

## Problems 4

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## Exercise 1

Let the simultaneous probability mass function (also called simultaneous probability density function or pdf) for the two discrete random variables  $X$  and  $Y$  be given by the table:

$y \backslash x$	1	2	3
5	$\frac{1}{12}$	0	0
6	$\frac{2}{12}$	0	$\frac{2}{12}$
7	$\frac{2}{12}$	$\frac{1}{12}$	$\frac{2}{12}$
8	0	$\frac{2}{12}$	0

- Find the marginal PMFs of  $X$  and  $Y$ .
- Find  $EX$ ,  $EY$ ,  $E[XY]$
- Specify whether  $X$  and  $Y$  are independent
- Find  $f_{X|Y}(x | y = 6)$

## Exercise 2

Let the simultaneous probability mass function (also called simultaneous probability density function or pdf) for the two discrete random variables  $X$  and  $Y$  be given by the table:

$y \backslash x$	4	5	7
-3	$k$	0	0
-1	$\frac{2}{10}$	0	$k$
0	$\frac{1}{10}$	0	$\frac{4}{10}$
5	0	$k$	0

- What is the value of  $k$ ?
- What are the marginal PMFs?
- Find
  - $E[X] =$
  - $E[Y] =$
  - $E[Y * X] =$
  - $E[X^2] =$
  - $E[Y^2] =$
  - $P(Y < 0) =$
  - $P(X = 5, Y > 0) =$
  - $P(X < 6, Y < 0) =$
  - $Var(X) =$

### Exercise 3

Consider the following PDF:

$$f_{Y|X}(y) = x \times e^{-xy} \quad \text{for } y > 0$$

Find  $P(Y < 2 | X = 2)$  and  $E(Y | X = 2)$

### Exercise 4

Consider two random variables  $X$  and  $Y$  with joint PMF given by

$$P_{XY}(k, l) = \frac{1}{2^{k+l}}, \quad \text{for } k, l = 1, 2, 3, \dots$$

Find  $P(X^2 + Y^2 \leq 10)$

### Exercise 5

Let  $X$  and  $Y$  be two jointly continuous random variables with joint PDF

$$f_{XY}(x, y) = \begin{cases} \frac{1}{2}e^{-x} + \frac{cy}{(1+x)^2} & 0 \leq x, \quad 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- Find the constant  $c$ .
- Find  $P(0 \leq X \leq 1, 0 \leq Y \leq \frac{1}{2})$ .
- Find  $P(0 \leq X \leq 1)$ .

### Exercise 6

Let  $X$  and  $Y$  be two jointly continuous random variables with joint PDF

$$f_{XY}(x, y) = \begin{cases} e^{-xy} & 1 \leq x \leq e, \quad y > 0 \\ 0 & \text{otherwise} \end{cases}$$

- Find the marginal PDFs,  $f_X(x)$  and  $f_Y(y)$ .
- Write an integral to compute  $P(0 \leq Y \leq 1, 1 \leq X \leq \sqrt{e})$ .

### Exercise 7

Let  $X$  and  $Y$  be two jointly continuous random variables with joint PDF

$$f_{XY}(x, y) = \begin{cases} \frac{1}{4}x^2 + \frac{1}{6}y & -1 \leq x \leq 1, \quad 0 \leq y \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- Find the marginal PDFs,  $f_X(x)$  and  $f_Y(y)$ .
- Find  $P(X > 0, Y < 1)$ .
- Find  $P(X > 0 \text{ or } Y < 1)$ .
- Find  $P(X > 0 | Y < 1)$ .
- Find  $P(X + Y > 0)$ .