The fate of the spin polaron in the 1D antiferromagnet: supplementary materials

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t–J MODEL IN THE STAGGERED FIELD IN THE MAGNON-HOLON LANGUAGE

The t-J model Hamiltonian reads,

$$H = -t\sum_{\langle i,j\rangle}\left(\tilde{c}^\dagger_{i\sigma}\tilde{c}_{j\sigma} + H.c\right) + J\sum_{\langle i,j\rangle}\left[\frac{1}{2}\left(S^+_i S^-_j + S^-_i S^+_j\right) + S^z_i S^z_j - \frac{1}{4}\tilde{n}_i\tilde{n}_j\right], \tag{1}$$

where $\tilde{c}^\dagger_{i\sigma} = c^\dagger_{i\sigma}(1-n_{i\bar{\sigma}})$ can create electrons only on unoccupied sites. Let us consider a t–J chain sandwiched between two Ising chains. The energy of this system is described by the stated below coupled chains Hamiltonian,

$$H_{cc} = H + H_{\|} + H_{\perp}, \tag{2}$$

where H stands for t–J Hamiltonian with $J = J_{\|}$ and,

$$H_{\|} = J_{\|}\sum_{\langle i,j\rangle}\left(S^z_{i+\hat{y}}S^z_{j+\hat{y}} + S^z_{i-\hat{y}}S^z_{j-\hat{y}}\right), \tag{3}$$

$$H_{\perp} = \frac{J_\perp}{2}\sum_{\langle i,j\rangle}\left(S^z_i S^z_{i+\hat{y}} + S^z_j S^z_{j+\hat{y}} + S^z_i S^z_{i-\hat{y}} + S^z_j S^z_{j-\hat{y}}\right). \tag{4}$$

Assuming that the presence of t–J chain does not perturb the perfect Ising ordering in the ground state of $H_{\|}$ we obtain,

$$H_{\perp} = \frac{J_\perp}{2}\sum_{\langle i,j\rangle}\left(S^z_i\langle S^z_{i+\hat{y}}\rangle + S^z_j\langle S^z_{j+\hat{y}}\rangle + S^z_i\langle S^z_{i-\hat{y}}\rangle + S^z_j\langle S^z_{j-\hat{y}}\rangle\right) \tag{5}$$

$$= \frac{J_\perp}{2}\sum_{\langle i,j\rangle}\left[(-1)^i S^z_i + (-1)^j S^z_j\right]. \tag{6}$$

In the end, the t–J chain feels an effective staggered field $B = J_\perp$ comming from additional chains in its proximity. Moreover, the above is in the same way valid also when there are more Ising chains weakly coupled to one another.

Now let investigate how does the additional staggered field pronounce itself when we consider t–J model in polaronic description. In order to introduce holes and magnons we start with rotation of spins in one sublattice of the system. This results in

$$H_{\rm rot} = -t\sum_{\langle i,j\rangle}\left(\tilde{c}^\dagger_{i\sigma}\tilde{c}_{j\bar{\sigma}} + H.c\right) + J\sum_{\langle i,j\rangle}\left[\frac{1}{2}\left(S^+_i S^+_j + S^-_i S^-_j\right) - S^z_i S^z_j - \frac{1}{4}\tilde{n}_i\tilde{n}_j\right]. \tag{7}$$

This allows for introduction of holes and magnons accoridng to the following transformations

$$\begin{aligned} \tilde{c}^\dagger_{i\uparrow} &= h_i, & \tilde{c}_{i\uparrow} &= h^\dagger_i(1-a^\dagger_i a_i), \\ \tilde{c}^\dagger_{i\downarrow} &= h_i a^\dagger_i, & \tilde{c}_{i\downarrow} &= h^\dagger_i a_i, \end{aligned} \tag{8}$$

$$\begin{aligned} S^+_i &= h_i h^\dagger_i(1-a^\dagger_i a_i)a_i, & S^z_i &= \left(\frac{1}{2} - a^\dagger_i a_i\right)h_i h^\dagger_i, \\ S^-_i &= a^\dagger_i(1-a^\dagger_i a_i)h_i h^\dagger_i, & \tilde{n}_i &= 1 - h^\dagger_i h_i = h_i h^\dagger_i. \end{aligned} \tag{9}$$

Here magnons can be understood as deviations from state that after the rotation has all the spins pointing up. In the end the model (up to the shift by a constant energy) reads,

$$\mathcal{H} = \mathcal{H}_t + \mathcal{H}_J, \tag{10}$$

where,

$$\mathcal{H}_t = t\sum_{\langle i,j\rangle}\left[h^\dagger_i h_j\left(a_i + a^\dagger_j(1-a^\dagger_i a_i)\right) + h^\dagger_j h_i\left(a_j + a^\dagger_i(1-a^\dagger_j a_j)\right)\right], \tag{11}$$

$$
\begin{aligned}
\mathcal{H}_J = & \frac{J}{2}\sum_{\langle i,j\rangle} h_i h_i^\dagger \left[(1-a_i^\dagger a_i)(1-a_j^\dagger a_j)a_i a_j + a_i^\dagger a_j^\dagger (1-a_i^\dagger a_i)(1-a_j^\dagger a_j)\right] h_j h_j^\dagger \\
& + \frac{J}{2}\sum_{\langle i,j\rangle} h_i h_i^\dagger \left[a_i^\dagger a_i + a_j^\dagger a_j - 2a_i^\dagger a_i a_j^\dagger a_j - 1\right] h_j h_j^\dagger .
\end{aligned}
\tag{12}
$$

Now let us investigate a staggered magnetic field term,

$$
H_B = \frac{B}{2}\sum_{\langle i,j\rangle}\left[(-1)^i S_i^z + (-1)^j S_j^z\right]. \tag{13}
$$

Performing the same set of transformations as to the Hamiltonian we obtain (up to the constant energy shift),

$$
\mathcal{H}_B = \frac{B}{2}\sum_{\langle i,j\rangle}\left[a_i^\dagger a_i h_i h_i^\dagger + a_j^\dagger a_j h_j h_j^\dagger\right] \approx \frac{B}{2}\sum_{\langle i,j\rangle} h_i h_i^\dagger \left[a_i^\dagger a_i + a_j^\dagger a_j\right] h_j h_j^\dagger , \tag{14}
$$

where ommited terms on the righthandside of the approximation sign are modifying the magnetic field only around the hole and they are equal to $\frac{B}{2}\left[a_i^\dagger a_i h_j h_j^\dagger + a_j^\dagger a_j h_i h_i^\dagger\right]$. In the end we obtain,

$$
\begin{aligned}
\mathcal{H}_{J+B} = & \mathcal{H}_J + \mathcal{H}_B \\
\approx & \frac{J}{2}\sum_{\langle i,j\rangle} h_i h_i^\dagger \left[(1-a_i^\dagger a_i)(1-a_j^\dagger a_j)a_i a_j + a_i^\dagger a_j^\dagger (1-a_i^\dagger a_i)(1-a_j^\dagger a_j)\right] h_j h_j^\dagger \\
& + \frac{J}{2}\sum_{\langle i,j\rangle} h_i h_i^\dagger \left[\left(1+\frac{B}{J}\right)\left(a_i^\dagger a_i + a_j^\dagger a_j\right) - 2a_i^\dagger a_i a_j^\dagger a_j - 1\right] h_j h_j^\dagger .
\end{aligned}
\tag{15}
$$

Let us introduce a new coupling constant $J_z = J + B$ and parameter $\lambda = J/J_z$. Then, in the limit of a single hole, we can write,

$$
\begin{aligned}
\mathcal{H}_{J+B} \approx & \frac{J_z\lambda}{2}\sum_{\langle i,j\rangle} h_i h_i^\dagger \left[(1-a_i^\dagger a_i)(1-a_j^\dagger a_j)a_i a_j + a_i^\dagger a_j^\dagger (1-a_i^\dagger a_i)(1-a_j^\dagger a_j)\right] h_j h_j^\dagger \\
& + \frac{J_z}{2}\sum_{\langle i,j\rangle} h_i h_i^\dagger \left[a_i^\dagger a_i + a_j^\dagger a_j - 2\lambda a_i^\dagger a_i a_j^\dagger a_j\right] h_j h_j^\dagger .
\end{aligned}
\tag{16}
$$

For $\lambda \neq 1$ coupling constants in xy plane and in z direction are different. In the end the resulting model can be undestood as the t–J model with XY anisotropy *and* rescaled magnon-magnon interaction.

In the end, we can relate the parameters of copuled chains to the parameters of the t–J model with rescaled magnon-magnon interactions and XY anisotropy,

$$
J_z = J + B = J_\parallel + J_\perp , \tag{17}
$$

$$
\lambda = \frac{J}{J_z} = \frac{1}{1+\frac{J_\perp}{J_\parallel}} . \tag{18}
$$

In the table below we present values used in the calculations for the Fig. 4b of the main text.

$J_\parallel$	$J_\perp$	J_z	λ
$0.4t$	$0.004t$	$0.404t$	$\frac{100}{101}$
$0.4t$	$0.04t$	$0.44t$	$\frac{10}{11}$
$0.4t$	$0.2t$	$0.6t$	$\frac{2}{3}$

TABLE I. Table presenting various values of parameters equivalent in coupled chains problem and t–J model with rescaled magnon-magnon interactions and XY anisotropy.

SU(2) SYMMETRY BREAKING IN t–J MODEL WITH TUNEABLE MAGNON-MAGNON INTERACTIONS

We start by expressing the magnon-magnon interaction term in the 'standard' (i.e. spin) language,

$$a_i^\dagger a_i a_j^\dagger a_j = -S_i^z S_j^z + \frac{1}{4}\tilde{n}_i\tilde{n}_j - \frac{1}{2}\left(\xi_i^{\mathcal{A}} S_i^z \ + \xi_j^{\mathcal{A}} S_j^z\right)\tilde{n}_i\tilde{n}_j, \tag{19}$$

where $\xi_i^{\mathcal{A}}$ equals -1 for $i \in \mathcal{A}$ and equals 1 otherwise, with $\mathcal{A}, \mathcal{B}$ denoting sublattices of the bipartite lattice. Thus, Hamiltonian (2) of the main text (i.e. the t–J model with tuneable magnon-magnon interactions) reads,

$$H = -t\sum_{\langle i,j\rangle}\left(\tilde{c}_{i\sigma}^\dagger \tilde{c}_{j\sigma} + \text{H.c.}\right) + J\sum_{\langle i,j\rangle}\left\{S_i S_j - \frac{1}{4}\tilde{n}_i\tilde{n}_j + (\lambda-1)\left[S_i^z S_j^z - \frac{1}{4}\tilde{n}_i\tilde{n}_j + \frac{1}{2}\left(\xi_i^{\mathcal{A}} S_i^z \ + \xi_j^{\mathcal{A}} S_j^z\right)\tilde{n}_i\tilde{n}_j\right]\right\}. \tag{20}$$

In the above Hamiltonian the term

$$\frac{1}{2}\left(\xi_i^{\mathcal{A}} S_i^z \ + \xi_j^{\mathcal{A}} S_j^z\right)\tilde{n}_i\tilde{n}_j, \tag{21}$$

can be understood as a staggered field acting on all spins although it is halved for the neighbors of the hole. This term contributes to the Hamiltonian once $\lambda \neq 1$ and explicitly breaks the SU(2) symmetry.