Quantum Information Flow

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May 31, 2024

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Recent developments

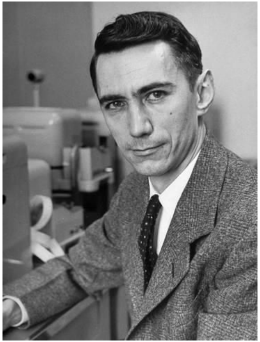
- Topological-quantum-simulator at room temp.
- Quantum-storage in 2-D material
- NWO research fundamental limits
- Australia progress
- Progress in China

Relevance quantum stack

1.) Noise sources: cross-talk, quantum noise
2.) Time and Space complexity of algorithms
3.) Thermal energy
4.) Communication and capacity
5.) Statistics
6.) Signal processing
7.) Foundations

Father Information theory

Claude Elwood Shannon (April 30, 1916 – February 24, 2001) was an American mathematician, electrical engineer, and cryptographer known as a father of information theory.
(Claude Shannon - The Bit Player Movie Trailer)



Information and communication theory

- Classic paper by Claude Shannon on communication theory starting point [1]
- Model communication in the presence of noise
- Concept of Information $H = \sum_{i=1}^{N} -p_i \log_2(p_i)$ (1 bit is tossing a coin)
- Entropy (wanorde) and Information use similar (minus sign)
- Relative entropy $KL(P||Q) = \sum_{x\epsilon X} -P(x) \log Q(x)/\log P(x)$ (Kullback Leiter Divergence/Distance, used in ML)
- Basis for calculation channel capacity, error correction, versions for analogue and digital systems

Physics concept entropy and information

ENTROPY

Boltzmann

$$S_B = k_B \log w_e$$

Gibbs

$$S_G(p) = -k_B \sum_i p_i \log p_i$$

Boltzmann constant $1.38065 \cdot 10^{-23} \frac{J}{K}$

Total number of configurations of the system

Rényi

$$H_\alpha(p) = \frac{1}{1-\alpha} \log \sum_i p_i^\alpha$$

$\alpha \to 1$

$\alpha \geq 0$

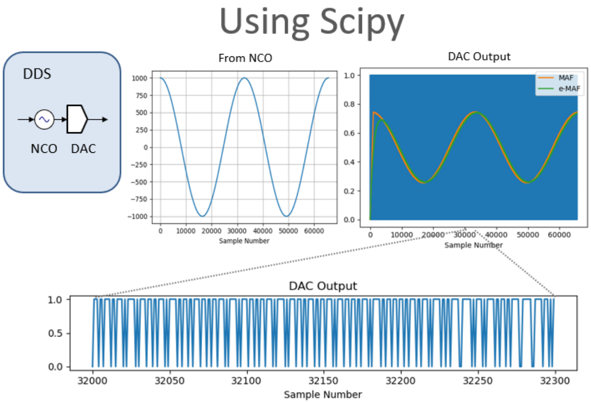
$\alpha \neq 1$

Shannon

$$H(p) = -\sum_i p_i \log p_i$$

Probability of each element of the alphabet

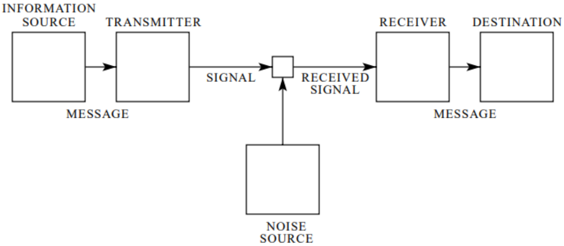
PyGSP. Scipy, Pyvisa Virtual instrument, Scipy examples, Scientific Python Lectures



Information theory applications

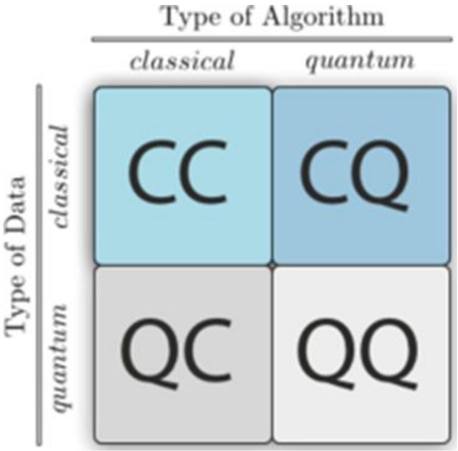
- Data compression
- Source coding
- Encoding in presence of noise
- Information content analogue and digital systems
- Machine Learning

Information and Channel capacity



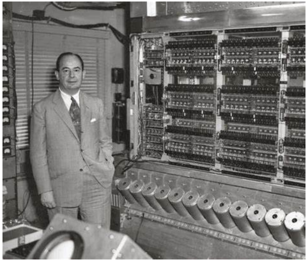
Channel capacity is related to information and noise in channel, $C = Blog_2(1 + \frac{S}{N})$ where C maximum bitrate/s, channel bandwidth, S/N signal to noise power (Shannon–Hartley theorem) and basis for encoding designs

Classic and quantum signal processing



Influence John von Neumann

Von Neumann made major contributions to many fields, including mathematics (foundations of mathematics, functional analysis, ergodic theory, group theory, representation theory, operator algebras, geometry, topology, and numerical analysis), physics (quantum mechanics, hydrodynamics, and quantum statistical mechanics), economics (game theory), computing (Von Neumann architecture, linear programming, self-replicating machines, stochastic computing), and statistics. He was a pioneer of the application of operator theory to quantum mechanics in the development of functional analysis, and a key figure in the development of game theory and the concepts of cellular automata, the universal constructor and the digital computer.



John von Neumann life

Quantum Information Science and Quantum Information Theory

Quantum information science is an interdisciplinary field that seeks to understand the analysis, processing, and transmission of information using quantum mechanics principles. It combines the study of Information science with quantum effects in physics. It includes theoretical issues in computational models and more experimental topics in quantum physics, including what can and cannot be done with quantum information.

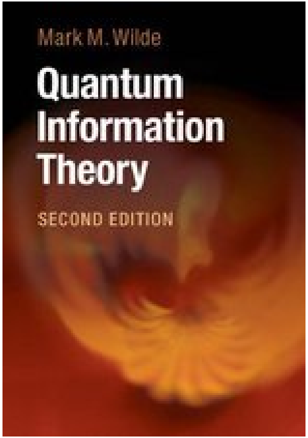
The term quantum information theory is also used, but it fails to encompass experimental research, and can be confused with a subfield of quantum information science that addresses the processing of quantum information. (overview Wikpedia)

Extension to quantum information theory

- Qubits (continuous) instead of bits(digital)
- Von Neuman entropy concept used in Quantum Computing based on density matrix
- Entropy of entanglement defines how states are related
- Quantum Information's Revolutionary Origins — Charles Bennett

Quantum Information theory references

- A mathemathical theory of communicatioon [1]
- Classical and quantum information theory [2]
- Quantum Information Science Youtube tutorial by Artur Eckert
- Quantum Information Theory and Foundationis of Quantum Mechanics [3]
- Quantum Information Processing and Error Correction[4]
- Quantum Information Theory [5]



Relaxion and dephasing

T1 is relaxation time i.e. time from $|1\rangle$ to $|\,|0\rangle$)
T2 is so-called dephasing time. It describes how long the phase of a qubit stays intact. In your words, it is time from $\langle +| = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ to $\langle -| = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ or conversely.
Both T1 and T2 are not actually "time from state x to state y" but rather decay constants. The corresponding probabilities after time t are given by formulas:

$$P_1(|1\rangle\,, t) = e^{-\frac{t}{T_1}}$$

$$P_2(|+\rangle\,, t) = e^{-\frac{t}{T_2}}$$

Cross-talk another source (electro-magnetic fields generate unwanted couplings between qubits.

Noise modelling physics

See Tutorials

- Device noise simulation
- qiskit-aer

Quantum sensing and measurement

- Quantum sensing and measurement
- https://qsm.quantumtinkerer.tudelft.nl/
- Basics of noise

Quantum Information theory topics

- Noiseless Quantum Theory
- Noisy Quantum Theory
- Purfied Quantum Theory
- Distance Measures
- Quantum Information and entropy
- Quantum Entropy inequalities and recoverability
- Information of Quantum channels
- The Packing Lemmas
- The Covering lemmas
- Schumachier Compression
- Entanglement Manipulation
- Coheren Communication with noisy resources
- Private Classical commmunication
- Quantum communication
- Trading resources for communication
- and more

Standard algorithms overview [6], [7]

Class	Problem/Algorithm	Paradigms used	Hardware	Simulation Match
Inverse Function Computation	Grover's Algorithm	GO	QX4	med
	Bernstein-Vazirani	n.a.	QX4, QX5	high
Number-theoretic Applications	Shor's Factoring Algorithm	QFT	QX4	med
Algebraic Applications	Linear Systems	HHL	QX4	low
	Matrix Element Group Representations	QFT	ESSEX	low
	Matrix Product Verification	GO	n.a.	n.a.
	Subgroup Isomorphism	QFT	none	n.a.
	Persistent Homology	GO, QFT	QX4	med-low
Graph Applications	Quantum Random Walk	n.a.	VIGO	med-low
	Minimum Spanning Tree	GO	QX4	med-low
	Maximum Flow	GO	QX4	med-low
	Approximate Quantum Algorithms	SIM	QX4	high
Learning Applications	Quantum Principal Component Analysis (PCA)	QFT	QX4	med
	Quantum Support Vector Machines (SVM)	QFT	none	n.a.
	Partition Function	QFT	QX4	med-low
Quantum Simulation	Schrödinger Equation Simulation	SIM	QX4	low
	Transverse Ising Model Simulation	VQE	none	n.a.
Quantum Utilities	State Preparation	n.a.	QX4	med
	Quantum Tomography	n.a.	QX4	med
	Quantum Error Correction	n.a.	QX4	med

Table 2. Overview of studied quantum algorithms. Paradigms include Grover Operator (GO), Quantum Fourier Transform (QFT), Harrow-Hassidim-Lloyd (HHL), Variational Quantum Eigenvalue solver (VQE), and direct Hamiltonian simulation (SIM). The simulation match column indicates how well the hardware quantum results matched the simulator results

Quantum Information Theory use

- Teleportation (transfer of qubit)
- Teleoperation (remote control)
- Error correction
- Encoding (compression)
- Machine learning
- Theoretical limitations

The *von Neumann Entropy* of a quantum state ρ (where ρ is a density matrix) is defined to be

$$S(\rho) = -Tr(\rho \log(\rho)). \tag{1}$$

When $\{\lambda_k\}$ is complete set of *eigenvalues* of the matrix ρ:

$$S(\rho) = -\sum_k \lambda_k \log(\lambda_k). \tag{2}$$

Notebook: Entropy jupyter notebook taken from Implementation in python

Density matrix

See Qiskit notebook on density: Qiskit density In the conventional state vector notation, an n-qubit pure state can be expressed as:

$$|\psi\rangle = \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_{N-1} \end{bmatrix},$$

where $N = 2^n$. An alternative way to express this pure quantum state is in the form of a matrix. This can be done by using the density operator representation, which is defined as:

$$\rho \equiv |\psi\rangle\langle\psi|$$

Here, the term $|\psi\rangle\langle\psi|$ represents the outer product of the state ψ with itself:

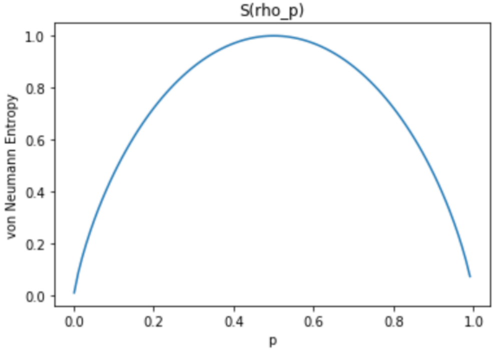
Density matrix

$$\rho = \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_N \end{bmatrix} \begin{bmatrix} \alpha_0^* & \alpha_1^* & \dots & \alpha_N^* \end{bmatrix}$$

$$\rho = \begin{bmatrix} |\alpha_0|^2 & \alpha_0\alpha_1^* & \dots & \alpha_0\alpha_N^* \\ \alpha_1\alpha_0^* & |\alpha_1|^2 & \dots & \alpha_1\alpha_N^* \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_N\alpha_0^* & \alpha_N\alpha_1^* & \dots & |\alpha_N|^2 \end{bmatrix}$$

Dependency on probabilities

$$\rho(p) = p|0\rangle\langle 0| + (1-p)|1\rangle\langle 1| \quad (3)$$



For p=0, p=1/2 and p = 1 same values as Shannon entropy

Physics and quantum information

- overview information sciences Wikpedia
- overview density matrix Wikipedia
- Eigenvectors and Quantum Information
- Axiom description physics
- No go theorems
- Reference Quantum Information Theory notes

Eigenvectors

Definition

An **eigenvalue** for $A \in M_n$ is a complex number $\lambda \in \mathbb{C}$ such that there is a nonzero vector $|x\rangle \in \mathbb{C}^n$ satisfying

$$A\,|x\rangle = \lambda\,|x\rangle \tag{4}$$

Definition

A matrix $A \in M_n$ is **diagnolizable** if and only if:

1.) There exists a diagonal matrix D and an invertable matrix P such that $A = PDP^{-1}$ if and only if

2.) there exists a basis for C^n consisting of eigenvalues for A.

Hermitian operators

1.) Self-Adjoint (Hermitian)

$$A \in M_n \text{ is Hermitian if}$$
$$A^\dagger = A \tag{5}$$
$$For\ Pauli\ matrices\ : {\sigma_x}^\dagger = \sigma_x, {\sigma_y}^\dagger = \sigma_y, {\sigma_z}^\dagger = \sigma_z$$

Additionally, all Hermitian matrices are diagnolizable, meaning their eigenvalues form a orthonormal basis. Dagger is equavalent to Complex conjugate, Hermitian conjugate or adjoint matrix includes conjugated entries next to transpose

2.) Normal Matrices (every normal matrix is diagonalizable)

$$A * A^\dagger = A^\dagger * A \leftrightarrow \text{eigenvectors form orthonormal basis} \tag{6}$$

Hermitian operators continued

3.) Positive semi-definite matrix M_n is a symmetric matrix with no negative eigen values:

$$\begin{aligned}&\forall \left|x\right\rangle \in \mathbb{C}^n, \text{we have}\\&\langle x|M_n|x\rangle \geq 0 \text{ where } M_n \left|x\right\rangle = y\end{aligned} \tag{7}$$

4.) (Unitary Matrices) The following are all equivalent:

$U \in M_n$ is unitary

$U^\dagger = U^{-1}$ or $(U^{-1} * U = U * U^{-1} = I_n)$

$U * U^\dagger = I_n$ or $U^\dagger * U = I_n$ *note: $U^\dagger$ not always equal to U, but they are normal.*

The column of U form an orthonormal basis for $\mathbb{C}^n$.

$\forall \left|x\right\rangle, \left|y\right\rangle \in \mathbb{C}^n$ and $\langle U_x|U_y\rangle = \langle x|y\rangle$ meaning unitary matrices are rotation matrices.

Quantum physics axiom 1

Classically, $|x\rangle$ should be polarized $\uparrow$ or $\rightarrow$, if this is the case we expect two outcomes:

1.) If $|x\rangle$ goes through filter, then $|x\rangle$ is $\uparrow$.

2.) If $|x\rangle$ is deflected, then $|x\rangle$ is $\rightarrow$.

Quantum physics axiom 1 continued

Physical States (Choices made by humans to describe quantum mechanics):

$\uparrow = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \rightarrow = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ (pure state)

Something of note, these two vectors form a basis for $\mathbb{C}^2$ and all combinations are:

$|x\rangle = \alpha \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ where $\|\alpha\|^2 + \|\beta\|^2 = 1$

To keep track of this choice, we make the matrix $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ representing vertically polarized filter.

Another note, the eigen-vals of that matrix are $|v\rangle, |h\rangle$.

Quantum physics axiom 2

An observable of a state $|x\rangle$ corresponds to a Hermitian matrix A. $|x\rangle$ is in state $|u\rangle$, an eigen-vect for A with probability $|\langle u|x\rangle|^2$

Definition

Expectation value of A (or mean value) of observable associated to A after measurements with respect to many copies of $|\psi\rangle$ is **the weighted average of the expected outcomes**.

$$\langle A\rangle_\psi = \sum_{i=1}^{n} |\langle u_i|\psi\rangle|^2$$

$$|\psi\rangle = \sum_{i=1}^{n} c_i\,|u_i\rangle \text{ where } u_i \text{ is an e-basis from A}$$

Quantum physics axiom 3

The time dependence of a state is governed by the Schrödinger equation:

$$i\bar{h}\frac{\delta|\psi|}{\delta t} = H\,|\psi\rangle \tag{8}$$

$\bar{h}$ is reduced Planck's constant
H is Hermitian matrix corresponding to energy of the system **Hamiltonian**.
When H is time invariant (constant), the Schrödinger equation becomes:

$$|\psi(t)\rangle = e^{\frac{-itH}{\bar{h}}}\,|\psi(0)\rangle \tag{9}$$

Applications eigen vectors

- Part of algorithm analysis (e.g. phase estimation, measurements)
- Principal Component Analysis transforms the covariance matrix into an ordering of feature combinations based on eigenvectors, eigenvalues
- The quantum algorithm for principal component analysis is called the LMR algorithm ([8])
- Application in von Neuman entropy
  - ✓ $S = \sum_{i=1}^{N} -\eta_i \ln(\eta_i)$ where η_i are eigenvalues.
  - ✓ $\rho = \sum_j \eta_j |j\rangle \langle j|$, $S = -\mathrm{tr}(\rho \ln \rho)$

Time evolution density

Axiom 1': A physical state of a system, whose Hilbert Space $\mathbb{H}$, is completely determined by it's associated density matrix.

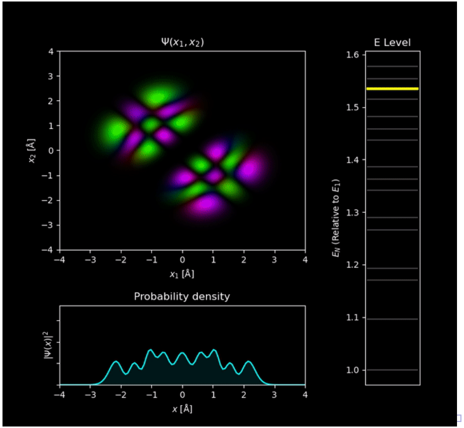
Axiom 2': The mean value of an observable A is $< A >_\psi = tr(\rho A)$

Axiom 3': The time evolution of density matrix is given by the Louisville - von Neumann equation

$$i\hbar \frac{d}{dt}\rho = [H, \rho] \text{ where commutator operator is defined as } [A, B] = AB - BA \tag{10}$$

Quantum physics

- Quantum Toolbox in Python
- Quantum visualizations Schrödinger equation



no go theorems

- No cloning: There is no unitary operator that can clone an arbitrary quantum state
- No Broadcasting: One cannot broadcast the ‘same’ quantum state to more than one receiver
- No Teleportation: One cannot determine an arbitrary quantum state by one measurement alone. I.e: the information carried in a single qubit cannot be converted to classical bits.
- No Hiding: If information is lost in a quantum system due to the interactions with its environment (through decoherence), then the information has simply moved from the system to the environment.
- No deletion: Given two copies of some arbitrary quantum state, it is impossible to delete one of the copies against the other
- No Signalling: It is not possible for one observer, by making a measurement of a subsystem of the total state, to communicate information to another

Einstein Podolsky and Rosen(EPR) Paradox

[9]

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

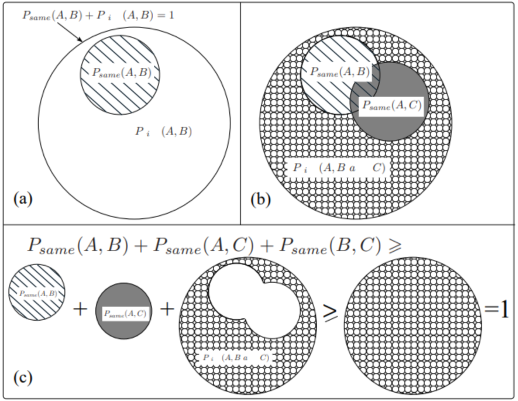
Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. Einstein, B. Podolsky and N. Rosen, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

Bell inequality proof



[10], see notebook on Bell inequalities, taken from github book [11]:Chapter 7 code (CSH inequality)

Conclusion

- Classical Information Theory
- Extension to Quantum Information Theory and Quantum Information Science
- Quantum Information Science and fundamentals
- Many research questions related to quantum stack

Options paper

Exerise using notebooks:

- Experiment with Entropy notebooks
- Check Bell inequality with notebook (CSH inequality
- Experiment with noise and qiskit libraries
- Experiment with quantum physics visualisation and libraries
- Study basic papers about paradoxes and Bell's inequality
- Study specific Quantum Information topics (consult teacher(s))

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