

Problems 4

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Exercise 1

Let the simultaneous probability mass function (also called simultaneous probability density function or pdf) for the two discrete random variables X and Y be given by the table:

$y \backslash x$	1	2	3
5	$\frac{1}{12}$	0	0
6	$\frac{2}{12}$	0	$\frac{2}{12}$
7	$\frac{2}{12}$	$\frac{1}{12}$	$\frac{2}{12}$
8	0	$\frac{2}{12}$	0

- Find the marginal PMFs of X and Y .
- Find EX , EY , $E[XY]$
- Specify whether X and Y are independent
- Find $f_{X|Y}(x | y = 6)$

Exercise 2

Let the simultaneous probability mass function (also called simultaneous probability density function or pdf) for the two discrete random variables X and Y be given by the table:

$y \backslash x$	4	5	7
-3	k	0	0
-1	$\frac{2}{10}$	0	k
0	$\frac{1}{10}$	0	$\frac{4}{10}$
5	0	k	0

- What is the value of k ?
- What are the marginal PMFs?
- Find
 - $E[X] =$
 - $E[Y] =$
 - $E[Y * X] =$
 - $E[X^2] =$
 - $E[Y^2] =$
 - $P(Y < 0) =$
 - $P(X = 5, Y > 0) =$
 - $P(X < 6, Y < 0) =$
 - $Var(X) =$

Exercise 3

Consider the following PDF:

$$f_{Y|X}(y) = x \times e^{-xy} \quad \text{for } y > 0$$

Find $P(Y < 2 | X = 2)$ and $E(Y | X = 2)$

Exercise 4

Consider two random variables X and Y with joint PMF given by

$$P_{XY}(k, l) = \frac{1}{2^{k+l}}, \quad \text{for } k, l = 1, 2, 3, \dots$$

Find $P(X^2 + Y^2 \leq 10)$

Exercise 5

Let X and Y be two jointly continuous random variables with joint PDF

$$f_{XY}(x, y) = \begin{cases} \frac{1}{2}e^{-x} + \frac{cy}{(1+x)^2} & 0 \leq x, \quad 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- Find the constant c .
- Find $P(0 \leq X \leq 1, 0 \leq Y \leq \frac{1}{2})$.
- Find $P(0 \leq X \leq 1)$.

Exercise 6

Let X and Y be two jointly continuous random variables with joint PDF

$$f_{XY}(x, y) = \begin{cases} e^{-xy} & 1 \leq x \leq e, \quad y > 0 \\ 0 & \text{otherwise} \end{cases}$$

- Find the marginal PDFs, $f_X(x)$ and $f_Y(y)$.
- Write an integral to compute $P(0 \leq Y \leq 1, 1 \leq X \leq \sqrt{e})$.

Exercise 7

Let X and Y be two jointly continuous random variables with joint PDF

$$f_{XY}(x, y) = \begin{cases} \frac{1}{4}x^2 + \frac{1}{6}y & -1 \leq x \leq 1, \quad 0 \leq y \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- Find the marginal PDFs, $f_X(x)$ and $f_Y(y)$.
- Find $P(X > 0, Y < 1)$.
- Find $P(X > 0 \text{ or } Y < 1)$.
- Find $P(X > 0 | Y < 1)$.
- Find $P(X + Y > 0)$.