We now know that for a point potential acting on x_0 on a Bethe lattice with $z = 3, 4$ we have a "phase transition" from a no-QP to a QP regime for a critical value of the potential $V_0^*(z)$. We now ask ourselves what happens when we change the shape of the potential, i.e., we allow for the potential to include more sites keeping a "step-box" shape. This change has three important cases:

1. Fixed area: this case amounts to fixing the area of the potential well A.
2. Fixed width: we can fix the width ℓ of the potential and change the depth V_0. Note that fixing ℓ is the same as fixing the number of steps s.
3. Fixed depth: we fix the depth of the well, denoted by V_0 and change the number of sites (steps) affected by it.

We denote with V_0 the value of the potential at the site x_0, V_1 will then be the value of the potential at the nearest neighbouring sites, V_2 at the next nearest and so on. Note that $|V_0| > |V_1| > |V_2| > \dots$

1 Fixing the area

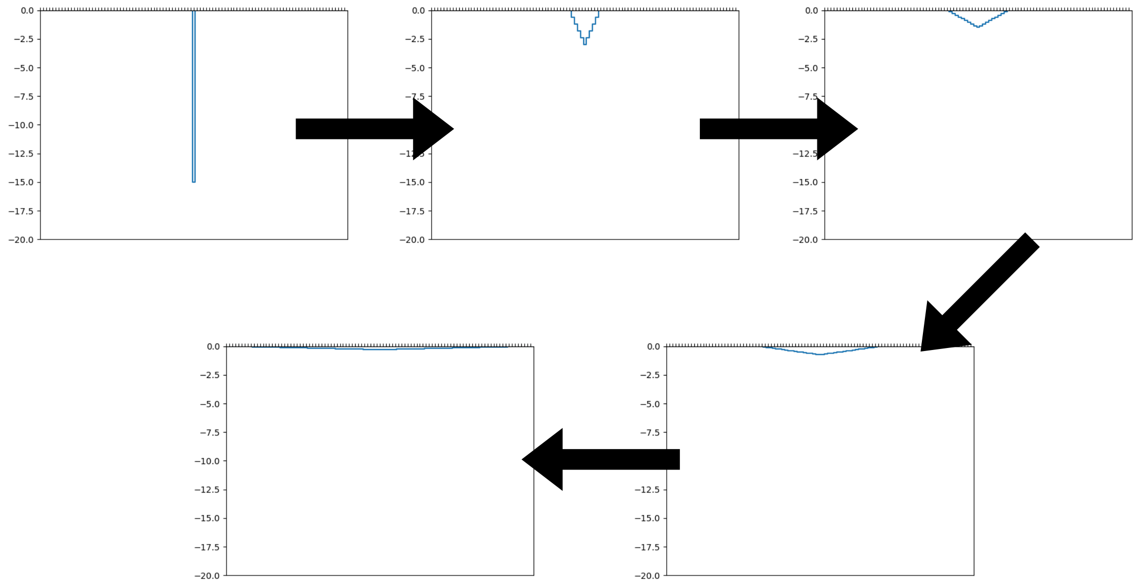

Figure 1: Fixing the area A leads to the limit of almost uniform potential across the lattice for every starting value of V_0.

The way the potential changes when we fix the area A is described in Fig.1. There are several ways to change A in terms of V_i when increasing ℓ. We decided to always make the increase in a "linear" way, meaning the difference $|V_i - V_j|$ is equal for all NN i and j. We expect to have disappearance of QP when starting with a point potential $V_0 > V_0*$ and "shallowing" the well to a uniform potential. This will happen for a certain ratio $r(A)$ with $r = n/N$, with n being the number of sites affected by the potential and N being the size of the system (can also be done in terms of ℓ).

Note that we are taking one step per site, so that the integral (area) of the potential curve is given by $A = \sum_i V_i$ and A is then equal to the initial value of V_0.

2 Fixing the width

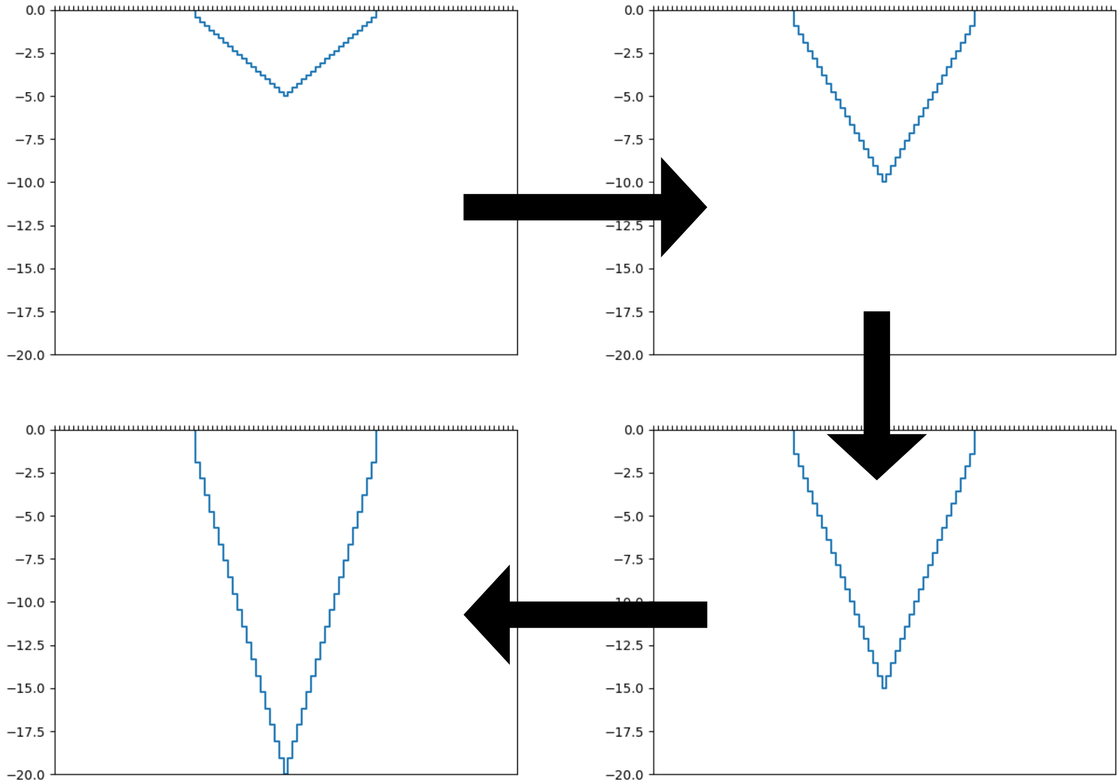

Figure 2: Fixing the width and having one step per site means we can only increase V_0.

When we fix the width ℓ, we expect the behaviour to be similar to that of the point potential, but with a change of the critical value V_0^*. Therefore, we want to study the behaviour of $V_0^*(\ell)$. We have decided to still increase the depth "linearly". If we find something interesting we might also consider how increasing the depth differently affects the properties of the transition.

3 Fixing the depth

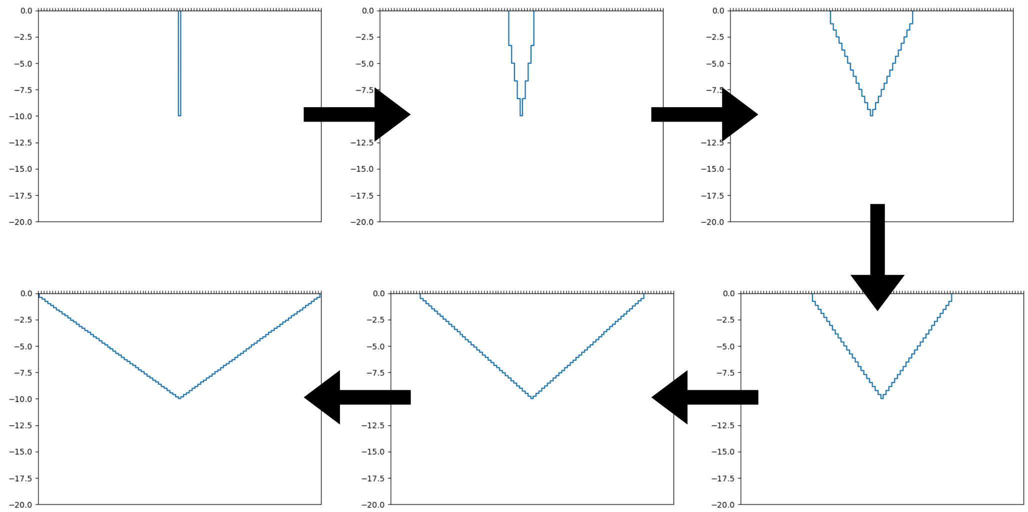

Figure 3: Fixing the depth leads to the limiting case of almost linear potential along the whole lattice.

Fixing the depth means fixing the value of V_0. We can then increase the width to include more sites. The interesting case is when starting from a value $V_0 < V_0^*$, then we should go from no-QP to QP for some critical value $r^*(V_0)$ (r defined above). Moreover, when starting from $V_0 > V_0^*$, we go from one QP to more. This might also be an interesting point if we can understand how it works as a function of r (or ℓ).