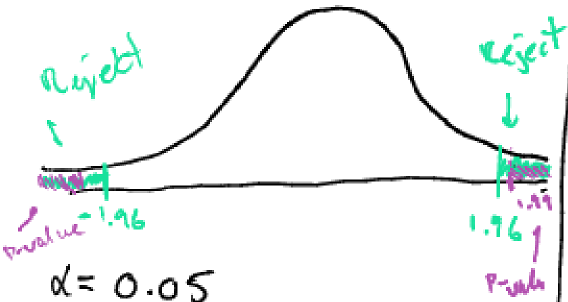
Example: $\bar{x} = 998$, $n = 30$, $\sigma = 5.5$:

Two-tailed test

H_0: $\mu = 1000$

H_1: $\mu \neq 1000$



$\alpha = 0.05$

Critical value:

$z_{1-\alpha/2} = 1.96 = z_{crit}$

Right-tailed

$\mu \leq 1000$

$\mu > 1000$

$\alpha = 0.05$

C.V: $z_{1-\alpha} = 1.64$

Left tailed

$\mu \geq 1000$

$\mu < 1000$

$\alpha = 0.05$

$C.V = z_\alpha = -1.64$

Test statistic:

$$z_{test} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{998 - 1000}{5.5/\sqrt{30}} = -1.99$$ } This is what differs between tests.

$|z| = 1.99 \rightarrow$ lies in rejection region

Rejection Criteria:

$|z_{crit}| < |z_{test}|$ reject

else fail to reject (accept).

P-Value:

The probability of obtaining an observation as extreme or more than the computed estimate (e.g. z), given that the null hypothesis is true (and based on evidence:

$$P(X_i > \bar{x} \mid H_0 \text{ is true})$$

So the p-value is a probability:

$$\begin{aligned} \text{P-value} &= 2\cdot(1-P(Z<|Z_{test}|) \\ &= 2\cdot(1-P(Z<|-1.99|) \\ &= 2\cdot(1-P(Z<1.99) = 0.047 \end{aligned}$$

Rejection Criteria:

p-value < α reject

- Is relative to problem/researcher
- 0.05 is the standard level of significance.

Types of Tests:

1) Independent samples : Test difference in two groups - often the mean.

2) A paired sample : Test compares the same group at different times - used to test change.

3) One sample test : Test a single group against a hypothesised parameter (μ, σ, p)

4) Two variables : Test whether there is independence between two things

What can you test:

One sample:

- Mean → t-test or z-test
- Proportion → z-test
- Variance → F-test / χ^2-test * taken out

Two samples:

- Compare means of distinct groups
- — " —— of same group at different times
- Compare two proportions
- Compare two std./variances.

Two Variables:

- Compare independence between two categorical variables

Two independent samples:

Hyp_1: Females are more intelligent than males. ✓

Hyp_2: Males are more intelligent than females.

Hyp_3: Males and females differ in intelligence

Hyp_1:	Hyp_2:	Hyp_3:
$H_0: \mu_f \leq \mu_m$ false ↓	$H_0: \mu_f \geq \mu_m$	$H_0: \mu_f = \mu_m$
$H_1: \mu_f > \mu_m$ True	$H_1: \mu_f < \mu_m$	$H_1: \mu_f \neq \mu_m$

Notice:

$H_0: \mu_f = \mu_m \iff \mu_f - \mu_m = 0$

↳ If things are equal their diff is 0

From book:

$\mu_f - \mu_m = D$, D is the difference
D = 0 means no diff.

$\mu_f - \mu_m = 10$

$\begin{cases} \mu_f \leq \mu_m \rightarrow \mu_f - \mu_m \leq 0 \\ \mu_f > \mu_m \rightarrow \underline{\mu_f - \mu_m} > 0 \end{cases}$ D is positive

$\begin{cases} \mu_f \geq \mu_m \rightarrow \mu_f - \mu_m \geq 0 \\ \mu_f < \mu_m \rightarrow \underline{\mu_f - \mu_m} < 0 \end{cases}$ → D is negative

Paired test

	w_1	w_2	Diff
Peter	78	76	2
Frida	72	72	0
Alice	79	70	9
Bob	84	85	-1

Is mean difference:

$\mu_D \neq 0$ weight change

$\mu_D > 0 \rightarrow$ weight loss

$\mu_D < 0 \rightarrow$ Weight gain

From Book:

$\bar{D}$ is mean difference in samples

Δ_0 is hypothesised difference in pop.

Test for Independence:

Discrete PDF:

$P(A \cap B) = P(A) \cdot P(B)$
$\downarrow$
Independent.

	$y=0$	$y=1$	$y=2$	Marginal pdf
$x=0$	a_{11}			$f_{x=0}$
$x=1$	a_{12}			$f_{x=1}$
$x=2$				$f_{x=2}$
Marginal	$f_{y=0}$	$f_{y=1}$	$f_{y=2}$	

$a_{11} = f(x=0) \cdot f(y=0)$

$a_{12} = f(x=1) \cdot f(y=0)$

$\vdots$

If I sum all these, what will I get?

If x and y are independent

I expect $a_{11} = f(x=0) \cdot f(y=0)$

H_0: X and Y are independent
H_1: X and Y are dependent

} always the hypotheses.

Davids Case:

Observed y

X	$y=B$	$y=R$	$y=G$	sum
$x=1$	$46/400$ $O_{11}=0.12$	$82/400$	$72/400$	$200/400$
$x=2$	$42/400$	$38/400$	$20/400$	$100/400$
$x=3$	$52/400$	$40/400$	$8/400$	$100/400$
Sum	$140/400$	$160/400$	$100/400$	$\frac{400}{400}$

Real observed values from sample!

Observed values

O_{ij}

$O_{11} = 46$

Expected: y

X	B	R	G	
1	0.18 $7/40$	$1/5$	$1/8$	$1/2$
2		P_{ij}	.	$1/4$
3	-	.	.	$1/4$
sum	$7/20$	$2/5$	$1/4$	

If X and Y are independent what is E_{ij}?

e.g. O_{11}

This table gives me all the expected values under the assumption that H_0 is true.

10.40