FINAL PROJECT REPORT
ON

Forge Roll Analysis

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ME0041: Engineering Design
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I. Introduction

Roll forging is a technique used to create preforms - a billet with redistributed mass to facilitate subsequent forging processes. The goal of this report is to analyze static and dynamic properties of the roll forging system and assess whether the given assumptions in conjunction with the deduced assumptions result in a functional system which will withstand static and dynamic loading conditions without failure. Reaction forces at bearings, failure analysis for static and dynamic loading, gear analysis, and deflection of the roll dies will be studied.

Minimum Bearing C_{10} value (kN)	2315.92
Static Failure Factor of Safety	4.096
First Cycle Yielding Factor of Safety	22.7
Goodman - Fatigue Factor of Safety	8.18
Train value	0.2
Maximum deflection (mm)	0.182
Deflection angle at bearings (rad)	-0.000822

Table 01. Executive Summary of project results

Above is a table of the important results from the report. There are a few particular numbers that should be noted. The lowest factor of safety obtained is 4.096 in static, single cycle failure. However, this number is well above 1 and it can be safely assumed that this roller will never fail due to single cycle static failure. Overall, this roll forge is designed with intended high safety factors in order to protect the investment from any damage and allow it to be reliably used longer than the minimum cycle.

Material property	Billet material (AISI 1055)	Forging rolls (H-13 Tool Steel)
Ultimate tensile strength (MPa)	660	1990
Yield strength (MPa)	560	1650
Modulus of elasticity (GPa)	190-210	210
Poisson’s ratio	0.27-0.30	0.3
Elongation at break (%)	10	9
Brinell hardness	197	--
Vickers hardness	207	--

Table 02. Material properties of AISI 1055 and H-13 tool steel

II. Assumptions

1. Forge rolling orientation is longitudinal.
2. The degrees of rotation for the roll forge dies is 180°.
3. The desired preform geometry is two connected rods.
4. The starting billet diameter is 0.070 m.
5. The starting billet length is 0.626 m.
6. The diameter and length of the billet are unchanged per forge pass.
7. The material of the billet is AISI 1055.
8. The material of the roll forging dies is H-13 tool steel.
9. The material of the bearings is H-13 tool steel.
10. The bearings on each side are identical.
11. Peak radial force is 1410 kN.
12. Peak torque is 25.7 kN*m.
13. The distributed loads are simplified to point forces.
14. The dimensions of the roll forge match those provided on the project assignment slides
15. The working temperature of the billet is 1100℃.
16. The temperature of the roll forges will be ignored as H-13 tool steel is designed for high temperatures.
17. The roll speed of the dies are 0.5 rev/s.
18. 4 passes are required per preform.
19. The cycle time of the roll forging dies is 360 preforms/h.
20. The dies run for 8 h a day.
21. 260 days are needed to produce the desired quantity of preforms.

III. Free body diagrams

Assumption: The dimensions of the roller match the image provided in the assignment slides.

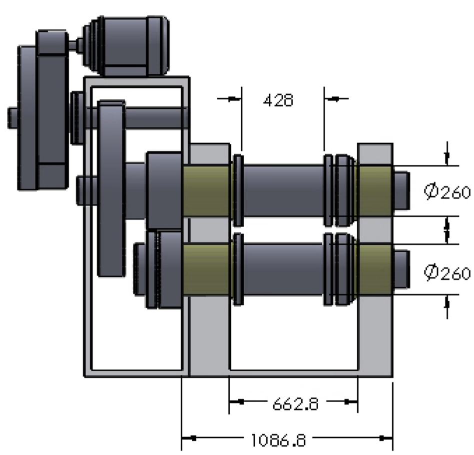

Figure 01. Dimensions of the roll forge as provided in the assignment slides

Assumption: The bearings are all identical in size.

The system was simplified to the region between the outer edges of the bearings. The region between the bearings was also simplified as a plain cylinder, without the ridges shown in the picture. The billet was placed at the center of the roller with a width of 0.07 m, and the identical bearings on either side counteracted the load by reacting in the opposite direction at half the magnitude of the total force. To simplify some calculations, a second diagram was made with the distributed loads condensed into point forces at the bearings.

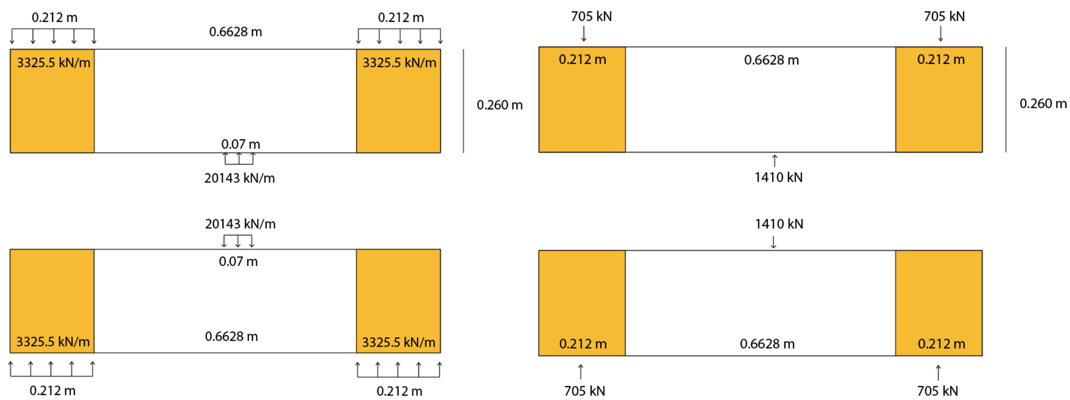

Figure 02. Simplified FBDs of the top and bottom forge rollers

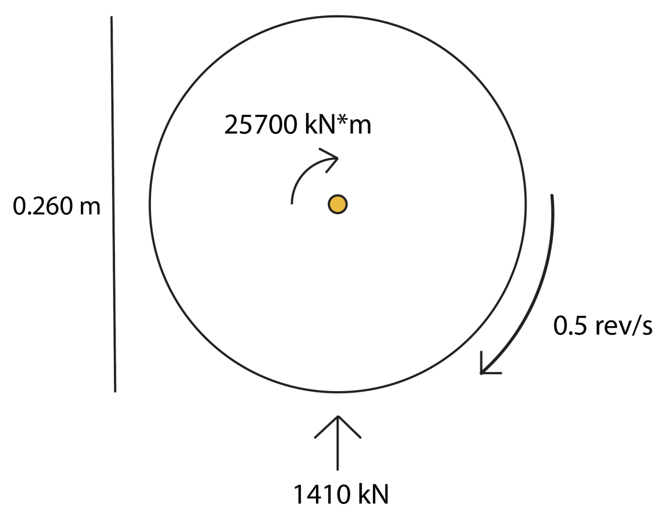

Figure 03. FBD of a single forge roller as seen at the cross-section

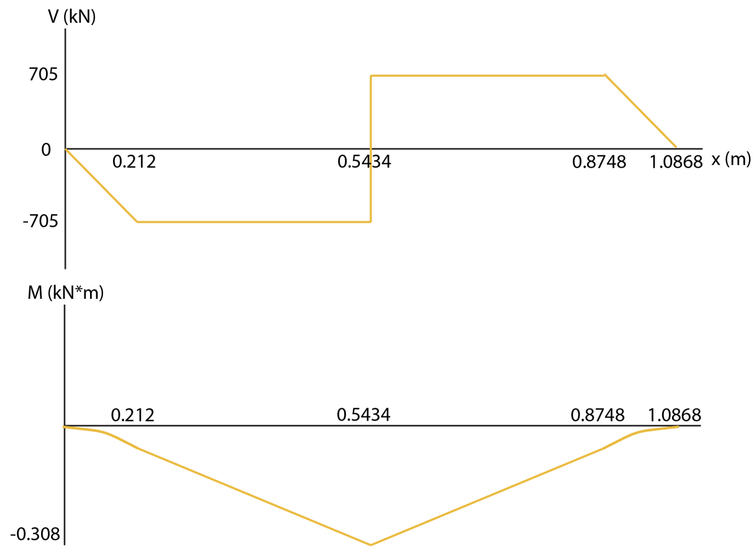

Figure 04. Shear and moment diagrams of the roller

IV. Bearing analysis

The first step in choosing bearings is to decide what type of bearings are going to be used, whether it be ball bearings or roller bearings or some other type.

Since the forge will be operating at low speeds, high radial forces, minimal to no axial load, and very low (to be discussed later) deflection, a roller bearing is the most practical type of bearing to be used in the forge.

In order to solve for the bearings of the roller, the Weibull distribution of bearing values is used to approximate a minimum C_{10} value required to support the forge for its entire lifetime.

Type of Application	Load Factor
Precision gearing	1.0–1.1
Commercial gearing	1.1–1.3
Applications with poor bearing seals	1.2
Machinery with no impact	1.0–1.2
Machinery with light impact	1.2–1.5
Machinery with moderate impact	1.5–3.0

Figure 05. Load factor for bearings. The load factor applied was 1.5, taken from Machinery with light impact on the higher end.

		Weibull Parameters Rating Lives		
Manufacturer	**Rating Life, Revolutions**	x_0	θ	b
1	$90(10^6)$	0	4.48	1.5
2	$1(10^6)$	0.02	4.459	1.483

Figure 06. Weibull Parameters. Manufacturer 2 was used due to the appropriate rating life.

$$x_0 = 0.02,\ R_D = 0.99, F_D = 705E3\ N,\ \theta = 4.459, a = \frac{10}{3}, b = 1.483,\ a_f = 1.5, x_D = \frac{2.9952E6}{1E6}$$

$$C_{10} = a_f F_D \left[\frac{x_D}{x_0 + (\theta - x_0)(ln\frac{1}{R_D})^{\frac{1}{b}}} \right]^{\frac{1}{a}} = (1.5)(705E3)\left[\frac{2.9952}{0.02 + (4.459 - 0.02)(ln\frac{1}{0.99})^{\frac{1}{1.483}}} \right]^{\frac{3}{10}} = 2315.92562\ kN$$

The next step in the process was to find a bearing from the catalog that appropriately fit the metric for rated life. In order to do this, a catalog was searched thoroughly for a roller bearing that best suited the forge in terms of size and life. The bearing found to best fit the forge is the NJG 2352 VH roller bearing. This bearing is ideal for the forge as its inner diameter is 0.260m, which is the same as the diameter of the roll forge cylinder, and has a C_{10} rating above the minimum value.

Figure 07. NJG 2352 VH bearing

The NJG 2352 VH bearing has a C_{10} rating of 3410 kN, which is well above the minimum C_{10} value, meaning the resulting reliability is actually above the desired 99%. Below is a calculation for the actual obtained reliability factor for this bearing.

$$R = 1 - \left\{ \frac{x_D \left(\frac{a_f F_D}{C_{10}} \right)^a - x_0}{\theta - x_0} \right\}^b = 1 - \left\{ \frac{2.9952 \left(\frac{(1.5)(705E3)}{3410E3} \right)^{\frac{10}{3}} - 0.02}{4.459 - 0.02} \right\}^{1.483} = 0.9991$$

$$0.9991 > 0.99$$

V. Static failure analysis of the roll forge dies

The static failure is to be analyzed for one rotation of the forge rollers. As such, it is also assumed that the roller applies the same force each pass. Only combined loading is to be considered, as the roller is most likely to fail due to static failure from combined loading.

Since the percent elongation at break is 9%, a ductile factor of safety measure will be used. In order to get a more realistic value from the factor of safety, it is best to use von Mises (or Distortion Energy) to calculate the factor of safety.

Assumption: At the moment of highest radial force, there is 5 cm of longitudinal contact between the billet and the roller.

Area of contact between the roller and the billet

$$A = .07m * .05m = .0035\ m^2$$

Assumption: Since the roller uses a semi-cylindrical die, assume that the billet length is half of the circumference of the entire forge circumference.

Look for the radius of the roll forge

$$\frac{C}{2} = .626 = \pi r$$

$$r = \frac{.626}{\pi} = .199\ meters$$

Assumption: The entire roller is made of H-13 steel, therefore the possible effect of having different materials throughout the die can be ignored.

In order to calculate the factor of safety of the die, only the maximum forces need to be considered because if it doesn't fail at the maximum, then it is definitely not going to fail when there is less force applied.

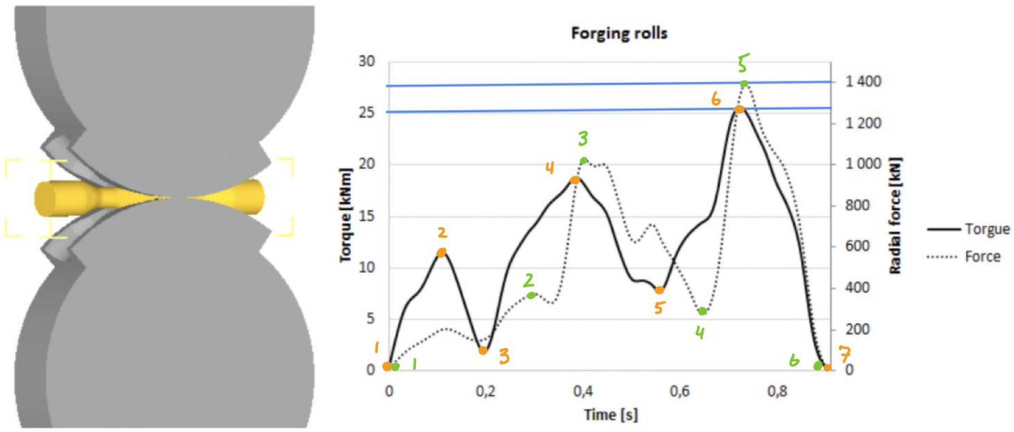

Figure 08. Plot of torque and radial load experienced by the forging rolls over 1s.

Assumption: The points of maximum torque and radial force (6 and 5 respectively) occur at the same exact time.

With all of the main assumptions out of the way, the calculations for static failure analysis and factors of safety can begin.

To begin, calculate the maximum stress applied at the moment of highest radial force.

$$\sigma_y = \frac{F}{A} = \frac{1410\times10^3}{.0035} = 402857142.857\,Pa = 402.86\,MPa$$

Then, calculate the maximum shear stress applied.

$$\tau_{xy} = \frac{16T}{\pi d^3} = \frac{16\cdot25.7\times10^3}{\pi\cdot.398^3} = 2076127.48\,Pa = 2.076\,MPa$$

Using these values, a Mohr's circle can be created.

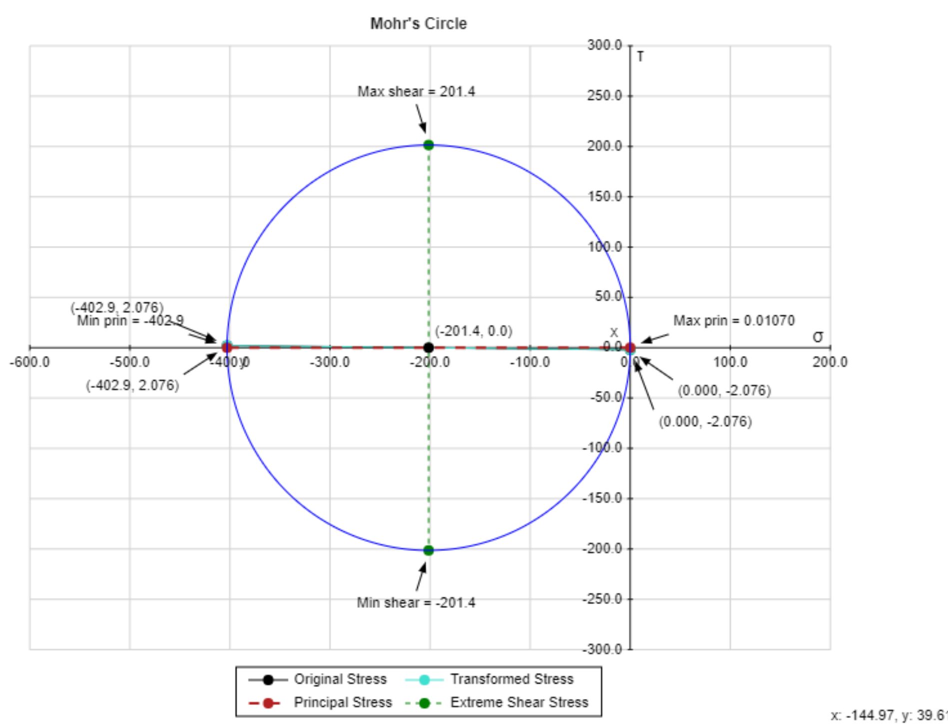

Figure 09. The figure above represents the Mohr's circle for the area of contact on the surface of the plate (estimated above to be .0035m^2). Note that the principal stress line and the original stress lines are almost collinear. This is because the torque applied to the surface of the roller is almost negligible when compared to the magnitude of the radial load.
Image Sourced: https://mechanicalc.com/calculators/mohrs-circle/

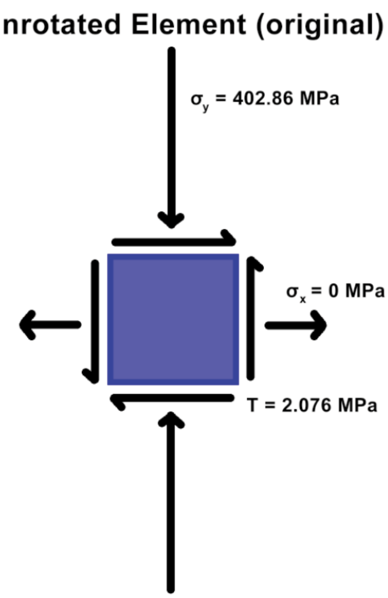

Figure 10. The figure above represents the plane stress element of the original orientation. The surface is under 402.86MPa of compression in the y-direction, 0MPa of tension in the x-direction, and is undergoing 2.076MPa shear stress.

Finally, combine the shear stress and normal stress to obtain the maximum stress.

$$\sigma' = (\sigma_y^2 + 3\tau_{xy}^2)^{\frac{1}{2}} = (402.86^2 + 3 \cdot 2.076^2)^{\frac{1}{2}} = 402.87\ MPa$$

Using the equated maximum shear stress and Mohr's circle, a new plane stress element can be drawn.

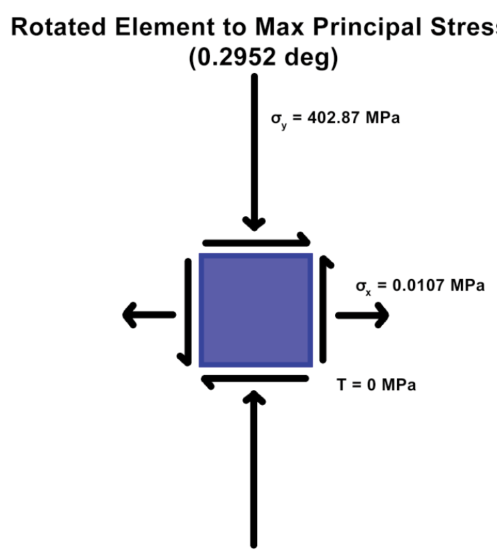

Figure 11. The figure above represents the plane stress element in the principal stress orientation. The surface is under 402.87MPa of compression in the y-direction, 0.0107MPa of tension in the x-direction, and 0MPa of shear stress.

Conceptually, this result makes sense. In the case of the roller, there are going to be some extreme forces radially, as the roller is attempting to mold and change the shape of a hot metal object. In comparison, the torque being applied to the machine by the billet and the gears is going to be relatively small.

The resulting (yet very small) stress in the x-direction also makes sense, as the internal forces of the roller will apply some tension stress in the x-direction as the roller is getting dragged along the billet. The value is extremely small because of the nature of a roller. Since it is circular in shape, it doesn't require much force to get it to roll along the axis, therefore the x-direction stress is very small and the torque required to get it to rotate is relatively small.

$$S_y = 1650\ MPa$$

With all of the values needed, calculate the factor of safety for stress.

$$\eta = \frac{S_y}{\sigma'} = \frac{1650}{402.87} = 4.096$$

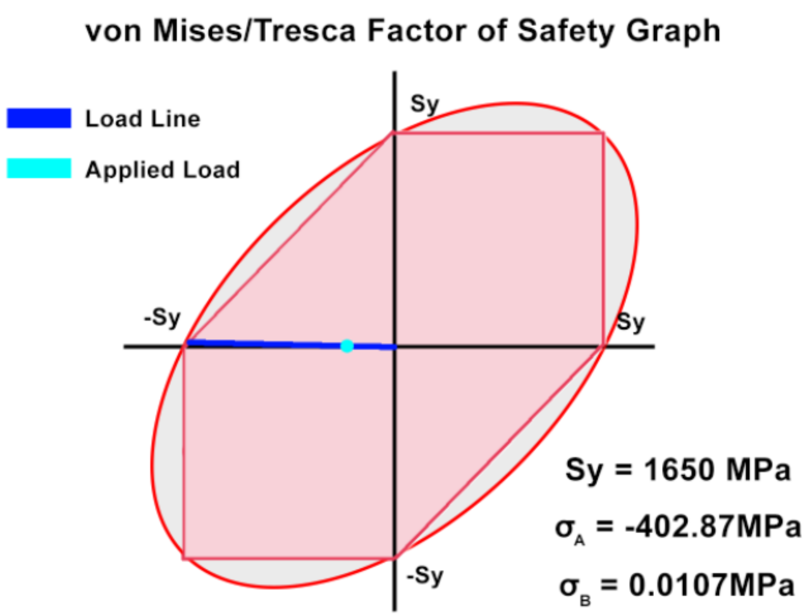

Figure 12. von Mises/Tresca factor of safety graph and load line.

The factor of safety for the stress on the roller is 4.096. While the maximum stress applied gets extremely close to the maximum allowed stress, it never goes over, so the roller is safe. The roller will not yield from the applied force. It makes sense that the designer of the roll forge would want to apply as much force as possible per pass to maximize the value that they can get out of the expensive roll forge.

Shear stress (unrotated) *(MPa)*	402.86
Shear stress (principal) *(MPa)*	402.87
Yield stress *(MPa)*	1650
Factor of safety (von Mises)	4.096

Table 03. Important values and results from static failure analysis.

VI. Roller deflection

During the forging process, it is crucial that there is no significant deflection at the center of the rollers. If significant deflection occurs, there will be an increased gap between the rollers' point of contact, decreasing the amount of force applied on the billet. This will greatly decrease the efficiency of the forge.

When the forge rollers compress together with the billet in between, forces are applied to the pair of rollers as demonstrated below.

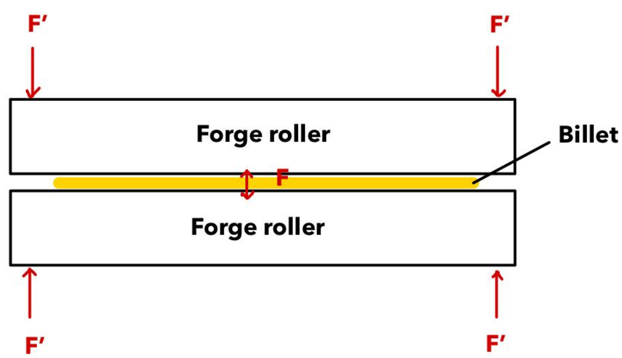

Figure 13. Internal and external forces on the pair of forge rollers with a billet in between

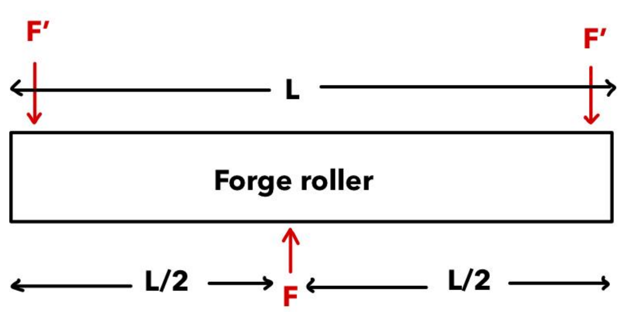

Figure 14. FBD for a single forge roller

The isolated roller shown in the FBD above is treated as a simple support model with a center load as demonstrated in the figure below. A simple support model has two end reaction forces, and one center load as shown below.

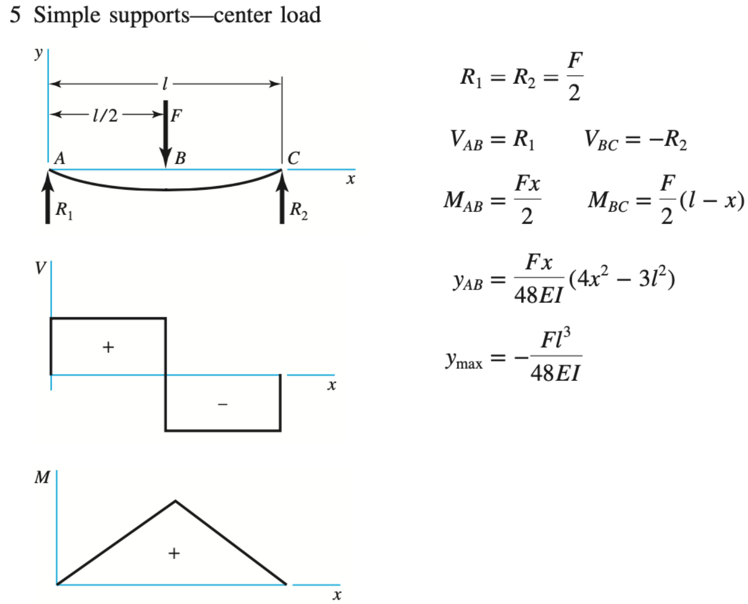

Figure 15. Simple support model from Table A-9 on p1031 in *Shigley's Mechanical Engineering Design.*

Substituting the appropriate values for the above equations in context of this application:

$$R_1 = R_2 = 3325.5(0.212) = 705.0\ kN$$

$$V_{AB} = R_1 \qquad V_{BC} = -R_2$$

$$V_{AB} = 705.0\ kN \qquad V_{BC} = -705.0\ kN$$

$$M_{AB} = \frac{Fx}{2} \qquad M_{BC} = \frac{F}{2}(l - x)$$

$$M_{AB} = \frac{1410E3(0.6628/2)}{2} = 233637\ Nm$$

$$M_{BC} = \frac{1410E3}{2}(0.6628 - 0.6628/2) = 233637\ Nm$$

$$y_{max} = \frac{Fl^3}{48EI}$$

$$y_{max} = \frac{1410E3(0.6628)^3}{48(2.1E11)\pi(0.13)^4/4} = 0.000182\ m \ = \ 0.182\ mm$$

Treating a single forge roller as a simply supported rigid object is an alright simplification of this system. However, the real system is likely closer to something in between a simply supported and fixed support system. The deflection is calculated again using Castigliano’s with the new assumption that the two ends of the forge rollers are fixed geometries.

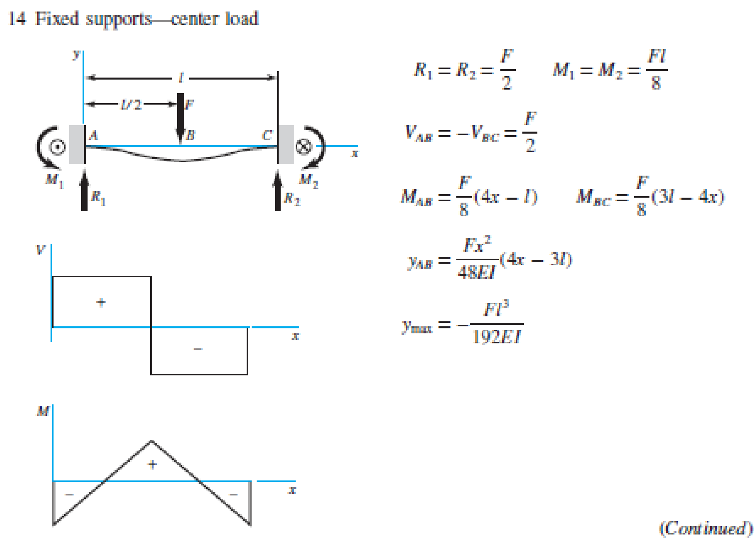

Figure 16. Fixed support model from Table A-9 on p1031 in *Shigley’s Mechanical Engineering Design.*

$$y_{max} = \frac{Fl^3}{192EI}$$

$$y_{max} = \frac{1410E3(0.6628)^3}{192(2.1E11)\pi(0.13)^4/4} = 0.0000454\ m \ = \ 0.0454\ mm$$

In order to validate Castigliano's calculations, a simplified CAD model of the roll forge was created in SOLIDWORKS. In the center of the roll forge, a rectangle the size of the assumed area of contact from a previous assumption (0.07m * 0.05m) was split out of the side. The radial force of 1410 kN was applied to the split. The sides were then set as fixed joints, the mesh was created, and the simulation was run.

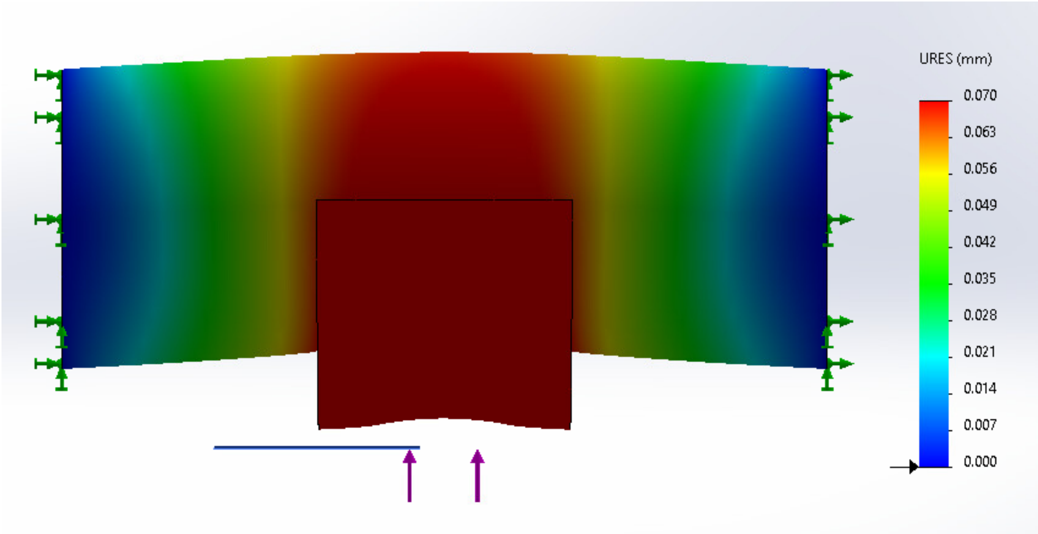

Figure 17. The deflection of the roller with an adjusted color gradient to be more representative of the displacement in the locations of the bearings. This graphic shows that the deflection of the left and right ⅓ of the roll forge is about 0.07 mm (or 0.00007m).

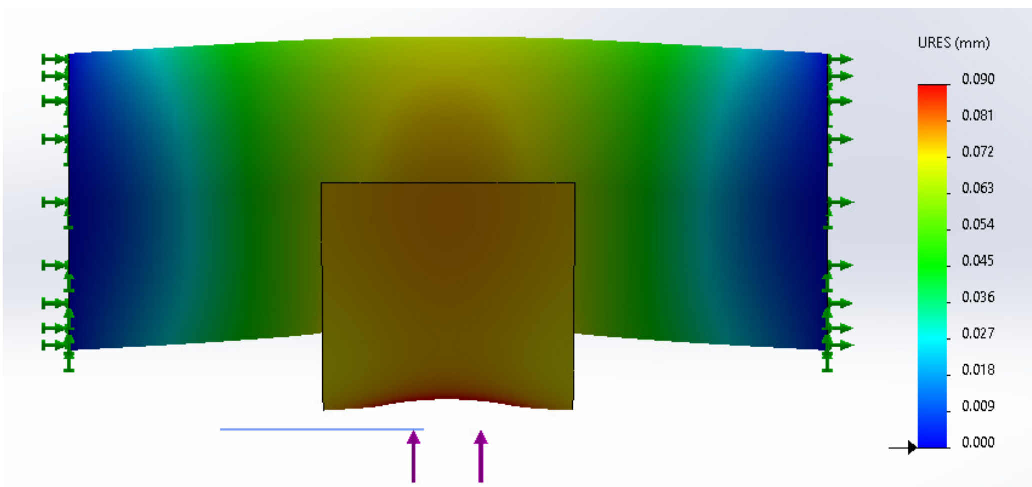

Figure 18. The deflection of the roller with an adjusted color gradient to be representative of the maximum deflection of the main section of the roller (roughly 0.090 mm).

The deflection measured by the three different methods gives an idea of the bounds of deflection from the roller. It makes sense that this deflection is less than the Castigliano's calculations, as the CAD model takes into account a thicker part for the semi-cylindrical shape of the roll forge which would increase the radius (and therefore the second moment of inertia, *I*), decreasing deflection. Intuitively, it is also more difficult to bend and thus deflect a thicker part.

To find the deflection angle along the beam, the deflection equation was differentiated.

$$y_{ab} = \frac{Fx}{48EI}(4x^2 - 3l^2)$$

$$E = 2.1E11$$

$$I = \frac{\pi(0.130)^4}{4}$$

$$\theta = \frac{d}{dx}\left(\frac{Fx}{48EI}(4x^2 - 3l^2)\right) = \frac{d}{dx}\left(\frac{F}{48EI}(4x^3 - 3l^2x)\right) = \frac{F}{48EI}(12x^2 - 3l^2)$$

To find the deflection angle at the bearings, 0 was substituted in for x.

$$\theta(0) = \frac{F}{48EI}(12(0)^2 - 3l^2) = \frac{1410E3}{48(2.1E11)(\frac{\pi(0.130)^4}{4})}(-1)(3)(0.6628)^2 = -0.000822\,rad$$

This deflection was verified with the table below.

Slopes	
Tapered roller	0.0005–0.0012 rad
Cylindrical roller	0.0008–0.0012 rad
Deep-groove ball	0.001–0.003 rad
Spherical ball	0.026–0.052 rad
Self-align ball	0.026–0.052 rad
Uncrowned spur gear	<0.0005 rad

Figure 19. Table 7-2: Typical maximum range for slopes

Simple support - center load (mm)	0.182
Fixed support - center load (mm)	0.0454
CAD (mm)	0.090
Deflection angle at bearings (rad)	-0.000822

Table 04. Calculated maximum deflections

VII. Fatigue failure analysis of the roll forge dies

From assumptions #18 - #21, the total number of passes which these forge rollers must withstand can be calculated.

$$\frac{4\,passes}{preform} \times \frac{360\,preforms}{active\,hour} \times \frac{8\,active\,hours}{day} \times 260\,days = 2,995,200\,passes$$

Active hours refers to the number of hours for which the forge rollers are running. With this large quantity of passes, it is crucial that the forge rollers do not fail by fatigue in order to ensure a secure and functional design. There are four main steps in determining the factor of safety for fatigue failure.

1. Identifying the loading condition
2. Determining mean and alternating stresses
3. Check for first cycle yielding
4. Select an appropriate failure criteria and calculate the fatigue factor of safety

i) Step 1: Identifying the loading condition

Consider a single point on the surface of one of the forging rolls. When this point is not tangentially in contact with the other forging roll (a), there is a load of 0. When the dies rotate enough so that the point makes contact with the other forging roll, there is an applied radial load (b), and when the rolls rotate so that the point of focus has surpassed the tangent point of contact, there is once again 0 load force on that point.

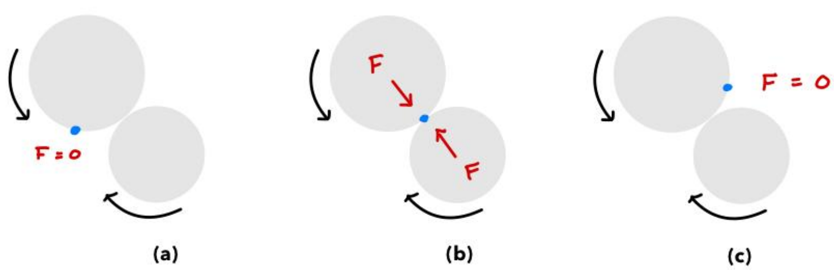

Figure 20. Demonstration of repeated load experienced by a single point on the roll forge die

The load experienced by this point during the preforming process is repeated.

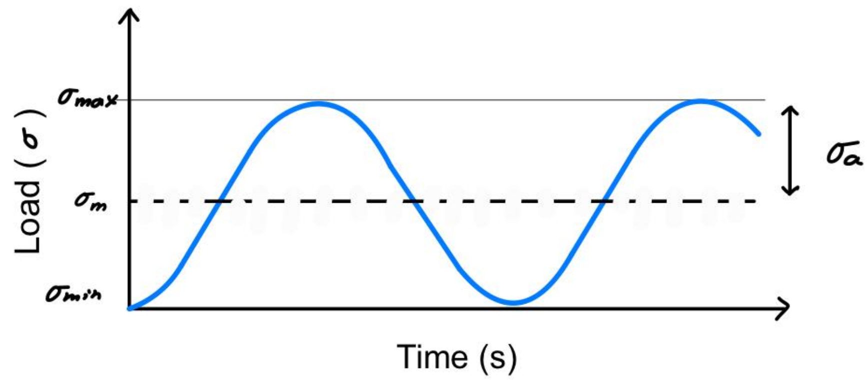

Figure 21. Repeated loading condition on a load vs. time plot

ii) Step 2: Mean and alternating stresses, endurance limit, fatigue strength

In order to perform a fatigue failure analysis on this system, the following quantities must be calculated: mean stress, alternating stress, mean torsion, and alternating torsion. A radial load and surface area of contact is necessary to calculate stress.

Assumption: The average radial load over 1 second is applied over 0.0035 m^2 as used in the static failure analysis section. The mean and alternating stresses are calculated from this average radial load and total covered surface area.

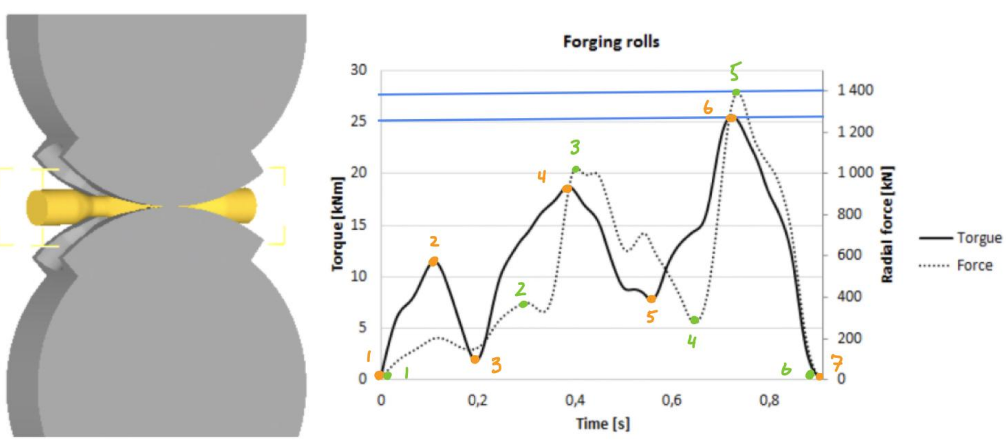

Figure 22. Plot of torque and radial load experienced by the forging rolls over 1s

In the above figure, the orange points 1-7 indicate the easiest identifiable extrema over which the torque is averaged. The green points 1-6 likewise indicate the easiest identifiable extrema over which the radial force is averaged. From these data points, the average torque and load stresses can be found.

Radial force (kN):	Torque (kNm):
1 = 0	1 = 0
2 = 400	2 = 12
3 = 1000	3 - 2.5
4 = 250	4 = 18
5 = 1400	5 = 8
6 = 0	6 = 25
F,avg = 508.3 kN	7 = 0
	T, avg = 9.36 kNm

From here, σ_a, σ_m, T_a, and T_m can be calculated.

Assumption: For each pass, the change in diameter along the preform is ignored, and the billet is assumed to have uniform diameter and length for each pass.

$$\sigma_{max} = \frac{F,avg}{A_{s,billet}} = \frac{508.3\times10^3\,[N]}{0.0035[m^2]} = 1.45 \times 10^8\ N/m^2 = 1.45 \times 10^8\ Pa$$

$$\sigma_{min} = 0\ N/m^2 = 0\ Pa$$

$$\sigma_a = \frac{\sigma_{max}}{2} = 7.25 \times 10^7\ Pa$$

$$\sigma_m = \frac{\sigma_{max}-\sigma_{min}}{2} = 7.25 \times 10^7\ Pa$$

$$T_a = 0\ kNm$$

$$T_m = 9.36 \times 10^3\ Nm$$

In order to find a factor of safety, the endurance limit and fatigue strength of the roll forge dies are calculated. The endurance limit S_e and fatigue strength f are found using appropriate correction factors.

$S_e = k_a k_b k_c k_d S'_e$

$S'_e = 700 \times 10^6 Pa$ because $S_{ut} \geq 1400 MPa$

$k_a = aS_{ut}^{\ b} = 1.38(1990[MPa])^{-0.067} = 0.830$ **Formula is made to be used with MPa.** → **Assumption:** The roll forge dies are a ground finish.

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.21	1.38	−0.067
Machined or cold-drawn	2.00	3.04	−0.217
Hot-rolled	11.0	38.6	−0.650
As-forged	12.7	54.9	−0.758

Figure 23. Curve fit parameters for surface factor

$k_b = 1$ → **Assumption:** The radius of the forge rollers is 0.199m as found in the static failure section.

$d = 0.199 \times 2 = 0.398m = 15.67"$. The diameters of the forge rollers is out of range of diameters for the size factor. Omit.

$k_c = 0.59$ for torsion because this correction factor is less than that for axial load.

$k_d = n/a$ because only the temperature of the billet is known but not of the forge rollers themselves.

$S_e = k_a k_b k_c S'_e = (0.830)(1)(0.59)(700 \times 10^6 [Pa]) = 3.43 \times 10^8\ Pa$

$f = 1.06 - (4.1 \times 10^{-4}) S_{ut} + (1.5 \times 10^{-7}) S_{ut}^2$ **Formula is made to be used with MPa.**

$= 1.06 - (4.1 \times 10^{-4})(1990) + (1.5 \times 10^{-7})(1990)^2 = 0.838$

Since this application is combined loading (torsion and radial forces), the Von Mises stresses must be used in place of the regular stresses. If the treat the two cylindrical forge rolls as shafts:

$$\tau_m = \frac{16T_m}{\pi d^3} = \frac{16(9.36 \times 10^3)}{\pi(0.199 \times 2)^3} = 756.1 \times 10^3\ Pa$$

$$\sigma'_{max} = (\sigma_a^2 + 3\tau_m^2)^{1/2} = [(7.26 \times 10^7)^2 + 3((756.1 \times 10^3)^2)]^{1/2} = 7.26 \times 10^7\ Pa$$

$$\sigma'_a = \sigma_a$$

$$\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2} = [(7.26 \times 10^7)^2 + 3(756.1 \times 10^3)^2]^{1/2} = 7.26 \times 10^7\ Pa$$

iii) Step 3: Check for first cycle yielding

In all applications, it is a good idea to check for first cycle yielding to ensure that the design will not break after a single use.

$$n_y = \frac{S_y}{\sigma'_{max}} = \frac{1650 \times 10^6 [Pa]}{7.26 \times 10^7 [Pa]} = 22.7$$

Evidently, since the design factor is much greater than 1, there is no first cycle yielding in this application - it is appropriate to move on to selecting a specific failure criterion. Notice that the factor of safety for first-cycle yielding in this section is greater than the calculated factor of safety in the static failure section of this project report. This is reasonable because a conservative failure criterion was selected for this section, while a non-conservative method was used in static failure. Additionally, the average force was used for this section as opposed to the maximum force which decreased the stress values.

iv) Step 4: Selecting a failure criterion and calculating the fatigue factor of safety

Most metals under fatigue will experience brittle-like failure (element.com). This, in conjunction with the high costs of these dies, urges the use of a brittle conservative fatigue failure criterion like the Goodman criterion.

$$n = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{7.25\times10^7\,Pa}{845.77\times10^6} + \frac{7.26\times10^7}{1990\times10^6}\right)^{-1} = 8.18$$

Once again, the design factor is greater than 1, and so there will be no failure.

v) Further analysis

In order to confirm that fatigue failure will occur instead of first cycle yielding, the load line slope r and critical slope r_{crit} are used. For a Goodman-Langer analysis (fatigue versus single cycle yielding):

$$S*m = \frac{(S_y - S_e)S_{ut}}{S_{ut} - S_e} = \frac{(1650\times10^6 - 3.43\times10^8)1990\times10^6}{1990\times10^6 - 3.43\times10^8} = 1.58 \times 10^9\,Pa$$

$$S*a = S_y - S*m = (1650 \times 10^6) - (1.58 \times 10^9) = 7 \times 10^7\,Pa$$

$$r_{crit} = \frac{S*a}{S*m}$$

$$r_{crit} = \frac{7\times10^7\,[Pa]}{1.58\times10^9\,[Pa]} = 0.044$$

$$r = \frac{\sigma_a}{\sigma_m}$$

$$r = \frac{7.25\times10^7}{7.25\times10^7} = 1$$

Since $r_{crit} < r$, it is confirmed that this system fails due to fatigue.

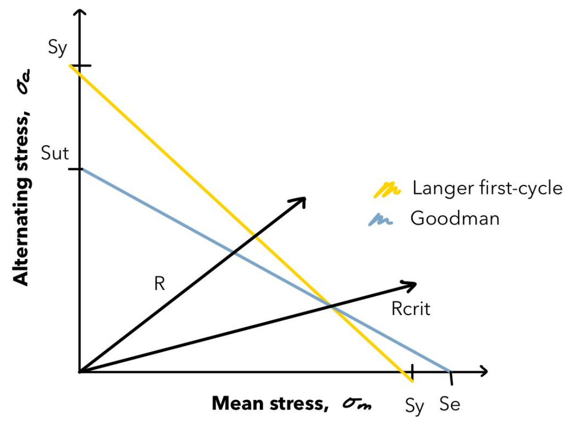

Figure 24. Mean stress vs. alternating stress for Goodman and Langer criterion with load and critical load lines.

It is also possible to calculate the maximum stress at which the forge roller can complete N passes before failure using the equation

$$S_f = aN^b$$
where
$$a = (fS_{ut})^2/S_e = (0.838 * 1990 \times 10^6)^2/3.43 \times 10^8 = 8.10 \times 10^9$$
$$b = -\frac{1}{3}log(fS_{ut}/S_e) = -\frac{1}{3}log(0.838 * 1990 \times 10^6/3.43 \times 10^8] = -0.229$$
$$S_f = 8.10 \times 10^9(2,995,200)^{-.229} = 2.66 \times 10^8\, Pa$$

The forge roller design can withstand $2.66 \times 10^8\, Pa$ at 2,955,200 cycles before failure by fatigue. This is a reasonable outcome as the maximum von mises stress calculated was $7.26 \times 10^7\, Pa$, and so the system will not fail.

Alternating stress, σ_a (Pa)	7.25×10^7
Mean stress, σ_m (Pa)	7.25×10^7
Alternating torque, T_a (kNm)	0
Mean torque, T_m (kNm)	9.36
Endurance strength, S_e (Pa)	$3.43 \times 10^8\, Pa$
Fatigue fraction, f	0.838
Maximum Von Mises stress, σ'_{max} (Pa)	7.26×10^7
Von Mises alternating stress, σ'_a (Pa)	7.26×10^7
Von Mises median stress, σ'_m (Pa)	7.26×10^7

Design factor for first cycle yield	22.7
Design factor for Goodman failure criterion	8.18

Table 05. Final updated values for fatigue failure analysis

VIII. General gear analysis

Overall main Assumptions:

- Max torque experienced by rollers (25kN-m) is what gears 9,8,16,17 experienced
- Diameters given in cad model to be pitch diameters for gears
- Full depth teeth with 20° pressure angle
- Material for all gears is ASTM A536 Ductile grade 120-90-02
- No temperature effect on gears

Gear #	D(mm)	N	m (mm/tooth)	n (rpm)	F(mm)	Torque Experienced (kNm)	W_t (kN)	Bending Stress	Wear Stress (Mpa)
10	175	10	17.6	150 cw	130	125	1428.6	13.9 kPa	-10.8 MPa
9	880	50	17.6	30 ccw	130	25	52.82	80.76 MPa	-.379 MPa
8	396	30	13.2	30 ccw	186	25	126.3	174.82 MPa	-.385 MPa
16	396	30	13.2	30 cw	107	25	126.3	303.89 MPa	-.507 MPa
17	396	30	13.2	30 cw	79	25	126.3	411.6 MPa	-.591 MPa
Gear train Value	.2								

Table 06. Executive summary of gear analysis values

Assumption: Pulley, flywheel, and v-belt were not considered a part of the gear system analysis mainly for simplicity and analyzing belt systems was not covered in class.

i) Gear train value evaluation

$e = \frac{n_L}{n_F} \Rightarrow e = \frac{30}{150} = .2$

The dimensions of each gear was estimated from the figures of roll machine design details. Specifically figure 5 was used to estimate the number of teeth of the gears.

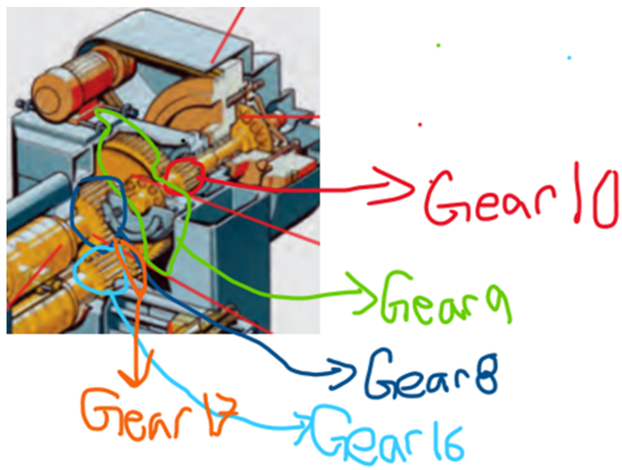

Figure 25. Depiction of Roll Forge machine with gears highlighted

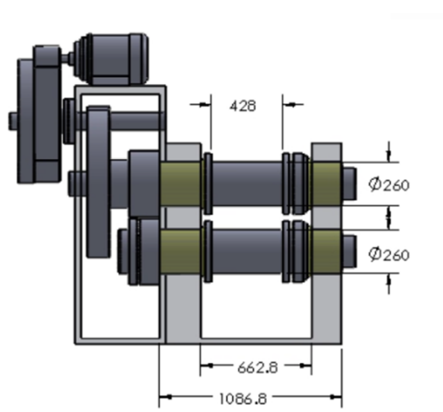

Figure 26. CAD Design of Roll Forge Machine with dimensions in mm.
Using dimensional analysis with Figure XX the face width of each gear was determined.

The max torque experienced by gears 8, 16, 17 was 25 kNm which was the max torque experienced by the forging rollers.

Assumption: Based on the previous assumption that the average radial load over 1 second is applied over the total covered surface area, the speed of gears 9, 8, 16, 17 is 60 rpm.

Since gear 9 and 8 are on the same shaft they experience the same speed and torque. Based on the torque of gear 9 and the number of teeth for gear 9 and 10, the torque of gear 10 was determined:

$$\frac{N_{10}}{N_9} = \frac{T_9}{T_{10}} \Rightarrow \frac{10}{50} = \frac{25}{T_{10}} \Rightarrow 10T_{10} = 1250 \Rightarrow T_{10} = 125\ kNm$$

Speed of gear 10 was determined (note: Gear 10 is driving Gear 9):

$$n_9 = \frac{N_{10}}{N_9} n_{10} \Rightarrow 60 = \frac{10}{50} n_{10} \Rightarrow \frac{50*30}{10} = n_{10} = 150\ rpm$$

Assumption: Since Pulley, flywheel are not a part of the analysis, gear 10 is the pinion.

The transmitted loads for each load were determined next, using the general equation:

$$T = W_t(\frac{d}{2}) \Rightarrow \frac{2*Torque}{d} = W_t$$

Gear 10:

$$W_t = \frac{2*125kNm}{.175m} = 1,428.57\ kN$$

Gear 9:

$$W_t = \frac{2*25kNm}{.880m} = 56.82\ kN$$

Gear 8:

$$W_t = \frac{2*25kNm}{.3965m} = 126.3\ kN$$

Gear16:

$$W_t = \frac{2*25kNm}{.396m} = 126.3\ kN$$

Gear17:

$$W_t = \frac{2*25kNm}{.396m} = 126.3\ kN$$

AGMA A536 Ductile grade 120-90-02 quenched & tempered was chosen as the material for all the gears because it has a brinell hardness of 269 HB which is much higher than the hardness of the billet material which was 197 HB. The properties of the gear material are greater than the billet material thus ensuring they will stand the stresses and strain experienced during the forging process.

Material Properties	Gear Material: A536 Ductile Grade 120-90-02	Billet Material (AISI 1055)
Ultimate Tensile Strength (MPa)	827	660
Yield Strength(MPa)	621	560
Modulus of elasticity(GPa)	170	190-210
Poisson's ratio	.28	0.27-0.30
Elongation at break (%)	2	10
Brinell hardness	269	197
Vickers hardness	272	207

Table 07. Comparison of Gear and Billet Material (Properties of the gear was determined from the book and online sources)

Gear 10 is the pinion so the AGMA approach of analysis was taken for this gear since it is the main driving gear so it is vital this gear does not fail. Using the AGMA approach was very time consuming due to the time it takes to determine the factors and getting used to the slight differences of the metric equations that are slightly different. To be more efficient with time the Lewis Bending equation and adapted Hertz theory equation were used to analyze the rest of the gears.

Assumption: Gear 10 also is the smallest gear and experiences the most torque therefore since it does not fail does then the rest of the gears that are of the same material and are larger won't fail either.

ii) Gear 10 analysis

Bending:

First allowable of bending stress ($O_{b\ all}$)was calculated : $\sigma_{b\,,all} = \frac{S_T Y_N}{S_F Y_0 Y_z}$ From the Textbook Table 14-4 : S_T =213.74 MPa

Assumption: S_F =1.5, Number of Load cycles: 10^7 , the gears are far enough from the rollers and billet such that there is no temperature effect ($Y_0 = 1$)

From Table 14-14 : $Y_n = 1.3558N^{-.0178} = 1.3558 * (10^7)^{-.0178} = 1.02$

From Table 14-10: $Y_z = 1$ **Assumption:** .99 reliability $\sigma_{b\ all} = \frac{(213.74MPa)(1.02)}{(1.5)(1)(1)} = 145.34MPa$

O_b was determined next using eq 14-15: $\sigma = W^t k_o k_v k_s \frac{1}{bm_t} \frac{k_H k_B}{Y_j}$

- K_o: **Assumption:** Uniform-Moderate shock
  From Figure 14-18: **K_o= 1.25**
- K_v: **Assumption:** Q_v = 8
  $@ .5\, rev/s = \frac{\pi d}{2} m/s$ = V = $\frac{\pi * .175m}{2} = .275m/s$

  From eq 14-27,28 : $k_v = (\frac{A+\sqrt{200V}}{A})^B where\ A = 50 + 56(1 - B) \quad B = .25(12 - Q_v)^{\frac{2}{3}}$

  B = $.25(12 - 8)^{2/3} = .63 \Rightarrow A = 50 + 56(1 -.63) = 70.72$

$k_v = (\frac{70.72+\sqrt{200*.275}}{70.72})^{.63} = 1.06$

- K_s: **Assumption:** k_s = 1(for simplicity)
- K_H: **Assumption:** Uncrowned Teeth

  $K_H = C_{mc}(C_{pf}C_{pm} + C_{ma}C_{ce})$ From eq 14-31: C_{mc}=1

  From eq 14-32: $C_{pf} = \frac{F}{10d_p} - 0.0375 + .0125F \Rightarrow \frac{130}{10*175} - 0.0375 + .0125 * 130 = 1.66$

  From eq 14-33: $C_{pm} = 1$

  From eq 14-34: $C_{ma} = A + BF + CF^2$ **Assumption:** Condition for Table 14-9 is Commercial,

enclosed units

A = .127, B = 0.0158, C = -.930(10^{-4}) ⇒

$C_{ma} = .127 + (.058 * 130) +-.930(10^{-4})(130)^2 = .61$

From eq 14-35: Ce = 1

$k_H = 1 + 1(1.66(1) + (.61)(1)) = 3.27$

- K_B : **Assumption:** $m_B \geq 1.2$ From eq 14-40: K_B = 1
- Y_J : From Figure 14-6: Y_J = 0.20

Plugging all the values in: $\sigma = (1428.6)(10^3)(1.25)(1.06)(1)\frac{1}{(130)(17.6)}\frac{(3.27)(1)}{.2} = 13526.6\,Pa \Rightarrow 13.9kPa$

$\eta = \frac{\sigma_{b,all}}{\sigma_b} = \frac{145.34\,MPa}{0.0139MPa} = 10456$ ⇒This gear essentially as an infinite life in regards to bending stress

It makes sense that gear 10 experiences a relatively low bending stress of 13.9 kPa since gear 10 is furthest away from the rollers. In terms of bending gear 10 has infinite life since it is magnitudes smaller than the allowable bending stress calculated.

Wear:

Allowable stress due to wear was calculated first using eq 14-18: $\sigma_{c,all} = \frac{S_c}{S_H}\frac{Z_N Z_W}{Y_\theta}$

- From the Textbook Table 14-5 : S_c=710.16 MPa
- S_H=1.5
- From previous assumptions and using Table 14-15: Z_N=1.1
- **Assumption:** $H_{BP} = H_{BG}$ (Hardness of all the Gears is the same) According to Figure 14-12

  $\Rightarrow if\ \frac{H_{BP}}{H_{BG}} < 1.2,\ Z_w = 1$
- $Y_\theta = 1$

$\sigma_{c,all} = \frac{710.16}{1.5}\frac{(1.1)(1)}{(1)} \Rightarrow 520.78\,MPa$

Actual Wear Stress: Eq 14-16 $\sigma_c = Z_e\sqrt{W^t k_o k_v k_s \frac{K_H}{d_{w1}b}\frac{Z_R}{Z_I}}$

- $W^t k_o k_v k_s k_H$ are all the same from bending stress calculation
- d_{w1}=175 mm b = 17.6 mm

- $Z_e = \left(\frac{1}{\pi\left(\frac{1-V_p^{\ 2}}{E_p}+\frac{1-V_G^{\ 2}}{E_G}\right)}\right)^{1/2}$ From table A-5: $V_p = V_G = .211$, $E_p = E_G = 100$ GPa $Z_e =$ $\left(\frac{1}{\pi\left(\frac{1-.211^2}{100(10^9)}+\frac{1-.211^2}{100(10^9)}\right)}\right)^{1/2} = 129062.3$
- **Assumption:** $Z_R = 1$ (not enough information given in textbook)
- $Z_I = \frac{2}{d_p \sin\phi_t}\frac{m_G+1}{m_G}$, where $m_G = \frac{d_G}{d_p}$ $Z_I = \frac{2}{175 sin20}\frac{\frac{880}{175}+1}{\frac{880}{175}} = .04$

Plugging all the factors $\sigma_c = (1.29 * 10^5\sqrt{(1428.57)(10^3)(1.25)(1.06)(1)\frac{3.27}{(175)(130)}\frac{1}{.04}} = 10.6\ MPa$

$\eta = \frac{\sigma_{c,all}}{\sigma_c} = \frac{710.16\ MPa}{10.6 MPa} = 67$ ⇒This is much lower than the η for bending so if Gear 10 wear to fail it would be due to wear not bending.

Since Gear 10 is the smallest gear ,experiences the largest transmitted load,and spins the fastest so the relatively high wear stress makes sense when compared to the other gears.

iii) Gear 9 analysis

Gear 9 is driven by Gear 10 and has the largest pitch diameter of 880 mm, 50 teeth, and a face width of 130 mm. Gear 9 shares a shaft with gear 8 meaning they have the same speed.

Bending

The Lewis bending equation was used to analyze this gear (eq 14-8): $\sigma = \frac{k_v W^t}{FmY}$

For Gear 9, W^t=56.82 kN which is the lowest transmitted load out of all the gears experiencing 25kNm of torque, a result of Gear 9 having the largest radius. This can easily be observed in the previous calculations of transmitted load for the gears.

To determine the factor k_v Eq 14-6c was used: $k_v = \frac{3.56+\sqrt{V}}{3.56}$ **Assumption:** The teeth of gears 9,8,16,17 are hobbed or shaped profile. : $@ .5\ rev/s = \frac{\pi d}{2} m/s = V = \frac{\pi * .880m}{2} = 1.38 m/s \Rightarrow k_v = \frac{3.56+\sqrt{1.38}}{3.56} = 1.33$

From Table 14-2 Y for N = 50, Y = .409

Plugging all the values: $\sigma = \frac{(1.33)(56.82)(10^3)}{(130)(17.6)(.409)} \Rightarrow \sigma_b = 80.76\ MPa$

Wear

The Hertz Theory equation was used to analyze the wear of Gear 9: (eq 14-14) $\sigma_c = -Z_e\sqrt{\frac{k_v W^t}{F\cos\phi}\left(\frac{1}{r_1}+\frac{1}{r_1}\right)}$

Since factor Ze depends strictly on material properties and the gears and the pinions are of the same material then Z_e is the same for all the gears. From earlier calculation, $Z_e = 1.29*10^5$. From earlier assumptions, $\phi = 20^\circ$. r_1 is the radius of Gear 10 and r_2 corresponds to the radius of Gear 9: r_1 =87.5 mm & r_2 = 440 mm.
Having all the values determined they are now plugged in to eq 14-14 to find wear stress:

$$\sigma_c = -(1.29 * 10^5)\sqrt{\frac{(1.33)(56.82)(10^3)}{(130)cos20}\left(\frac{1}{87.5}+\frac{1}{440}\right)} \Rightarrow \sigma_c = -378838.23\ Pa = -.379\ MPa$$

iv) Gear 8 analysis

Gear 8 is on the same shaft as Gear 9 so it is indirectly driven by Gear 10.

Bending

The Lewis bending equation was used to analyze this gear (eq 14-8): $\sigma = \frac{k_v W^t}{FmY}$

For Gear 8, W^t=126.3 kN. Gear 8 and 9 both travel at 30 rpm but Gear 8 has a much smaller radius so V will be different and as a result so will the factor k_v.

Eq 14-6c was used: $k_v = \frac{3.56+\sqrt{V}}{3.56}$ $: @ .5\, rev/s = \frac{\pi d}{2} m/s$ = V = $\frac{\pi * .396m}{2} = .622 m/s$

$\Rightarrow k_v = \frac{3.56+\sqrt{.622}}{3.56} = 1.22$

From Table 14-2 Y for N = 30, Y = .359

Plugging all the values: $\sigma = \frac{(1.22)(126.3)(10^3)}{(186)(13.2)(.359)} \Rightarrow \sigma_b = 174.82\, MPa$

Wear

The Hertz Theory equation was used to analyze the wear of Gear 8: (eq 14-14) $\sigma_c = -Z_e\sqrt{\frac{k_v W^t}{F\cos\phi}\left(\frac{1}{r_1} + \frac{1}{r_1}\right)}$

Since factor Ze depends strictly on material properties and the gears and the pinions are of the same material then Z_e is the same for all the gears. From earlier calculation, Z_e = 1.29*10^5. From earlier assumptions, $\phi = 20^\circ$. r_1 is the radius of Gear 8 and r_2 corresponds to the radius of Gear 16 since they mesh with each other: r_1=198 mm & r_2 = 198 mm. Having all the values determined they are now plugged in to eq 14-14 to find wear stress:

$$\sigma_c = -(1.29 * 10^5)\sqrt{\frac{(1.22)(126.3)(10^3)}{(186)\cos 20}\left(\frac{1}{198} + \frac{1}{198}\right)} \Rightarrow \sigma_c = -384950\, Pa = -.385\, MPa$$

It's interesting that Gear 9 and 8 experience very similar wear stress despite being different in size and in transmitted load.

v) Gear 16 analysis

Gear 8 meshes with gear 16, and they are pretty much the same gear. The only difference is Gear 16 has a smaller face width(F= 107mm) so it will experience more bending and wear stress.

Bending

The Lewis bending equation was used to analyze this gear (eq 14-8): $\sigma = \frac{k_v W^t}{FmY}$

For Gear 16, W^t=126.3 kN. Gear 8 and 16 both travel at 30 rpm and have the same radius so V will be the same and so will the factor k_v. $\Rightarrow k_v = 1.22$

Gear 16 and Gear 8 also have the same number of teeth and module so Y = .359 as well.

Plugging all the values: $\sigma = \frac{(1.22)(126.3)(10^3)}{(107)(13.2)(.359)} \Rightarrow \sigma_b = 303.89\, MPa$

Since Gear 16 has a smaller face width it makes it more susceptible to bending stress, hence why it experiences a higher value than Gear 8 which has the same exact loading conditions.

Wear

The Hertz Theory equation was used to analyze the wear of Gear 8: (eq 14-14) $\sigma_c = -\ Z_e \sqrt{\frac{k_v W^t}{F cos\phi} (\frac{1}{r_1} + \frac{1}{r_1})}$

Since factor Z_e depends strictly on material properties and the gears and the pinions are of the same material then Z_e is the same for all the gears. From earlier calculation, Z_e = 1.29*10^5. From earlier assumptions, $\phi\ =\ 20^{\circ}$. r_1 is the radius of Gear 8 and r_2 corresponds to the radius of Gear 16 since they mesh with each other: r_1 =198 mm & r_2 = 198 mm. Having all the values determined they are now plugged in to eq 14-14 to find wear stress:

$$\sigma_c = -\ (1.29\ *\ 10^5)\sqrt{\frac{(1.22)(126.3)(10^3)}{(107)cos20} (\frac{1}{198} + \frac{1}{198})} \Rightarrow \sigma_c\ =\ -\ 507538\ Pa\ =\ -\ .507 MPa$$

Like for bending stress since Gear 16 has a smaller face width it experiences higher wear stress.

vi) Gear 17 analysis:

Assumption: Gear 17 is analyzed as if it is always engaged with gear 8.

Gear 8 meshes with gear 17. Gear 17 has the smallest face width(F=79 mm) so it will experience the most bending and wear stress out of gears 9,8,and 16.

Bending

The Lewis bending equation was used to analyze this gear (eq 14-8): $\sigma\ =\frac{k_v W^t}{FmY}$

Gear 17 has the same radius as gear 16 so it experiences the same transmitted load, W^t=126.3 kN. Gear 16 and 17 both travel at 30 rpm and have the same radius so V will be the same and so will the factor k_v. $\Rightarrow k_v\ =\ 1.22$

Gear 16 and Gear 17 also have the same number of teeth and module so Y = .359 as well.

Plugging all the values: $\sigma\ =\frac{(1.22)(126.3)(10^3)}{(79)(13.2)(.359)} \Rightarrow \sigma_b\ =\ 411.6\ MPa$

Since Gear 16 has a smaller face width it makes it more susceptible to bending stress, hence why it experiences a higher value than Gear 8 which has the same exact loading conditions.

Wear

The Hertz Theory equation was used to analyze the wear of Gear 8: (eq 14-14) $\sigma_c = -\ Z_e \sqrt{\frac{k_v W^t}{F cos\phi} (\frac{1}{r_1} + \frac{1}{r_1})}$

Since factor Ze depends strictly on material properties and the gears and the pinions are of the same material then Z_e is the same for all the gears. From earlier calculation, Z_e = 1.29*10^5. From earlier assumptions, $\phi\ =\ 20^{\circ}$. r_1 is the radius of Gear 8 and r_2 corresponds to the radius of Gear 17 since they mesh with each other: r_1 =198 mm & r_2 = 198 mm. Having all the values determined they are now plugged in to eq 14-14 to find wear stress:

$$\sigma_c = -\ (1.29\ *\ 10^5)\sqrt{\frac{(1.22)(126.3)(10^3)}{(79)cos20} (\frac{1}{198} + \frac{1}{198})} \Rightarrow \sigma_c\ =\ -\ 590673\ Pa\ =\ -\ .591 MPa$$

Summary

When compared to all the gear besides Gear 10, Gear 17 experiences the most bending and wear stress. This result is reasonable because it has the same loading conditions as gears 9,8,and 16, but the smallest face width. All the gears experience really low wear stress compared to the $\sigma_{c,all}$ calculated for the material($520.78\ MPa$) and this is due to materials high S_c(710.16 MPa).

Gears 9,16,17 all experience more bending stress than wear stress, so if the gears were to fail it would be via bending. None of the gears experience stress greater than the ultimate tensile strength or the yield strength, so it can be concluded the gears will withstand the entire roll forging process.

IX. Results summary

Static Failure Factor of Safety	4.096
First Cycle Yielding Factor of Safety	22.7
Goodman - Fatigue Factor of Safety	8.18
Train value	0.2
Maximum deflection (mm)	0.182
Deflection angle at bearings (rad)	-0.000822

Table 08. Final results for failure analysis

IX. Discussion

To ensure the functionality of the forge roller design and its ability to withstand system demands, failure from static and dynamic loading conditions were calculated. Different force assumptions were used between static and dynamic loading. While the maximum applied force was used for static failure analysis, fatigue failure analysis utilized the average applied roll over a single pass. Due to this condition, the factor of safety corresponding to first-cycle yielding as calculated in the fatigue failure analysis section was greater than that of static failure. For future work, it is a good idea to use the same applied force value for both failure analyses. While the justification for using the average applied load for fatigue is to include forces at all points of a single pass, using the same force value for both failure analyses will help maintain uniformity in data.

Assessing the deflection of the rollers was critical in confirming the efficiency of the rollers. Too much deflection will decrease the contact stress between the rollers, dropping the effectiveness of each pass. In this study, two Castigliano's values for deflection were calculated - one treating the rollers as a simply supported structure, and one treating it as a structure with two fixed ends. By calculating the deflection under both circumstances, a bound was defined wherein the actual deflection will appear. As a supplementary cross-check to Castigliano's calculations, an FEA was run via SOLIDWORKS with a single model of the forge roller geometry. A difference between Castigliano's calculations and the FEA was the geometry itself. Castigliano's calculations treated the roller as a single uniform cylinder. However, the FEA included an extra geometrical detail and modeled the forge rollers as the cylindrical roller and the forge die which will come in contact with the billet itself. The three results of Castigliano's of simple supports, Castigliano's of fixed supports, and the FEA model provided results which estimate the approximate window of solutions for maximum deflection. Given more time, Castigliano's could have been calculated to include the difference in diameter at the center of the roller where contact with the billet occurs to better match with the CAD model.

X. Citations

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