Recap and Exercise

Conditional Probability:

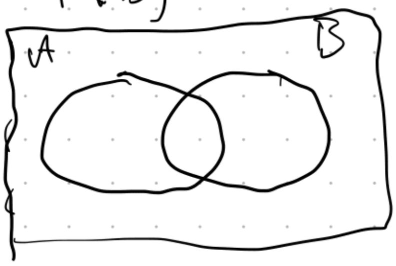
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A'|C) = 1 - P(A|C)$$

%. $P(A|C') = 1 - P(A|C)$ %.

$$P(A|B') = \frac{P(A \cap B')}{P(B')} = \frac{P(A \cap B')}{1 - P(B)}$$

$$= \frac{P(A) - P(A \cap B)}{1 - P(B)}$$



Independence

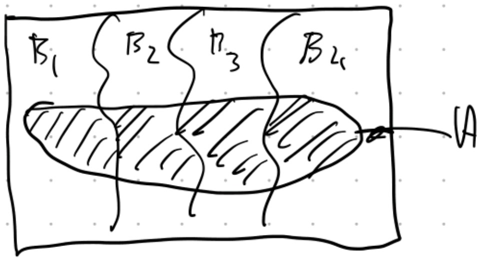
$$P(A) = P(A|B)$$

$$P(A \cap B) = P(A) \cdot P(B)$$

Law of total Probability

$$P(A) = \sum P(A \cap B_i)$$

$$= \sum P(A|B_i) \cdot P(B_i)$$



Bayes' Rule:

$$P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A)}$$

$$= \frac{P(A|B) \cdot P(B)}{P(A|B) \cdot P(B) + P(A|B') \cdot P(B')}$$

Example: False positive Paradox

- Disease affects 1 out of 10K
- Prob. of positive test given no disease is 0.02
- Prob of negative test given has disease is 0.01

$P(D) = \frac{1}{10{,}000}$ $\qquad P(T' \mid D) = 0.01$

$P(T \mid D') = 0.02$

$$P(D \mid T) = \frac{P(T \mid D) \cdot P(D)}{P(T \mid D) \cdot P(D) + P(T \mid D') \cdot P(D')}$$

$$= \frac{(1-0.01) \cdot 1/10000}{(1-0.01) \cdot 1/10000 + 0.02 \cdot (1 - 1/10000)}$$

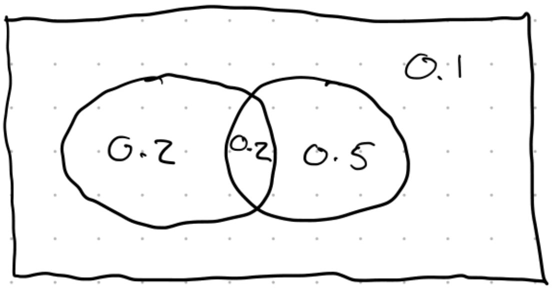
$$= 0.00492 \approx 0.492\%$$

Assignment 3 (10%)

The probability that a regularly scheduled flight departs on time is 0.81; the probability that it arrives on time is 0.80; and the probability that it departs and arrives on time is 0.76. Find the probability that a plane arrives on time, given that it did *not* depart on time

$P(D) = 0.81$, $P(A) = 0.8$, $P(D \cap A) = 0.76$

$$P(A \mid D') = \frac{P(A) - P(A \cap D)}{P(D')} = \frac{0.8 - 0.76}{0.19} = \underline{\underline{0.21}}$$



Exercise 4

Let A and B be two events such that:

$$P(A) = 0.4, \quad P(B) = 0.7, \quad P(A \cup B) = 0.9$$

a. Find $P(A \cap B)$.

b. Find $P(A^c \cap B)$.

c. Find $P(A - B)$.

d. Find $P(A^c - B)$.

e. Find $P(A^c \cup B)$.

f. Find $P(A \cap (B \cup A^c))$.

a. $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.4 + 0.7 - 0.9$

$= \underline{\underline{0.2}}$

b. $P(A^c \cap B) = P(B) - P(A \cap B)$

$= \underline{\underline{0.5}}$

c. $P(A - B) = 0.4 - 0.2 = \underline{\underline{0.2}}$

b' $P(A^c \cap B) = P(A \cup B) - P(A)$

$= 0.9 - 0.4 = 0.5$

d. $P(A' - B) = P(A') - P(A' \cap B)$

$= 0.6 - 0.5 = \underline{\underline{0.1}}$

e. $P(A' \cup B) = P(A') + P(B) - P(A' \cap B)$

$= 0.6 + 0.7 - 0.5 = \underline{\underline{0.8}}$

f. $P(A \cap (B \cup A^c)) = P(A \cap B) \cup P(A \cap A')$

$= P(A \cap B) = \underline{\underline{0.2}}$

Exercise 5

Consider a random experiment with a sample space.

$$S = \{1, 2, 3, \cdots\}.$$

Suppose that we know:

$$P(k) = P(\{k\}) = \frac{c}{3^k} \quad \text{for} \quad k = 1, 2, \cdots$$

where c is a constant number.

a. Find c.

b. Find $P(\{2, 4, 6\})$.

c. Find $P(\{3, 4, 5, \cdots\})$.

a. $1 = \sum \frac{c}{3^k} = c \sum \frac{1}{3^k} = c \sum \left(\frac{1}{3}\right)^k$

$1 = \frac{1/3}{1-1/3} = \frac{1/3}{2/3}$ $\Big\}$ $P(k) = \frac{2}{3^k}$

$\underline{c = 2}$

b. $P(\{2,4,6\}) = 2\left(\frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6}\right) = 2 \cdot \frac{3^4+3^2+1}{3^6} \approx \underline{0.25}$

c. $P(\{3,4,5,\dots\}) = 1 - P(X \le 2) = 1 - \left(\frac{2}{3^1} + \frac{2}{3^2}\right)$

$= 1 - 8/9 = \underline{1/9}$

Exercise 6

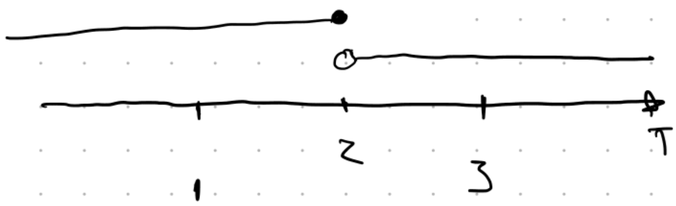
Let T be the time needed to complete a job at a certain factory. By using the historical data, we know that

$$P(T \le t) = \begin{cases} \frac{1}{16}t^2 & \text{for} \quad 0 \le t \le 4 \\ 1 & \text{for} \quad t > 4 \end{cases}$$

a. Find the probability that the job is completed in less than one hour, i.e., find $P(T \le 1)$.

b. Find the probability that the job needs more than 2 hours.

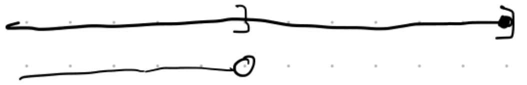
c. Find the probability that $1 \le T \le 3$.



a. $P(T \le 1) = \underline{\frac{1}{16}}$

b. $P(T > 2) = 1 - P(T \le 2)$

$= 1 - \frac{1}{16} \cdot 2^2 = 3/4$



c. $P(1 \le T \le 3) = P(T \le 3) - P(T < 1)$

$= \frac{1}{16} \cdot 3^2 - \frac{1}{16} \cdot 0^2 = \underline{\frac{9}{16}}$

$$\boxed{P(1 \le T \le 3) \qquad \int_1^3 \frac{1}{8} \cdot t \, dx = F(3) - F(1) = \frac{1}{16} \cdot 3^2 - \frac{1}{16} \cdot 1^2}$$

If T were continuous