Quantum Annealing

Quantum Capita Selecta

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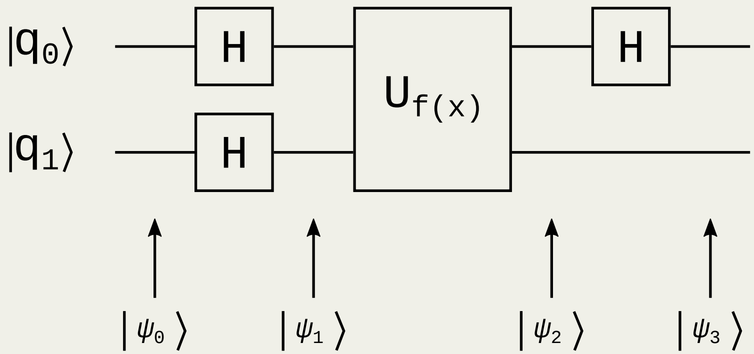
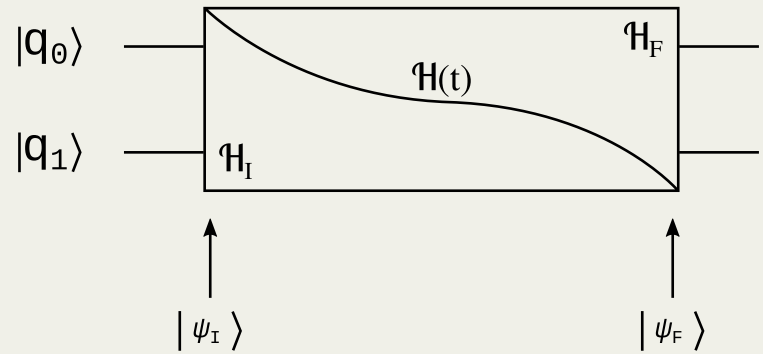
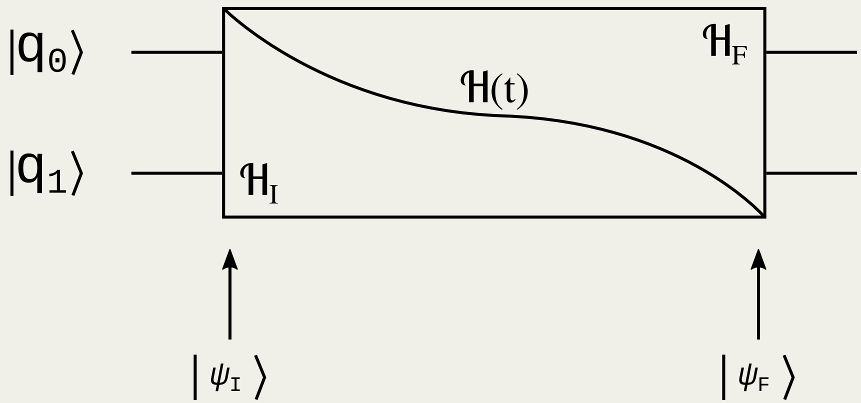
Adiabatic Model of Computation





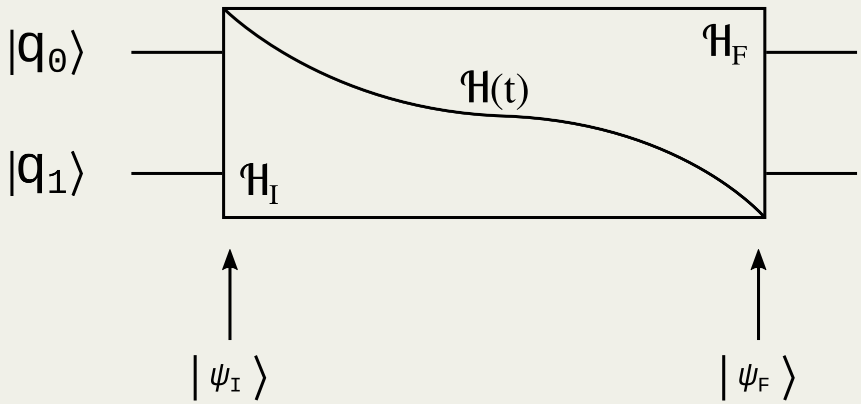
- Alternative to the gate model
  - Polynomially equivalent
  - Discrete versus analog nature
- Not full CPUs
  - Smart memory accelerators
  - Solve NP–hard optimization problems
  - Using Quantum Annealing

Adiabatic Model of Computation



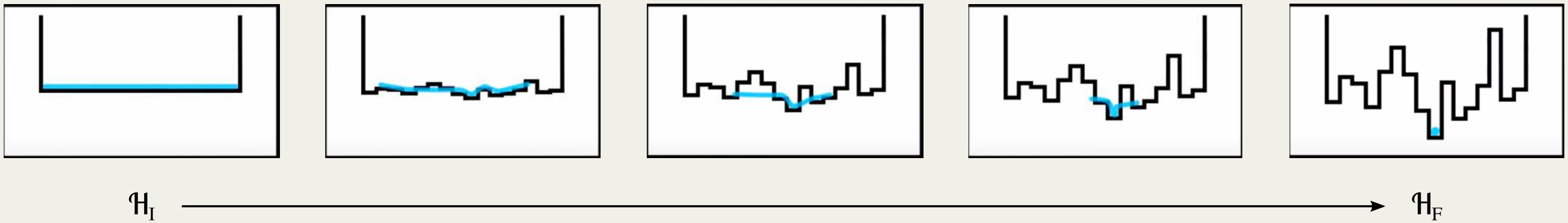
- States gradually evolve according to certain forces
  - Represented by Hamiltonians
- Transition from initial to final Hamiltonian
  - $\mathcal{H}_I \rightarrow \mathcal{H}_F$
- State is read at the end of the transition
- Each observable state has an associated energy
- Ground state $\rightarrow$ lowest energy over all states

Adiabatic Model of Computation



- Adiabatic Theorem:
  - If the qubits start in ground state, and
  - the transition is slow enough; then,
  - the system, with high probability, will finish in ground state

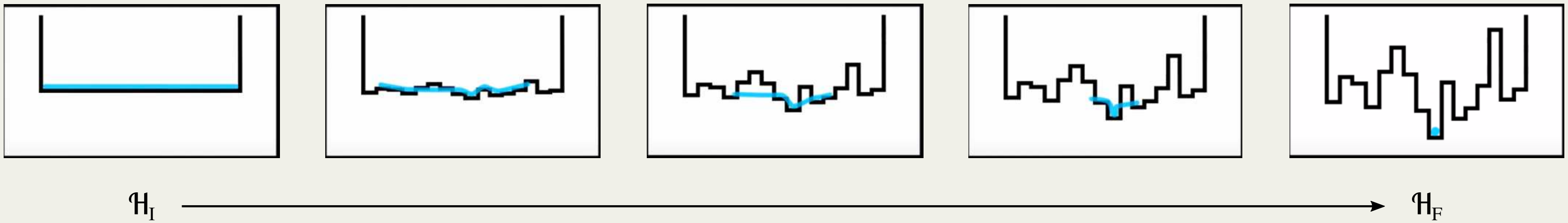
Adiabatic Model of Computation



- Key components:
  - Initial Hamiltonian ($\mathcal{H}_I$): ground state
  - Final Hamiltonian ($\mathcal{H}_F$): optimal solution
  - Adiabatic path ($s(t)$): transition from $\mathcal{H}_I$ to $\mathcal{H}_F$

$$s(t) = 1 - t$$

Adiabatic Model of Computation



- Time–varying Hamiltonian:
  - Creates a gradual transition: $\mathcal{H}_I \to \mathcal{H}_F$

$$\begin{aligned}\mathcal{H}(t) &= s(t)\mathcal{H}_I + (1 - s(t))\mathcal{H}_F \\ &= (1-t)\mathcal{H}_I + t\mathcal{H}_F\end{aligned}$$

- This Hamiltonian is an AQC algorithm for solving the problem

Advantages

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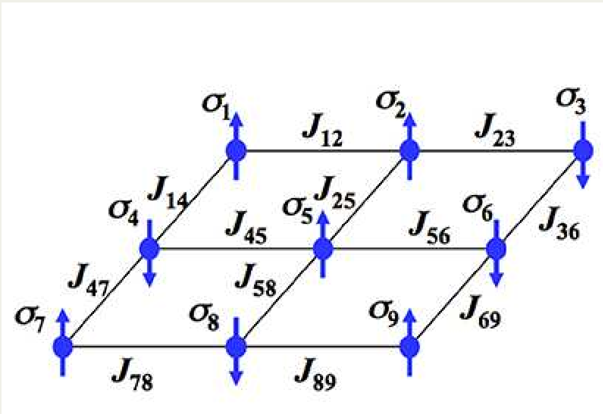
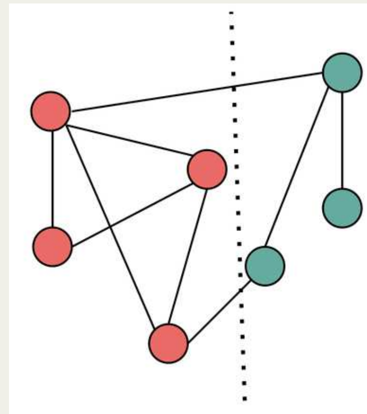
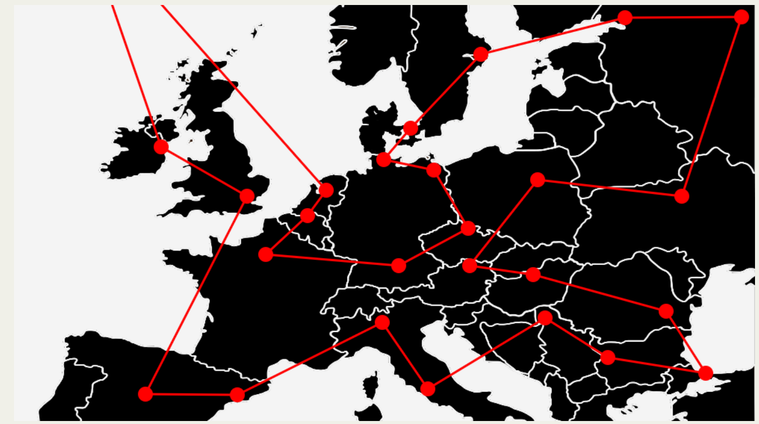
- Robust against decoherence:
  - Computation takes place in ground state
- Relaxed computational goal:
  - Designed to solve one NP–hard optimization problem (the Ising Model)
    - Find an assignment to N variables that minimizes a given objective function
    - Feasible solutions can be verified
    - Optimal solutions cannot
  - Using a specific solution approach (Quantum Annealing)

The Ising Model



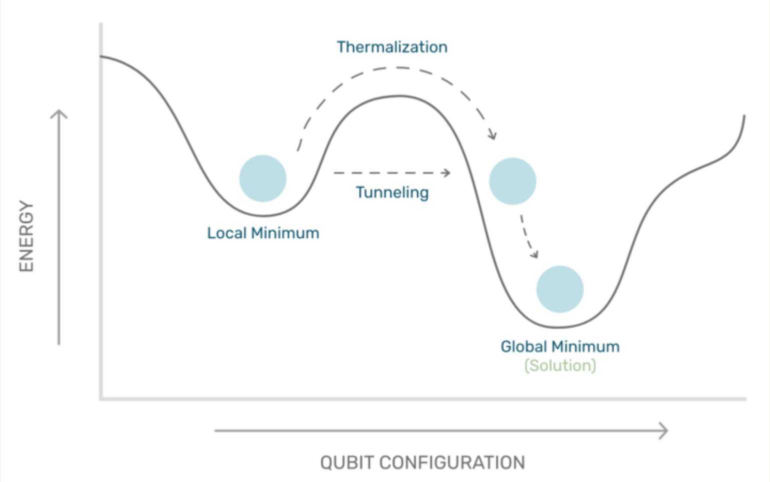



- A spin system: up and down
- Graph–like arrangement
  - Allowing spin interaction
- The energy of the system is given by:

$$\mathcal{H}(\sigma) = -\sum_{\langle ij \rangle} J_{ij}\sigma_i\sigma_j - \sum_j h_j\sigma_j$$

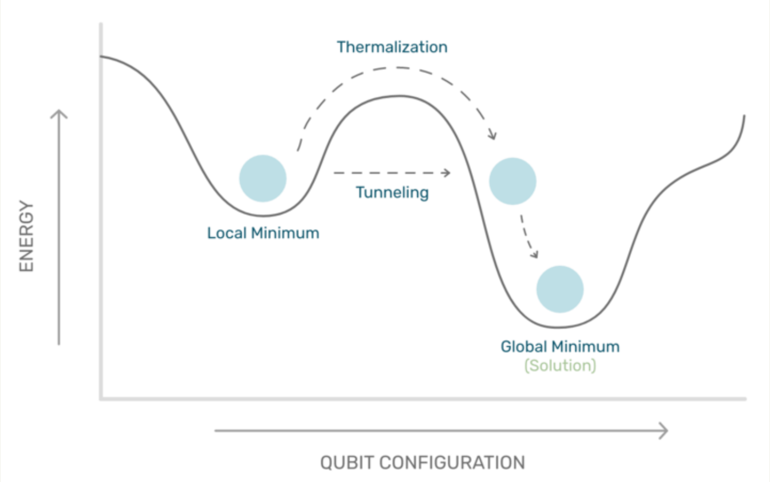
- Reformulated to many interesting NP–hard problems

The Ising Model



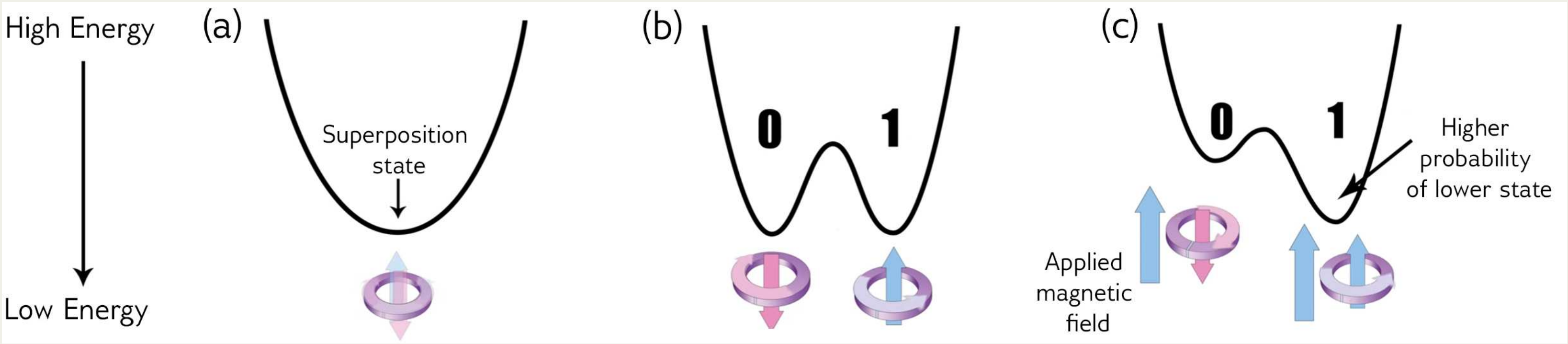
- Each spin configuration produces an energy value
  - Energy landscape
- Goal: find the global minimum
  - What about local minimum?
    - Simulated annealing
    - Quantum annealing

Quantum Annealing



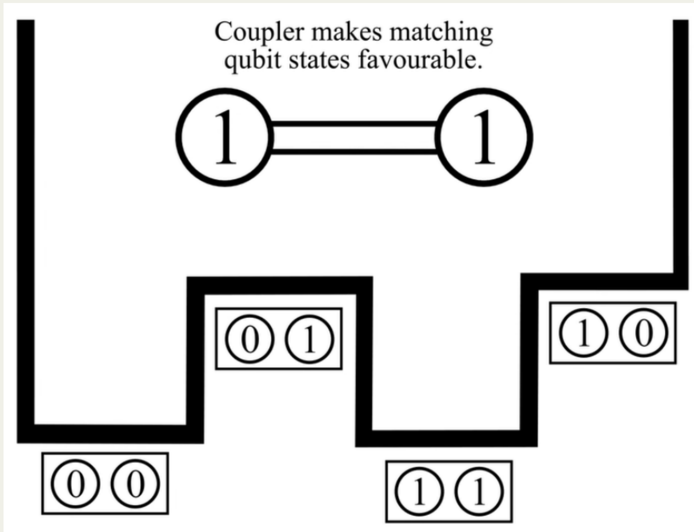
- Heuristic approach to solving combinatorial optimization problems
  - Restriction on final Hamiltonian $\mathcal{H}_F$
- Similar to simulated (thermal) annealing
- Can “tunnel” through hills
- Can run on classical or adiabatic platforms

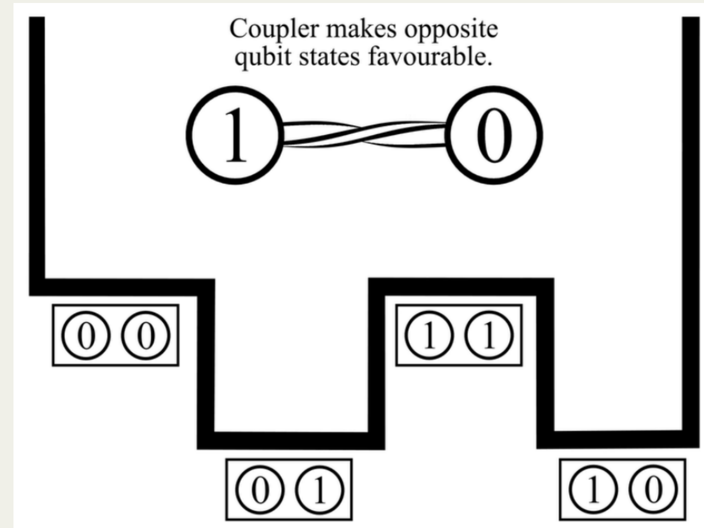
Energy diagram



- The system is initially in superposition
  - Lowest point in the valley
- Quantum annealing runs, a barrier is raised
  - Double–well potential
- Apply an external magnetic field (bias)
  - Tilting the double–well potential

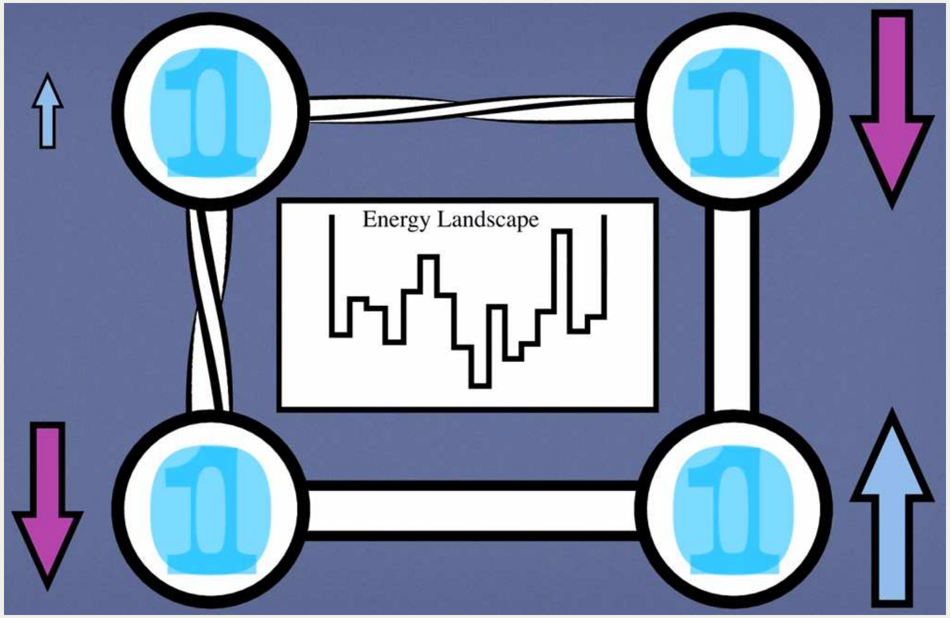
Couplers





- Defines how qubits influence each other
- Sets the correlation between two interacting qubits
  - Qubits end up in the same state
  - Qubits end up in opposite states
- Effectively enabling entanglement

Programming in Quantum Annealing



- Choose the values for biases and couplers
  - Direction and strength
- Defines an energy landscape
- The quantum annealing solves and finds the minimum energy

Programming in Quantum Annealing

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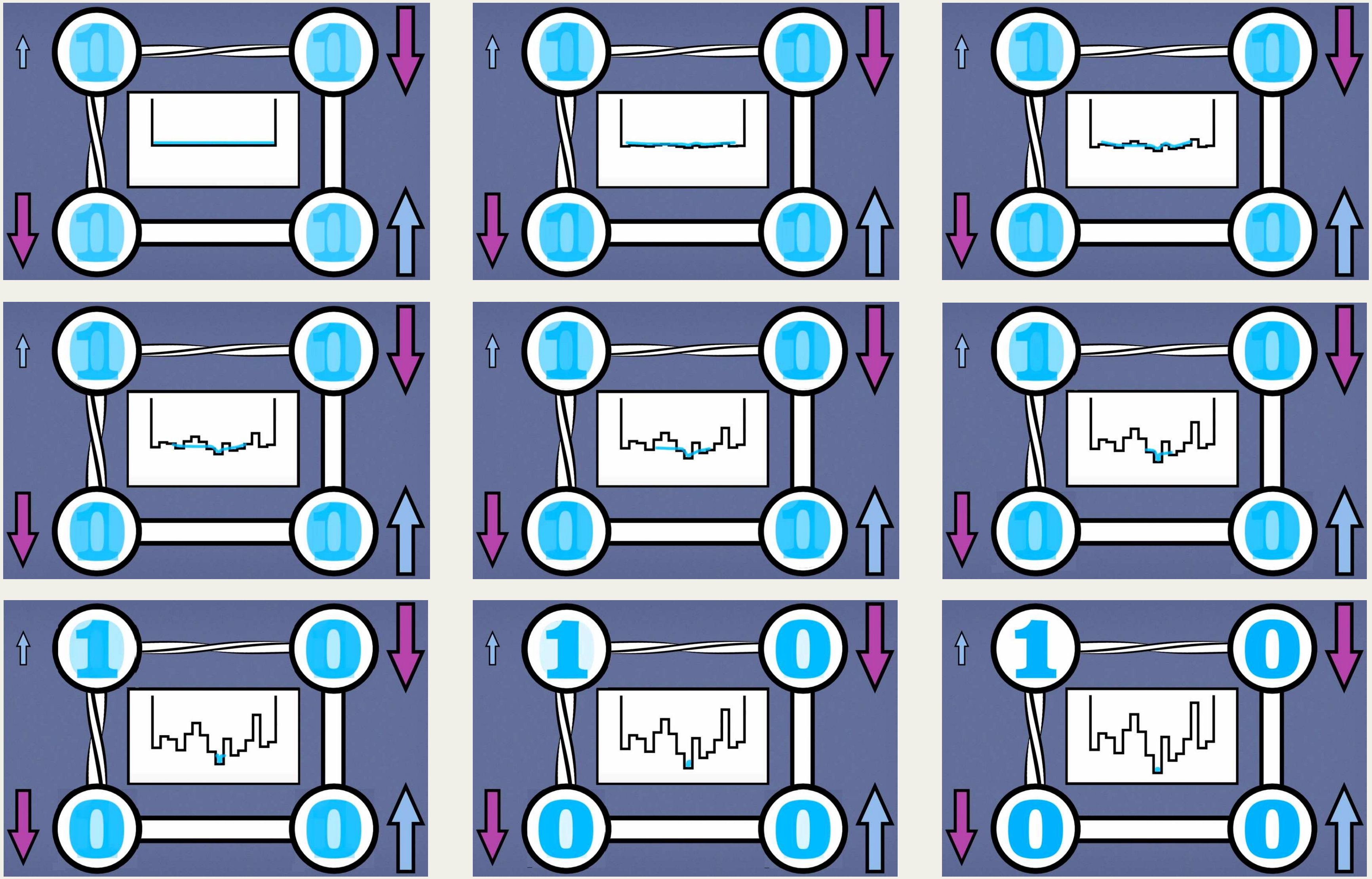
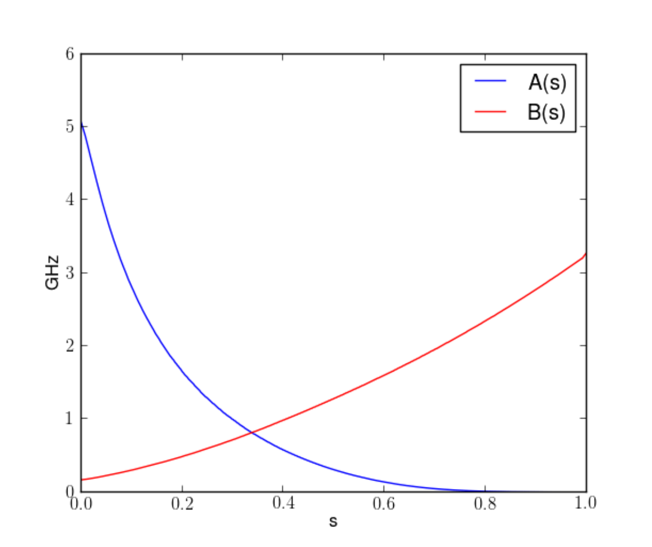


Hamiltonian



- The D–Wave's quantum annealing system incorporates:

$$\mathcal{H}_I = \sum_i \sigma_i^x$$

$$\mathcal{H}_F = \sum_i h_i \sigma_i^z + \sum_{ij} J_{ij} \sigma_i^z \sigma_j^z$$

$$\mathcal{H}(s) = A(s)\mathcal{H}_I + B(s)\mathcal{H}_F$$

D–Wave

- Advantage2
  - Announced in 2021 (Released ???)
  - 7000+ qubits
  - Zephyr graph topology


Questions?