MECA0029–1 Theory of vibration

Analysis of the dynamic behaviour of a steel canopy

Academic year 2025 – 2026

In industrial areas, some equipment and materials are often stored outside. In order to protect them from the rain or snow, steel canopies can be used, consisting of a truss structure on which a roof is attached. In this work, the canopy between buildings B52 and B53 will be studied. The structure is presented in Fig. 1.

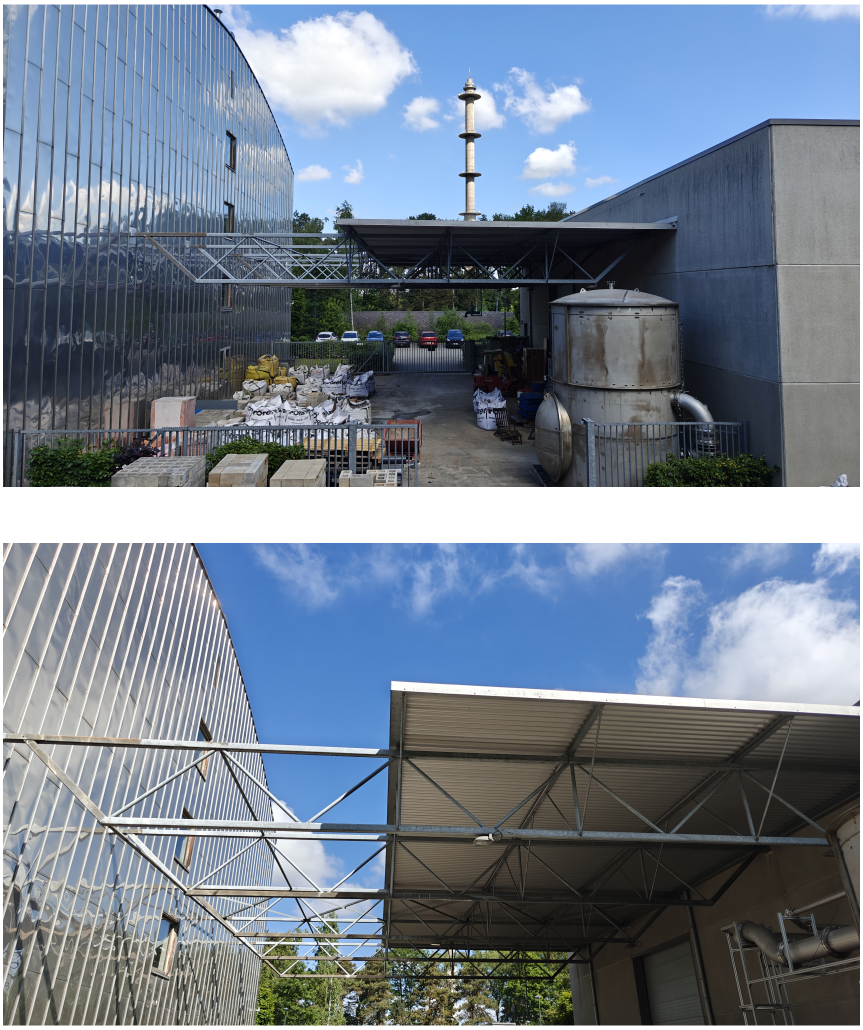


Figure 1 – Steel canopy between building B52 and B53 at the University of Liège.

Work instructions

1. The work will be done **by groups of two students maximum**. A report will be submitted in which all the **descriptions**, **results** and **discussions** have to figure out. More information on the form of the report and evaluation criteria can be found on eCampus under "Objectives of the project".
   Reminder: a report must be written as a standalone work.

   - The report and the implementation (MATLAB/Python codes, NX files) have to represent **your own work**. The use of artificial intelligence must comply with the University charter (link). **Any plagiarism, or failure to comply with this charter will be considered as fraud (0 grade with a fraud report to the registrar's office)**.
   - **40 pages** maximum are tolerated. You will be penalized if the report is too long (or extremely short).

2. The deadline for the submission of your work is fixed to **November 27, 2025** at **11:59 PM**. No additional delay will be tolerated.

   Your work must be uploaded on eCampus. The report (.pdf) and an archive (.zip) containing the following must be provided:

   - The MATLAB/Python codes
   - The Siemens NX files (.prt, .fem and .sim only)

   For sorting purposes, name your main files as follows:

   - Report: `MECA0029_Group_XX.pdf`.
   - Siemens NX file (.sim): `MECA0029_Group_XX.sim`.
   - MATLAB/Python files: the main files must be named `MECA0029_Group_XX_Y.py`, where XX is the number of your group and Y is the file number. You can use and name other files and subfunctions at your own convenience.

1 Modeling of the structure

Recommended deadline: October 16, 2025

The structure to study represents a section of a simplified version of the canopy. It is built according to Fig. 2. It is composed of two frames, colored in red in Fig. 2a, spanning 15 [m] in the X-direction, and of support beams, colored in blue in Fig. 2a, forming a zig-zag pattern every 1.5 [m] in the X-direction. The two frames are 4 [m] apart in the Y-direction and are linked by transverse support beams.

All the beams of the structure are made of steel with the following material properties:

- Density: $\rho = 7800$ [kg/m^3].
- Poisson's ratio: $\nu = 0.3$ [-].
- Young's modulus: $E = 210$ [GPa].

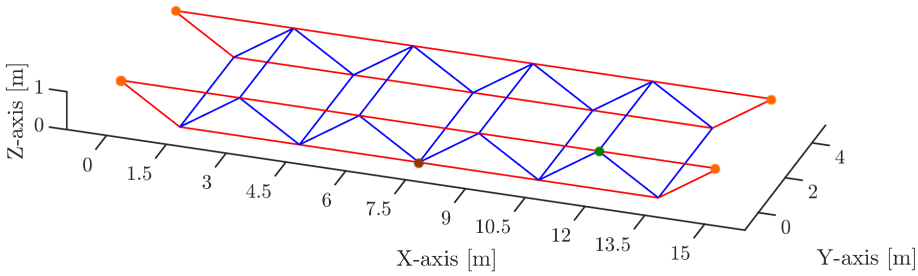
All the beams are assumed to have a **circular hollow cross-section**. The **frame beams** (red) have an outer diameter of 120 [mm] and a wall thickness of 5 [mm], while the **support beams** (blue) have an outer diameter of 70 [mm] and a wall thickness of 3 [mm].

The frames are assumed to be **clamped** to the building B52 and B53. The corresponding nodes are colored in orange in Fig. 2a.

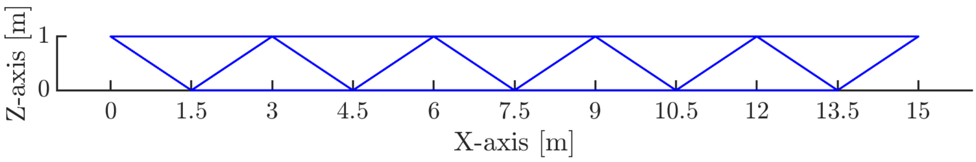
In order to model the roof, lumped masses are added to some nodes to represent its weight distribution. We will assume that the total mass of the roof is 500 [kg] and that it is **uniformly distributed over the 8 pink nodes** in Fig. 3. The rotational inertia of the roof is neglected.

It is asked to

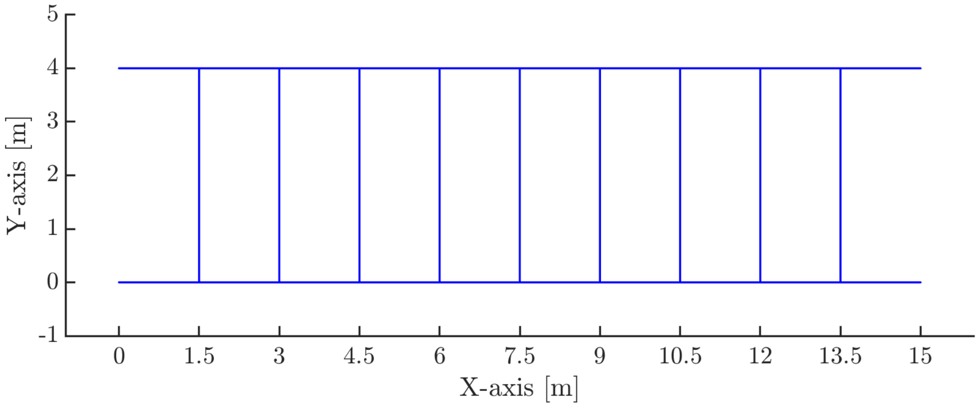
- Build the canopy model in **NX** based on 3D beam elements and extract the first **six natural frequencies** along with their corresponding **mode shapes**.
- Build the canopy model in **MATLAB/Python** using the 3D beam model seen in the theory, extract the first **six natural frequencies** and draw the corresponding **mode shapes** of the structure.
- Make a **convergence study** (as a function of the number of elements per beam) in **both software**.
- Compute the **total mass** of the system composed of the truss and the roof using a rigid-body mode in translation.



(a) 3D view. The clamped nodes are the orange dots, the excitation location for the stationary response is the green dot, and the excitation location for the transient response is the brown dot.



(b) XZ (side) view.



(c) XY (top) view.

Figure 2 – Canopy section model.

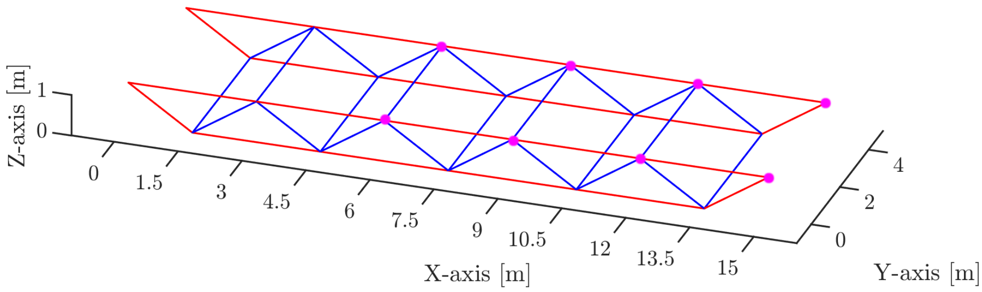


Figure 3 – Nodes where the concentrated masses are applied.

2 Stationary response

Recommended deadline: October 30, 2025

The roof of the canopy sustained damage due to violent wind gusts. Construction workers are dispatched to the site to repair it with the help of heavy machinery. The resulting force of the machine on the canopy is assumed to be a **cosine** wave with a frequency of **2.4 [Hz]** and an amplitude of **500 [N]**. The force is **purely transverse** along the Y-axis and is applied to the **green node** in Fig. 2a, that we will refer to as "excitation node".

It is asked to (in MATLAB/Python)

- Compute the "exact" stationary response to the excitation using the **FRF matrix** of the system. Plot the **time** evolution **in the Y-direction** and its corresponding **discrete Fourier transform** at the excitation node.
- Compute an approximate solution using the **mode displacement method** (use the special case of the forced harmonic response, see Appendix A). Plot the **time** evolution **in the Y-direction** at the excitation node.
- Compute an approximate solution using the **mode acceleration method** (use the special case of the forced harmonic response, see Appendix A). Plot the **time** evolution **in the Y-direction** at the excitation node.
- **Compare and discuss** the results of **both** approximation methods and discuss the **convergence** in terms of the number of modes included in the superposition.
- Based on the reference solution, compute and plot the **power spectral density (PSD) of the acceleration of the response** at the excitation node and determine the **RMS** value of the **lateral acceleration** (see Appendix B).

- Evaluate the **comfort level** for the workers on the canopy at the excitation node according to the **ISO 2631-1** standard by estimating the maximum time of exposure which would cause them noticeable fatigue (see Fig. 5).

3 Transient response

Recommended deadline: November 13, 2025

For their integrated project, a group of students of the Aerospace engineering master built a small plane prototype with a working jet engine. The final step is to test it in flight conditions. Unfortunately, during flight, the remote controller fails and the plane violently crashes in the canopy in a spectacular explosion. Luckily, the workers were not on the bridge that day! We will consider an **impact** of **7000** [**N**] for a duration of **0.01** [**s**] at the point colored in **brown** in Fig. 2a, which will become the new "excitation node". The direction of the force is supposed **purely transverse** along the Y-direction. **Damping** will be introduced in this part.

It is asked to (in MATLAB/Python)

- Compute the **damping matrix** using the **proportional damping** assumption such that the damping ratio of the **first two** modes is equal to **1 %**. **List** the damping ratios for modes 1 to 6.
- Compute the solution by time integration using the **Newmark** algorithm. **Justify** the choice of the time step and integration parameters. Plot the **time** evolution **in the Y-direction** and the amplitude of its corresponding **discrete Fourier transforms** at the excitation node.
- **Discuss** the results.

4 Reduction methods

Recommended deadline: November 27, 2025

To decrease the number of degrees of freedom, a part of the structure is condensed and **only the translation degrees of freedom** of the **3** nodes highlighted in purple in Fig. 4 are retained.

It is asked to (in MATLAB/Python)

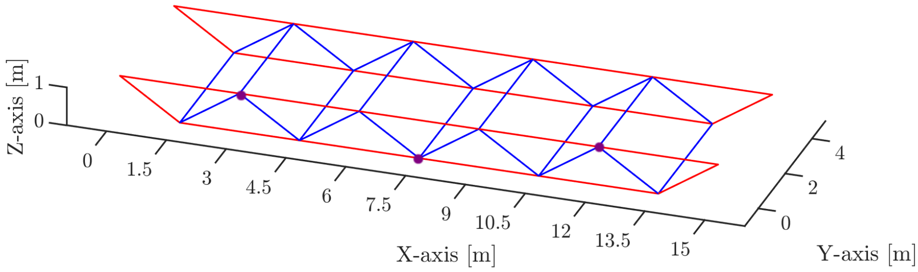


Figure 4 – Nodes retained for the reduction.

- Use **Guyan-Irons** method and **compare** the natural frequencies with the **initial** finite element model.
- Use **Craig-Bampton** method and choose an appropriate number of modes such that the maximum relative error on the six previously identified natural frequencies is **less than 1 %**. **Compare** the results with the **initial** model and with **Guyan-Irons** method.
- Compute again the transient response to the plane crash using **Newmark** algorithm and the reduced models. Show the interest of reduction methods by comparing the responses at **the excitation node** in terms of **accuracy**, **size of the problem** and **computation time**.

A Mode displacement and acceleration methods for a periodic excitation

In the special case of a harmonic excitation of the form $\mathbf{s}\cos(\Omega t)$, the solution for the mode displacement method yields a synchronous response

$$\mathbf{q}(t) = \mathbf{x}\cos(\Omega t)$$

with

$$\mathbf{x} = \left(\sum_{s=1}^{k} \frac{\mathbf{x}_s \mathbf{x}_s^T}{(\omega_s^2 - \Omega^2)\mu_s} \right) \mathbf{s}$$

where $\mathbf{x}_s$ and μ_s, the eigenvector and generalized mass related to the s^{th} natural frequency ω_s, respectively.

For the mode acceleration method, it becomes

$$\mathbf{x} = \left(\mathbf{K}^{-1} + \Omega^2 \sum_{s=1}^{k} \frac{\mathbf{x}_s \mathbf{x}_s^T}{(\omega_s^2 - \Omega^2)\omega_s^2 \mu_s} \right) \mathbf{s}$$

B PSD and RMS of a cosine signal

For a cosine excitation of the form $f = A\cos(\Omega t)$ $(A \geq 0)$, the mean square value is given by

$$\bar{f^2} = \frac{1}{2} A^2$$

and the root mean square (RMS) is thus

$$\sqrt{\bar{f^2}} = \frac{A}{\sqrt{2}}.$$

The PSD of the signal can be defined from the mean square value as

$$G_f(\omega) = \frac{1}{2} A^2 \delta(\omega - \Omega).$$

where $\delta(\bullet)$ is the Dirac function.

C ISO 2631-1

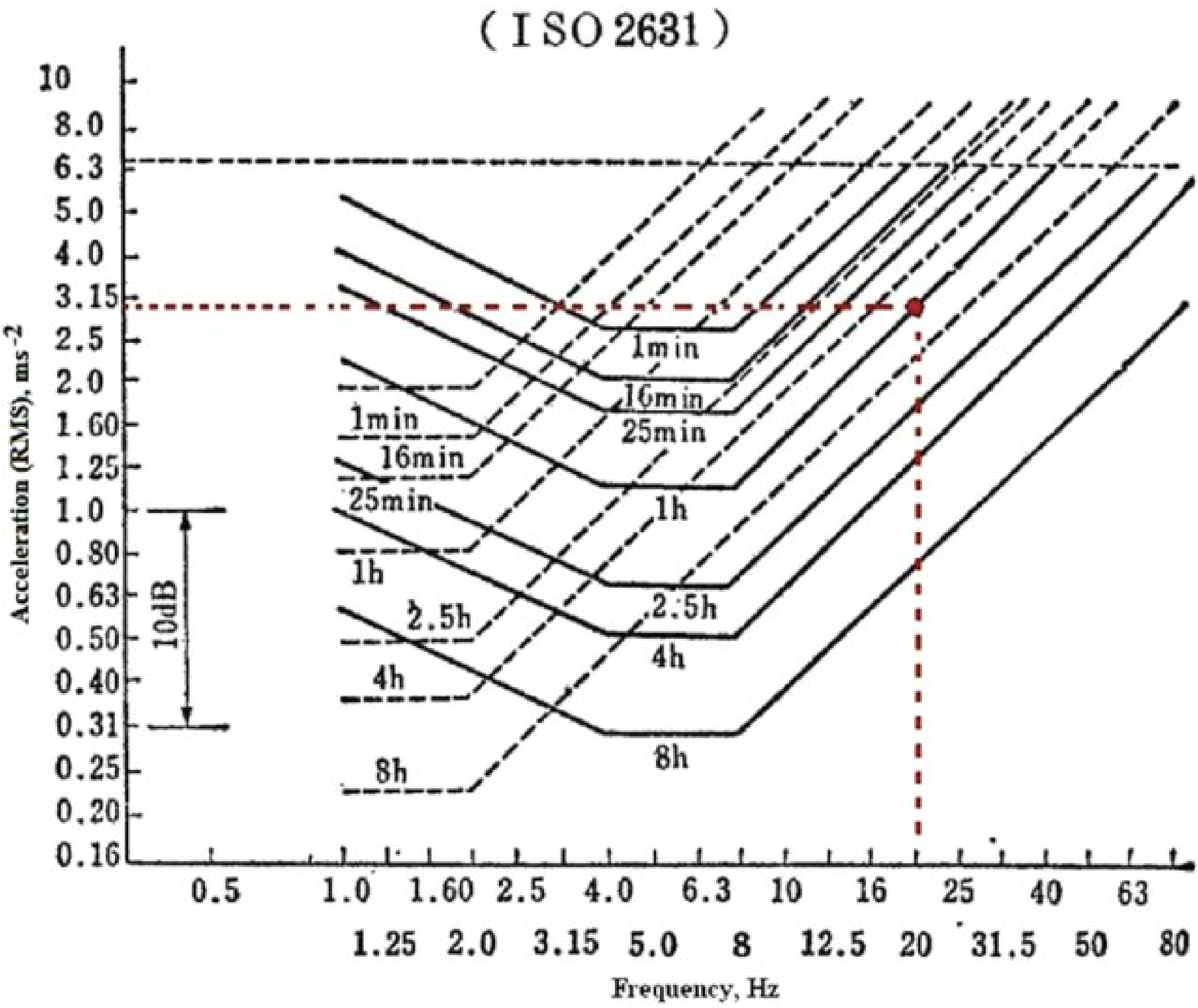


Figure 5 – ISO 2631 curves defining equal fatigue-decreased proficiency boundaries for a vertical acceleration (solid line) and a lateral acceleration (dashed line).