Homework2

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1.Download the population data from the GauchoSpace website. Open the data set in MATLAB.

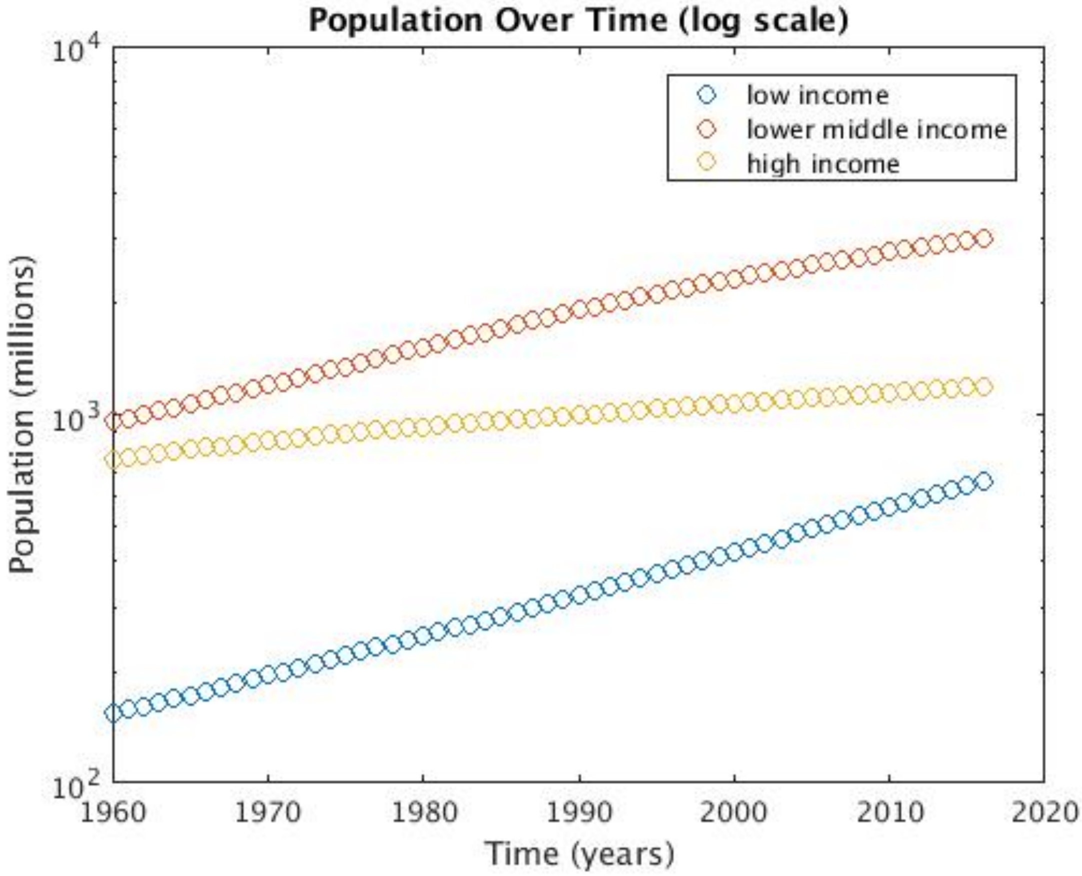
```
% open data
M = importdata('API_SP.POP.TOTL_DS2_en_csv_v3.csv');
data = M.data;
headers = M.textdata;

% the year is the first row
% the rest of the rows are the population data
year = data(1,:);

% find the relevant countries (the index of the row for each country)
ilow = find(strcmp(M.textdata,'Low income'));
ilowmid = find(strcmp(M.textdata,'Lower middle income'));
ihigh = find(strcmp(M.textdata,'High income'));
ijapan = find(strcmp(M.textdata,'Japan'));
ichina = find(strcmp(M.textdata,'China'));
ius = find(strcmp(M.textdata,'United States'));
```

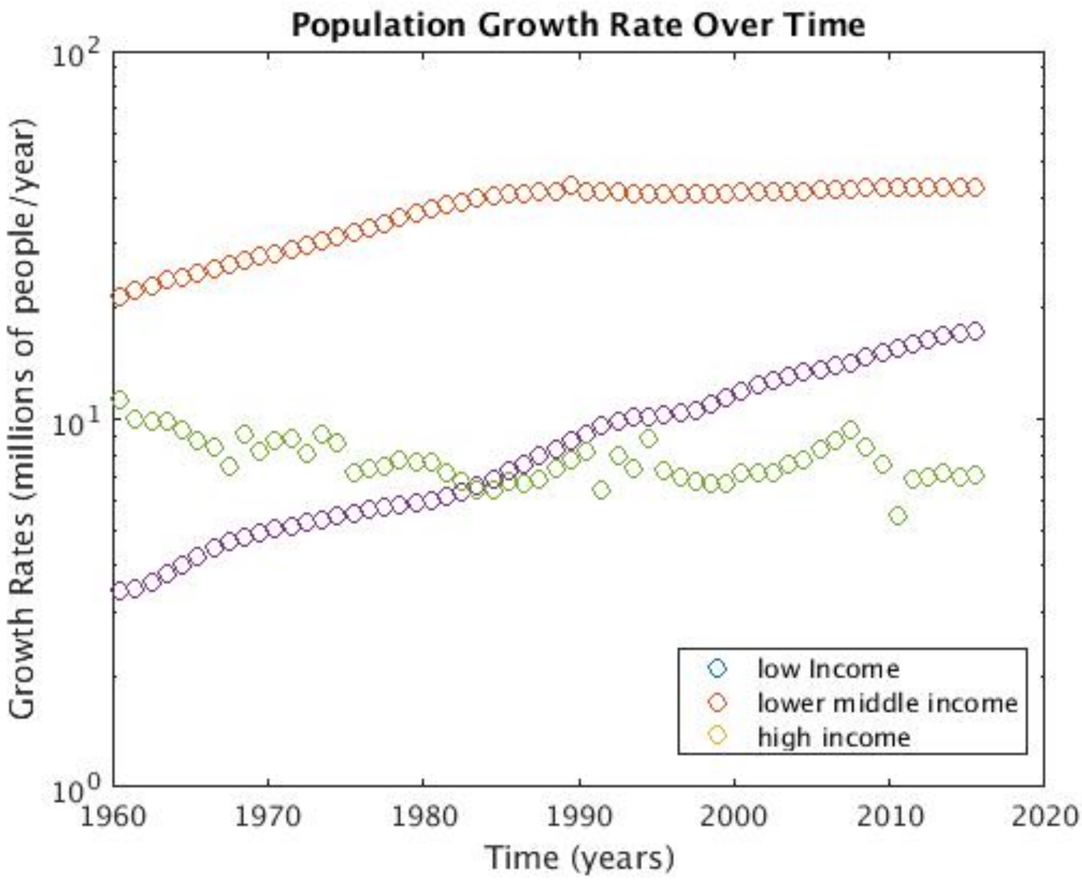
a) Find the columns corresponding to the population of the following three sets of countries: Low income, lower middle income, and high income. Plot the population of these countries (in millions of people) on a semi-log plot (population on y-axis on log scale, time on x-axis on linear scale). Label the plot appropriately.

```
low_country = data(ilow,:);
lowmid_country = data(ilowmid,:);
high_country = data(ihigh,:);
figure(1)
plot(year,low_country./(10^6),'o')
hold on
plot(year, lowmid_country./(10^6),'o')
hold on
plot(year, high_country./(10^6),'o')
xlabel('Time (years)')
ylabel('Population (millions)')
set(gca, 'YScale','log')
title('Population Over Time (log scale)')
```



b) Calculate the growth rates (in millions of people/year) of each set of countries in (a) and plot them as a function of time on a separate semi-log plot. Label the plot appropriately.

```
nt = length(year);
d_low = low_country(2:nt) - low_country(1:nt-1);
d_lowmid = lowmid_country(2:nt) - lowmid_country(1:nt-1);
d_high = high_country(2:nt) - high_country(1:nt-1);
dt = year(2:nt)-year(1:nt-1);
d_low_dt = d_low./dt;
d_lowmid_dt = d_lowmid./dt;
d_high_dt = d_high./dt;
tmid = .5*(year(2:nt)+year(1:nt-1));
figure(2)
plot(tmid, d_low_dt./(10^6),'o')
hold on
plot(tmid, d_lowmid_dt./(10^6),'o')
hold on
plot(tmid, d_high_dt./(10^6),'o')
xlabel('Time (years)')
ylabel('Growth Rates (millions of people/year)')
set(gca, 'YScale','log')
title('Population Growth Rate Over Time')
%c
% when you plot a derivetive and it's a linear, if fits for the exponential
```



c) Examine the plots. Which set of countries (low income, lower middle income, or high income) is growing exponentially over the past 5 decades? How can you tell?

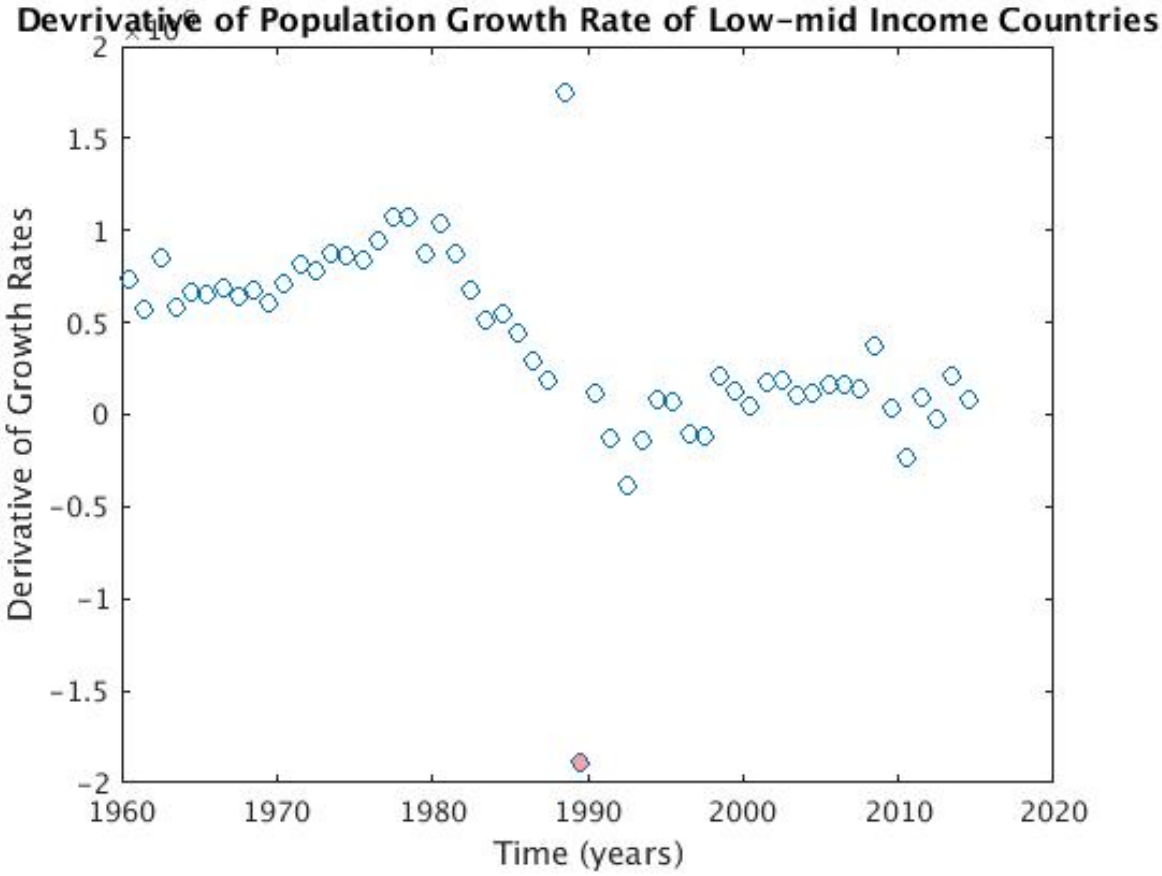
Low income and lower middle income countries are growing exponentially over the past 5 years because the data sets are linear in the plot. When plot a derivative and it's a linear, if fits for the exponential.

d)When did the population growth rate of low-mid income countries slow down?

```
d_lowmid = lowmid_country(2:nt) - lowmid_country(1:nt-1);
dt = year(2:nt)-year(1:nt-1);
d_lowmid_dt = d_lowmid./dt;

ddt = year(2:nt-1) - year(1:nt-2);
dd_lowmid = d_lowmid_dt(2:nt-1) - d_lowmid_dt(1:nt-2);
dd_lowmid_dt = dd_lowmid./ddt;
tmidd = .5*(year(2:nt-1)+year(1:nt-2));

figure(3)
plot(tmidd, dd_lowmid_dt,'o')
xlabel('Time (years)')
ylabel('Derivative of Growth Rates')
title('Devrivative of Population Growth Rate of Low-mid Income Countries')
```



I plot derivative of population growth rate of lower middle income countries over the past 5 decades. In the plot, starting from 1989 (pink point, the derivative is negative), the population growth rate slow down.

e) Assume an exponential growth model of the form P(t) = Poe(a*t) for the population of low-income countries since 1960. Find the values of the constants Po and a by fitting a linear model to the log-transformed population data (use the MATLAB function polyfit).

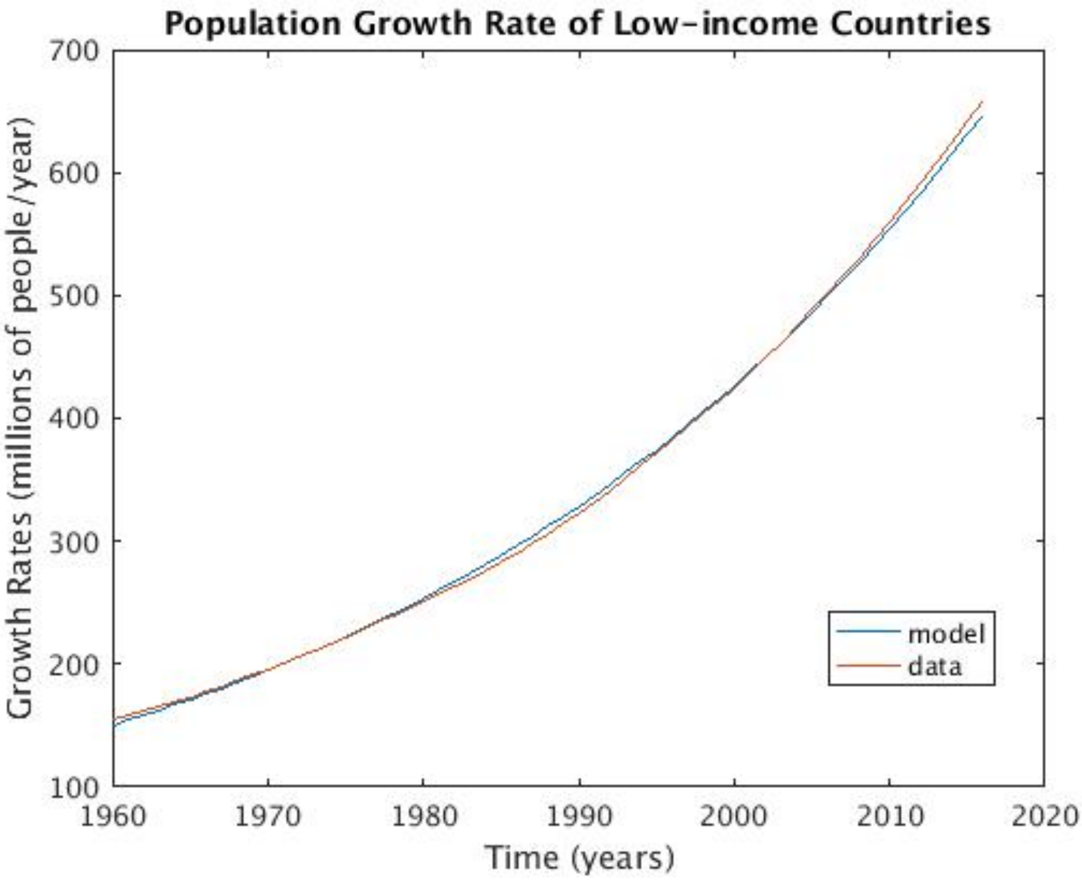
```
log(P) = logP0 + a*t
P= polyfit(year, log(low_country),1);
a = 0.0261
P0 = -32.2768
```

f) Write an expression for the growth rate of low-income countries based on the model you derived in (e).

```
Low_Growth = exp(P(2)+P(1)*year)
```

g) Compare the actual population of low-income countries with that predicted by the model you derived in (e) (i.e. plot both model and data on the same plot). Using the exponential growth model, predict what the population of low-income countries will be in 2050.

```
figure(4)
% exponential model
plot(year, Low_Growth./(10^6), '-')
hold on
% actual income
plot(year, low_country./(10^6), '-')
xlabel('Time (years)')
ylabel('Growth Rates (millions of people/year)')
title('Population Growth Rate of Low-income Countries')
% the two plots are pretty close
Low_Country_50 = exp(P(2)+P(1)*50)
ans = 3.5363e-14
```

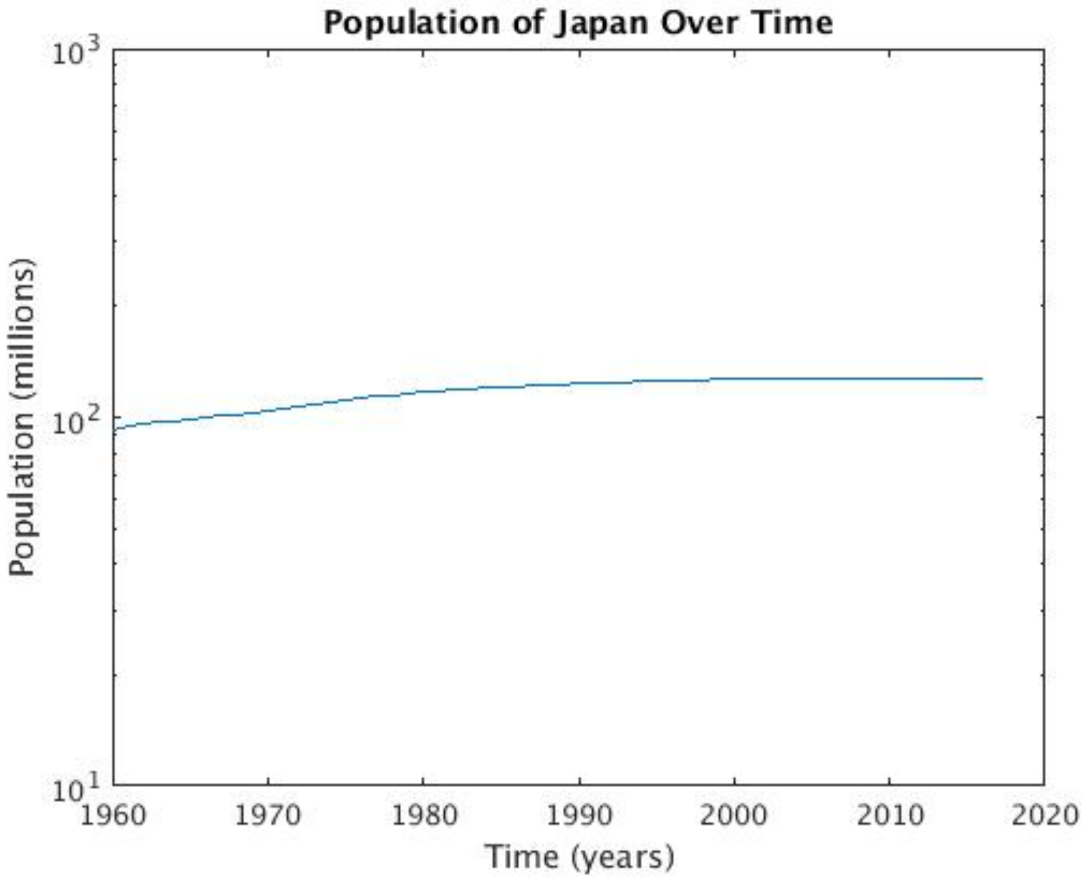


The two plots are pretty close. The population of low income country will be (3.5363e-14) million in 2050.

2. Analyze the population of Japan.
a) Plot the population of Japan (in millions of people) over time.

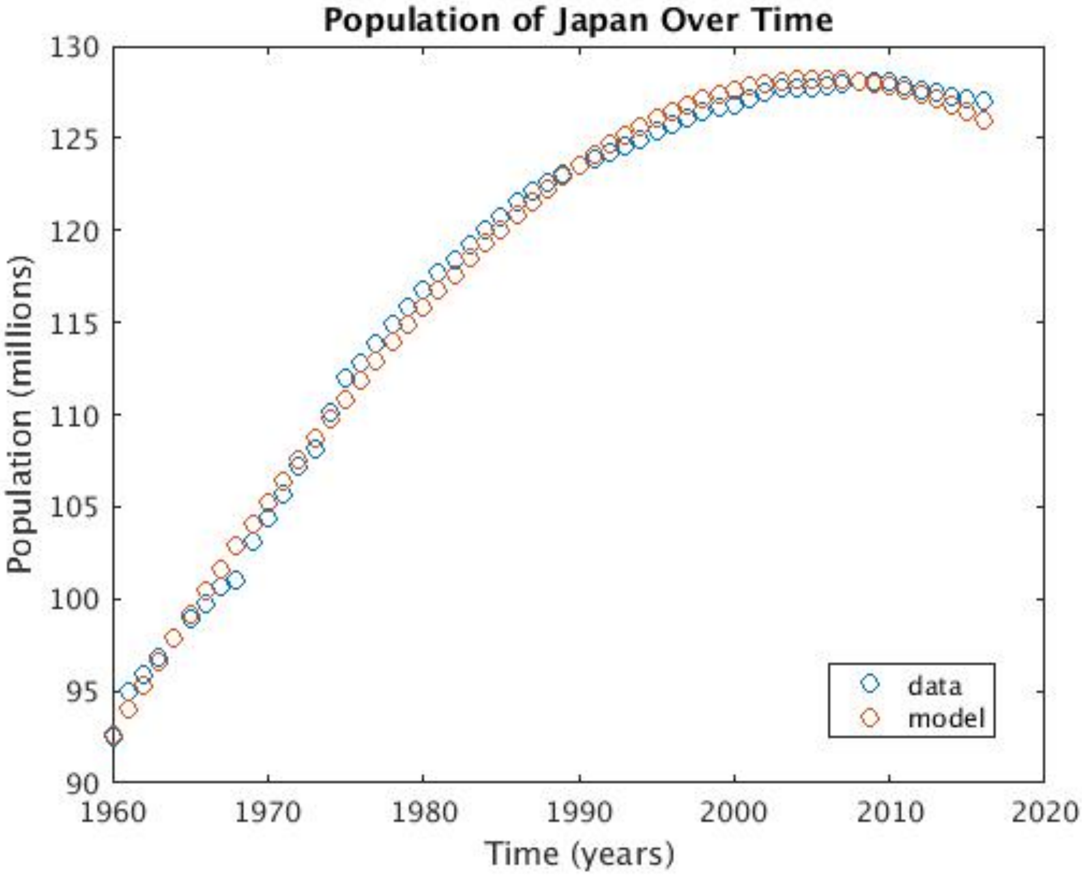
```
pop_japan = data(ijapan,:);
figure(5)
plot(year, pop_japan/(10.^6),'-')
xlabel('Time (years)')
ylabel('Population (millions)')
set(gca, 'YScale','log')
```

```
title('Population of Japan Over Time')
```



b) Fit a 2nd-degree polynomial to describe the population of Japan as a function of time since 1960 (use the MATLAB function polyfit). Write the equation. Plot the modeled population of Japan on the same plot as (a).

```
P= polyfit(year, log(pop_japan),2);
Growth_Japan = exp(P(1)*year.^2 + P(2)*year+P(3));
figure(10)
plot(year, pop_japan/(10.^6),'o')
hold on
plot(year, Growth_Japan/(10.^6), 'o')
xlabel('Time (years)')
ylabel('Population (millions)')
```



c) Expand the polynomial you determined in (b) in a Taylor series at year 2016. Write the expression.

```
t= year
Taylor1 = exp(P(1)*t.^2 + P(2).*t + P(3));
Taylor2 = 2.*t.^3*(P(1)).^2 + 3.*(t.^2).*P(1).*P(2) + 2.*t.*P(1).*P(3) + t.*(P(2)).^2 +
P(2)*P(3);
Taylor3 = 6.*(P(1).^2).*t.^2 + 6.*P(1).*P(2).*t + 2.*P(1).*P(3) + P(2).^2;

Taylor_3_term = (Taylor1)./(10.^6) + Taylor2.*(50-16) + Taylor3.*(50-16).^2.*(1/2);
```

d) Predict the population of Japan in 2050 in 3 different ways: (i) Using the polynomial model you determined in (b), (ii) using only the first 2 terms in the Taylor series you determined in (c), and (iii) using the first 3 terms in the Taylor series you determined in (c). Discuss how the answers compare.

```
t= 2050;
Taylor1 = exp(P(1)*t.^2 + P(2).*t + P(3));
Taylor2 = 2.*t.^3*(P(1)).^2 + 3.*(t.^2).*P(1).*P(2) + 2.*t.*P(1).*P(3) + t.*(P(2)).^2 +
P(2)*P(3);
Taylor3 = 6.*(P(1).^2).*t.^2 + 6.*P(1).*P(2).*t + 2.*P(1).*P(3) + P(2).^2;

Taylor_term = Taylor1./(10.^6);
```

```
Taylor_term = 93.9109

Taylor_2_term = (Taylor1 )./(10.^6)+ Taylor2.*(50-16);
Taylor_2_term = 85.1759

Taylor_3_term = (Taylor1)./(10.^6) + Taylor2.*(50-16) + Taylor3.*(50-16).^2.*(1/2);
Taylor_3_term = 81.9546
```

The result of first 3 terms in the Taylor series is more accurate than the result of first 2 terms in the Taylor series. The first 2 terms in the Taylor series is more accurate than the polynomial model.