Problems 10

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Exercise 1

Consider the Markov chain with three states, $S = \{1, 2, 3\}$, that has the following transition matrix

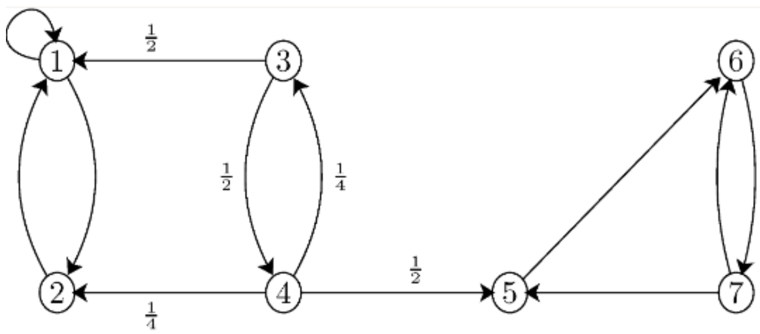
$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

a. Draw the state transition diagram for this chain.

b. If we know $P(X_1 = 1) = P(X_1 = 2) = \frac{1}{4}$, find $P(X_1 = 3, X_2 = 2, X_3 = 1)$. 1/12

Exercise 2

Consider the Markov chain in the figure below. There are two recurrent classes, $R_1 = \{1, 2\}$, and $R_2 = \{5, 6, 7\}$.

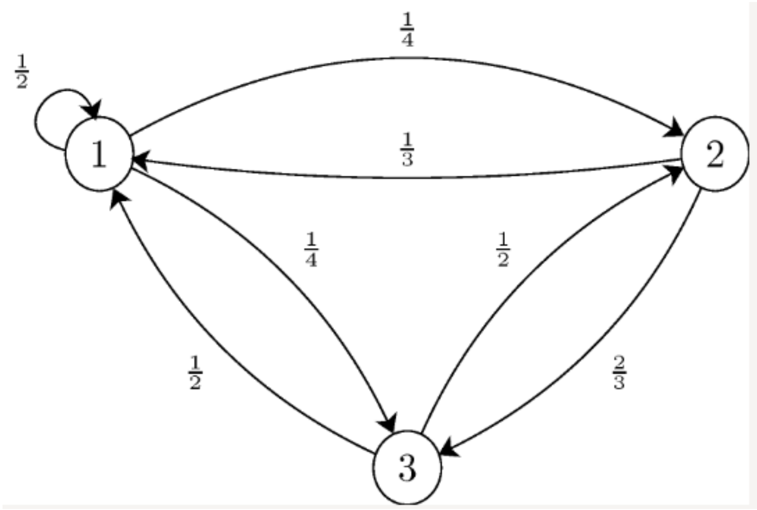


a. Assuming $X_0 = 3$, find the probability that the chain gets absorbed in R_1. $a_3 = \frac{5}{7}$

b. Find the expected time (number of steps) until the chain gets absorbed in R_1 or R_2. More specifically, let T be the absorption time, i.e., the first time the chain visits a state in R_1 or R_2, so find $E[T \mid X_0 = 3]$ $t_3 = \frac{12}{7}$

Exercise 3

Consider the following Markov chain



a. Is this chain irreducible? yes

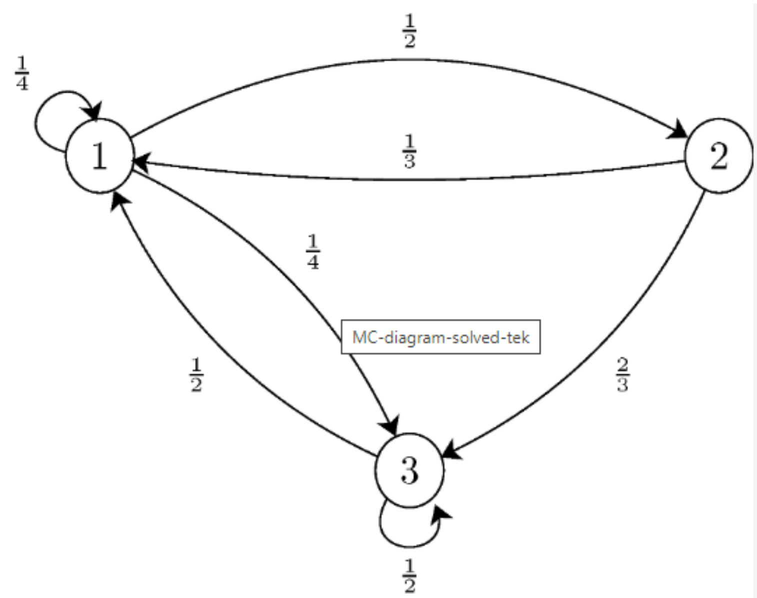
b. Is this chain aperiodic? yes

c. Find the stationary distribution for this chain. $\pi_1 \approx 0.457, \pi_2 \approx 0.257, \pi_3 \approx 0.286$

d. Is the stationary distribution a limiting distribution for the chain? yes

Exercise 4

Consider the following Markov chain



Assume $X_0 = 1$, and let R be the first time that the chain returns to state 1. Find $E[R \mid X_0 = 1]$. 8/3