OBHPC - maths
CM2

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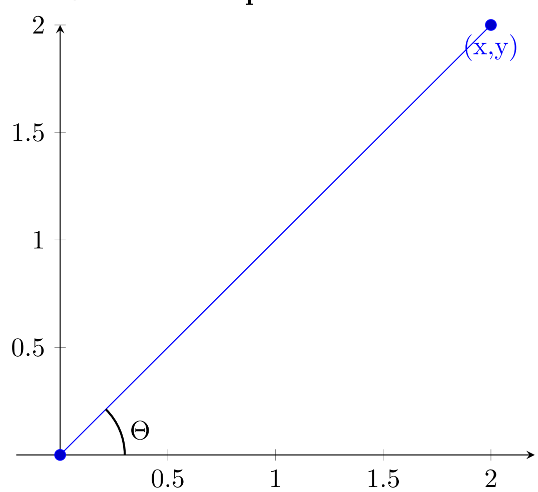
- Coordonnées polaires



$\boxed{x = R\cos(\Theta)\ y = R\sin(\Theta)} \rightarrow \frac{y}{x} = tan(\Theta)$

$R = \sqrt{x^2 + y^2}\ \Theta = arctan(\frac{y}{x})$

$F : \mathbb{R} \rightarrow \mathbb{R} \qquad G : \mathbb{R} \rightarrow \mathbb{R} \qquad \boxed{(F \circ G)' = G' * F' \circ G}$

$u : \mathbb{R}^2 \rightarrow \mathbb{R} \quad u(x, y)$

$\phi : (x, y) \quad \theta(x, y)$

$\phi = \alpha y + \beta x \qquad \begin{pmatrix} \phi \\ \theta \end{pmatrix} = \begin{pmatrix} \beta & \alpha \\ \delta & \gamma \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

$\theta = \gamma y + \delta x$

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$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \phi}\underbrace{\frac{\partial \phi}{\partial x}}_{\beta} + \frac{\partial u}{\partial \theta}\underbrace{\frac{\partial \theta}{\partial x}}_{\delta}$$

$$\frac{\partial^2 u}{\partial^2 x} = \frac{\partial}{\partial x}(\frac{\partial u}{\partial \phi})\beta + \frac{\partial}{\partial \theta}(\frac{\partial u}{\partial \theta})\delta$$

$$= \beta\frac{\partial}{\partial \phi}(\frac{\partial u}{\partial x}) + \frac{\partial}{\partial \theta}(\frac{\partial u}{\partial x})$$

$$= \beta\frac{\partial}{\partial \phi}(\beta\frac{\partial u}{\partial \phi} + \delta\frac{\partial u}{\partial \theta}) + \delta\frac{\partial}{\partial \theta}(\beta\frac{\partial u}{\partial \phi} + \delta\frac{\partial u}{\partial \theta})$$

$$= \beta^2(\frac{\partial^2 u}{\partial^2 \phi}) + \delta^2(\frac{\partial^2 u}{\partial^2 \theta}) + 2\beta\delta(\frac{\partial^2 u}{\partial \phi \partial \theta})$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial \phi}\underbrace{\frac{\partial \phi}{\partial y}}_{\alpha} + \frac{\partial u}{\partial \theta}\underbrace{\frac{\partial \theta}{\partial y}}_{\gamma}$$

$$\frac{\partial^2 u}{\partial^2 \partial x \partial y} = \alpha\frac{\partial}{\partial x}(\frac{\partial u}{\partial \phi}) + \gamma\frac{\partial}{\partial x}(\frac{\partial u}{\partial \theta})$$

$$= \alpha\frac{\partial}{\partial \phi}(\frac{\partial u}{\partial x}) + \gamma\frac{\partial}{\partial \theta}(\frac{\partial u}{\partial x})$$

$$= \alpha\frac{\partial}{\partial \phi}(\beta\frac{\partial u}{\partial \phi}) + \delta\frac{\partial}{\partial \theta}(\frac{\partial u}{\partial \theta}) + \gamma\frac{\partial}{\partial \theta}(\beta\frac{\partial u}{\partial \phi} + \delta\frac{\partial u}{\partial \theta})$$

$$= \alpha\beta\frac{\partial^2 u}{\partial^2 \phi} + \gamma\delta\frac{\partial^2 u}{\partial^2 \theta} + (\alpha\delta + \gamma\beta)\frac{\partial u}{\partial \phi \partial \theta}$$

- Changement variable linéaire

$u : \mathbb{R}^2 \to \mathbb{R}$

$u(x,y) \quad \phi(x,y) \quad \theta(x,y)$

$\phi = \alpha y + \beta x \qquad \theta = \gamma y + \delta x$

$$\begin{pmatrix}\phi\\ \theta\end{pmatrix} = \begin{pmatrix}\beta & \alpha\\ \delta & \gamma\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}$$

$\beta\gamma - \alpha\delta \neq 0$

$$A\frac{\partial^2 u}{\partial^2 x} + B\frac{\partial^2 u}{\partial y \partial x} + C\frac{\partial^2 u}{\partial^2 y} = 0$$

$$\frac{\partial^2}{\partial^2 \phi} \to \underbrace{A\beta^2 + B\alpha\beta + C\alpha^2}_{E}(= 0) \Rightarrow A\begin{pmatrix}\beta\\ \alpha\end{pmatrix}^2 + B\begin{pmatrix}\beta\\ \alpha\end{pmatrix} + C$$

$$\frac{\partial^2 u}{\partial^2 \theta} \to \underbrace{A\delta^2 + B\gamma\delta + C\gamma^2}_{F}(= 0) \Rightarrow A\begin{pmatrix}\delta\\ \gamma\end{pmatrix}^2 + B\begin{pmatrix}\delta\\ \gamma\end{pmatrix} + C$$

$$\frac{\partial^2 u}{\partial \phi \partial \theta} \to \underbrace{A2\beta\delta + B(\alpha\delta + \beta\gamma) + C(2\alpha\gamma)}_{G}$$

on suppose $A \neq 0$ et $C \neq 0$

$\Delta = B^2 - 4AC \qquad \alpha = \gamma = 2A$

$\beta = -B - \sqrt{\Delta} \qquad \delta = -B - \sqrt{\Delta}$

$\frac{\beta}{\alpha} = \frac{-B+\sqrt{\Delta}}{2A}$

$2(-B + \sqrt{\Delta})A - (2A(-B - \sqrt{\Delta}))$

$$\Downarrow$$

$\cancel{-2AB} + 2A\sqrt{\Delta} + \cancel{2AB} + 2A\sqrt{\Delta}$

$E\frac{\partial^2 u}{\partial^2 \phi} + F\frac{\partial^2 u}{\partial\phi\partial\theta} + G\frac{\partial^2 u}{\partial^2\theta}$

- $\boxed{\Delta > 0}$ $\qquad \underbrace{\beta = -B - \sqrt{\Delta} | \delta = -B + \sqrt{\Delta} | \alpha = \gamma = 2A}_{E = 0 \text{ et } G = 0 \qquad \frac{\partial^2 u}{\partial\phi\partial\theta} = 0}$ **- HYPERBOLIQUE**

- $\boxed{\Delta > 0}$ **- ELLIPTIQUE**

- $\boxed{\Delta = 0}$ $\Delta = 0 \quad B^2 = 4AC \quad \frac{\beta}{\alpha} = \frac{-B+(\sqrt{\Delta})=0}{2A} \quad B = -\beta$ **- PARABOLIQUE**

$\alpha = 2A \quad \gamma = 0 \quad \delta = 1 \quad E = 0 \quad G = 0 \quad 2A \neq 0 \quad \frac{\partial^2 u}{\partial^2\theta} = 0$