Hypothesis Testing

What is a hypothesis?

↳ A statement about the world that has a definite truth value:

Normative
- "It is wrong to kill another human being"
- "The prettiest city in Europe is Prague"

Descriptive
- "The Capital of Denmark is Horsens" F
- "The accused is guilty"
- "The accused is innocent"

A hypothesis can either be rejected or not rejected

A hypothesis can either be True or False

Decision	Actual state of affairs: Hypothesis is True	Actual state of affairs: Hypothesis is false
fail to reject (accept)	Correct True positive (TP) $(1-\alpha)$	Error (Type II error) False Negative (FN) β
Reject	Error (Type I error) False Positive (F.P.) α	Correct True Negative (TN) $1-\beta$ (Power)

Null hypothesis (H_0): Assumed true until disproved

Alternative (H_1) : True if H_0 is False

H_0: Person is innocent
H_1: Person is not innocent (= guilty)

H_0: Machine is adequately calibrated
H_1: Machine is not adequately calibrated

Must quantify our hypothesis

H_0: $\mu = 250$

H_1: $\mu \neq 250$

What can we test:

- Mean
- Independence
- Proportion
- Variance/St.Dev.
- Distribution

$\left.\begin{array}{l}\text{Mean}\\ \text{Independence}\\ \text{Proportion}\end{array}\right\}$ Curriculum

One vs. Two samples

One vs. two tails

Examples

H_0: $\mu = 250$ $\quad$ H_0: $\mu \leq 250$ $\quad$ H_0: $\mu \geq 250$

H_1: $\mu \neq 250$ $\quad$ H_1: $\mu > 250$ $\quad$ H_1: $\mu < 250$

H_0: Smoking and lung cancer are independent

H_1: Smoking and lung cancer are not independent

H_0: $\mu_1 = \mu_2 \rightarrow \mu_1 - \mu_2 = 0$ $\quad$ OR $\quad$ $\mu_1 - \mu_2 \leq D$

H_1: $\mu_1 \neq \mu_2 \rightarrow \mu_1 - \mu_2 \neq 0$ $\quad\quad\quad$ $\mu_1 - \mu_2 > D$

H_0: $p \geq 0.7$ $\quad$ H_0: $p_1 - p_2 \geq 0.1$

H_1: $p < 0.7$ $\quad$ H_1: $p_1 - p_2 < 0.1$

Overall Method:

1. State H_0 and H_1 → Try to disprove H_0
2. Determine α
3. Calculate a ~~test~~ test statistic
4. Based on test statistic, Reject or fail to reject H_0
5. Conclude

You cannot prove H_0; only reject or fail to reject

Errors:

Which is worse?

$$\begin{array}{c|cc} & 1 & 0 \\ \hline 1 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$

"Defendent is innocent"

F.P. (Type I) — Innocent in jail

F.N. (Type II) — Guilty set free

"Patient has Ebola"

F.P. (Type I) — Well person given treatment

F.N. (Type II) — Ill person not treated.

"Patient has Corona"

F.P

F.N.

$P(\text{Reject } H_0 \mid H_0 \text{ is true}) = \alpha$

$P(\text{Not reject } H_0 \mid H_0 \text{ is false}) = \beta$

$1 - \alpha = \text{Confidence level}$

$= P(\text{Not reject } H_0 \mid H_0 \text{ is true})$

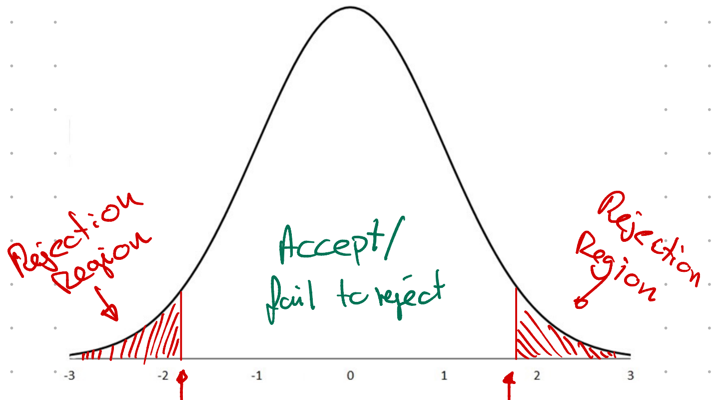
Example: CUP

1. $H_0: \mu = 250$

   $H_1: \mu \neq 250$

2. $\alpha = 0.05$

3. What is z-score associated with $\alpha = 0.05$ for two-tails: $\boxed{z_{\alpha/2} = 1.96}$



Critical Value   Critical Value

z_{crit}

4. Test statistic:

$$z_{test} = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{250.2 - 250}{2.5/\sqrt{25}} = 0.4$$

If $z_{crit} < 0$

Reject if $z_{test} < z_{crit}$

If $z_{crit} > 0$

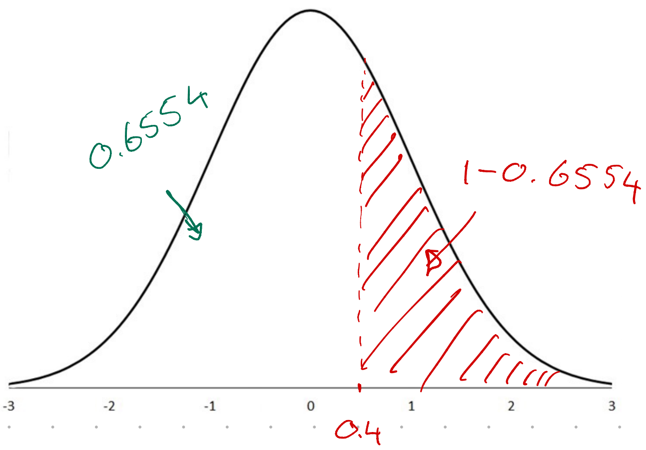
Reject if $z_{test} > z_{crit}$

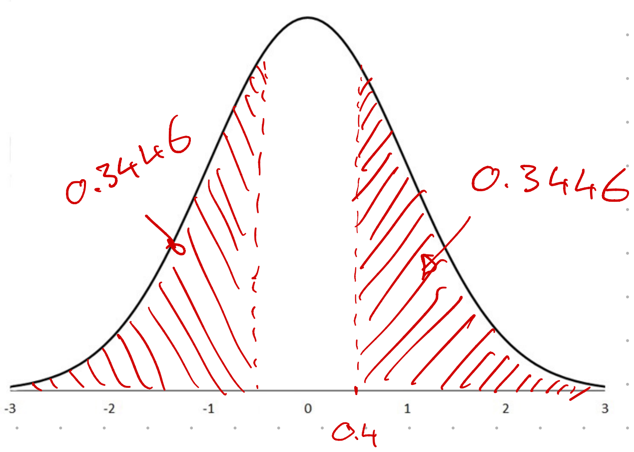
Since $z_{crit} > 0$ and $z_{test} < z_{crit}$ we fail to reject H_0.

5. We do not have sufficient evidence to support the claim that the machine does not fill 250 mL into the cup, and conclude that the cup is adequately filled.

Probability of Z_{test}:

$$\Phi(0.4) = \text{stats.norm.cdf}(0.4)$$
$$= 0.6554$$





$$1 - \Phi(0.4) = 0.3446$$

$2 \cdot (1 - \Phi(0.4)) = 0.6892$ } P-Value for two-tailed test

What is the p-value?

The probability of observing an observation more extreme given that H_0 is true.

$P(\text{Data observed} \mid H_0 \text{ is true})$

Two methods for hypothesis testing!

① Using critical value and ~~test~~ statistic like we did in example

② Finding the p-value from the test statistic and comparing it to α.

If p-value $< \alpha$
Reject