GEOG210A Assignment Week8

Question1

Write system of differential equations above in matrix-vector form:

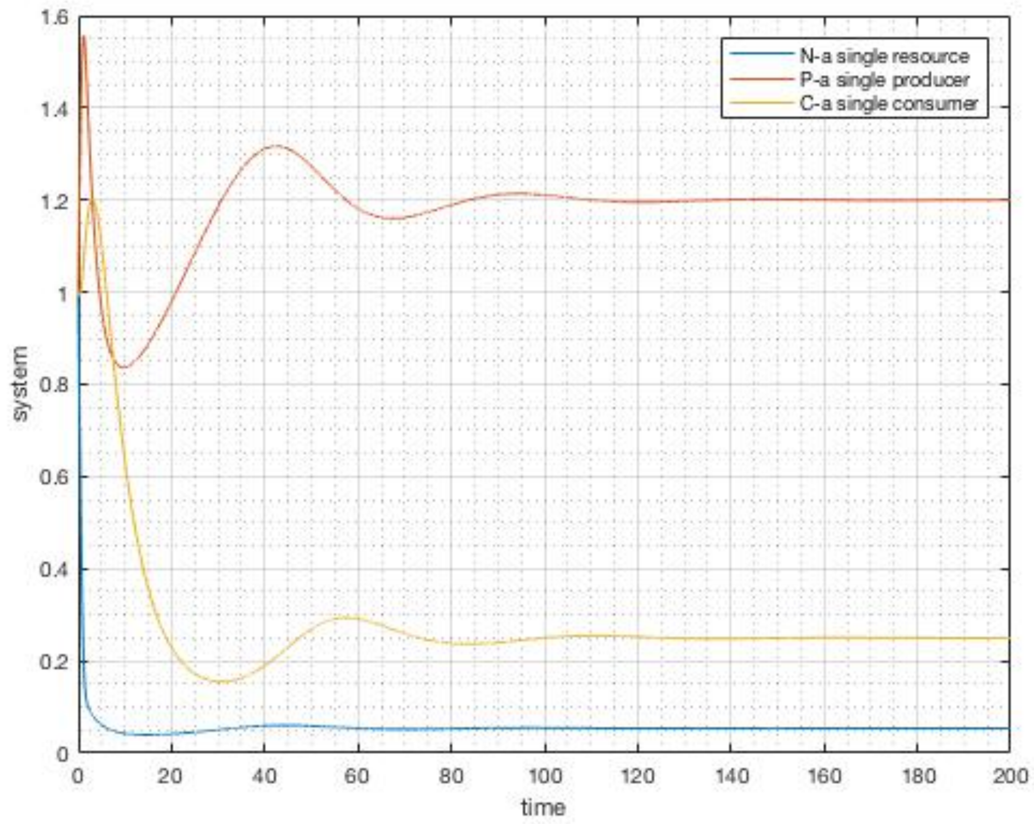
```
A = [0, qp, qc; 0, -qp, 0;0,0,-(w+qc)];
q = @(N,P,C)[- kR*(N./(N+kN));kR*(N./(N+kN)) - a.*P.*C;a.*P.*C];
r = [S;0;0];
```

Question2

a) Plot N, P, and C as a function of time as the system approaches steady-state.

```
qp = 1/2;
qc = 1/5;
kR = 2;
kN = 1/10;
a = 1/3;
S = 1/20;
w = 1/5;

A = [0, qp, qc; 0, -qp, 0;0,0,-(w+qc)];
q = @(N,P,C)[- kR*(N./(N+kN));kR*(N./(N+kN)) - a.*P.*C;a.*P.*C];
r = [S;0;0];

% time domain
dt = .01;
t = [0:dt:200];
nt = length(t);

% initial conditions
v0 = [1 1 1]';

v(:,1) = v0;

% pass through
for i = 2:nt
    v(:,i) = v(:,i-1) + dt*(A*v(:,i-1)+q(v(1,i-1),v(2,i-1),v(3,i-
1))+r);
end

figure(1)
plot(t,v')
legend('N-a single resource','P-a single producer','C-a single
consumer')
xlabel('time')
ylabel('system')
```

```
grid on
grid minor
```

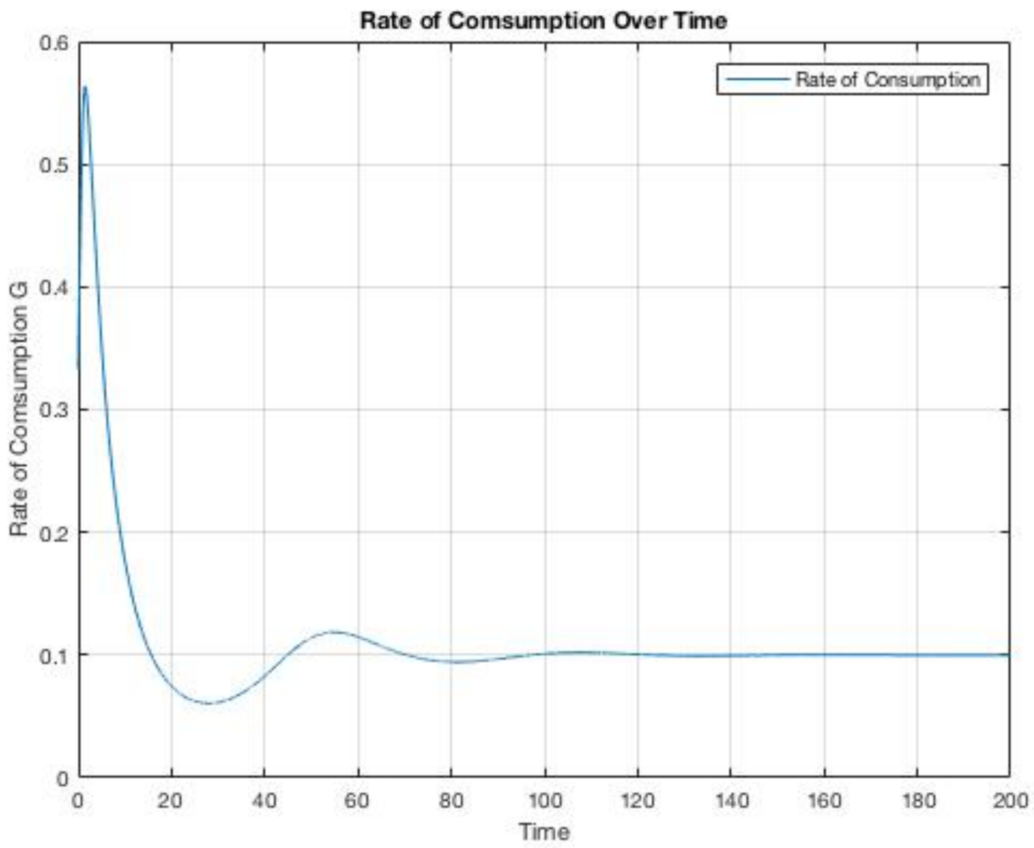


Plot #1

b) Plot the rate of consumption (G) as a function of time as the system approaches steady-state. What is the rate of consumption at steady-state?

```
for i = 1:nt
    G(i) = a*v(2,i)*v(3,i);
end
figure(2)
plot(t,G)
legend('Rate of Consumption');
set(gca,'YGrid','on')
set(gca,'XGrid','on')
xlabel('Time')
ylabel('Rate of Comsumption G')
title('Rate of Comsumption Over Time')
```

From plot2, we could tell that the rate of consumption at steady-state is around 0.1.



Plot #2

Question3

Write the Jacobian for the system of differential equations

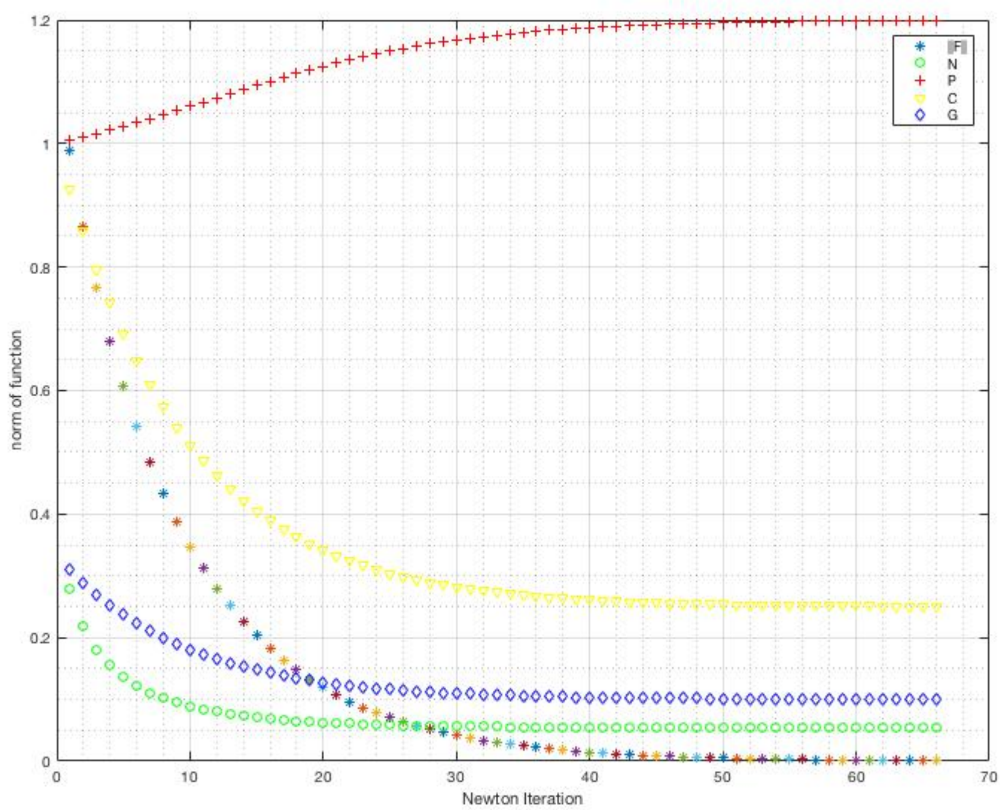
```
function [F,J] = jacobian(v, A, r)
qp = 1/2;
qc = 1/5;
kR = 2;
kN = 1/10;
a = 1/3;
S = 1/20;
w = 1/5;

N = v(1);
P = v(2);
C = v(3);
q = [- kR*(N./(N+kN));kR*(N./(N+kN)) - a.*P.*C;a.*P.*C];
F = A*v + q +r;

dqdv = [-kR*kN*((N+kN).^(-2)),0,0; kR*kN*((N+kN).^(-2)), -
a*C,-a*P;0,a*C,a*P];
J = A + dqdv;
```

a) Plot ||F(v)|| and the value of C, N, and P at each Newton iteration. See plot3.

b) Compare the steady-state values of N, P, C, and G determined by the Newton method to those determined by numerical integration. Are they the same?

```
% linear part of system (A)
A = [0, qp, qc; 0, -qp, 0;0,0,-(w+qc)];

% constants
r = [S;0;0];

% initial guess
v = [1 1 1]'; % v = [N P C]'

% iterate to solution with Newton's method
F = jacobian(v,A,r); % initial function evaluation
itc = 0; % iteration counter
while norm(F)> 1e-3 % 1e-3 tolerance

    % evaluate function and Jacobian
    [F,J] = jacobian(v,A,r);

    % update v with Newtion step
    v = v - .1*(J\F);

    % update iteration counter
    itc = itc + 1;

    % collect (N,P,C) values
    Ni(itc)=v(1);
    Pi(itc)=v(2);
    Ci(itc)=v(3);
    Gi(itc) = a*v(2)*v(3);
    % evaluate function and Jacobian
    [F,J] = jacobian(v,A,r);


    % plot the norm of the function
    figure(3)
    plot(itc,norm(F),'*','DisplayName',['Iteration',
num2str(itc)])
    hold on
    plot(itc,v(1),'-o','DisplayName',['Iteration',
num2str(itc)])
    hold on
    plot(itc,v(2),'o','DisplayName',['Iteration',
num2str(itc)])
    hold on
    plot(itc,v(3),'-','DisplayName',['Iteration',
num2str(itc)])
    hold on
```

```
    plot(itc,a*v(2)*v(3),'bd','DisplayName',['Iteration',
num2str(itc)])
    legend('||F||','N','P','C','G')
    drawnow % tell matlab to make plot
    set(gca,'YGrid','on')
    set(gca,'XGrid','on')
    xlabel('Newton Iteration')
    ylabel('norm of function')
end
grid minor
```



Plot#3

Numerical Integration	N =0.05	P=1.2	C=0.25	G=0.1
Newton's Method	N = 0.05	P=1.2	C=0.25	G=0.1

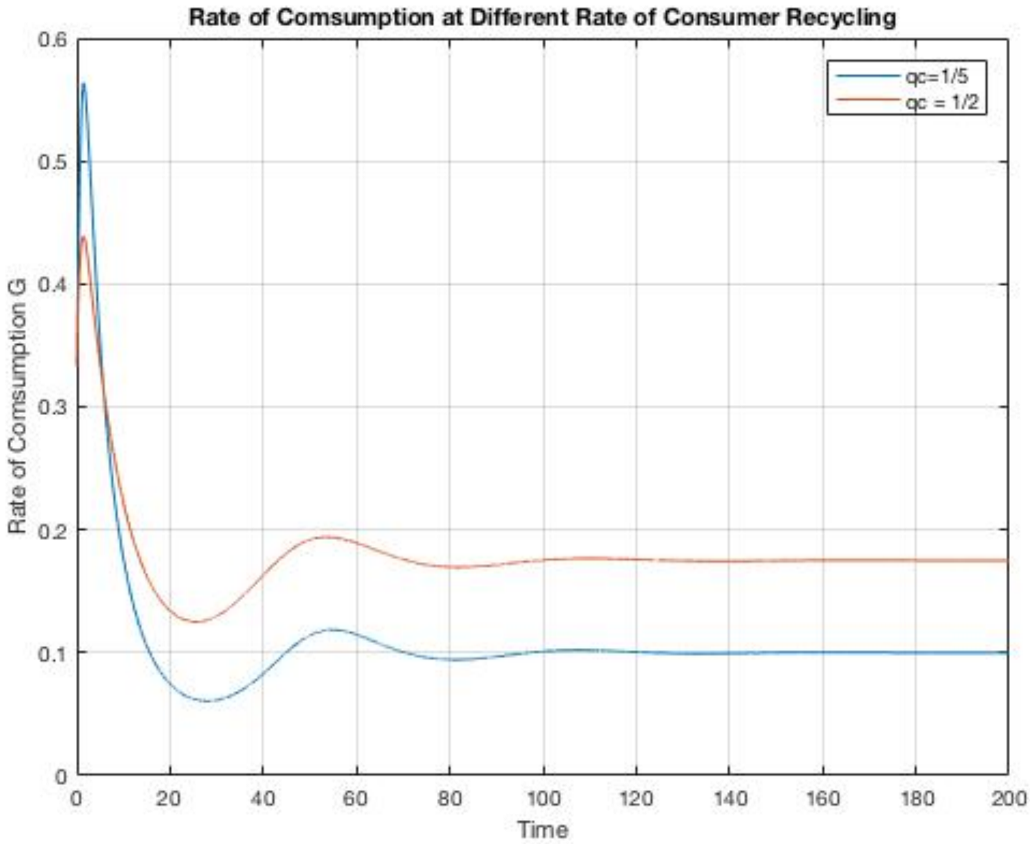
The steady-state values of N, P, C, and G determined by the Newton method and those determined by numerical integration are the same.

Question4 (Extra Credit)

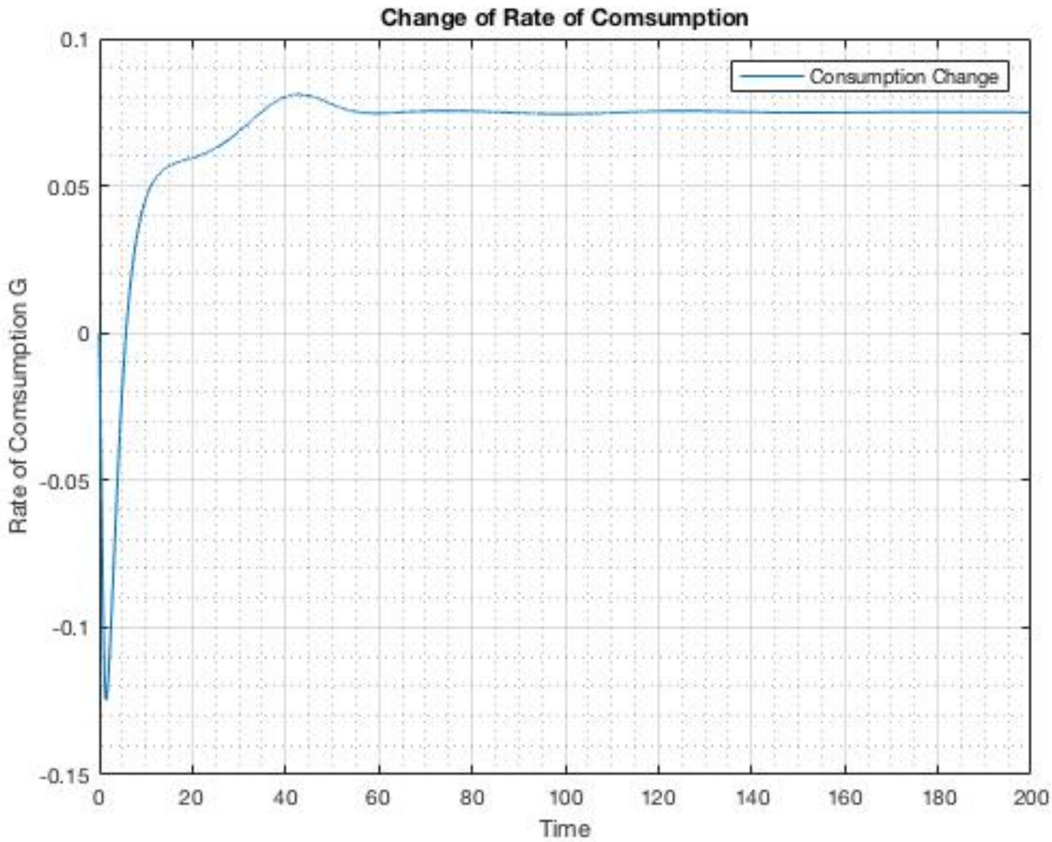
Starting at the steady-state you found in Question 2, repeat Question2 assuming the rate of consumer recycling increases to qc = 1/2. How much more (or less) consumption is allowed at steady-state compared to the case when qc =1/5?

```
qp = 1/2;
qc = 1/5;
qc1 = 1/2;
kR = 2;
kN = 1/10;
a = 1/3;
S = 1/20;
w = 1/5;

A = [0, qp, qc; 0, -qp, 0;0,0,-(w+qc)];
A1 = [0, qp, qc1; 0, -qp, 0;0,0,-(w+qc1)];
q = @(N,P,C)[- kR*(N./(N+kN));kR*(N./(N+kN)) -
a.*P.*C;a.*P.*C];
r = [S;0;0];

% time domain
dt = .01;
t = [0:dt:200];
nt = length(t);

% initial conditions
v0 = [1 1 1]';

v(:,1) = v0;
v1(:,1) = v0;

% pass through
for i = 2:nt
    v(:,i) = v(:,i-1) + dt*(A*v(:,i-1)+q(v(1,i-1),v(2,i-
1),v(3,i-1))+r);
    v1(:,i) = v1(:,i-1) + dt*(A1*v1(:,i-1)+q(v1(1,i-
1),v1(2,i-1),v1(3,i-1))+r);
end

for i = 1:nt
    G(i) = a*v(2,i)*v(3,i);
    G1(i) = a*v1(2,i)*v1(3,i);
end
figure(4)
plot(t,G)
hold on
plot(t,G1)
legend('qc=1/5','qc = 1/2');
set(gca,'YGrid','on')
set(gca,'XGrid','on')
xlabel('Time')
ylabel('Rate of Comsumption G')
```

```
title('Rate of Comsumption at Different Rate of Consumer
Recycling')

figure(5)
plot(t,G1-G)
legend('Consumption Change');
set(gca,'YGrid','on')
set(gca,'XGrid','on')
grid minor
xlabel('Time')
ylabel('Rate of Comsumption G')
title('Change of Rate of Comsumption')
```



Plot#4



Plot#5

From plot4, we could tell that consumption has increased during steady state after change qc to ½. From plot 5 which represent the difference between the rate of consumption of qc = ½ and qc = 1/5, we could tell that the rate of consumption has increased by 0.075 each time unit.