Quantum Approximate Optimization Algorithm (QAOA)

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Contents

Energy in a quantum system

- Hamiltonian: operator corresponding with the total energy of a quantum system described by a Hermitian matrix
- Expectation value: the energy in a system when in state $|\psi\rangle$: $E(|\psi\rangle) = \langle\psi| H |\psi\rangle$
- Ground state: the lowest energy state $|\psi^*\rangle$ of a quantum system: $|\psi^*\rangle = argmin_{|\psi\rangle \in H} E(|\psi\rangle)$

The variational method

1. Choose an initial state (ansatz) parameterized by $\theta : |\psi(\theta)\rangle$
2. Vary parameter θ to minimise the energy level:
   $E(\theta) = \langle\psi(\theta)| H |\psi(\theta)\rangle$ and $\theta^* = argmin_\theta E(\theta)$

QAOA

The Quantum Approximate Optimization Algorithm (QAOA) is

- a hybrid algorithm.
- a heuristic algorithm.
- an iterative algorithm.
- an algorithm which tries to solve combinatorial optimization problems.
- an approximation.

Combinatorial Optimization Problems

- The travelling salesman problem
- The minimum spanning tree
- The knapsack problem
- The max-cut problem

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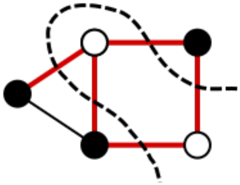
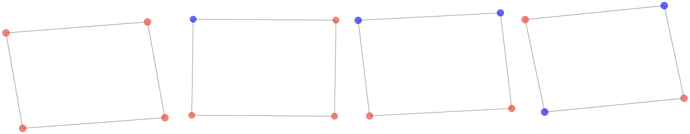
Application

- logistics
- supply chain optimisation
- water distribution networks
- manufacturing of microchips

Max-Cut

What is the maximum number of edges when have to sets of nodes?





Increase of possibilities to check is exponential: S^n where S is the number of sets and n the number of nodes.

QAOA is hybrid

Figure 1 shows the procedure for the hybrid algorithm QAOA.

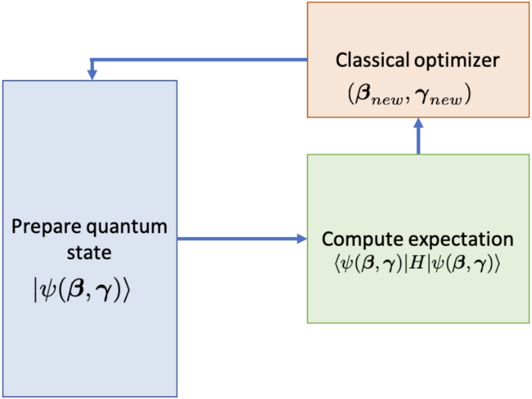


Figure: QAOA is hybrid (from IBM)

QAOA circuit

The QAOA circuit is composed of three elements:

1. Prepare an initial state, QAOA prepares a parametrised 'trial' state or 'ansatz' state of the form:
$$|\phi(\theta)\rangle = |\phi(\beta,\gamma)\rangle = e^{-i\beta_p B} e^{-i\gamma_1 C} \dots e^{-i\beta_1 B} e^{-i\gamma_p C} H^{\otimes n} |0\rangle \tag{1}$$
2. Apply the unitary $U(H_p) = e^{-i\gamma H_p}$ corresponding to the problem Hamiltonion, e.g. for the Max-Cut:
$$C = \frac{1}{2} \sum_{ij \in E} (1 - Z_i Z_j) \tag{2}$$
3. Apply the mixing unitary $U(H_b) = e^{-i\beta H_b}$ and B the mixer Hamiltonian:
$$B = \sum_i X_i \tag{3}$$
4. A classical optimizer is used to optimize the parameters β and γ:
$$f(\beta,\gamma) = \langle \phi(\beta,\gamma| C |(\beta,\gamma)\rangle \tag{4}$$

Procedure

The procedure is as follows:

1. Initialise β and γ using the QAOA circuit
2. Measure the state in the standard basis
3. Compute $\langle\phi(\beta,\gamma)| H_p |\phi\beta,\gamma\rangle$
4. Find a new set of parameters $(\beta_{new}, \gamma_{new})$
5. Set current parameters (β, γ) equal to the new parameters $(\beta_{new}, \gamma_{new})$.

QAOA circuit