Main simulation loop

Here are discribed the main steps of the simulation loop in the order they are applied. Given the i-th simulated particle, let us define its position has $\mathbf{p}_i$, its velocity as $\mathbf{v}_i$ and its mass as m_i.

Update velocities Velocities are first updated based on external forces such as gravity :

$$\mathbf{v}_i := \mathbf{v}_i + \frac{dt}{m} F_{ext}(...)$$

Note that during debugging it is sometime useful to deactivate the gravity.

Compute expected positions Expected position $\tilde{\mathbf{p}}_i$ are then computed based on velocities:

$$\tilde{\mathbf{p}}_i := \mathbf{p}_i + dt\mathbf{v}_i$$

Expected position can lead to overlap between particles, those overlaps are resolved during the next step.

Manage constraints Contacts are managed using a constraint system (using equality and inequality constraints). Detected collisions are used to generate constraints. Once all constraints have been generated they are resolved one by one by updating the expected position $\tilde{\mathbf{p}}_i$ of the particles defining the constraint. This step is described in more detail in next section.

Update velocities and positions Finally, after the constraint have been resolved, the velocity is computed from the difference in position:

$$\mathbf{v}_i := \frac{1}{dt}(\tilde{\mathbf{p}}_i - \mathbf{p}_i)$$

and the particle position becomes its expected position.

$$\mathbf{p}_i := \tilde{\mathbf{p}}_i$$

Constraint management

For all constraints, the objective is to compute δp_i such that

$$\tilde{\mathbf{p}}_i := \tilde{\mathbf{p}}_i + \delta_i$$

verify the constraint.

Particle/Static ground collisions A constraint with a plane $(\mathbf{p}_c, \mathbf{n}_c)$ is detected if:

$$(\tilde{\mathbf{p}}_i - \mathbf{p}_c)^T \mathbf{n}_c - r_i < 0$$

It can then be resolved by using :

$$\delta_i = -C\mathbf{n}_c$$

where

$$C = (\tilde{\mathbf{p}}_i - \mathbf{q}_c)^T \mathbf{n}_c - r_i$$

and

$$\mathbf{q}_c = \tilde{\mathbf{p}}_i - (\tilde{\mathbf{p}}_i - \mathbf{p}_c)^T \mathbf{n}_c \mathbf{n}_c$$

is the point of the plane that is closest to the particle center.

Constraint with other signed distance fields can be linearized by evaluating the distance to the obstacle and the gradient of the distance to deduce the value of the closest point on the surface $\mathbf{p}_c$ and the associated normal $\mathbf{n}_c$.

For a sphere of center $\mathbf{c}_i$ and radius R_i, we have:

$$sdf = \|\tilde{\mathbf{p}}_i - \mathbf{c}_i\| - r_i$$

a collision occure if $sdf < 0$.

Then the linearized constraint is defined by

$$\mathbf{n}_c = \frac{\tilde{\mathbf{p}}_i - \mathbf{c}_i}{\|\tilde{\mathbf{p}}_i - \mathbf{c}_i\|}$$

and

$$\mathbf{p}_c = \tilde{\mathbf{p}}_i - sdf * \mathbf{n}_c$$

Particle/Particle collisions The constraint describing the collision between two particles i and j is generated if the distance bewteen the particle center is smaller than the sum of particle radius.

It can then be resolved by using :

$$\delta_i = -\sigma_i \frac{\mathbf{x}_{ji}}{\|\mathbf{x}_{ji}\|}$$

where

$$\mathbf{x}_{ji} = \tilde{\mathbf{p}}_i - \tilde{\mathbf{p}}_j$$

$$\sigma_i = \frac{1/m_i}{(1/m_i) + (1/m_j)} C$$

$$C = \|\mathbf{x}_{ji}\| - (r_i + r_j)$$

The computed displacement δ_i is only applied if $C < 0$.

Similarly we have:

$$\delta_j = \sigma_j \frac{\mathbf{x}_{ji}}{\|\mathbf{x}_{ji}\|}$$

with

$$\sigma_j = \frac{1/m_j}{(1/m_i) + (1/m_j)} C$$

Particle/Particle link This type of constraint is similar to a string and can be used to represent deformable solids. It is described by a target distance L between two particles and a stiffness parameter γ. The value of γ should be in the range $[0; 1]$, it describes how fast the constraint will be resolved, a value smaller than 1 allow to represent deformable solids.

The constraint is created once when creating the solid from a set of particles, then it remains active as long as the solid exist.

It can be resolved by using:

$$\delta_i = -\frac{1/m_i}{1/m_i + 1/m_j} \beta C \frac{\mathbf{x}_{ji}}{\|\mathbf{x}_{ji}\|}$$

where

$$\mathbf{x}_{ji} = \tilde{\mathbf{p}}_i - \tilde{\mathbf{p}}_j$$

$$C = \|\mathbf{x}_{ji}\| - L$$

$$\beta = 1 - (1 - \gamma)^{1/n}$$

with n the number of constraint resolution step during one simulation loop.

For the second particle, we have:

$$\delta_j = \frac{1/m_j}{1/m_i + 1/m_j} \beta C \frac{\mathbf{x}_{ji}}{\|\mathbf{x}_{ji}\|}$$