Amsterdam University of Applied Sciences

FACULTY OF DIGITAL MEDIA AND CREATIVE INDUSTRIES
HBO – Information and Communication Technologies

Assignment # 6

Quantum Computing Introduction

Quincy Koelman, Lilith Bertens

February 28, 2024

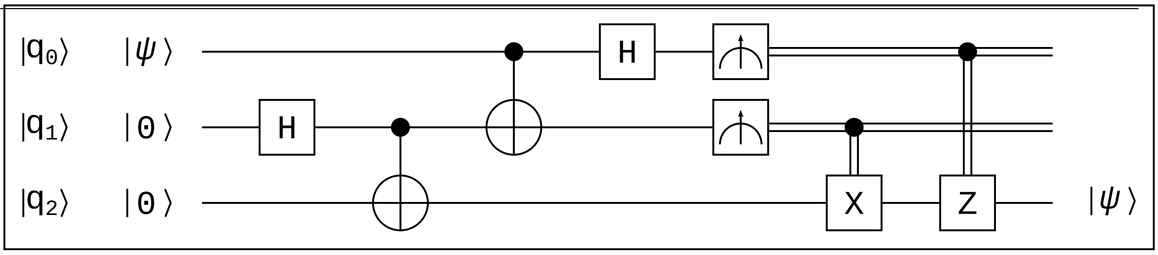

Figure 1: An arbitrary quantum circuit.

Welcome to the sixth assignment! During this assignment, you will practice your math skills in an environment more similar to the one that you will face during the course assessment. Although the questions will not be the same, they will follow the same structure and you will have to solve the exercises using the Dirac notation or matrix–vector multiplication.

Together with this assignment, you will find the LaTeX template for you to use when solving the exercises and writing down your answers. Remember to upload a single pdf file with the full development of the assignment and the answers.

Question 1 *Let* $|\psi\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$*. What will you always obtain when measuring two of the three qubits (any of them)?*

`Write down your solution here:`

You will always obtain the knowledge of what the last qbit's state is, as when you measure 11 its 1 and if you measure 00, its 0.

Question 2 *The quantum teleportation process, depicted in Figure 1, transmits quantum information from one location to another using two previously entangled qubits. Knowing that when measuring these entangled qubits, the system immediately collapses to the resulting state. Does the quantum teleportation process also enable faster–than–light communication between the two locations? Explain your answer.*

`Write down your solution here:`

wel, if some articles are to be believed, *yes it would be possible*, but after reffering to the source of those articles, *it does not explicitly mention FTL communication*
After refferencing wikipedia (yes really), it states that ***"classical information needs to be sent from sender to receiver. Because classical information needs to be sent, quantum teleportation cannot occur faster than the speed of light."***, so no, it does not enable FTL communication.

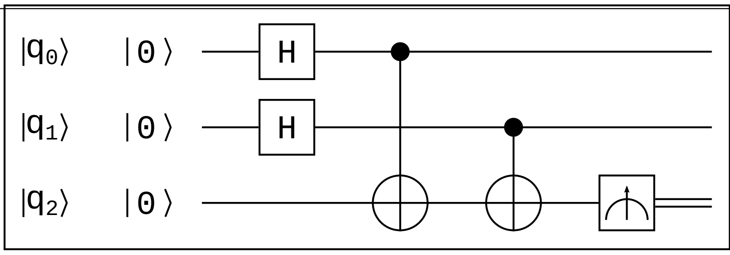

Figure 2: An arbitrary quantum circuit.

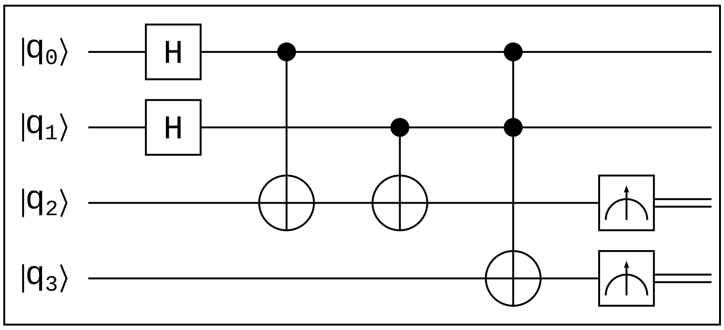

Figure 3: An arbitrary quantum circuit.

Question 3 *Consider the quantum circuit presented in Figure 2. What is the state of the qubits $|q_1q_0\rangle$ after measuring $|q_2\rangle$ with result 1 ($M(|q_2\rangle) = 1$)?*

`Write down your solution here:`

so lets see:
$|\psi_{in}\rangle = |000\rangle$
$H + H : |0++\rangle \rightarrow \frac{1}{2}(|000\rangle + |001\rangle + |010\rangle + |011\rangle)$
$CX : \frac{1}{2}(|000\rangle + |101\rangle + |010\rangle + |111\rangle)$
$CX : \frac{1}{2}(|000\rangle + |101\rangle + |110\rangle + |011\rangle)$
So when we measure $q_2 = 1$ we know that q_0 and q_1 must either be 01 or 10.

Question 4 *Consider the quantum circuit presented in Figure 3 and assume $|q_3q_2q_1q_0\rangle = |0000\rangle$. Determine, by using the Dirac notation, what is the state vector of the quantum circuit just before the measurement?*

`Write down your solution here:`

We will copy the last state of Question 3 due to the circuit and input being identical right before the ccnot in Figure 3.
Copied with a concatenated 0: $\frac{1}{2}(|0000\rangle + |0101\rangle + |0110\rangle + |0011\rangle)$
$CCX : \frac{1}{2}(|0000\rangle + |0101\rangle + |0110\rangle + |1011\rangle)$

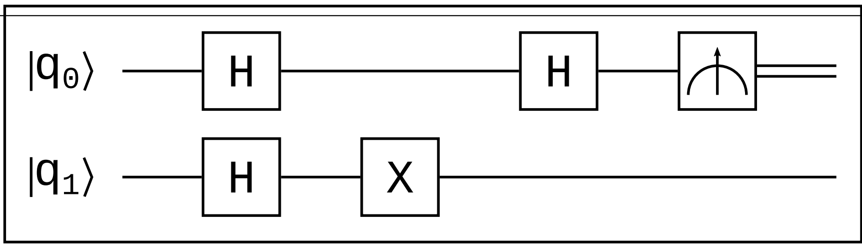

Figure 4: An arbitrary quantum circuit.

Question 5 *Considering the state vector obtained in Question 4. What is the probability* $\mathbf{Pr}\left\{M(|q_3\rangle) = 0\right\}$*?*

`Write down your solution here:`

that probability is 3/4 states, aka 75%.

Question 6 *Consider the quantum circuit presented in Figure 4 and assume* $|q_1 q_0\rangle = |10\rangle$*. Determine, by using the matrix–vector multiplication, what is the state vector of the quantum circuit just before the measurement?*

`Write down your solution here:`

Due to the circuit being compact-able i will put H and X in the same matrix.
$X \otimes H * H \otimes H * |10\rangle = |\psi_{out}\rangle$

$$\frac{1}{\sqrt{8}} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} * \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = |\psi_{out}\rangle$$

$$\frac{1}{\sqrt{8}} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = |\psi_{out}\rangle$$

$$\frac{1}{\sqrt{8}} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} = |\psi_{out}\rangle$$

$$\frac{1}{\sqrt{8}} \begin{bmatrix} -2 \\ 0 \\ 2 \\ 0 \end{bmatrix} = |\psi_{out}\rangle$$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = |\psi_{out}\rangle$$

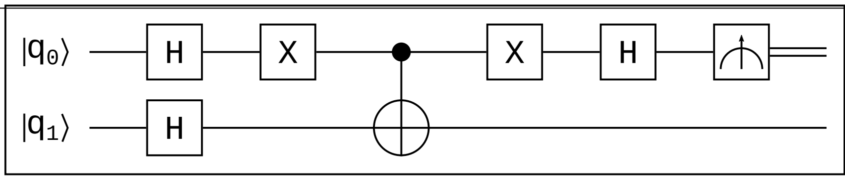

Figure 5: An arbitrary quantum circuit.

Question 7 *Confirm, by using the Dirac notation, the state vector obtained in Question 6.*

`Write down your solution here:`

Alright
$\psi_{in} = |10\rangle$
$H + H : |-+\rangle \rightarrow \frac{1}{2}(|00\rangle + |01\rangle - |10\rangle - |11\rangle)$
$H + X : \frac{1}{2\sqrt{2}}(|1+\rangle + |1-\rangle - |0+\rangle - |0-\rangle) \rightarrow$
$\frac{1}{2\sqrt{2}}(|10\rangle + |11\rangle + |10\rangle - |11\rangle - |00\rangle - |01\rangle - |00\rangle + |01\rangle) \rightarrow$
$\frac{1}{2\sqrt{2}}(2\,|10\rangle - 2\,|00\rangle) \rightarrow$
$\frac{1}{\sqrt{2}}(|10\rangle - |00\rangle)$
Which, indead, is equal to the state we found earlier.

Question 8 *At this point, I assume that you noticed that Figure 4 represents the Deutsch circuit for a constant function. Likewise, the quantum circuit presented in Figure 5 represents the Deutsch circuit for a balanced function. Assume $|q_1 q_0\rangle = |10\rangle$ and corroborate, by using the Dirac notation, that $M(|q_0\rangle) = 1$ with 100% probability.*

`Write down your solution here:`

I, uhm, didn't really, but lets calculate it anyway!
$\psi_{in} = |10\rangle$
$H + H : |-+\rangle \rightarrow \frac{1}{2}(|00\rangle + |01\rangle - |10\rangle - |11\rangle)$
$X : \frac{1}{2}(|01\rangle + |00\rangle - |11\rangle - |10\rangle)$
$CX : \frac{1}{2}(|11\rangle + |00\rangle - |01\rangle - |10\rangle)$
$X : \frac{1}{2}(|10\rangle + |01\rangle - |00\rangle - |11\rangle)$
$H : \frac{1}{2\sqrt{2}}(|1+\rangle + |0-\rangle - |0+\rangle - |1-\rangle) \rightarrow$
$\frac{1}{2\sqrt{2}}(|10\rangle + |11\rangle + |00\rangle - |01\rangle - |00\rangle - |01\rangle - |10\rangle + |11\rangle) \rightarrow$
$\frac{1}{2\sqrt{2}}(2\,|11\rangle - 2\,|01\rangle) \rightarrow$
$\frac{1}{\sqrt{2}}(|11\rangle - |01\rangle)$
So yes, indead q_0 is always 1.

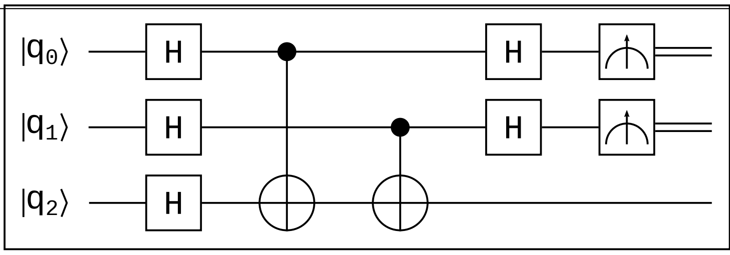

Figure 6: An arbitrary quantum circuit.

Question 9 *Consider the quantum circuit presented in Figure 6 and assume* $|q_2q_1q_0\rangle = |100\rangle$*. Can you determine, by using the Dirac notation, whether the implemented function is constant or balanced?*

`Write down your solution here:`

$|\psi_{in}\rangle = |100\rangle$
$H + H + H : |-++\rangle \rightarrow \frac{1}{2\sqrt{2}}(|000\rangle + |001\rangle + |010\rangle + |011\rangle - |100\rangle - |101\rangle - |110\rangle - |111\rangle)$
$CX : \frac{1}{2\sqrt{2}}(|000\rangle + |101\rangle + |010\rangle + |111\rangle - |100\rangle - |001\rangle - |110\rangle - |011\rangle)$
$CX : \frac{1}{2\sqrt{2}}(|000\rangle + |101\rangle + |110\rangle + |011\rangle - |100\rangle - |001\rangle - |010\rangle - |111\rangle)$
$H+H : \frac{1}{4\sqrt{2}}(|0++\rangle + |1+-\rangle + |1-+\rangle + |0--\rangle - |1++\rangle - |0+-\rangle - |0-+\rangle - |1--\rangle) \rightarrow$
$\frac{1}{4\sqrt{2}}(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle - |101\rangle + |110\rangle - |111\rangle + |100\rangle + |101\rangle - |110\rangle - |111\rangle + |000\rangle - |001\rangle - |010\rangle - |011\rangle - |100\rangle - |101\rangle - |110\rangle - |111\rangle - |000\rangle + |001\rangle - |010\rangle + |011\rangle - |000\rangle - |001\rangle + |010\rangle + |011\rangle - |100\rangle + |101\rangle + |110\rangle - |111\rangle) \rightarrow$
$\frac{1}{4\sqrt{2}}(4\,|011\rangle - 4\,|111\rangle) \rightarrow$
$\frac{1}{\sqrt{2}}(|011\rangle - |111\rangle)$
So, if i did that right, god i hope so, that calculation was far too long, it should be constant, because you measure 11 both times at the places the you measure ofcourse: $q_0 + q_1$

Question 10 *Refering to Question 9. Are you 100% sure about the type of the implemented function? Why?*

`Write down your solution here:`

uhm, no, like i said, long calculation, and it wasn't done with $|111\rangle$ too, so i can't be even remotely sure. as you would need the full truth table to know if its constant or balanced. but due to both q_0 and q_1 being 11 i can take a guess.