Exam-style problem

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Assignment

Two containers are connected by pipes, as in the figure below. Container A holds 300 l and container B 100 l. As a starting point, there is 90 g of salt in container A and 30 g salt in container B. Through pipe a 30 l/min of pure water is added to A and through pipe b 15 l/min of a salt-water mixture is added to container A.
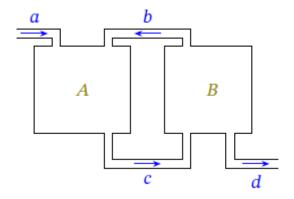
    a. How much mixture must pass through pipe c per minute if the amount of liquid in A is to be constant? How much must pass through pipe d if the amount of liquid in B is to be constant?

    b. Show that the amount of salt $y_0(t)$ in A and $y_1(t)$ in B satisfies the system

$$y_0'(t) = -0,15y_0(t) + 0,15y_1(t),$$
$$y_1'(t) = 0,15y_0(t) - 0,45y_1(t)$$

    In c) and d) you need to explain the concepts. This you will not need to do for the exam, but it will help you understand the concepts.

    c. Explain how the eigenvalues and eigenvectors of the coefficient matrix are calculated.

    d. Explain how the solutions to $y_0(t)$ og $y_1(t)$ can be determined by these eigenvalues and eigenvectors and the initial condition.

    e. Using Python, find $y_0(t)$ og $y_1(t)$, plot both functions against t. Also, plot $y_0(t)$ against $y_1(t)$. What is the limit of $y_1(t)/y_0(t)$ when $t \to \infty$?