7.11. Let $x_c(t)$ be a continuous-time signal whose Fourier transform has the property that $X_c(j\omega) = 0$ for $|\omega| \geq 2{,}000\pi$. A discrete-time signal

$$x_d[n] = x_c(n(0.5 \times 10^{-3}))$$

$T = 0.5 \times 10^{-3} s$ $\quad w_s = \frac{2\pi}{T} = 4000\pi$

is obtained. For each of the following constraints on the Fourier transform $X_d(e^{j\omega})$ of $x_d[n]$, determine the corresponding constraint on $X_c(j\omega)$:

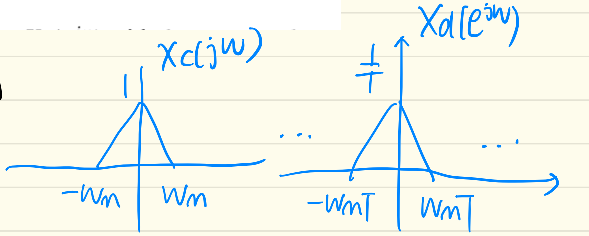
(a) $X_d(e^{j\omega})$ is real.

(b) The maximum value of $X_d(e^{j\omega})$ over all ω is 1.

(c) $X_d(e^{j\omega}) = 0$ for $\frac{3\pi}{4} \leq |\omega| \leq \pi$.

(d) $X_d(e^{j\omega}) = X_d(e^{j(\omega-\pi)})$.

$$X_c(t) \xrightarrow{\ \otimes\ p(t)\ } X_p(t) \rightarrow \boxed{} \rightarrow X_d[n]$$



$$X_p(jw) = \frac{1}{T}\sum_{n\in\mathbb{Z}} X_c\left(j\left(w - \frac{2n\pi}{T}\right)\right)$$

$$X_d(e^{jw}) = X_p\left(j\frac{w}{T}\right) = \frac{1}{T}\sum_{n\in\mathbb{Z}} X_c\left(j\left(\frac{w}{T} - \frac{2n\pi}{T}\right)\right) = \frac{1}{T}\sum_{n\in\mathbb{Z}} X_c(j(w-2\pi n)/T)$$

(a) Since $X_d(e^{jw})$ is formed by shifting and summing replicas of $X_c(jW)$
$\Rightarrow X_c(jW)$ is real

(b) $\max\{X_d(e^{jw})\} = \frac{1}{T}\max\{X_c(jw)\} = 1$

$\Rightarrow \max\{X_c(jW)\} = T = 0.5\times10^{-3}$

(c) $\frac{3\pi}{4} \leq |W| \leq \pi$ in the discrete-time domain
$\Leftrightarrow \frac{3\pi}{4T} \leq |W| \leq \frac{\pi}{T}$ in the continuous-time domain

$\Leftrightarrow X_c(jW)=0$ for $\frac{3\pi}{4T} \leq |W| \leq \frac{\pi}{T}$, that is $1500\pi \leq |W| \leq 2000\pi$
$\Rightarrow X_c(jW) = 0$ for $|W| \geq 1500\pi$

(d) $\pi \rightarrow 2000\pi$ $\Rightarrow$ $X_c(jW) = X_c(j(W-2000\pi))$ for $0 \leq W \leq 2000\pi$
$\downarrow$ discrete-time $\quad \downarrow$ continuous time

7.13. With reference to the filtering approach illustrated in Figure 7.24, assume that the sampling period used is T and the input $x_c(t)$ is band limited, so that $X_c(j\omega) = 0$ for $|\omega| \geq \pi/T$. If the overall system has the property that $y_c(t) = x_c(t-2T)$, determine the impulse response $h[n]$ of the discrete-time filter in Figure 7.24.

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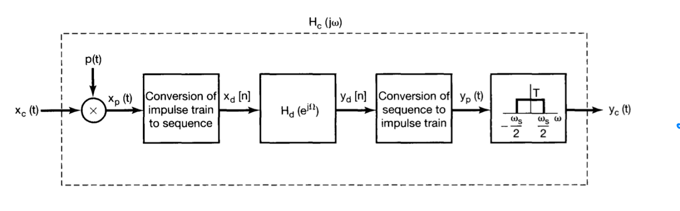
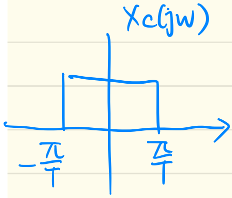




We can assume that $x_c(t) = \dfrac{\sin(\pi t/T)}{\pi t} \xrightarrow{F} X_c(jw) = \begin{cases} 1 & |w| < \frac{\pi}{T} \\ 0 & |w| > \frac{\pi}{T} \end{cases}$

The overall output is: $y_c(t) = x_c(t-2T) = \dfrac{\sin(\frac{\pi}{T}(t-2T))}{\pi(t-2T)}$

From $x_c(t)$, $x_d[n] = x_c(nT) = \dfrac{\sin(\frac{\pi nT}{T})}{\pi nT} = \dfrac{1}{T}\dfrac{\sin(\pi n)}{\pi n} = \frac{1}{T}\cdot\delta[n]$

Also, from $y_c(t)$, $y_d[n] = y_c(nT) = \dfrac{\sin(\frac{\pi}{T}(nT-2T))}{\pi(nT-2T)} = \dfrac{1}{T}\dfrac{\sin(\pi(n-2))}{\pi(n-2)}$

Note that $y_d[n] = 0$ when $n \neq 2$, when $n=2$, $y_d[n] = \frac{1}{T}$

$\Rightarrow y_d[n] = \frac{1}{T}\delta[n-2]$

$X_d(e^{jw})\cdot H_d(e^{jw}) = Y(e^{jw}) \Rightarrow \frac{1}{T}\cdot H_d(e^{jw}) = e^{-jw2}\cdot\frac{1}{T}$

$\Rightarrow H_d(e^{jw}) = e^{-jw2} \Rightarrow h_d[n] = \delta[n-2]$

7.16. The following facts are given about the signal $x[n]$ and its Fourier transform:

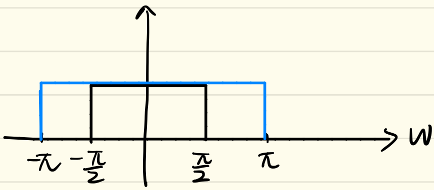
1. $x[n]$ is real.
2. $X(e^{j\omega}) \neq 0$ for $0 < \omega < \pi$.
3. $x[n]\sum_{k=-\infty}^{\infty}\delta[n-2k] = \delta[n]$.

Determine $x[n]$. You may find it useful to note that the signal $(\sin \frac{\pi}{2}n)/(\pi n)$ satisfies two of these conditions.

$$x_1[n] = \frac{2\sin(\frac{\pi n}{2})}{\pi n} = \text{sinc}[\tfrac{1}{2}n] = \begin{cases} 1 & n=0 \\ 0 & n=2k \end{cases}$$ is real

$$x_1[n]\cdot\sum_{k=-\infty}^{\infty}\delta[n-2k] = \sum_{k=-\infty}^{\infty} x[2k]\cdot\delta[n-2k] = \delta[n]$$ $\rightarrow$ satisfies the third condition

$$X_1(e^{jw}) = \begin{cases} 2 & |w| < \frac{\pi}{2} \\ 0 & |w| > \frac{\pi}{2} \end{cases}$$ $\rightarrow$ does not satisfy the second condition.



$X_1(e^{jw}) * X_1(e^{jw})$
$\Longleftrightarrow X_1[n]\cdot X_1[n]$

let $x[n] = X_1[n]\cdot X_1[n] = 4\left(\frac{\sin(\frac{\pi n}{2})}{\pi n}\right)^2$

$$= \text{sinc}^2[\tfrac{1}{2}n] = \begin{cases} 1 & n=0 \\ 0 & n=2k \end{cases}$$

$\rightarrow$ satisfies all condition

7.17. Consider an ideal discrete-time bandstop filter with impulse response $h[n]$ for which the frequency response in the interval $-\pi \le \omega \le \pi$ is

$$H(e^{j\omega}) = \begin{cases} 1, & |\omega| \le \frac{\pi}{4} \text{ and } |\omega| \ge \frac{3\pi}{4} \\ 0, & \text{elsewhere} \end{cases}.$$

Determine the frequency response of the filter whose impulse response is $h[2n]$.

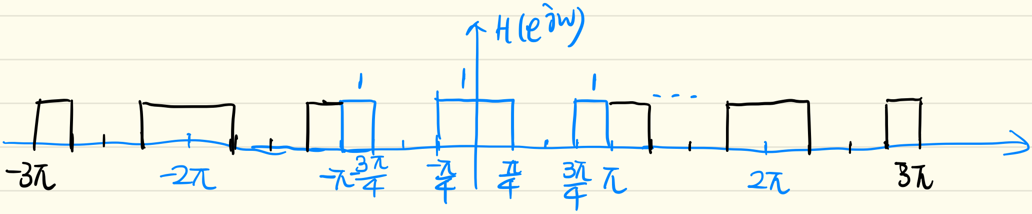
- Decimation (denote: $h_b[n] = h[2n]$)

$$h[n] \longrightarrow h_p[n] \longrightarrow h_b[n]$$

Step 1: impulse train sampling, $N=2$, $w_s = \frac{2\pi}{N} = \pi$

$$h_p[n] = \sum_{k\in\mathbb{Z}} h[n]\cdot\delta[n-2k] = \sum_{k\in\mathbb{Z}} h[2k]\,\delta[n-2k]$$

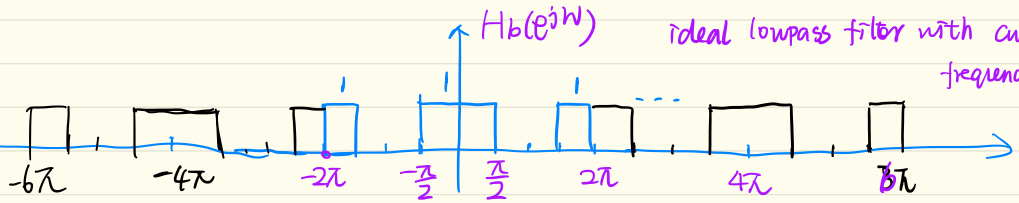
$$H_p(e^{jw}) = \frac{1}{2}\sum_{n=0}^{1} H(e^{j(w-nw_s)}) = \frac{1}{2}H(e^{jw}) + \frac{1}{2}H(e^{j(w-\pi)})$$
$$= H(e^{jw})$$



Step 2:

$$H_b(e^{jw}) = H_p(e^{j\frac{w}{N}}) = H_p(e^{j\frac{w}{2}})$$

$\Rightarrow H_b(e^{jw})$ is $H_p(e^{jw})$ expanded by a factor of 2



ideal lowpass filter with cutoff frequency $\frac{\pi}{2}$