

2. A time invariant continuous-time system has the frequency response

$$H(j\omega) = \begin{cases} 1, & 2 \leq |\omega| \leq 7, \\ 0, & \text{otherwise.} \end{cases}$$

Compute the output response y when the input v is

(a) $v(t) = 2 + 3 \cos(3t) - 5 \sin(6t - \frac{\pi}{6}) + 4 \cos(13t - \frac{\pi}{9})$

(b) $v(t) = 1 + \sum_{k=1}^{\infty} \frac{1}{k} \cos(2kt)$

Determine whether the system is stable. (BIBO)

Since the system only apply scaling to the input \Rightarrow Stable

(a) $H(j\omega) = 1$, when $2 \leq \omega \leq 7$ and $-7 \leq \omega \leq -2$

$$\begin{array}{ccccccc} v(t) = & 2 & + & 3 \cos(3t) & - & 5 \sin(6t - \frac{\pi}{6}) & + & 4 \cos(13t - \frac{\pi}{9}) \\ & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ & \omega=1 & & \omega=3 & & \omega=6 & & \omega=13 \end{array}$$

$$y(t) = 3 \cos(3t) - 5 \sin(6t - \frac{\pi}{6})$$

(b) $v(t) = 1 + \sum_{k=1}^{\infty} \frac{1}{k} \cos(2kt)$

$$= 1 + \frac{1}{1} \cos(2t) + \frac{1}{2} \cos(4t) + \frac{1}{3} \cos(6t) + \frac{1}{4} \cos(8t) + \dots$$

$$\Rightarrow y(t) = \cos(2t) + \frac{1}{2} \cos(4t) + \frac{1}{3} \cos(6t)$$

3. A linear time invariant continuous-time system has the frequency response

$$H(j\omega) = \frac{1}{j\omega + 1}.$$

Compute the output response y when the input v is

(a) $v(t) = \cos(t), -\infty < t < \infty.$

(b) $v(t) = \cos(t + \frac{\pi}{4}), -\infty < t < \infty.$

Step 1: whether the system is stable

$$H(s) = \frac{1}{s+1} \Rightarrow p = -1 \Rightarrow \text{stable}$$

(b) Show that the output of the system from the input $v(t) = \cos(\omega_0 t)$ under zero initial conditions is

$$y(t) = |H(j\omega_0)| \cos(\omega_0 t + \angle H(j\omega_0))$$

(a) $\omega_0 = 1$ $H(j1) = \frac{1}{j+1} = \frac{\sqrt{2}}{2} \angle -\frac{\pi}{4} = |H(j1)| \angle H(j1)$

$$y(t) = \frac{\sqrt{2}}{2} \cos(t - \frac{\pi}{4}) = 0.707 \cos(t - \frac{\pi}{4})$$

(b) $\omega_0 = 1$

$$y(t) = \frac{\sqrt{2}}{2} \cos(t + \frac{\pi}{4} - \frac{\pi}{4}) = \frac{\sqrt{2}}{2} \cos(t)$$

5. A complex signal v with period T has complex Fourier series

$$v(t) = \sum_{k=-\infty}^{\infty} c_k^v e^{jk\omega_0 t}$$

This signal is applied to the LTI continuous-time system with frequency response function

$$H(j\omega) = \begin{cases} 10e^{-j5\omega}, & |\omega| > \frac{\pi}{T}, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Let the resulting output response y have complex Fourier series

$$y(t) = \sum_{k=-\infty}^{\infty} c_k^y e^{jk\omega_0 t}.$$

Express the coefficients c_k^y in terms of the coefficients c_k^v .

(b) Suppose $c_0^v = 2$. Find the constants a , b , and c such that

$$y(t) = av(t-b) + c$$

(c) Suppose the input signal is

$$v(t) = \begin{cases} 1, & -0.5 < t < 0.5, \\ 0, & 0.5 < t < 1.5, \end{cases} \quad \text{and} \quad v(t) = v(t+2)$$

Compute and plot $y(t)$ for this input signal.

(a) the system only apply time shift and scaling to the input \Rightarrow stable

$$V(t) = \sum_{k=-\infty}^{\infty} C_k^v \cdot e^{jk\omega_0 t} \quad \omega_0 = \frac{2\pi}{T}$$

$$y(t) = V(t) \cdot H(jk\omega_0) = \begin{cases} \sum_{k=-\infty}^{\infty} C_k^v \cdot e^{jk\omega_0 t} \cdot 10 \cdot e^{-j5k\omega_0} & |k\omega_0| > \frac{\pi}{T} \\ 0 & \text{otherwise.} \end{cases}$$

$$\text{since } \omega_0 = \frac{2\pi}{T}, \quad |k\omega_0| = |k \frac{2\pi}{T}| > \frac{\pi}{T} \Rightarrow k \neq 0$$

$$\Rightarrow C_k^y = \begin{cases} C_k^v \cdot 10 \cdot e^{-j5k\omega_0} & k \neq 0 \\ 0 & k = 0 \end{cases}$$

$$(b) \quad C_0^v = 2, \quad V(t) = \sum_{k=-\infty}^{\infty} C_k^v \cdot e^{jk\omega_0 t}$$

$$y(t) = \sum_{k=-\infty}^{\infty} C_k^y e^{jk\omega_0 t}$$

$$= \sum_{k \neq 0} C_k^v \cdot 10 \cdot e^{-j5k\omega_0} \cdot e^{jk\omega_0 t} = 10 \cdot \sum_{k \neq 0} C_k^v \cdot e^{jk\omega_0 (t-5)}$$

$$= 10[V(t-5) - C_0^v \cdot 1 \cdot 1] = 10V(t-5) - 20$$

$$\Rightarrow a = 10, \quad b = 5, \quad c = -20$$

(c) Suppose the input signal is

$$v(t) = \begin{cases} 1, & -0.5 < t < 0.5, \\ 0, & 0.5 < t < 1.5, \end{cases} \quad \text{and} \quad v(t) = v(t+2)$$

Compute and plot $y(t)$ for this input signal.

(c) $C_0^v = ?$

$$C_0^v = \frac{1}{2} \int_{-0.5}^{0.5} 1 \cdot dt = 0.5$$

$$y(t) = 10 \cdot [v(t-5) - 0.5] = 10v(t-5) - 5$$

6. Consider the RL series circuit below:

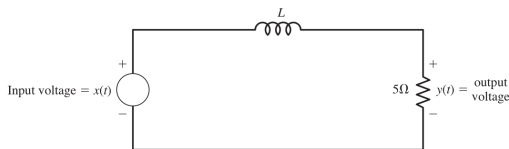


Figure 1: Diagram for Problem 6

- (a) Use circuit laws to derive a differential equation for the output y in terms of R , L and the input x . Hence obtain $H(j\omega)$, the frequency response of the circuit.
 (b) Suppose the input x is the periodic signal

$$x(t) = 10|\sin(377t)|, \quad x(t) = x(t + \pi/377)$$

Obtain c_0^x , the constant term in the complex Fourier series for x .

- (c) Let y be the output from this input x . Use the frequency response $H(j\omega)$ to obtain c_0^y , the constant term in the complex Fourier series for y .

$$(a) \quad \frac{Y(j\omega)}{X(j\omega)} = \frac{5 \cdot I}{(5 + j\omega L)I} = \frac{5}{5 + j\omega L} = H(j\omega)$$

$$\begin{aligned} (b) \quad C_0^x &= \frac{1}{T} \int_0^T 10|\sin 377t| dt \\ &= \frac{10}{T} \int_0^T \sin 377t dt \\ &= \frac{10}{T} \left[\frac{\cos 377t}{-377} \right]_0^T = \frac{10}{T} \left[\frac{\cos 377T - 1}{-377} \right] \\ T &= \frac{\pi}{377} \\ \Rightarrow C_0^x &= \frac{10 \times 377}{\pi} \left[\frac{-1 - 1}{-377} \right] = \frac{20}{\pi} \end{aligned}$$

$$(c) \quad H(s) = \frac{5}{s + 5} \Rightarrow p = -5 \Rightarrow \text{the system is stable}$$

$$C_0^y = C_0^x \cdot H(jk\omega_0) = C_0^x \cdot H(0) = C_0^x \cdot 1 = \frac{20}{\pi}$$