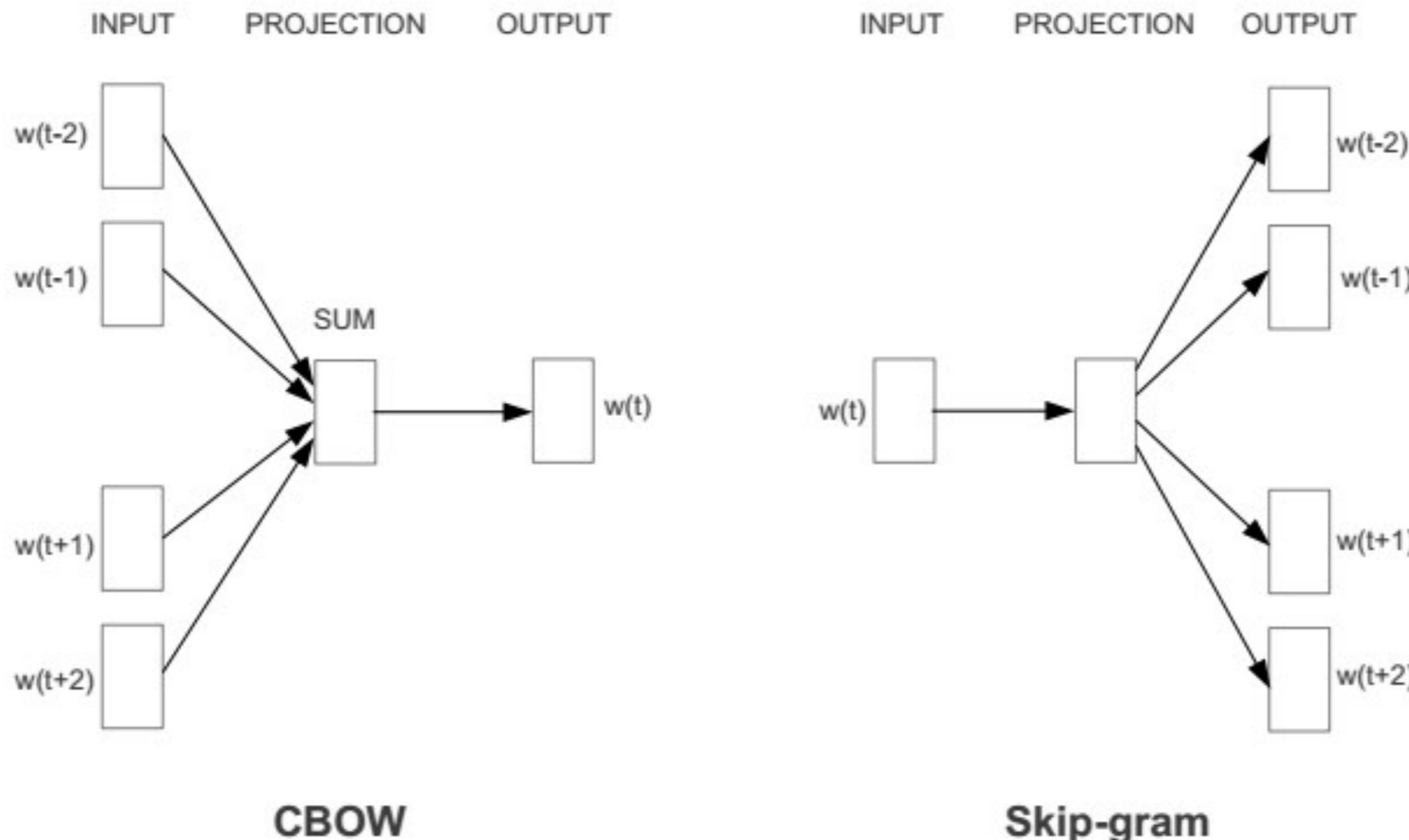
[4] Yunchuan Chen, Lili Mou, Yan Xu, Ge Li, Zhi Jin. "Compressing neural language models by sparse word representations." In ACL, 2016.

**Deep neural networks:  
To be, or not to be? That is the question.**



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# CBOW, SkipGram (word2vec)



Mikolov T, Chen K, Corrado G, Dean J. Efficient estimation of word representations in vector space. arXiv preprint arXiv:1301.3781. 2013



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# Hierarchical Softmax and Negative Contrastive Estimation

- HS

$$p(w|w_I) = \prod_{j=1}^{L(w)-1} \sigma \left( \llbracket n(w, j+1) = \text{ch}(n(w, j)) \rrbracket \cdot {v'_{n(w,j)}}^\top v_{w_I} \right)$$

- NCE

$$\log \sigma({v'_{w_O}}^\top v_{w_I}) + \sum_{i=1}^k \mathbb{E}_{w_i \sim P_n(w)} [\log \sigma(-{v'_{w_i}}^\top v_{w_I})]$$

Mikolov T, Chen K, Corrado G, Dean J. Efficient estimation of word representations in vector space. arXiv preprint arXiv:1301.3781. 2013



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# Tricks in Training Word Embeddings

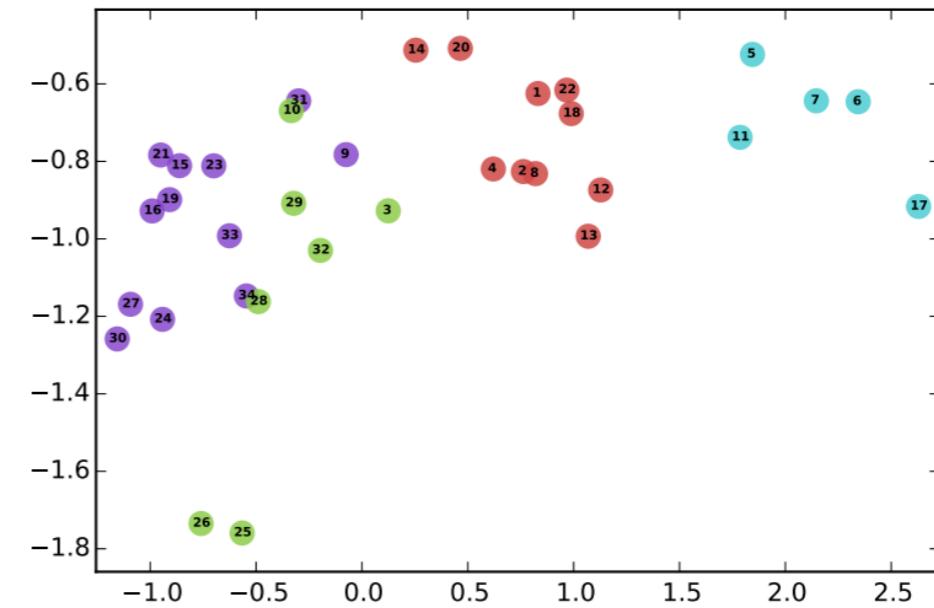
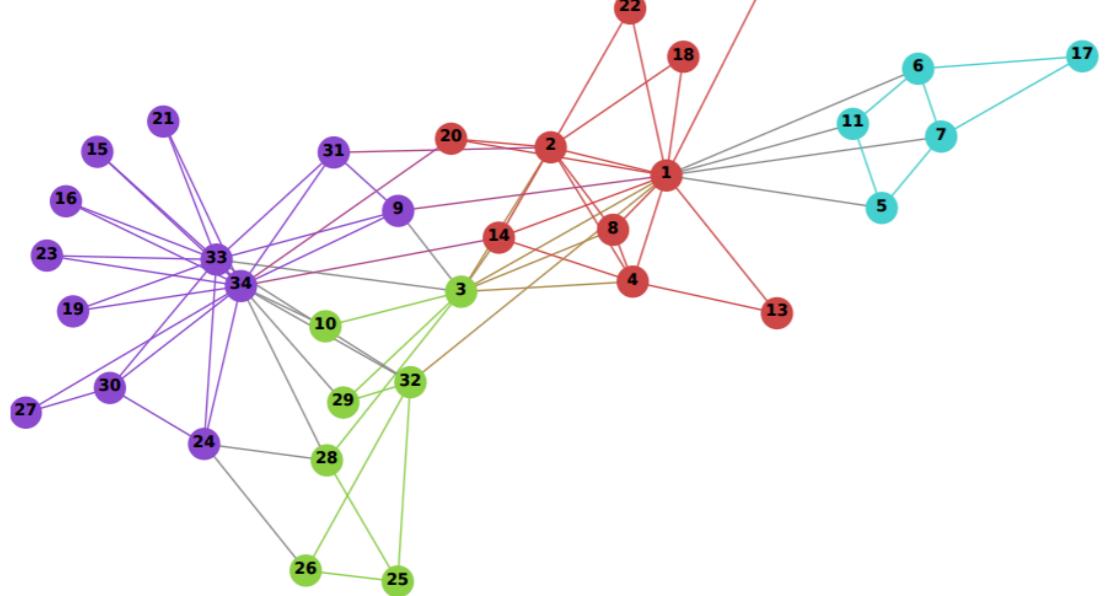
- The # of negative samples?
  - The more, the better.
- The distribution from which negative samples are generated? Should negative samples be close to positive samples?
  - The closer, the better.
- Full softmax vs. NCE vs. HS vs. hinge loss?

# Recent Advances in Pretraining

- Pretraining the embedding mapping for words is not enough
  - $E: \text{Vocabulary} \rightarrow \mathbb{R}^n$
- Context info?
  - Why not pre-train follow-up layers as well?
  - E.g., ELMo, BERT
  - Represent a word in a context, with LM-like pretraining
  - Factorization of  $p(\mathbf{w}) = p(w_1)p(w_2 | w_1)\cdots p(w_n | w_1\cdots w_{n-1})$  is unnecessary

# Learning Embeddings of Other Stuff

- Node embeddings of a network



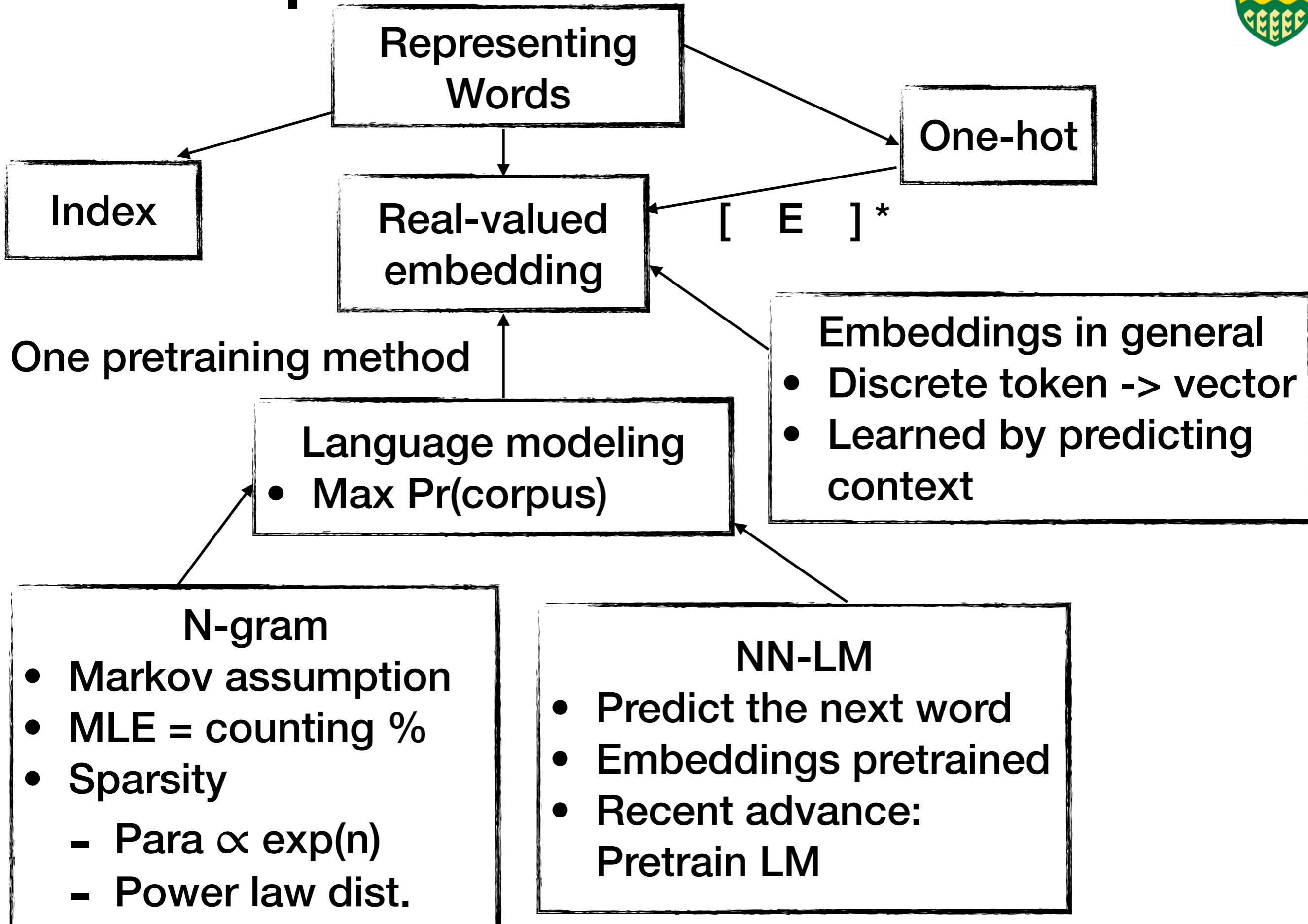
[DeepWalk, KDD 2014]

- General criteria of embedding learning
  - Atomic token represented by an embedding
  - Training embeddings by predicting “context”



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# Mindmap





# Suggested Reading

- **Neural LM:** Bengio, Yoshua, et al. "A Neural Probabilistic Language Model." JMLR. 2003.
- **word2vec:** Mikolov T, Chen K, Corrado G, Dean J. Efficient estimation of word representations in vector space. arXiv preprint arXiv:1301.3781. 2013
- **ELMo:** Peters, M.E., Neumann, M., Iyyer, M., Gardner, M., Clark, C., Lee, K. and Zettlemoyer, L., 2018. Deep contextualized word representations. In *NAACL*, 2018.
- **BERT:** Devlin, J., Chang, M.W., Lee, K. and Toutanova, K., 2018. Bert: Pre-training of deep bidirectional transformers for language understanding. In *NAACL*, 2019.
- **DeepWalk:** Perozzi, B., Al-Rfou, R. and Skiena, S. DeepWalk: Online learning of social representations. In *KDD*, 2014.



# More References

- Graves, A., Abdel-rahman M., and Geoffrey H. Speech recognition with deep recurrent neural networks. In *ICASSP*, 2013.
- Mikolov, T., Sutskever, I., Chen, K., Corrado, G. S., & Dean, J. (2013). Distributed representations of words and phrases and their compositionality. In *NIPS*, 2013.
- Li, W. Random texts exhibit Zipf's-law-like word frequency distribution. *IEEE Transactions on Information Theory*, 38(6), 1842-1845, 1992.
- Bengio, Yoshua, et al. A Neural Probabilistic Language Model. *JMLR*. 2003.
- Mikolov T, Karafiát M, Burget L, Cernocký J, Khudanpur S. Recurrent neural network based language model. In *INTERSPEECH*, 2010.
- Devlin, J., Chang, M.W., Lee, K. and Toutanova, K., 2018. Bert: Pre-training of deep bidirectional transformers for language understanding. In *NAACL*, 2019.
- Mikolov, T., Chen, K., Corrado, G. and Dean, J., 2013. Efficient estimation of word representations in vector space. arXiv preprint arXiv:1301.3781.
- Mikolov, T., Yih, W.T. and Zweig, G., June. Linguistic regularities in continuous space word representations. In *NAACL*, 2013.