Lecture Notes 2: Distribution

Professor: Zhihua Zhang

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2 Discrete Distribution Examples

2.1 Uniform Discrete Distribution

Random variable $\mathbf{X} \in \{x_1, x_2, ..., x_n\}$ has a uniform discrete distribution pmf f if

$$f(x) = \begin{cases} \frac{1}{n} & x = x_i, i = 1, 2, ..., n \\ 0 & \text{otherwise} \end{cases}$$

2.2 Point Mass Distribution

Random variable $\mathbf{X}$ has a point mass distribution pmf f if

$$f(x) = \begin{cases} 1 & x = a \\ 0 & \text{otherwise} \end{cases}$$

2.3 Bernoulli Distribution

Random variable $\mathbf{X}$ has a bernoulli distribution pmf f if

$$f(x) = \begin{cases} p & x = a \\ 1 - p & \text{otherwise} \end{cases}$$

where $p \in [0, 1]$.

2.4 Poisson Distribution

A discrete random variable $\mathbf{X}$ is said to have a Poisson distribution with parameter $\lambda > 0$, if, for $k = 0, 1, 2, ...$, the probability mass function of $\mathbf{X}$ is given by:

$$f(k; \lambda) = \Pr(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

.

Remark: If $\mathbf{X}_1 \sim Poisson(\lambda_1)$, $\mathbf{X}_2 \sim Poisson(\lambda_2)$, then $\mathbf{X}_1 + \mathbf{X}_2 \sim Poisson(\lambda_1 + \lambda_2)$.

2.5 Binomial Distribution

A discrete random variable $\mathbf{X}$ is said to have a binomial distribution with parameter n and p, we write $\mathbf{X} \sim Binomial(n,p)$). The probability mass function is given by:

$$f(k;n,p) = \Pr(\mathbf{X} = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

for $k = 0, 1, 2, ..., n$, where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ is the binomial coefficient. It can be interpreted that the probability of exact k successes after n trials.

[need to add more ...]

2.6 Negative Binomial Distribution

Suppose there is a sequence of independent Bernoulli trials, each trial having two potential outcomes called "success" and "failure". In each trial the probability of success is p and of failure is $1p$. We are observing this sequence until a predefined number r of failures has occurred. Then the random number of successes we have seen, $\mathbf{X}$, will have the negative binomial (or Pascal) distribution:

$$\mathbf{X} \sim NB(r,p).$$

The probability mass function of the negative binomial distribution is:

$$f(k;r,p) = Pr(\mathbf{X} = k) = \binom{k+r-1}{k} p^k (1-p)^r$$

for $k = 0, 1, 2, ...$

2.7 Geometric Distribution

3 Continuous Distribution Examples

3.1 Continuous Uniform Distribution

A continuous random variable $\mathbf{X}$ is said to have a uniform distribution in $[a,b]$, if the probability density function is given by:

$$f(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

3.2 Normal(Gaussian) Distribution

A continuous random variable $\mathbf{X}$ is said to have a Gaussian distribution with parameter μ and σ, if the probability density function of $\mathbf{X}$ is given by:

$$f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

The cumulative distribution function of Gaussian random variable $\mathbf{X}$ with parameter $\mu = 0$ and $\sigma = 1$ is:

$$\Phi(z) = Pr(\mathbf{X} < z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$$

3.3 Disac Distribution

3.4 Exponential Power Distribution

3.5 Generalized Inverse Gaussian Distribution

A continuous random variable **X** is said to have generalized inverse Gaussian distribution with parameters α, β, r, if the probability density function of **X** is given by:

$$f(x) = \frac{(\alpha/\beta)^{r/2}}{2K_r(\sqrt{\alpha\beta})} x^{r-1} e^{-(\alpha x + b/x)/2}, x > 0$$

where K_r is a modified Bessel function of third kind with index r, $\alpha > 0$, $\beta > 0$.

3.6 Chi-Squared Distribution

A continuous random variable **X** is said to have chi-squared distribution, if the probability density function of **X** is given by:

$$f(x) = \frac{1}{\Gamma(\frac{p}{2}) 2^{\frac{p}{2}}} x^{\frac{p}{2}-1} e^{-\frac{x}{2}}$$