Gør tanke til handling

VIA University College

# Probability theory 1

# Important definitions

In the following slides, three very central concepts from probability theory will be defined:

- Experiments
- Samples spaces
- Events

# Experiments

In probability theory, an **experiment** is a procedure that can be repeated any number of times in order to produce an outcome.

An experiment could for example be to role a dice, flip a coin three times, or draw a sample from a batch of 10 electronic devises for inspection.

# Sample space

We call the set of all possible outcomes of an experiment the **sample space** of that experiment.

heads

When you flip a coin, the sample space is $\{H, T\}$.

tails

When you role a dice, the sample space is $\{1,2,3,4,5,6\}$.

When you flip a coin two times, the sample space is $\{HH, HT, TH, TT\}$ (where e.g. $HT$ denotes the event of getting heads in the first flip and getting tails in the second flip).

# Event

An event is a subset of the sample space.

In the "role a die" - experiment, an event could be $A = \{1,2,3\}$.

The probability for this event is $P(A) = 0.5$.

# Examples

- What is the probability of getting "heads" when you flip a coin?

  The sample space is $\{\mathrm{H}, \mathrm{T}\}$.

  Since there are two possible outcomes, and we assume that they are equally likely, the probability for each must be $\frac{1}{2}$.

  Therefore, $P(\mathrm{H}) = \frac{1}{2}$.

- What is the probability of getting "4" when you role a dice?

  The sample space is $\{1,2,3,4,5,6\}$.

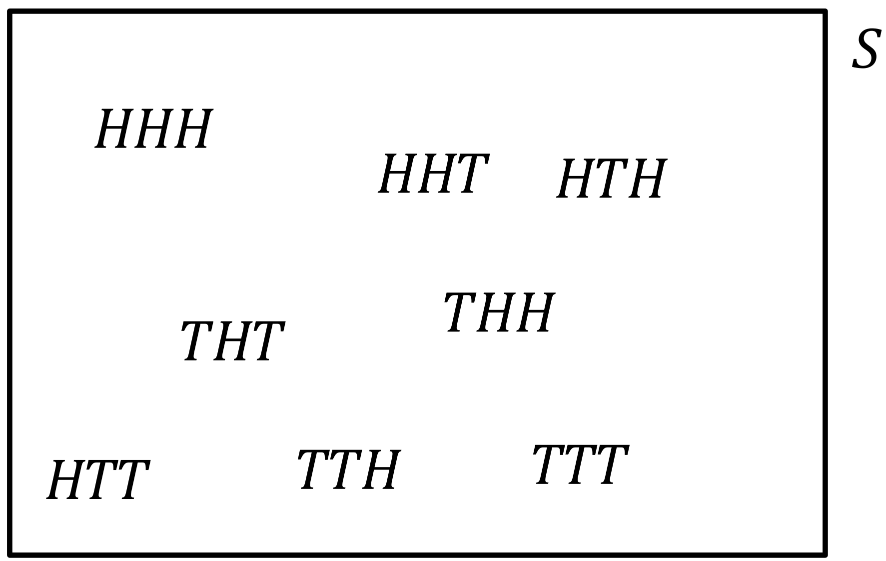
  In this case, there are 6 possible outcomes, and assuming they are equally likely, the probability for each must be $\frac{1}{6}$.

  Therefore, $P(4) = \frac{1}{6}$.

# Rules of probability

In the following slides, we will state and give examples of a set of useful rules and concepts for calculating probabilities.

In all cases, the rules are illustrated by considering the experiment consisting of flipping a coin three times, which has the sample space




where e.g. $THT$ denotes the event of getting tails first, then heads, and then tails again.

Most of the rules can easily be deduced from the Venn-diagram.

# The probability of an event

We can calculate the probability of an even $A$ in a sample space $S$ as

$$P(A) = \frac{|A|}{|S|}.$$

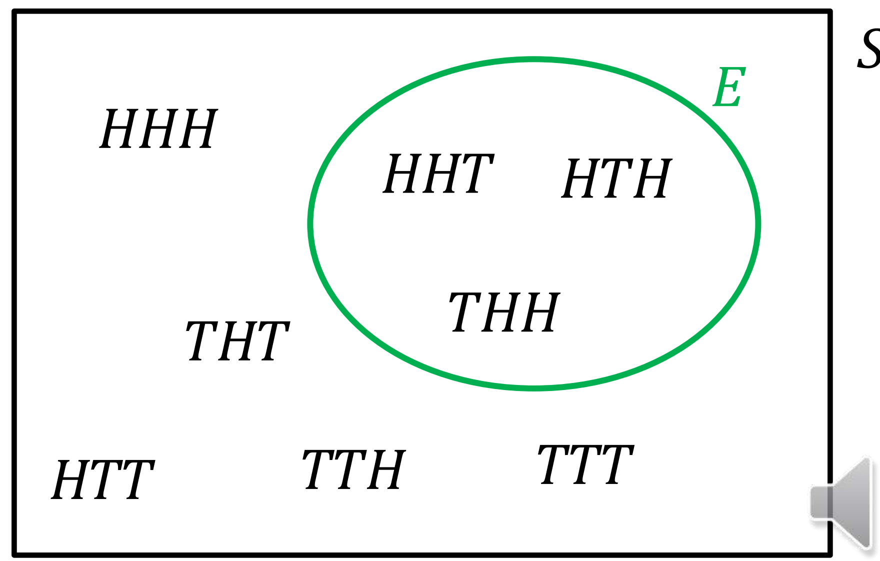
Size of the set $A$ (i.e. number of elements in $A$).

Size of the set $S$.

<u>Example:</u>

Consider the event $E$ corresponding to getting tail exactly once. In this case, $|E| = 3$ and $|S| = 8$ so

$$P(E) = \frac{|E|}{|S|} = \frac{3}{8}.$$



# Probability of an event – example 1

A woman has two children. What is the probability that she has two girls? (In general, we assume that the probability of getting a boy and getting a girl are both 50%).

Let $\mathrm{B}$ denote boy and $\mathrm{G}$ denote girl, and let e.g. $(\mathrm{G}, \mathrm{B})$ denote the event that her first child is a girl and her second child is a boy. The sample space is then $\{(\mathrm{B}, \mathrm{B}), (\mathrm{B}, \mathrm{G}), (\mathrm{G}, \mathrm{B}), (\mathrm{G}, \mathrm{G})\}$. Since there are four equally likely outcomes in the sample space, the probability for the event $(\mathrm{G}, \mathrm{G})$ is $\mathrm{P}((\mathrm{G}, \mathrm{G})) = \frac{1}{4}$.

# Probability of an event – example 2

What is the probability that a hacker guesses your 6-digit passwords with only lowercase letters in the first attempt?

Answer: Let $S$ be the sample space, which consists of all possible passwords. From combinatorics ("permutations with replacement") we know that $|S| = 26^6$. Let $A$ be the event consisting of the correct password – that means $|A| = 1$. The probability that the hacker guesses correctly is then $P(A) = \frac{|A|}{|S|} = \frac{1}{26^6} = 3.2 \cdot 10^{-9}$.

# Probability of complement

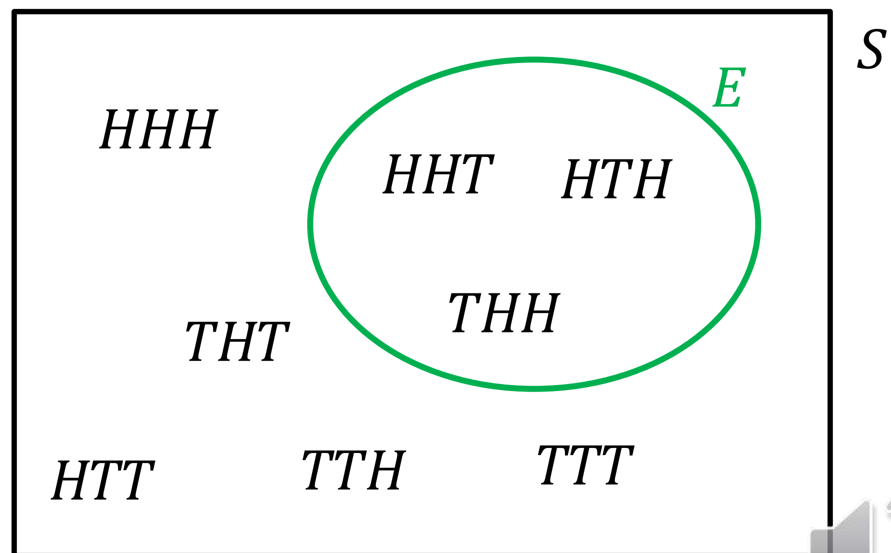
The probability for an event $A$ *not* happening is written as $P(A')$ ("the probability for not $A$") and can be calculated as

$$P(A') = 1 - P(A)$$

<u>Example:</u>

Consider again the event $E$ corresponding to getting tails exactly once. From the last slide we know that $P(E) = \frac{3}{8}$, so the probability of *not* getting tails exactly once is

$$P(E') = 1 - P(E) = \frac{5}{8}.$$



# Probability of complement - example

"When flipping a coin three times, what is the probability of getting heads one or more times?"

Let $A$ denote the event of getting heads one or more times. The easiest way to answer this question is to recognize that the *complement* of $A$ is getting tails three times in a row. The probability of this happing is $P(A') = \frac{1}{8}$, and therefore we can deduce that $P(A) = 1 - \frac{1}{8} = \frac{7}{8}$.

# Probability of union

The probability of the union of two events $A$ and $B$ is the probability of either $A$ happening, $B$ happening or both. It can be calculated as

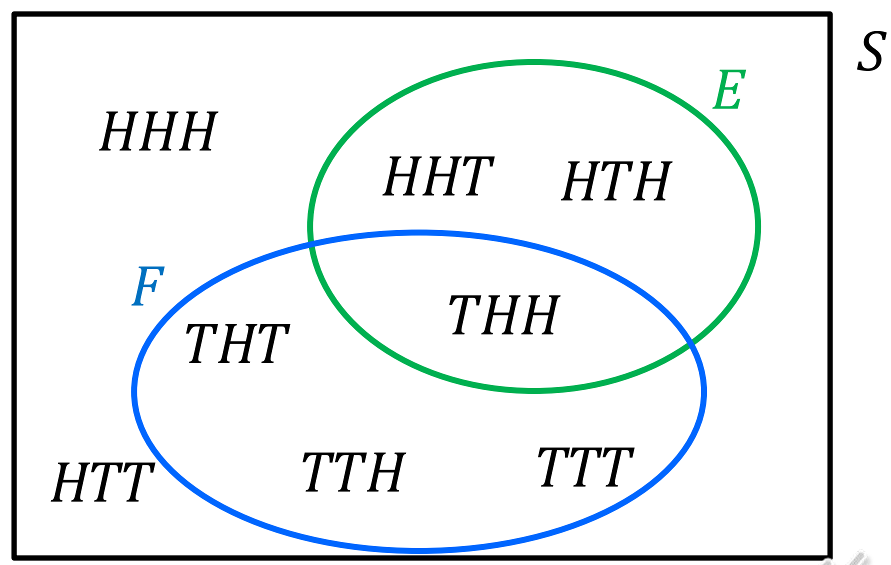
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

<u>Example:</u>

Consider the event $E$ from the last slide and the event $F$ corresponding to getting flip in the first flip. The probability of getting either tails exactly once *or* the getting tails in the first flip is *or* both is

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$= \frac{3}{8} + \frac{4}{8} - \frac{1}{8} = \frac{6}{8}.$$



# Probability of union - example

"A woman has two children. What is the probability that the oldest *or* the youngest is a girl?"

Let $A$ denote the event that the oldest child is a girl and let $B$ denote the event that the youngest child is a girl. We then has to determine $P(A \cup B)$. We know that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{2}$. The intersection between $A$ and $B$ is the event that she has two girls, and from earlier we know that this is $P(A \cap B) = \frac{1}{4}$. So we can use the rule from the last slide to calculate

$$P(A \cup \mathrm{B}) = P(A) + P(B) - P(A \cap B) = \frac{1}{2} + \frac{1}{2} - \frac{1}{4} = \frac{3}{4}.$$