Control Systems Lab Presentation

Inverted Double Pendulum

TU Delft

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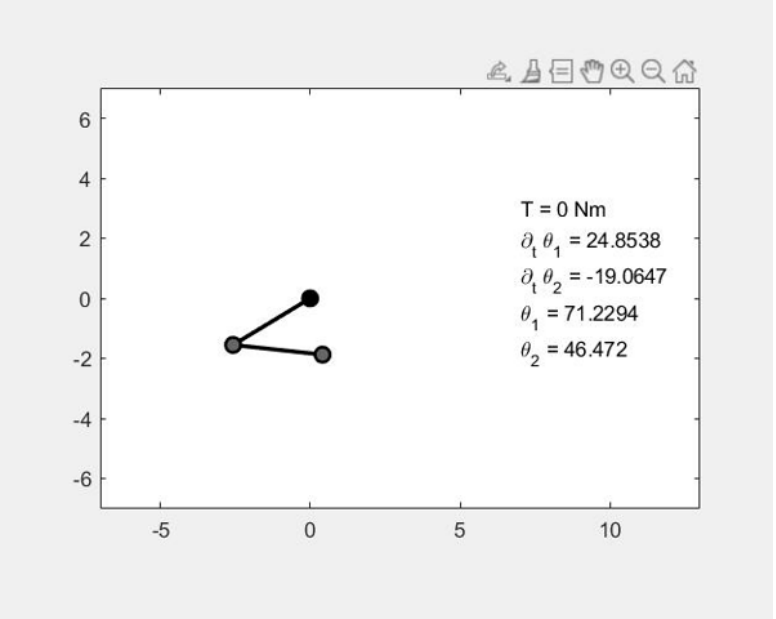
Outline

- Modelling
  - Full Non Linear Model
  - State Space Linearization
- Identification
  - Interfacing
  - Nonlinear White Box model
  - Linear Black box model
- Linear Control
  - LQR Regulator
  - PID Controller



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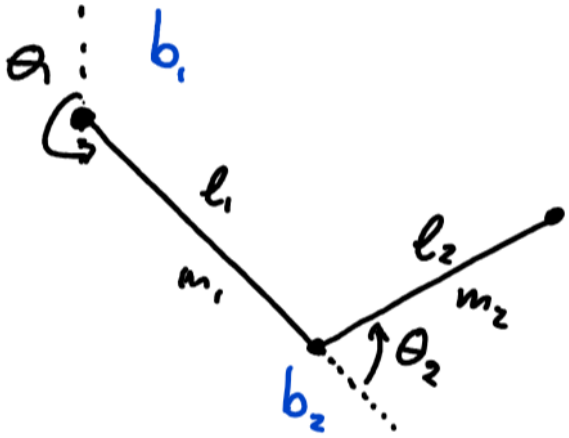
Plant - Inverted Double Pendulum



Modelling - Ideal Nonlinear SIMO Model

- Single Input

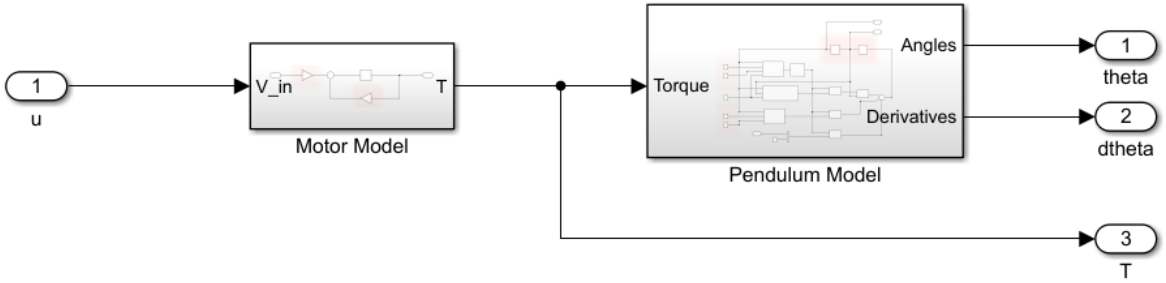
$$u(t) = v_{in}(t)$$

- Chosen State (5th order system)

$$\vec{x}(t) = (\frac{d\theta_1}{dt}, \frac{d\theta_2}{dt}, \theta_1, \theta_2, T)^T$$

- Available Outputs

$$\vec{y}(t) = (\theta_1, \theta_2)^T$$



=

$$\frac{d\vec{x}}{dt}(t) = \vec{F}(\vec{x}(t), v_{in}(t), t)$$

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Modelling - Linearization

- Matlab Manual Symbolic Linearization
  - Longer implementation
  - Longer execution
  - More control (Educational)
- Final State Space
  - Continuous Time

$$\frac{d\vec{x}}{dt}(t) = \vec{F}(\vec{x}(t), v_{in}(t), t)$$

$$\downarrow$$

$$\frac{d\vec{x}}{dt} = \mathbf{A}\vec{x} + \mathbf{B}u$$

$$\vec{y} = \mathbf{C}\vec{x}$$

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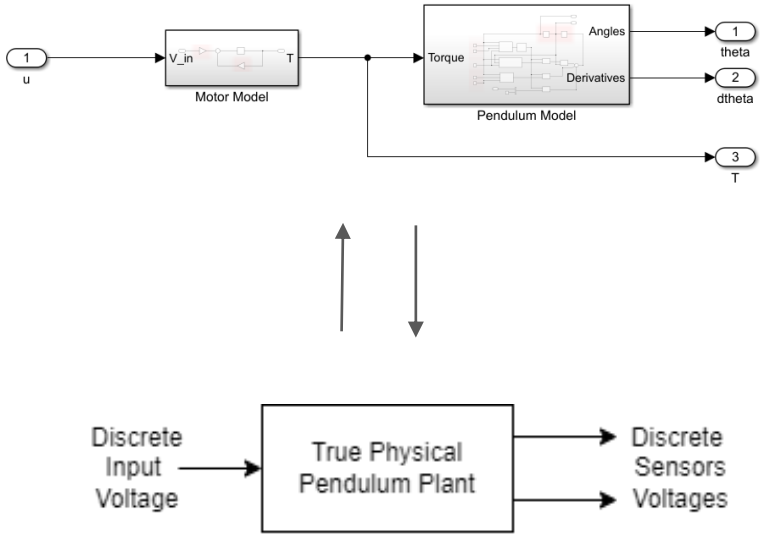
Identification - Interfacing

- Sampling Period

  h ~ Fastest Dynamic / 2 ~ τ_e / 2

  Design in CT

- Sensor Calibration



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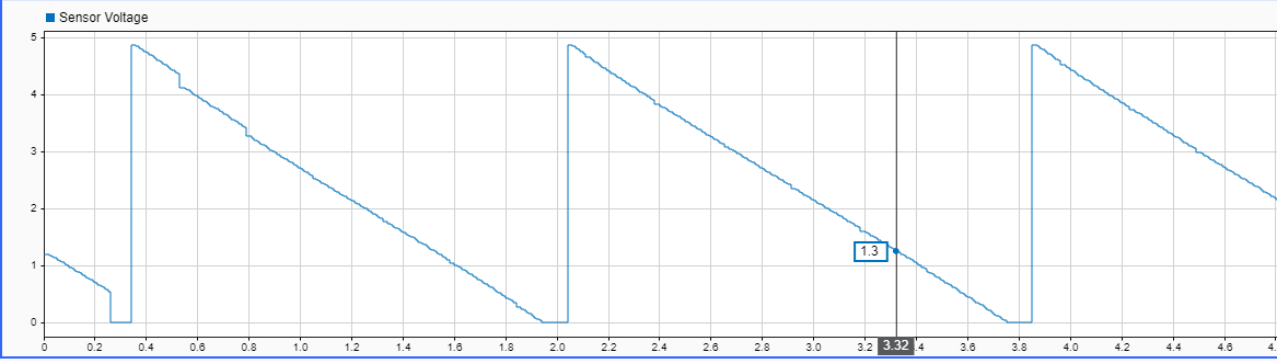
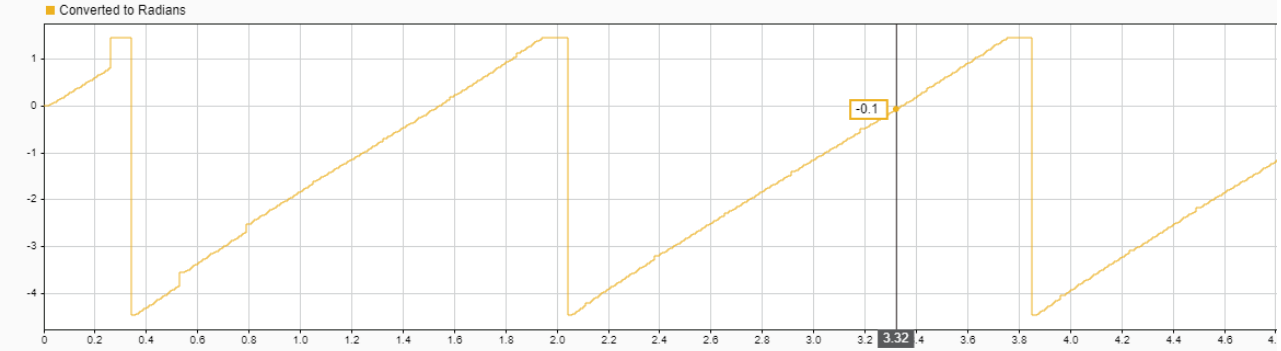
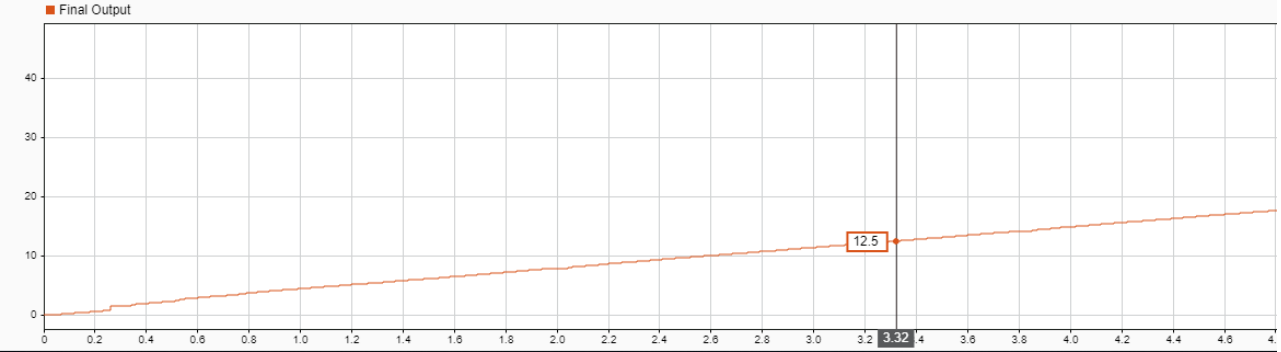


Probed Sensor Voltage



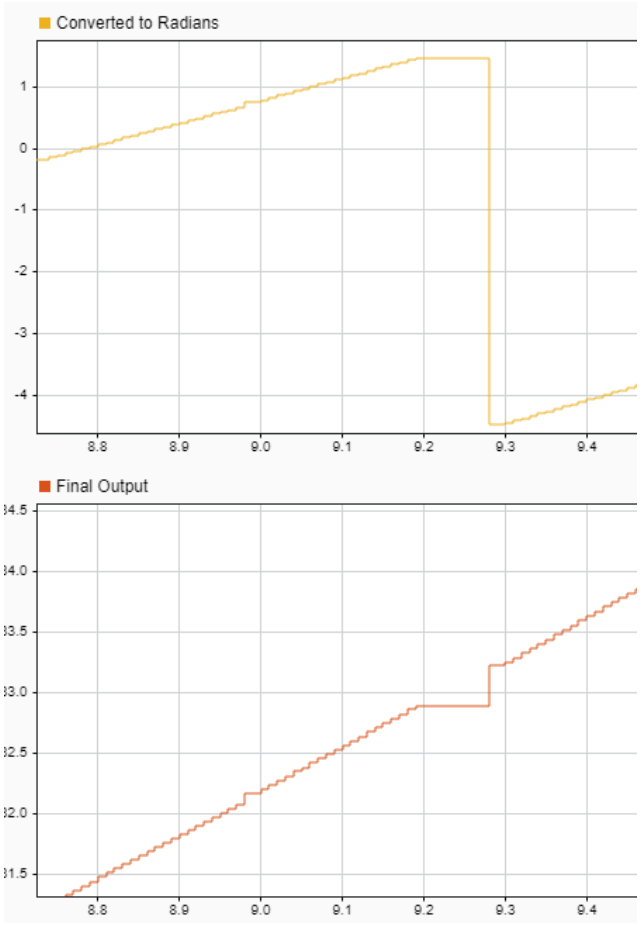
Converted in Radians



Unwrapped Measurement

Dead-zones / Compensation

- Dead-zones
  - ~5% range loss
  - At pi/2 for both links
- Range Stretching
  - Reduces discontinuity
  - Spreads error



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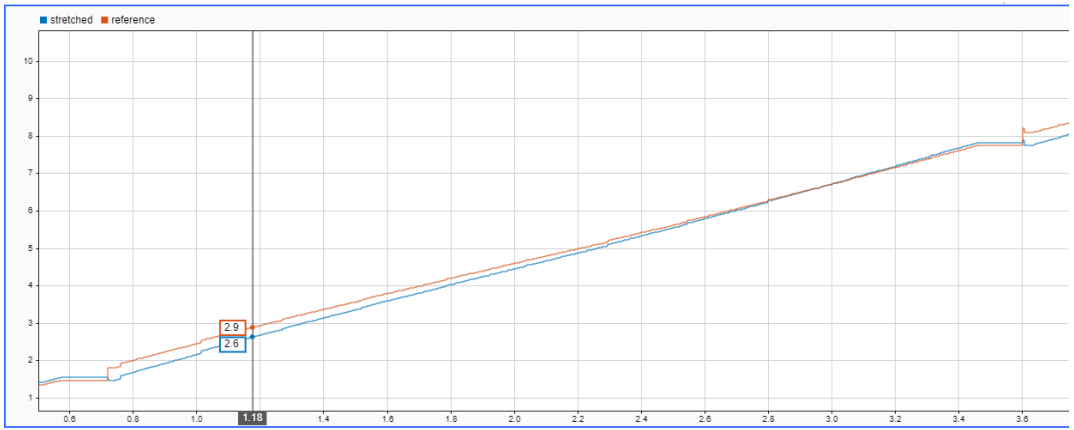
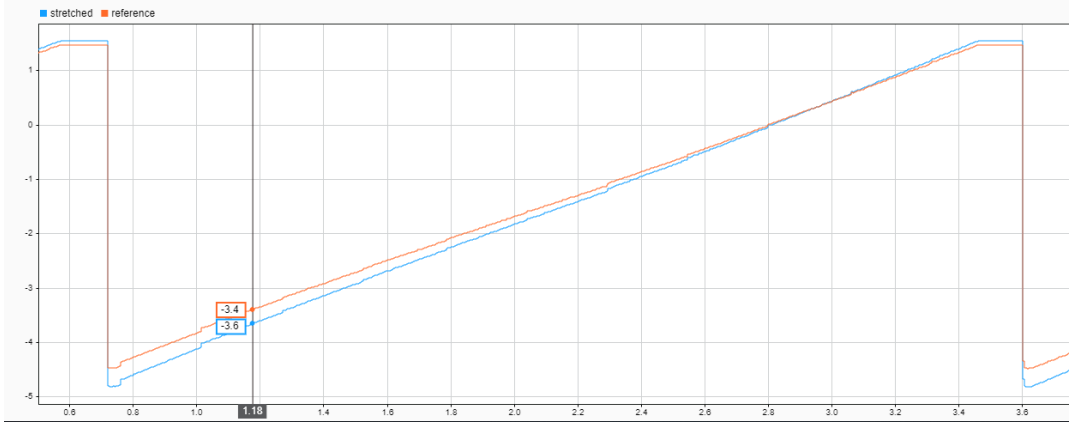
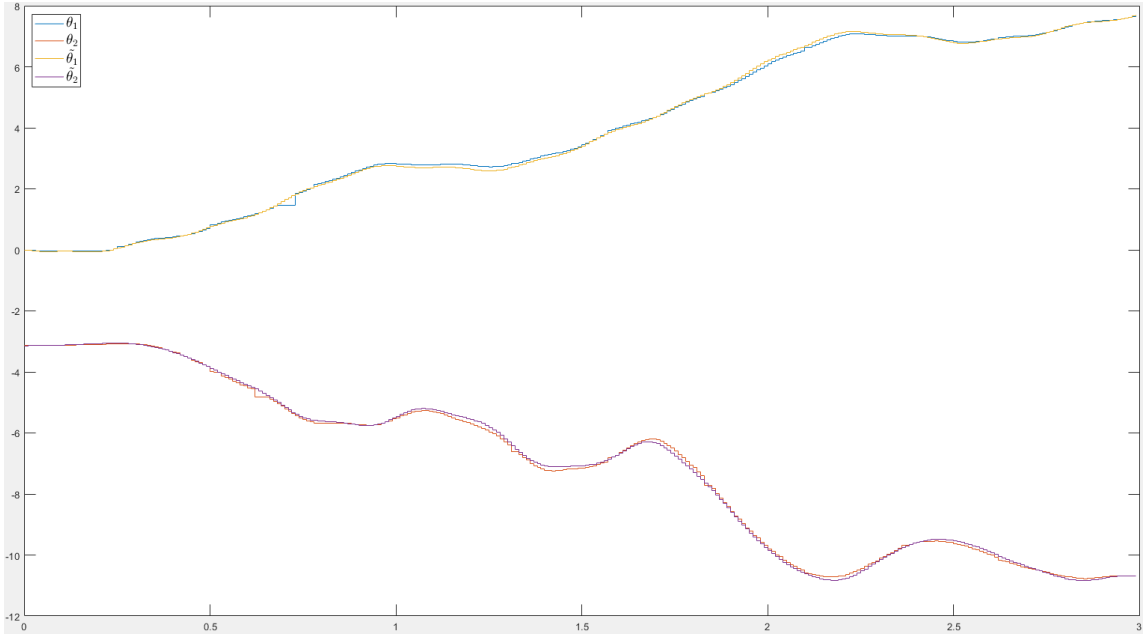




Identification - Nonlinear White Box model

- Method
  - Simdata
  - lsqnonlin
- Experiments
  - Second link
  - Motor
  - First link
- Troubles



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Identification - Nonlinear White Box model

Symbol	Predetermined	Estimated
l1 [m]	0.1	0.0897
l2 [m]	0.1	0.1
m1 [kg]	0.125	0.2
m2 [kg]	0.05	0.087972
c1 [m]	-0.04	0.01
c2 [m]	0.06	0.07
I1 [kg.m^2]	0.074	0.01076
I2 [kg.m^2]	0.00012	0.0001
b1 [kg.s^{-1}]	4.8	5.33518
b2 [kg.s^{-1}]	0.0002	0.00004
k_m *[Nm]*	50	39.2954
τ_e *[s]*	0.03	0.01

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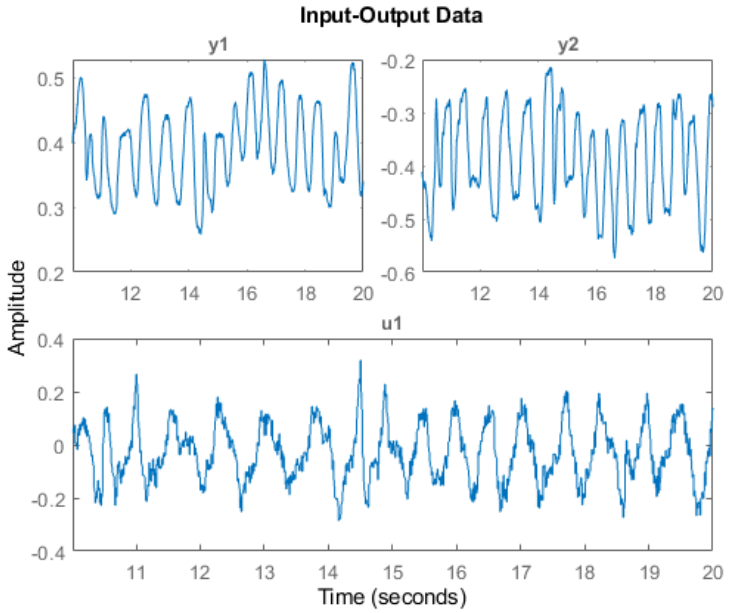
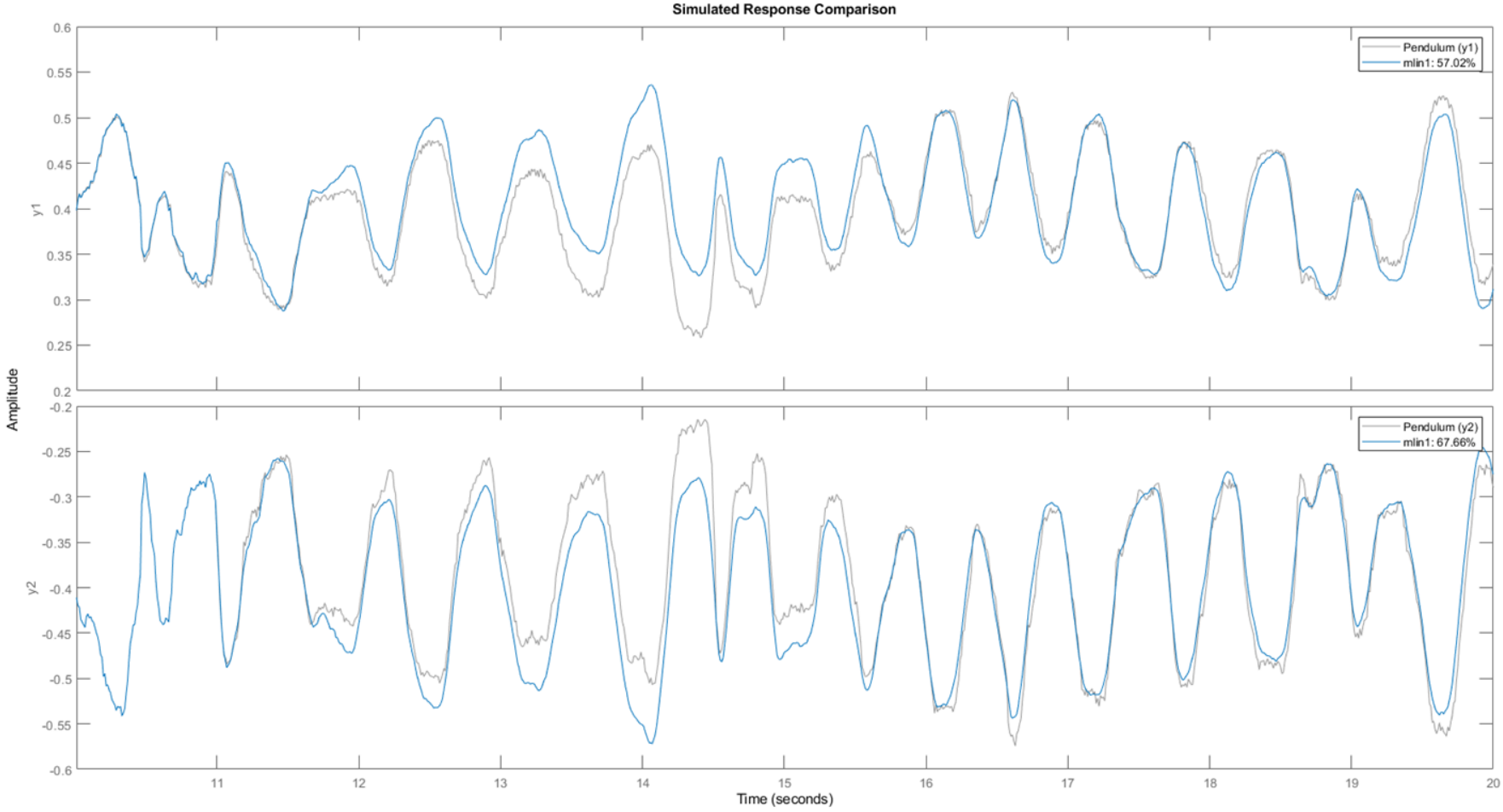
Identification - Linear Black box model

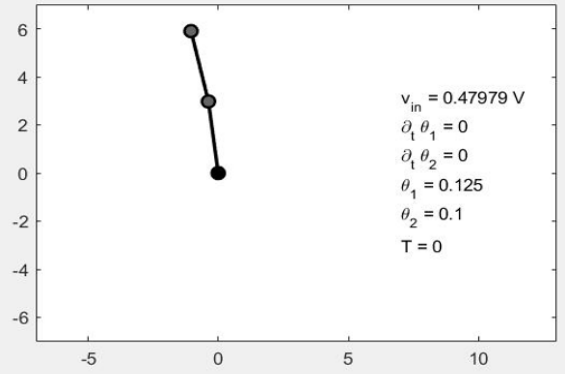
- ARX linear discrete time model
  - Data set: up - up position
  - Not used in practice
  - Educational

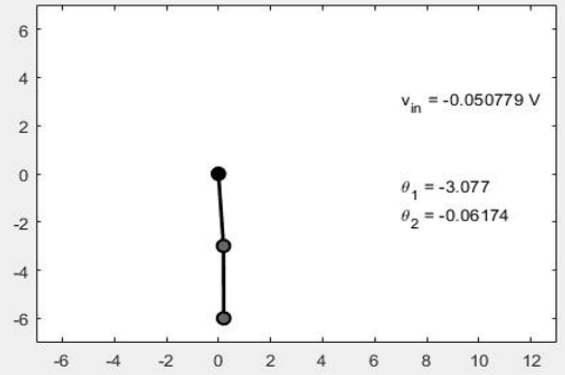



Simulated Response Comparison
Pendulum (y1)
mlin1: 57.02%
Pendulum (y2)
mlin1: 67.66%
Amplitude
y1
y2
Time (seconds)
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
-0.2
-0.25
-0.3
-0.35
-0.4
-0.45
-0.5
-0.55
-0.6
11
12
13
14
15
16
17
18
19
20

Linear Control - Objectives

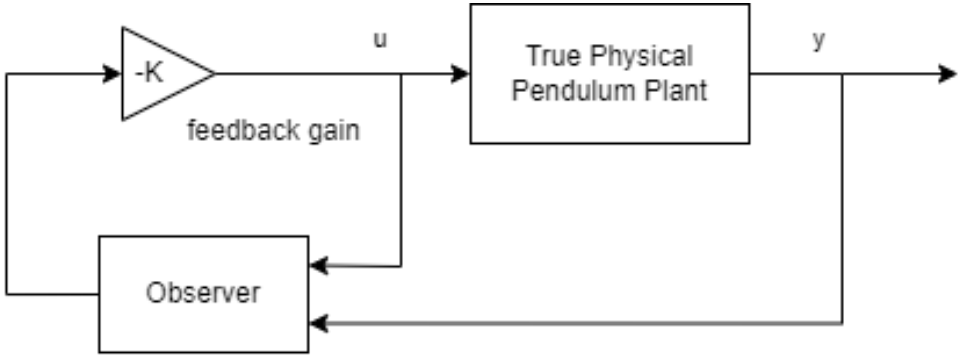
- Equilibrium Stabilization
  - Up up position
- Input Disturbance Rejection
  - Down Down position
- Motivation for Positions
  - Intuition
  - Simulation ‘eig(ctrb(A, B))’





Up Up Position - LQR Regulator

- Motivation w.r.t Objective
  - Easy pole placement
- Combined Layout
  - Luenberg Observer
  - Output Feedback
- Design
  - Decoupling of the eigenvalues
  - Feedback first, observe faster



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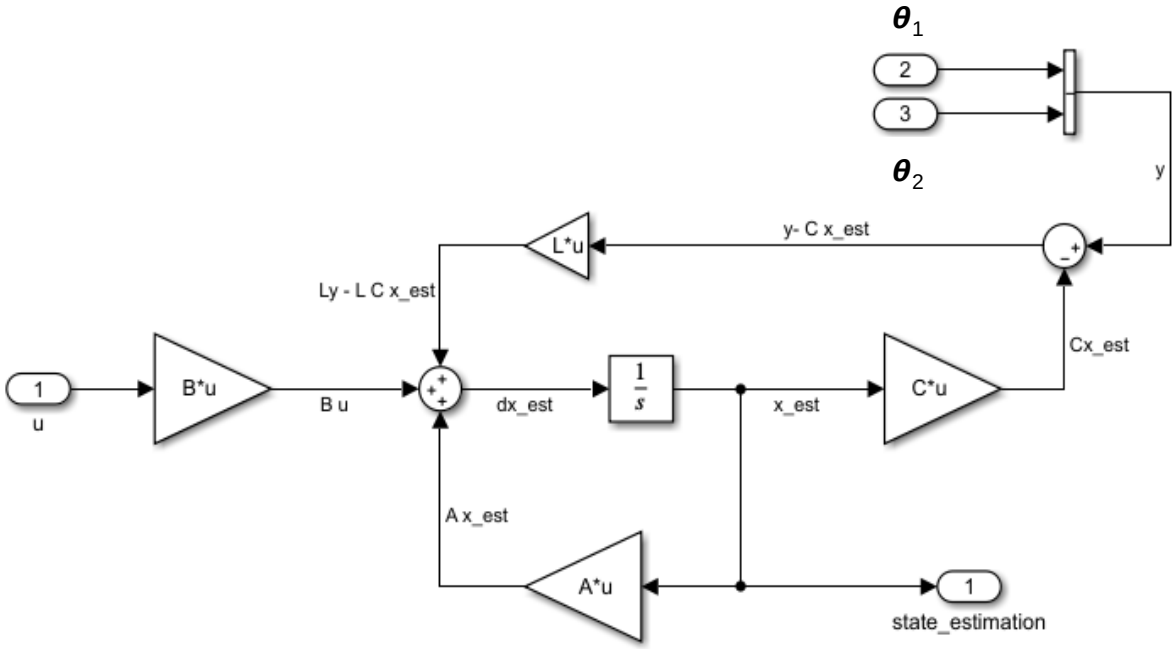
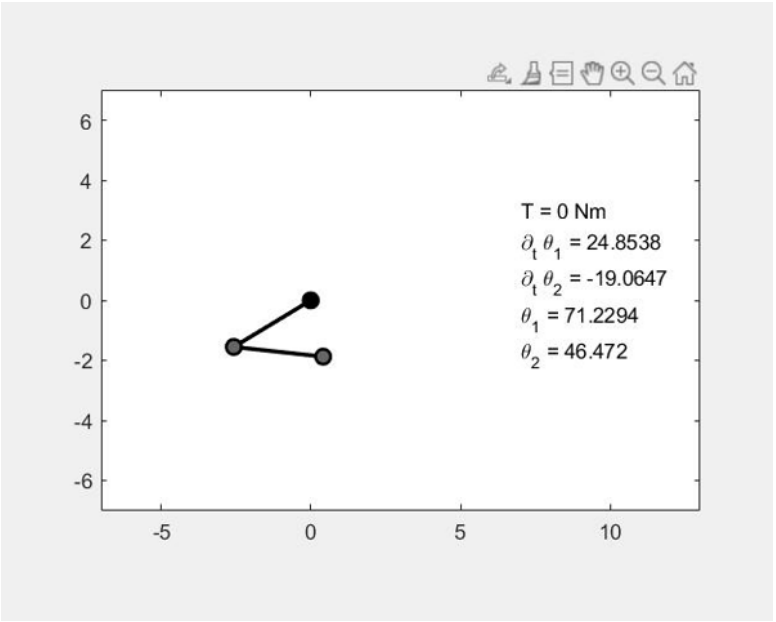
LQR Regulator - Luenberg Observer



Demo



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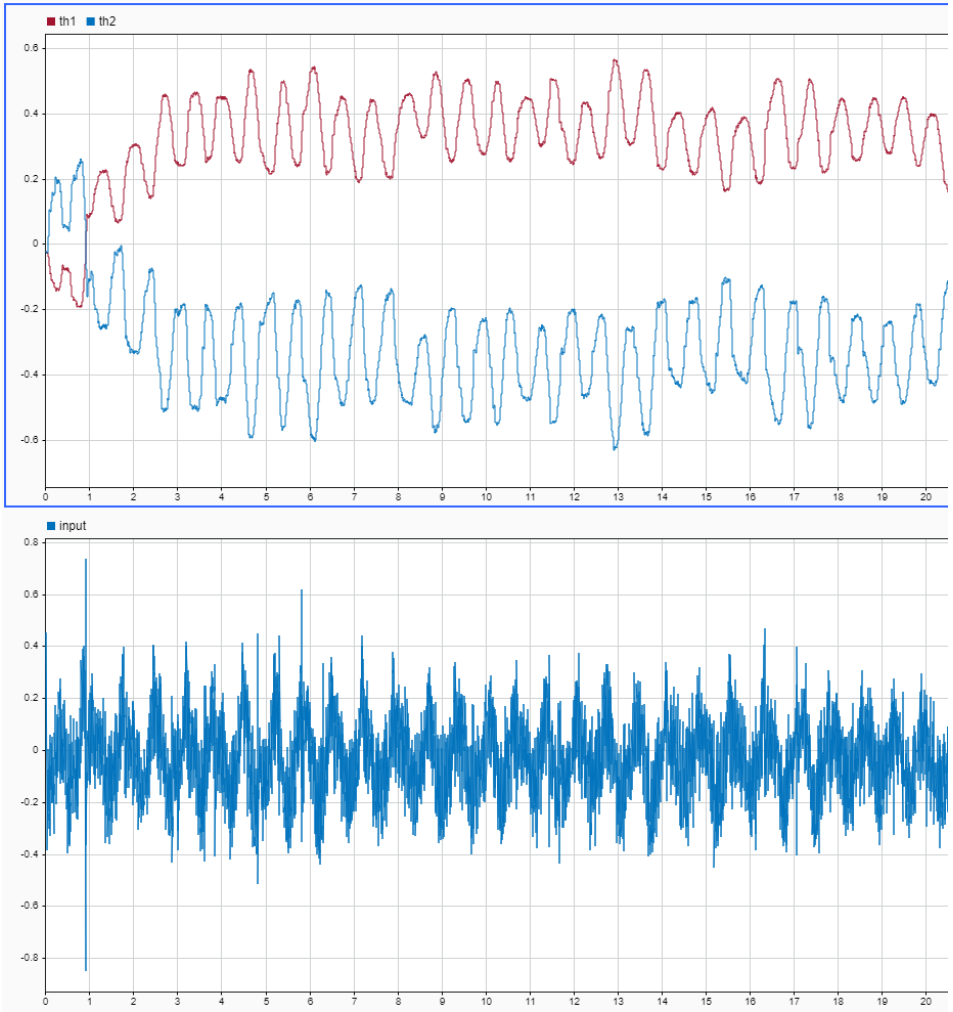
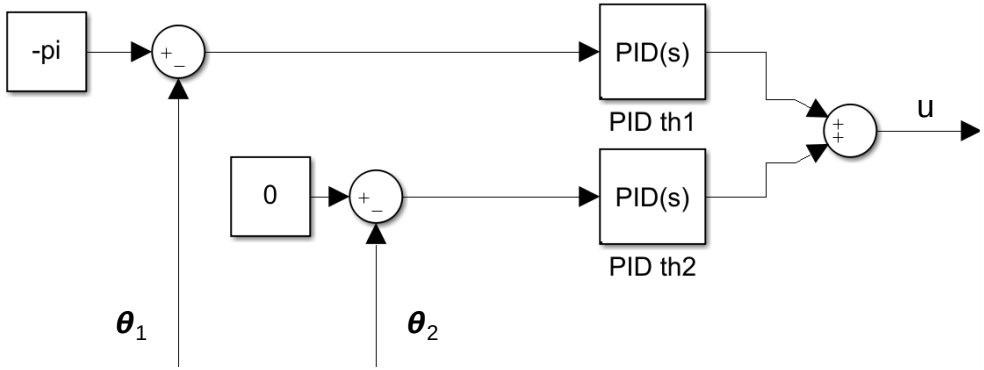
Results and Problems

- Objective
- Control input
  - Noisy observer
  - Non Saturated
- Backlash / Friction
  - Small Oscillations
  - Steady State Error



Down Down - PID Controller

- Motivation w.r.t Objective
  - Tune response to disturbance
  - Derivative control w/o observer
- Layout Motivation
  - Avoid unwanted stable equilibriums
- Problems
  - Many parameters
  - Long settling time



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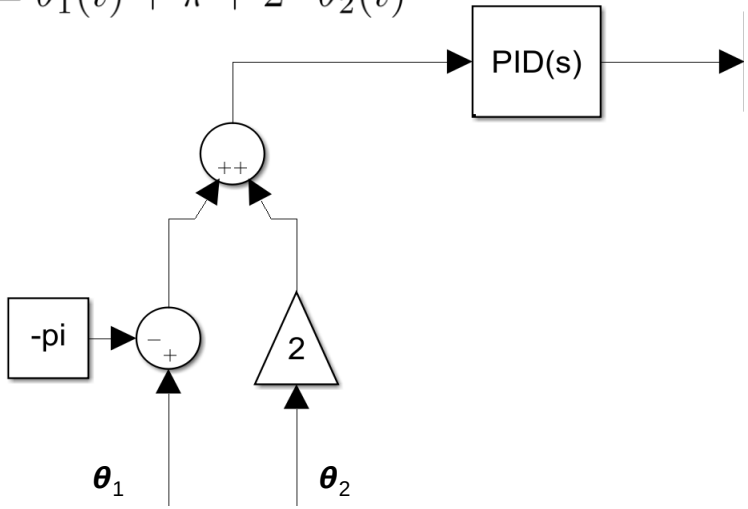
Down Down- Improved PID Controller

- Improvement
  - Single controller
  - Less parameters
  - Faster Settling time
- Layout Motivation
  - Avoid unwanted stable points

$$e(t) = \theta_1(t) + \pi + 2 \cdot \theta_2(t)$$



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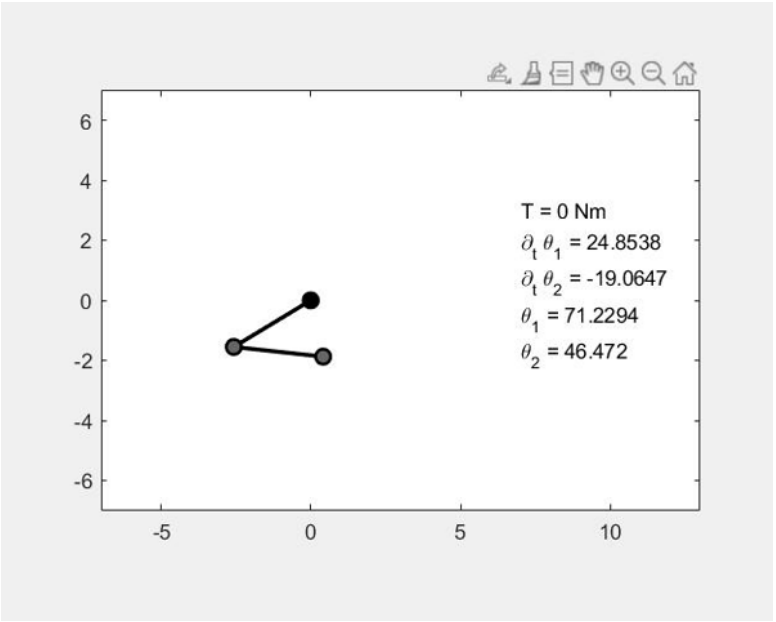
Demo



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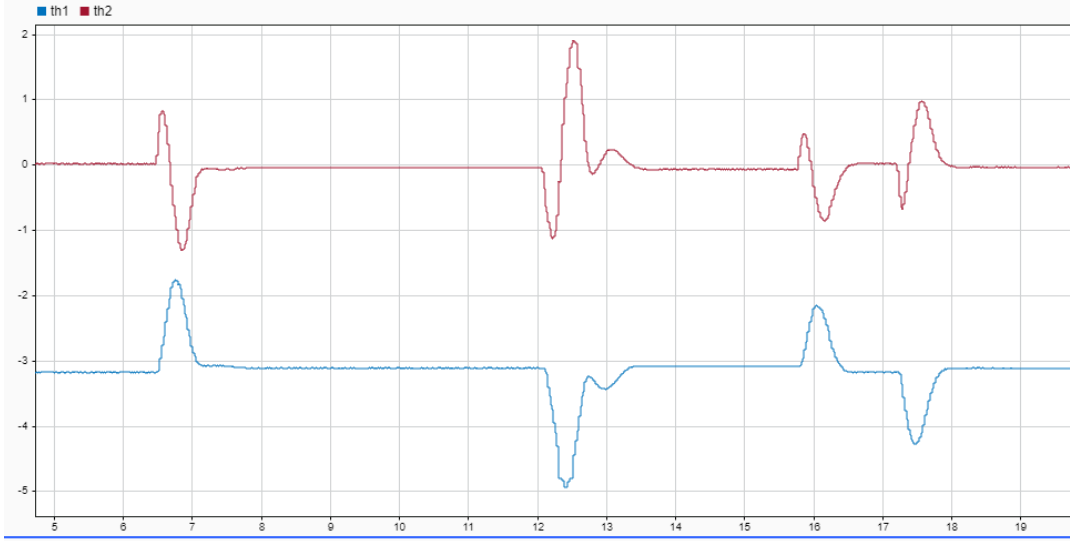
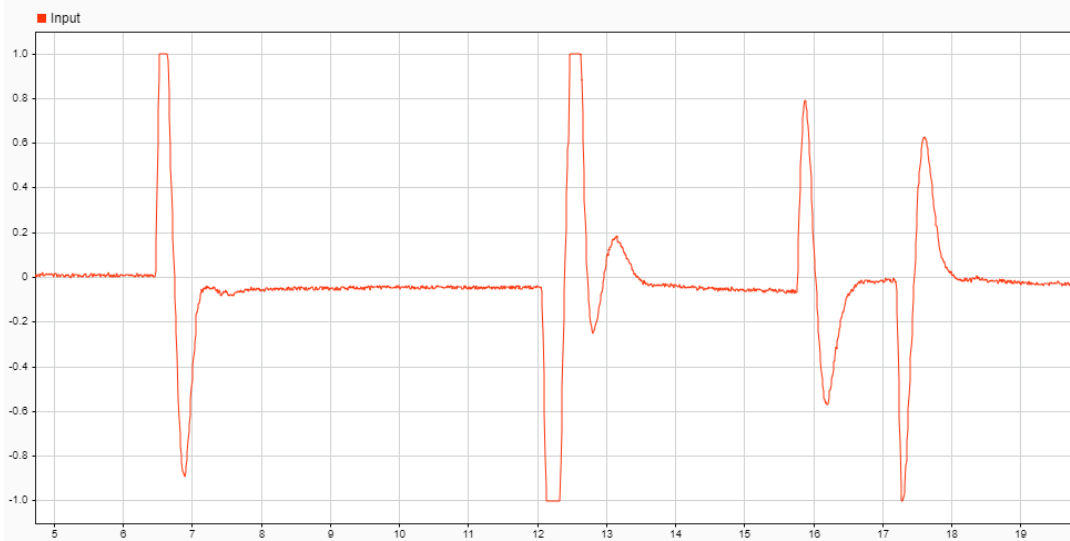
Results and Problems

- Objective
  - Fast Settling
  - Low Overshoot
  - Stability and backlash
- Control input
  - Saturates
  - Wind up

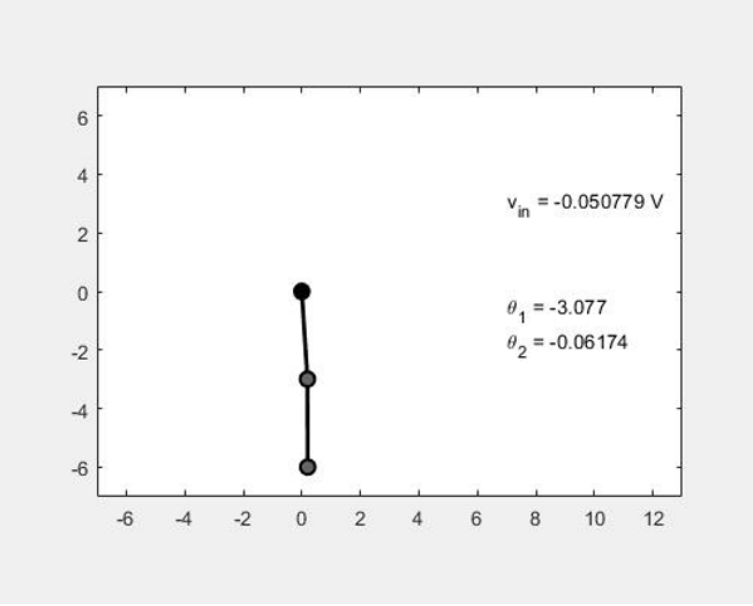


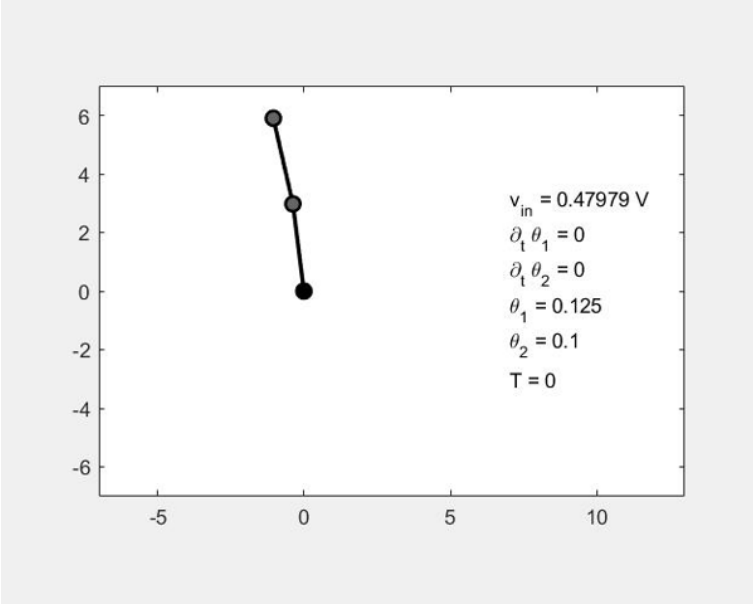


Improvements

- Time spent on white box
- Observer
- Filtering
- Swing up ?

Questions?





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