DESIGN FOR DEFLECTION

ME0041: Engineering Design II

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Executive Summary.

The goal of Phase II was to design a structure that would deflect 0.5in under a 200lb load and have a spring constant of 400. We redesigned our structure to incorporate new filets, tapering and additional support structure in an attempt to balance the weight of the structure, and the total deflection under load while also minimizing yielding. Our numerical simulation via SolidWorks predicted a deflection of 0.5008in under a 200lb load with minor yielding. We found it difficult to entirely remove yielding while keeping the resulting deflection close to 0.5 in and thus prioritized reaching the desired deflection at the potential risk of the minimal yielding. When tested using the Instron testing rig, the structure reached the 0.5in deflection mark at just above 180 lbs with a total mechanism weight of 95.85g. However the structure had already begun to yield when the deflection was at 0.448 in under a load of 166 lbs. Continuing on to undergo the full 200lb load, the mechanism deflected a total of 0.56825in.

Overall, when comparing the deflection predictions of Castigliano's and SolidWorks to that collected through Instron testing, the deflection value of Castigliano's was less than that of the desired deflection, 0.5", as well as both the FEA analysis and the testing results. Both the SolidWorks and Castigliano's estimates were less than the testing results, however, with estimated deflections of 0.4871in and 0.5008in, respectively. The discrepancy between the Castigliano's estimate, the Solidworks estimate and the actual testing values is suspected to be due to several factors, including differences in material thickness, and assumptions made during Castigliano's analysis.

	Castigliano Estimate	SolidWorks Estimate	Testing Results	Target Values
Spring Constant	N/A	364 at yield point, 400 at 200lb load	373	400
Load at 0.5 in deflection (lbs)	200	200	200	200
Deflection (in)	0.4871	0.5008	0.56825	0.5

Table 1: Castigliano's vs. SOLIDWORKS FEA solutions.

Introduction.

The purpose of Phase I of the project was to simulate the deflection of a given structure under a vertical force of 25 lbf at an indicated load point and determine its maximum deflection. Students were tasked with mathematically modeling this deflection using analytical techniques like the Strain Energy Method with Castignliano's Theorem, and independently calculating the distance of beam deflection before comparing those values to numerically derived ones using the Finite Element Analysis (FEA) SolidWorks simulation. Throughout the process, students learned how to apply analytical and numerical techniques for designing a durable aluminum alloy structure. Students were also instructed to make key assumptions in the structure when completing calculations, such as assuming the structure is simply supported at the two hole locations at the bottom of its base. The material for the structure, purchased through McMaster-Carr, is noted to be an extruded aluminum alloy (6061-T6511), with a modulus of elasticity of 10 x 10^6 psi and a yield strength of 46.50 ksi.

The purpose of Phase II of the project was to give students the opportunity to redesign the original structure from Phase 1 in SolidWorks and predict its performance when supporting a 200lb load to achieve a desired deflection value. The design's goal was to achieve a 0.5" vertical deflection at the rightmost end from the center of the hole at point A without yielding. In addition, students were tasked with designing an efficient structure, defined as providing the target performance with the least material possible. It was specified that students must have three holes drawn in their model with 0.385" diameter at locations identical to the original structure. The remainder of the 4" x 8" area is left to students to design their original structure. The material, an extruded aluminum alloy (6061-T6511), was identified to have the following properties:

- Elastic modulus, E = 10e6 psi
- Yield strength, Sy = 46.50 kpsi
- Material = AI6061-T6511

Approach. The general approach to achieve the goals of Phase II was to begin with the provided structure from Phase I and iterate various versions of this model with intermediate testing. Before contributing any changes to the provided structure design, a 200 lbf was applied in place of the 25 lbf from phase I to determine the resulting deflection pre-redesigns. It was found that the resulting deflection was too large, and so efforts to mitigate this vertical deflection were taken.

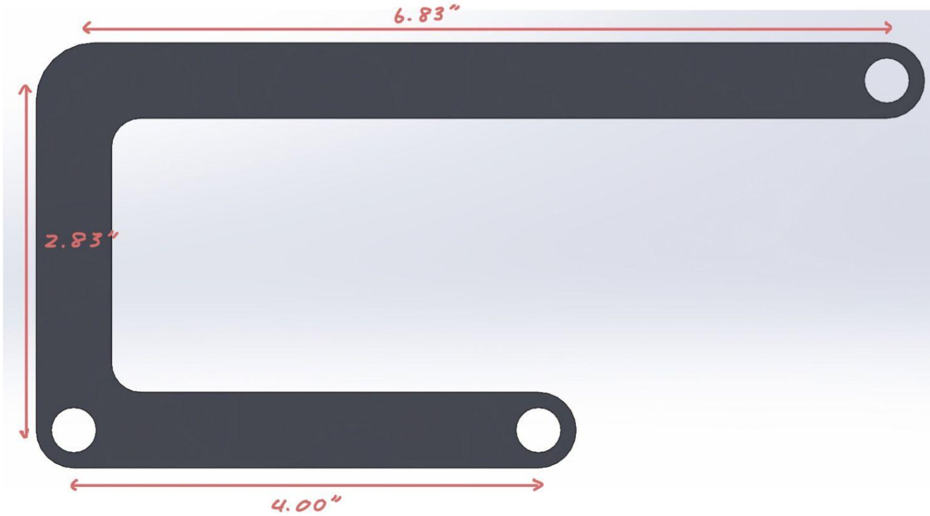


Figure 1. Testing the original provided structure from phase I with a 200lb load.

The first redesign made to lessen the vertical deflection at the 200 lbf load point was a diagonal member between point C and member AB. By adding this diagonal member, an additional counterclockwise moment was added about point C to counteract the clockwise moment about point C caused by the 200 lbf load. The inserted rod would be attached to the structure as pins to make it a *two-force* member, meaning it will only have two forces at their end locations and no moments.

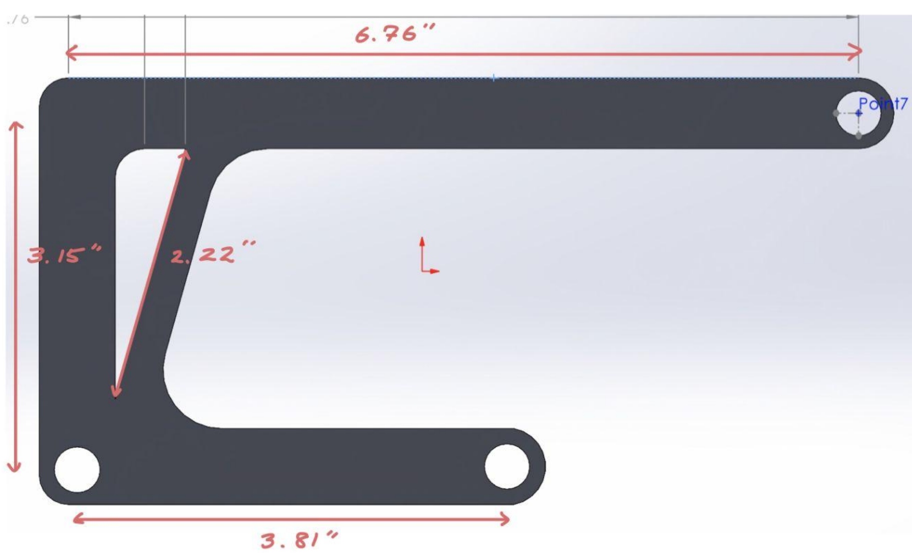


Figure 2. First iteration re-design incorporating the diagonal member.

The length - and thus implicitly the location at which the diagonal member connects with member AB - was adjusted to various lengths to best achieve the desired deflection of 0.5”. To change the resulting deflection the location of this point was changed. In instances where deflection at point A needed to be increased, the length of the diagonal member could be decreased to enable more movement of the top beam in the vertical direction and thus more deflection. Alternatively, to decrease the deflection at point A, the length of the diagonal member could be increased. Fillets were also added and adjusted to mitigate yielding in critical locations such as at locations of acute angles within the overall geometry, particularly the area underneath the diagonally inserted bar.

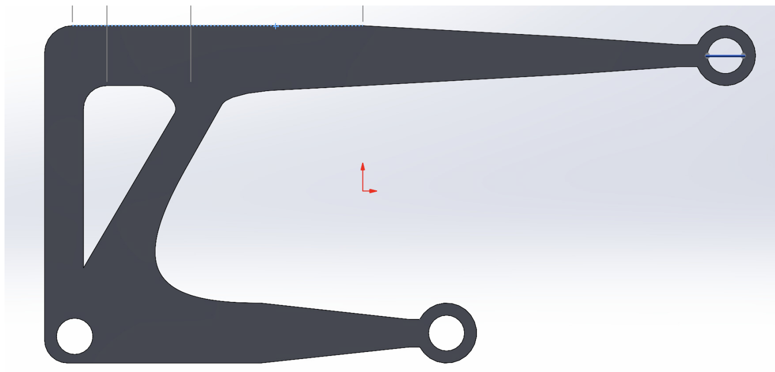


Figure 3. Final iteration re-design with tapered ends, fillets, and diagonal support member.

While the addition of the diagonal beam was effective in deflection adjustment, significant yielding still occurred even with the fillets. This was addressed by tapering the top and bottom beams toward the right, where stresses were lower, then reinforcing the structure by widening beams and fillets in weaker areas. This created a more consistent stress measurement down the length of the beam, increasing deflection within the elastic range.

Analysis.

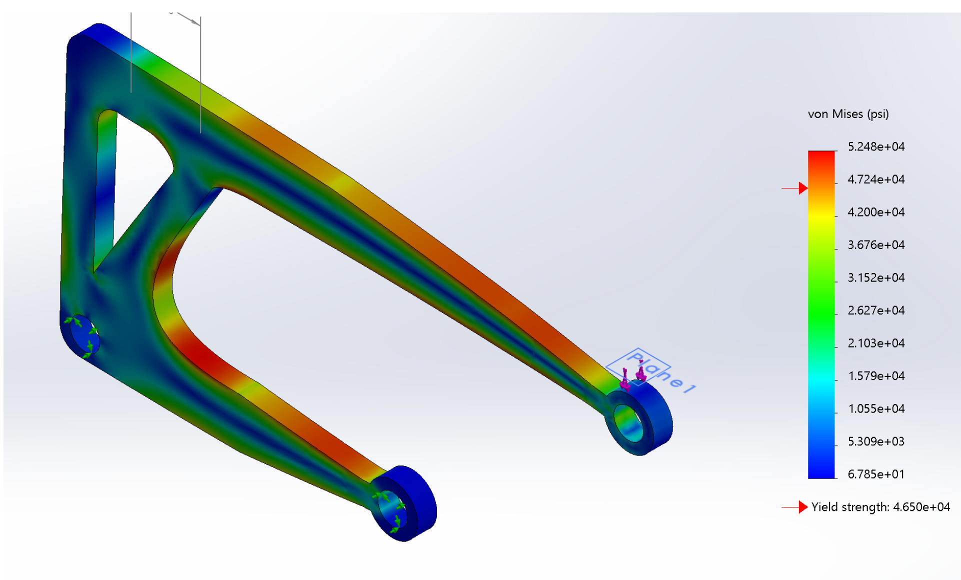


Figure 4. Von Mises stress plot.

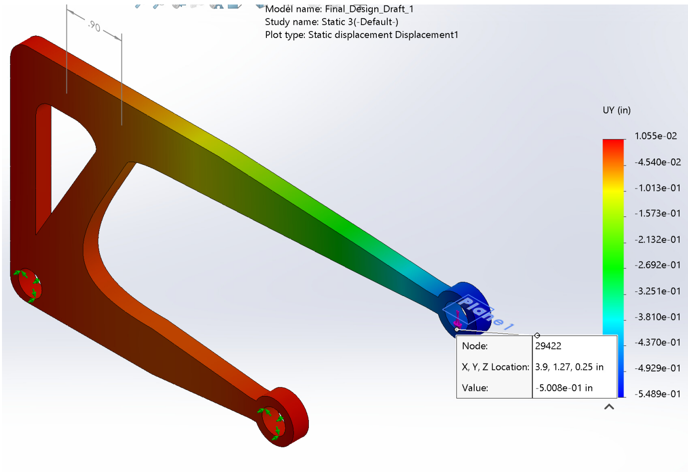


Figure 5. Y-direction deflection with probing.

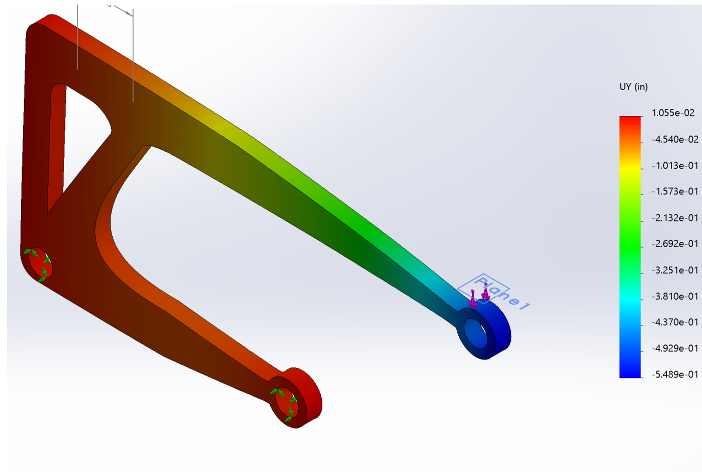


Figure 6. Y-direction deflection without probing.

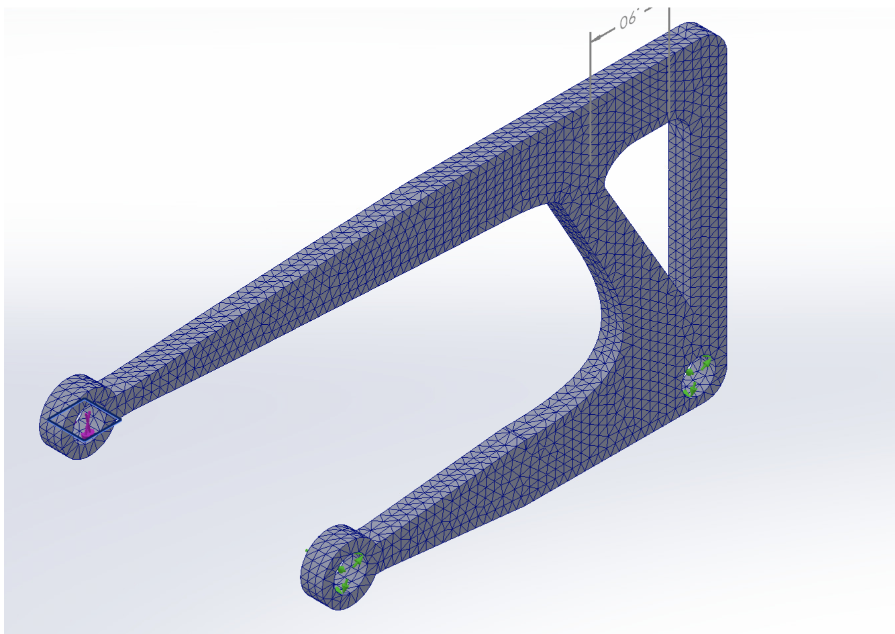


Figure 7. Mesh analysis with fixtures and loads.

Castigliano's Analysis Calculations:

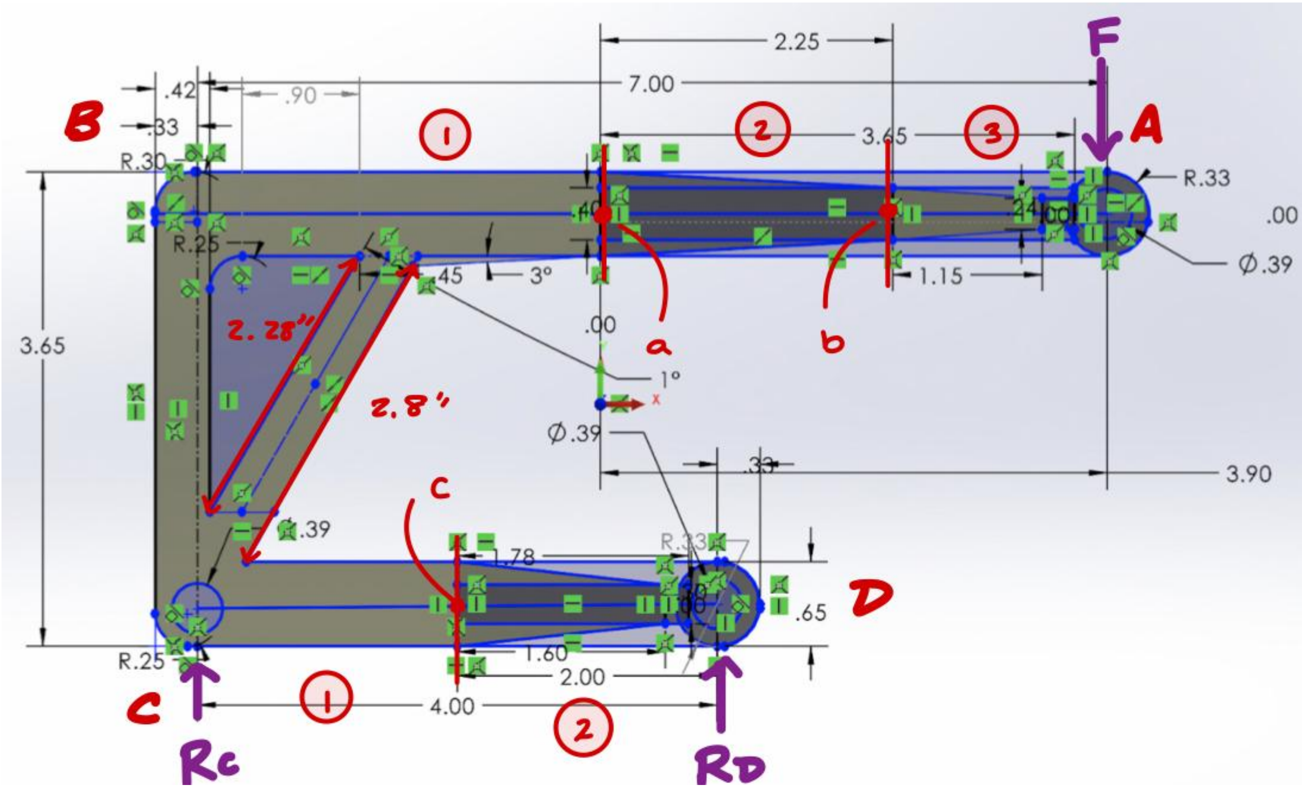
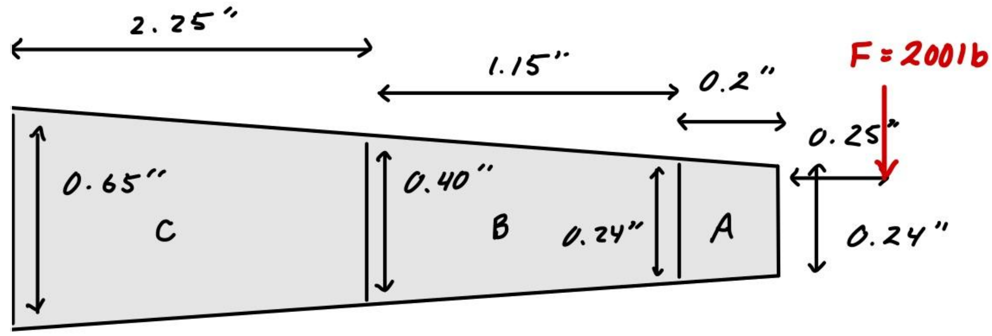


Figure 8. Simplified figure without fillets used to calculate Castigliano's analysis.

Prior to performing any calculations using Castigliano's Theorem, it is important to take into account the provided conditions and variables, as well as any key assumptions that were made about the experimental setup in order to make the calculations manageable with the material we have learned so far in the course:

- The part of the structure to the left of the diagonal member forms a rigid triangle with little deflection.
- Large fillets do not have significant deflection.
- Deflection is elastic.
- Angular deflection is small, so sin(θ) = θ and cos(θ) = 1.

The Free Body Diagrams of the structure are illustrated below, integrated within the Castigliano's calculations.



$$\delta = \int_0^L \frac{M\, dM/dF}{EI} \; ; \quad E = 10^7 \text{ psi}$$

$$I = bh^3/12 \,, \quad bh^3(x)/12$$

$$\theta = \frac{d\delta}{dx} = \left. \frac{M\, dM/dF}{EI} \right|_{x=L}$$

Section A

$$M = -F(0.25 + x) \; ; \quad dM/dF = -x - 0.25$$

$$b = 0.25, \quad h = 0.24, \quad L = 0.20$$

$$I = 2.88E-4$$

$$\delta_A = 0.00174" \; ; \quad \theta_A = 0.014 \text{ rad}$$

Section B

$$M = -F(0.45 + x) \; ; \quad dM/dF = -x - 0.45$$

$$b = 0.25", \quad h(x) = 0.24 + \left(\frac{0.4 - 0.24}{L} \right) x, \quad L = 1.15"$$

$$I = \frac{1}{12}(0.25) h^3(x)$$

$$\delta_B = 0.03363 \; ; \quad \theta_B = 0.0384 \text{ rad}$$

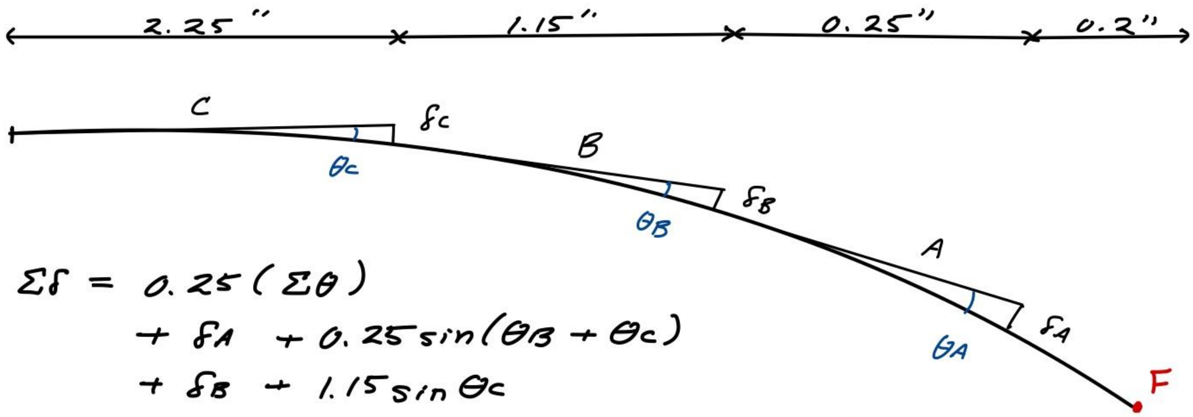
Section C

$$M = -F(1.6 + x) \; ; \quad dM/dF = -x - 1.6$$

$$b = 0.25", \quad h(x) = 0.4 + \left(\frac{0.65 - 0.4}{L} \right) x, \quad L = 2.25"$$

$$I = \frac{1}{12}(0.25) h^3(x)$$

$$\delta_C = 0.10780 \; ; \quad \theta_C = 0.05181 \text{ rad}$$



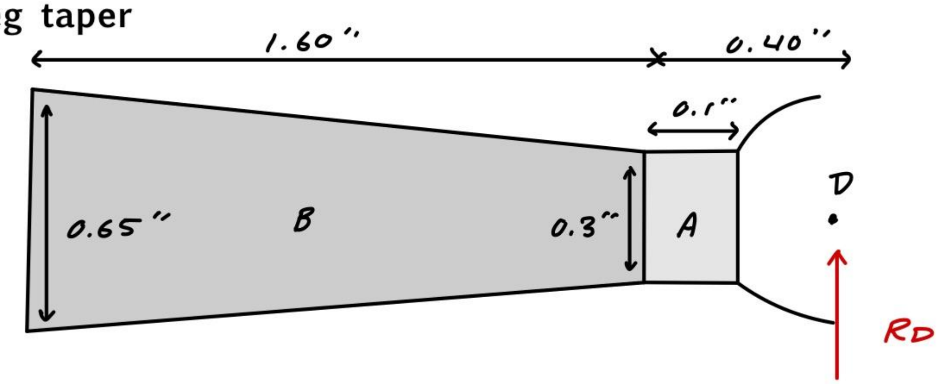
$$\Sigma\delta = 0.25(\Sigma\theta) + \delta_A + 0.25\sin(\theta_B + \theta_C) + \delta_B + 1.15\sin\theta_C + \delta_C = 0.24617''$$

Assume $\sin\theta = \theta$

$\cos\theta = 1$

(deflection of horiz. beam is vertical.)

Bottom leg taper



Section A

$M = -R_D(0.3 + x)$; $dM/dF = -x - 0.3$

$b = 0.25''$, $h = 0.3''$, $L = 0.1$

$I = 5.625E-4$

$\delta_A = 0.00077''$; $\theta_A = 0.00996$ rad

Section B

$M = -R_D(0.4 + x)$; $dM/dF = -x - 0.4$

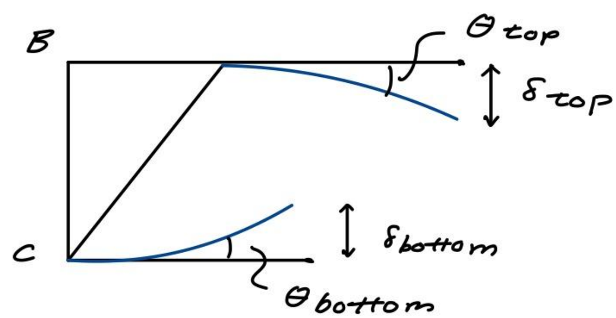
$b = 0.25''$, $h(x) = 0.3 + \left(\frac{0.65 - 0.3}{L}\right)x$, $L = 1.6''$

$I = \frac{1}{12}\, 0.25\, h^3(x)$

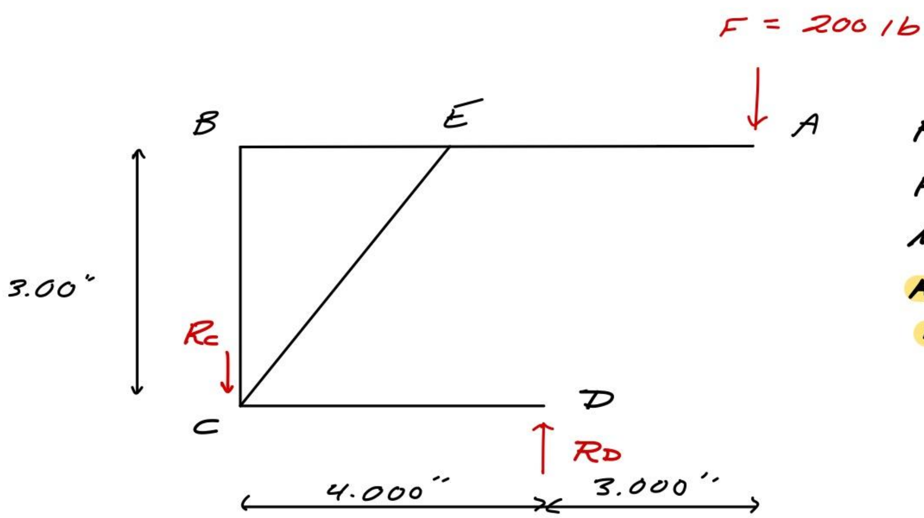
$\delta_B = 0.03335''$; $\theta_B = 0.02447$ rad

$$\Sigma\delta = 0.3\sin(\theta_A + \theta_B) + \delta_A + 0.1\sin\theta_B + \delta_B$$
$$= \delta_A + \delta_B + 0.4\sin\theta_B + 0.3\sin\theta_A$$
$$= 0.04689''$$
$$\Sigma\theta = \theta_A + \theta_B = 0.03443 \text{ rad.}$$

TOTAL DEFLECTION



$$\delta_{overall} = \delta_{bottom} + 7\sin\theta_{bottom} + \delta_{top}$$
$$= 0.04689'' + 7''\sin 0.03443$$
$$+ 0.24617''$$
$$= 0.48710''$$



$$R_D - R_C - F = 0$$
$$R_D - R_C = 200 \text{ lb}$$
$$M_A = 3R_D - 7R_C = 0$$
$$R_D = 350 \text{ lb} \uparrow$$
$$R_C = 150 \text{ lb} \downarrow$$

Top beam δ_A (in)	0.00174
Top beam θ_A (rad)	0.01400
Top beam δ_B (in)	0.03363
Top beam θ_B (rad)	0.03840
Top beam δ_C (in)	0.10780
Top beam θ_C (rad)	0.05181
Total δ of top beam (in)	0.24617
Bottom beam δ_A (in)	0.00077
Bottom beam θ_B (rad)	0.02447
Total δ of bottom beam (in)	0.04689
Total θ of bottom beam (rad)	0.03443
R_D (lb)	350
R_C (lb)	-150

Table 2. Compilation of Castigliano's value results.

Testing Results:

Figure 9.

Figure 10.

Figure 11.

Figure 9: Experimental testing setup
Figure 10: Mechanism deflecting under 200lb load
Figure 11: Mechanism after 200lb load with negligible yielding

A load-deflection analysis was performed on the mechanism using an Instron tensile strength tester to apply a 200lbs load. A cylindrical supporting fixture was used when applying the load our mechanism to enable rolling and distribution of horizontal forces and ensure the mechanism was only undergoing forces along the vertical axis.

As illustrated by Figure 5 and Figure 6 above, following being loaded with the total 200lbs, the mechanism reverted back to its initial state with minimal yielding once the load was removed, suggested by the lack of space between the instron force application head and the tip of the mechanism. In fact, when comparing Figure 4, which was the initial state of the mechanism prior to loading, to Figure 6, which was the mechanism following loading, both conditions appear to be fairly similar, with the mechanism tip and outer diameter of the supporting fixture touching the instron base.

Below is a table of the final results as measured via the Instron.

	Actual	**Desired**
Spring constant (N/m)	373	400
Total deflection at mechanism tip at yield point(in)	0.448	0.500
Total force applied at yield point (lbs)	166	200

Table 3: Final results as measured on the Instron vs. desired results as specified in the project description.

While the Instron test was running, the deflection vs. applied force was plotted as shown in Figure 7. A perfect elasticity would have resulted in a completely linear line from (0,0) to (200, 0.5). As evident by Figure 7, the final redesign fell slightly short of this perfect elasticity value k = 400 N/m.

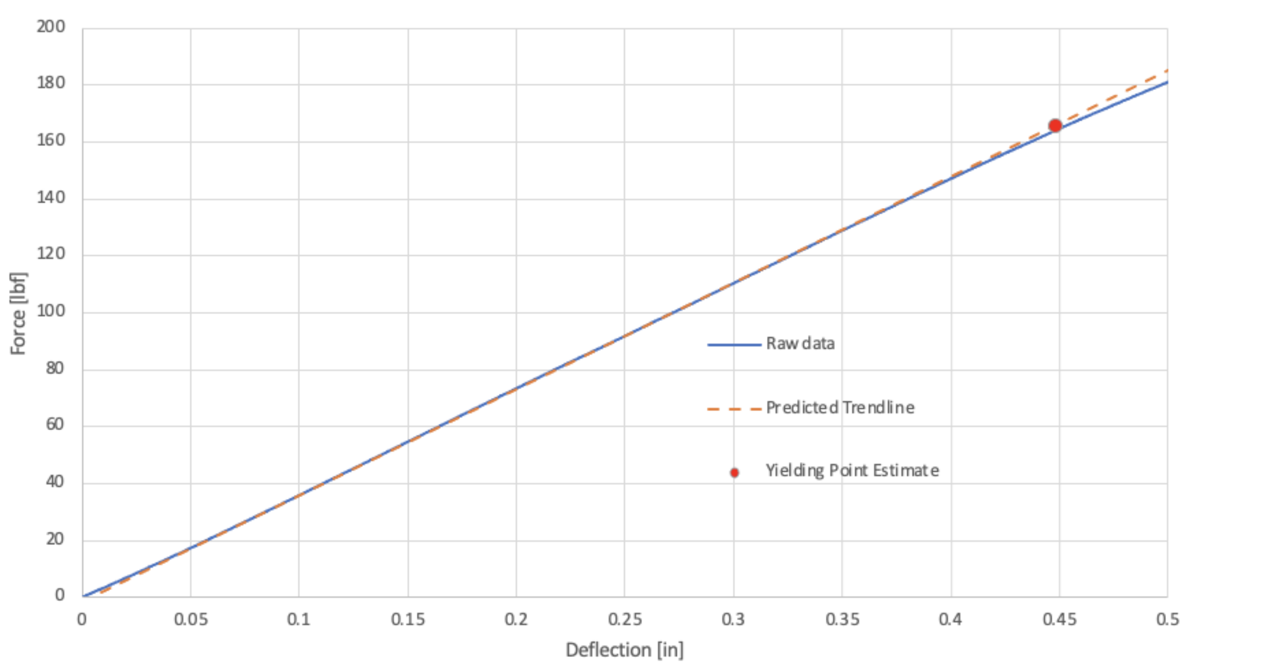


Figure 12:. Deflection of mechanism with respect to force applied over time.

It was possible to roughly estimate the yielding point force in SolidWorks. To do this, the load was gradually decreased from 200 lb and the FEA study was run for each new, lesser point load until the yield strength arrow in the stress color plot best aligned with the top of the Von Mises color scale, indicating at the maximum stress present on the model was the yield strength. First, a new load of 190 lbf was attempted. This still presented higher-than-yield-strength stresses, so 170 lbf was attempted next. The 170 lbf resulted in no yielding, and so intermediate load values were attempted. A load of 179 lbf was the highest achievable load in SolidWorks before visible yielding occurred.

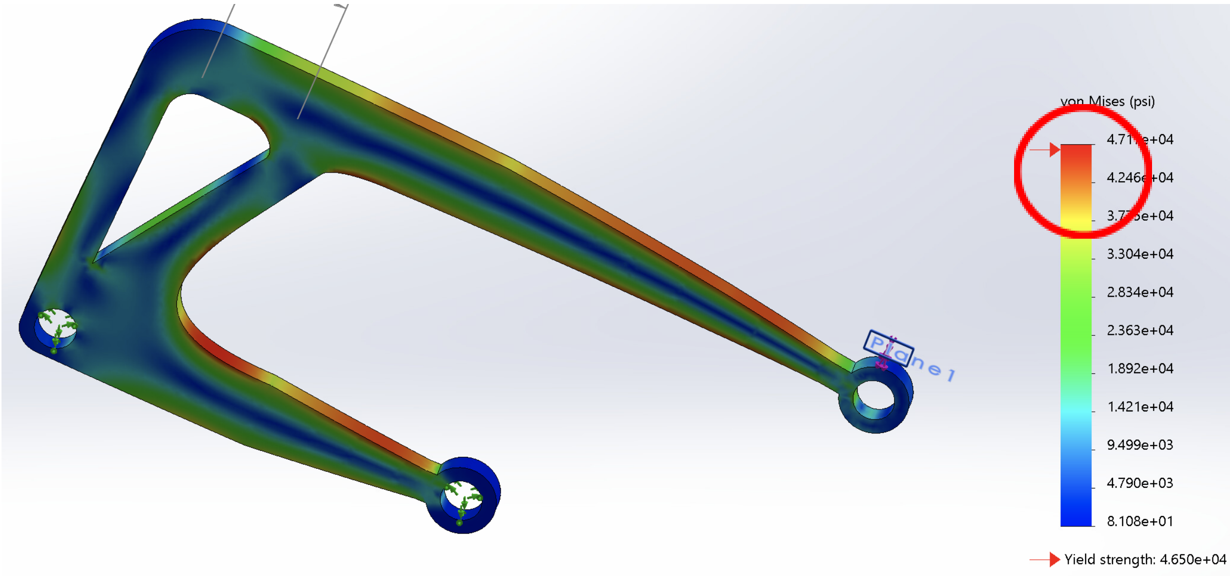


Figure 13. Von-Mises stress plot with a 179 lbf load.

The estimated load at the yielding point as predicted via the SOLIDWORKS FEA was 179 lbf, while the recorded load at the yielding point via the Instron was 165.6 lbf.

Discussion:

Our redesigned structure utilized new fillets, tapering and an additional support structure to ideally achieve the desired total deflection of 0.5" while minimizing yielding and the total weight of the mechanism. When performing an FEA analysis of our mechanism via SolidWorks, the simulation predicted a deflection of 0.5008in under a 200lb applied force at the tip of the mechanism with minor yielding. We found it challenging to entirely remove yielding while also maintaining a deflection close to the desired deflection, so the deflection measurement was ultimately prioritized over the risk of minimal yielding. In comparison, when performing numerical calculations using Castigliano's theorem, a deflection of 0.4871in was predicted when undergoing a load of 200lbs. Finally, when performing testing on our mechanism to observe the actual deflection of our structure under an applied load via Instron testing, the structure reached the 0.5in deflection mark at just above 180 lbs with a total mechanism weight of 95.85g. However the structure had already begun to yield when the deflection was at 0.448 in under a load of 166 lbs. Continuing on to undergo the full 200lb load, the mechanism deflected a total of 0.56825in.

When comparing the overall results of all three predictions of deflection for the mechanism to the actual results of Instron testing, it was found that both the Castigliano's estimate as well as the SolidWorks estimate were less than the actual deflection value found after testing with the Instron machine. However, Castigliano's calculation was predicted to be less than the desired deflection of 0.5" while the SolidWorks analysis predicted to be slightly greater with a 0.5008in deflection. Ultimately, both values under-estimated the actual deflection value of 0.56825in measured from Instron testing.

An important decision made while designing the project was how to balance the yielding and deflection of the design while also reducing the mechanism weight. A solution used to achieve this was tapering the upper and lower arms of the design where the stress was low. These tapers helped manage the deflection and stress level while also keeping the weight of the design minimal. Filets at corners were also used to minimize stress. It was noted that adding the extra member would increase the weight of the mechanism, however we hoped that the tradeoff of utilizing this member would enable for deflection closer to 0.5” while also reducing yielding at certain points in the mechanism by providing additional support.

Several assumptions were made in the Castigliano analysis that caused discrepancies between the calculations and the FEA results. Since the diagonal member created triangle BCE, that triangle was considered rigid, as were the large filets between the triangle and the tapered portions of AB and CD. One of the reasons for this assumption was that member CE is under primarily axial stress due to its positioning. This means that its deflection will be mainly compressive, which would be much smaller than bending deflection. Secondly, filets were approximated as regular corners in Castigliano's analysis. The removal of the filets in Castigliano’s analysis likely resulted in the calculated deflection to be greater than the FEA simulation, as the addition of the filets in the CAD model decreased stress for the cost of decreasing deflection simultaneously.

Another major reason for the discrepancies between Castigliano's analysis and the FEA predictions is the approximation of member CE as a two-force member. A two-force member is one which only has forces acting at two points of the beam and those forces are in an axial direction - this would only be achieved if member CE was connected to the rest of the structure via pins which restrict motion in the horizontal and vertical directions only. However, because member CE was connected to the structure via fixed connections, the true structure experiences not only vertical and horizontal restrictions, but also rotational restrictions. This results in an additional moment caused by the fixed connections at both ends of member CE. If Castigliano’s analysis was completed using the appropriate fixed geometries at these specified locations, the additional moment would likely decrease the final calculated deflection, as the top and bottom members would not be allowed to rotate about their connections to beam CE, decreasing their angles of deflection.

Finally, another factor that could have influenced the differences between Castigliano's analysis and the FEA results from what was actually observed following Instron testing may have been a result of the varying thickness of the material. While the CAD model was built with a specified 0.25” thickness, this was not the true thickness of the Al 6061-T6511, which varied in thickness. A thinner actual material likely led to a lower stiffness and strength of the overall material. Also, the calculations assumed fully elastic deflection, whereas yielding would have increased the deflection produced by the 200 lb force applied. Since many portions of the CAD model were extremely close to the yield point, a very small change in thickness could have caused the structure to yield sooner than expected. Because of this, both Castigliano's analysis and FEA underestimated the deflection in comparison to the actual deflection observed from Instron testing.