Quantum Phase Estimation

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Quantum Phase Estimation

Quantum Phase Estimation (QPE) is a very important subroutine because it's being used in a lot of algorithms, e.g. HHL.
QPE estimates Θ in

$$U|\phi\rangle = e^{2\pi i\theta}|\phi\rangle \tag{1}$$

where $|\phi\rangle$ is an eigenvector and $e^{2\pi i\theta}$ us the corresponding eigenvalue.

Eigenvector and eigenvalue:

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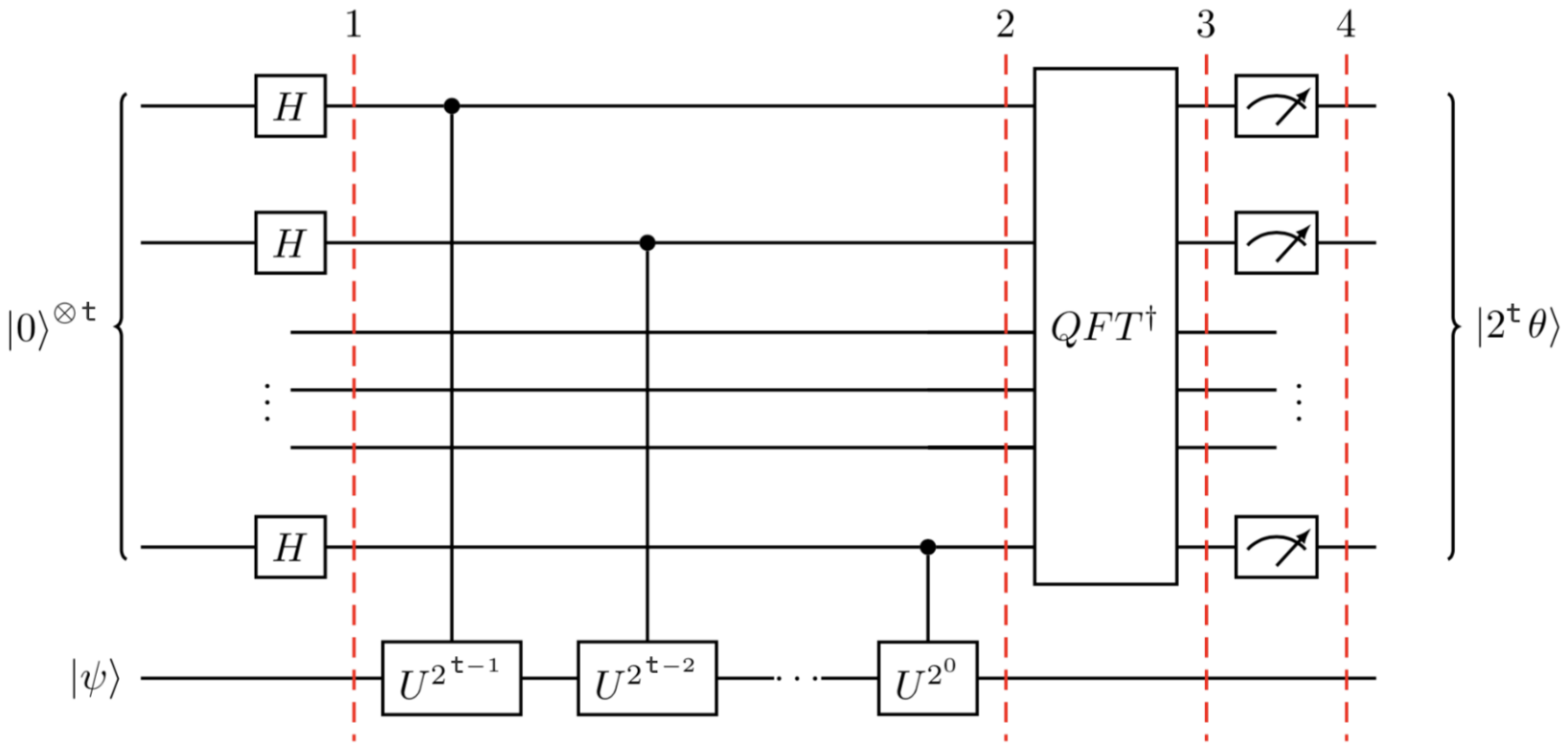
Phase-kickback:
The mechanism that effects the phase of the control-qubit.


1
2
3
4
$|0\rangle^{\otimes t}$
H
H
H
$|\psi\rangle$
$U^{2^{t-1}}$
$U^{2^{t-2}}$
U^{2^0}
$QFT^\dagger$
$|2^t\theta\rangle$

T-gate

$$T|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = e^{\frac{i\pi}{4}} |1\rangle \tag{2}$$

Since:

$$T|1\rangle = e^{2\pi i\theta} |1\rangle \tag{3}$$

The estimate will be:

$$e^{\frac{i\pi}{4}} = e^{2\pi i\theta} \iff \frac{i\pi}{4} = 2\pi i\theta \iff \frac{i\pi}{8} = \pi i\theta \iff \frac{i}{8} = i\theta \iff \frac{1}{8} = \theta \tag{4}$$