# Quantum search

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# Contents

# Assignment 1

- basis encoding: one number $\rightarrow$ several qubits needed
- angle encoding: one number $\rightarrow$ one qubit
- amplitude encoding: $n$ numbers $\log_2(n)$ qubits needed

# Assignment 2

- Quantum Fourier Transform function generalised
- Inverse QFT

# Assignment 3

- Making the T-gate $\frac{1}{3} \rightarrow \frac{3}{11}$
- Making it more precise by adding two qubits $\rightarrow \frac{11}{32}$
- Experiment with CNOT $\rightarrow$ 0 and 1

# Assignment 4

- Make a graph
- Make a max-cut $\rightarrow$ 01101 or 10010 NB: node numbers $4, 3, 2, 1, 0$
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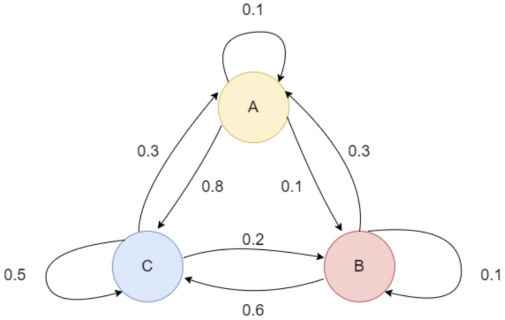
- Optimise:
  - method='Nelder-Mead'
  -

# Markov chains




Transition matrix:

$$P = \begin{pmatrix} 0.1 & 0.3 & 0.3 \\ 0.1 & 0.1 & 0.2 \\ 0.8 & 0.6 & 0.5 \end{pmatrix} \tag{1}$$

# States and operators

Two states:

- $\mathbb{H}_l$: state which represents the position of ‘the walker’
- $\mathbb{H}$: state which represents the coin and where ‘the walker’ should move next

$$\mathbb{H} = \mathbb{H}_l \otimes \mathbb{H} \tag{2}$$

# States and operators

Two states:

- $\mathbb{H}_{|}$: state which represents the position of ‘the walker’
- $\mathbb{H}$: state which represents the coin and where ‘the walker’ should move next

$$\mathbb{H} = \mathbb{H}_{|} \otimes \mathbb{H} \tag{2}$$

Two operators:

- coin operator $C$: acts on $\mathbb{H}_{|}$ and puts ‘the walker’ in superposition so it walks all possible paths simultaneously
- shift or step operator $S$: acts on $\mathbb{H}$ and moves ‘the walker’ to the next position

One step of the walker then becomes: $U = SC$.

# The algorithm

We have good and bad states:

$$|G\rangle = \frac{1}{\sqrt{|M|}} \sum_{x \in M} |x\rangle |p_x\rangle, |B\rangle = \frac{1}{\sqrt{N - |M|}} \sum_{x \notin M} |x\rangle |p_x\rangle \tag{3}$$

Other parameters:

$$\epsilon = |M|/N \text{ and } \theta = arcsin(\sqrt{\epsilon}) \tag{4}$$

1. Set up the initial state, a uniform superposition over all edges, with Hadamard gates

$$|U\rangle = \frac{1}{\sqrt{N}} \sum_{x} |x\rangle |p_x\rangle = sin\theta |G\rangle + cos\theta |B\rangle \tag{5}$$

2. Repeat $\mathbb{O}(\frac{1}{\sqrt{\epsilon}})$ times:
   1. Reflect through $|B\rangle$, by using a phase oracle
   2. Reflect through $|U\rangle$, by using Quantum Phase Estimation
3. Do a measurement in the computational basis

# Diagram of algorithm