Problems 6

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The first five exercises er from today's topics. Do these first. The rest is a bit of everything, including covariance and correlation which we did not do exercises in yet.

Exercise 1

ASPE: 8.1.7

Exercise 2

ASPE: 8.1.8

Exercise 3

ASPE: 8.2.10

Exercise 4

ASPE: 8.3.5

Exercise 5

ASPE: 8.4.1

Exercise 6

Let X be a continuous random variable with PDF

$$f_X(x) = \begin{cases} \frac{5}{32}x^4 & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

and let $Y = X^2$.

a. Find CDF of Y.

b. Find PDF of Y.

c. Find EY.

Exercise 7

Consider two random variables X and Y with joint PMF given in the Table

	$Y=0$	$Y=1$	$Y=2$
$X=0$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{8}$
$X=1$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{6}$

Find $\mathrm{Cov}(X,Y)$ and $\rho(X,Y)$

Exercise 8

Let X and Y be two independent $N(0,1)$ random variables and

$$Z = 11 - X + X^2 Y$$
$$W = 3 - Y.$$

Find $\text{Cov}(Z, W)$.

Exercise 9

Let X be a continuous random variable with PDF

$$f_X(x) = \begin{cases} x^2 + \frac{2}{3} & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

a. Find $E(X^n)$, for $n = 1, 2, 3, \cdots$. Note, you must find an expression involving n.

b. Find variance of X.

Exercise 10

Challenge Exercise.

The following exercise comes from the 2020 exam in the 10 ECTS course in Probability Theory and Stochastic Modelling at Aarhus University taken by students of Math, Data Science and Computer Science (i.e. not the course taken by Engineers that is similar to SMP). That is, this exercise is one level above the level of the SMP course. The assignment had a weighting of 30% of the total exam which is a 4 hour written exam. These students are, however, expected to state all relevant calculations (i.e. not just use solve or sp.integrate). Naturally, you should use these tool. But if you are able to do this type of assignment, in about an hour or so, things are looking bright for you.

Let X denote the stochastic variable with the following PDF:

$$f_X(x) = \begin{cases} \frac{3}{8}x^2 & 0 < x < 2 \\ 0 & \text{otherwise.} \end{cases}$$

a. Find $P(X > 3/4)$ and $P(X < 3/4)$.

b. Find $E[(X+2)/3]$ and $E\left[X^2\right]$

c. Let $Z = e^{2X}$ and find the probability density function of Z for all $z \in \mathbb{R}$.

Let Y denote the stochastic variable that is independent to X and has the distribution as X, i.e. Y has the following PDF:

$$f_Y(y) = \begin{cases} \frac{3}{8}y^2 & 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

d. Find $\text{Cov}(2X + 3Y, X - 4Y + 9)$.

e. Find $P(X \cdot Y < 1)$.