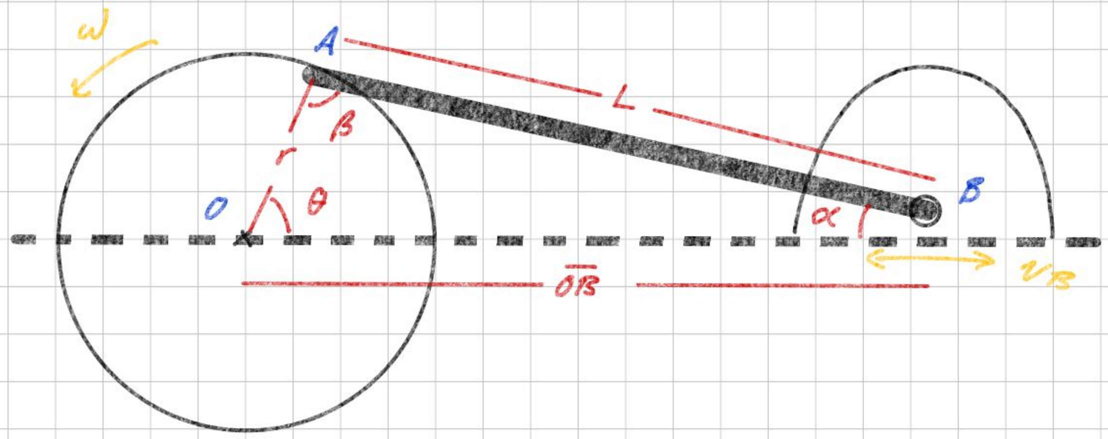
"MECHANISMS MODELS" PHASE 2A



Assumptions

Fixed: r, L

Variable: $\phi, \overline{OB}, \omega, \alpha$

Trigonometric relations

① $\frac{\sin\phi}{L} = \frac{\sin\alpha}{r} = \frac{\sin\beta}{\overline{OB}}$

② $r^2 = L^2 + \overline{OB}^2 - 2L(\overline{OB})\cos\alpha$

③ $L\cos\alpha = L^2 - r^2 + \overline{OB}^2 / 2\overline{OB}$

④ $L^2 = r^2 + \overline{OB}^2 + 2r(\overline{OB})\cos\phi$

$$V_B = V_A + V_{B/A}$$

$$= (\omega_{OA} \times \vec{r}_{A/O}) + (\omega_{BA} \times \vec{r}_{A/B})$$

$$= [\omega_{OA}\hat{k} \times (r\cos\phi\hat{i} + r\sin\phi\hat{j})] + [\omega_{BA}\hat{k} \times (L\cos\alpha\hat{i} + L\sin\alpha\hat{j})]$$

$$= \omega_{OA} r\cos\phi\hat{j} - \omega_{OA} r\sin\phi\hat{i} + \omega_{BA} L\cos\alpha\hat{j} - \omega_{BA} L\sin\alpha\hat{i}$$

$$= (-\omega_{OA} r\sin\phi - \omega_{BA} L\sin\alpha)\hat{i} + (\omega_{OA} r\cos\phi + \omega_{BA} L\cos\alpha)\hat{j}$$

$$= \left(-\frac{d\phi}{dt} r\sin\phi - \boxed{\frac{d\alpha}{dt}} L\sin\alpha\right)\hat{i} + \left(\frac{d\phi}{dt} r\cos\phi + \boxed{\frac{d\alpha}{dt}} L\cos\alpha\right)\hat{j}$$

$\frac{d\alpha}{dt} = \frac{d\alpha}{d\phi} \cdot \frac{d\phi}{dt} \rightarrow$ Need α in terms of ϕ; $\alpha(\phi)$

$r\sin\phi = L\sin\alpha$

$\alpha = \sin^{-1}\left(\frac{r}{L}\sin\phi\right) \Rightarrow \frac{d\alpha}{d\phi} = \frac{r\cos\phi}{\sqrt{(1 - r^2/L^2 \sin^2\phi)}}$

$$= \left(-\frac{d\phi}{dt} r\sin\phi - \frac{r\cos\phi}{\sqrt{1 - r^2/L^2 \sin^2\phi}} \frac{d\phi}{dt} L\sin\alpha\right)\hat{i} + \left(\frac{d\phi}{dt} r\cos\phi + \frac{r\cos\phi}{\sqrt{1 - r^2/L^2 \sin^2\phi}} \cdot \frac{d\phi}{dt} \left(L^2 - r^2 + \overline{OB}^2 / 2\overline{OB}\right)\right)\hat{j}$$

$\overline{OB}^2 - 2r\cos\phi(\overline{OB}) + r^2 - L^2$; Let $r^2 - L^2 = a =$ known const.

$\overline{OB}^2 - 2r\cos\phi(\overline{OB}) + a = 0$

$\overline{OB} = \frac{2r\cos\phi \pm \sqrt{4r^2\cos^2\phi - 4a}}{2}$ ← Quadratic Formula

$= \frac{\cancel{2}r\cos\phi \pm \cancel{2}\sqrt{r^2\cos^2\phi - r^2 + L}}{\cancel{2}}$

$= r\cos\phi \pm \sqrt{L^2 - r^2\sin\phi}$ ← Comes from Pythag's ID.

$$v_B = \left[-\frac{d\phi}{dt}\, r\sin\phi \left(1 + \frac{r\cos\phi}{\sqrt{1 - r^2/L^2 \sin^2\phi}} \right) \right] \hat{i}$$

$$+ \cancel{\left\{ \frac{d\phi}{dt}\, r\cos\phi \left\langle 1 + \frac{1}{\sqrt{1 - r^2/L^2 \sin^2\phi}} \; \frac{d\phi}{dt} \left[\frac{L^2 - r^2 + (r\cos\phi + \sqrt{L^2 - r^2 \sin^2\phi})}{2(r\cos\phi + \sqrt{L^2 - r^2\sin^2\phi})} \right] \right\rangle \right\}} \hat{j}$$

↳ $\emptyset$; No vertical movement

$\phi \rightarrow$ Some funct'n acting as an input for v_B.

$v_B \rightarrow$ 1 function dependent on ϕ & t

r_sin_phi $\rightarrow r\sin\phi$

r_cos_phi $\rightarrow r\cos\phi$

sqrt_second_term $\rightarrow r^2/L^2 \sin^2\phi$

sqrt $\rightarrow \sqrt{1 -}$ sqrt_second_term

a $\rightarrow L^2 - r^2$

second_sqrt $\rightarrow \sqrt{L^2 - r^2 \sin^2\phi}$

phi_diff = sym.diff(phi_function)