2. A linear time invariant continuous-time system has the frequency response

$$H(j\omega) = \begin{cases} 1, & 2 \le |\omega| \le 7, \\ 0, & \text{otherwise.} \end{cases}$$

Compute the output response y when the input v is

(a) $v(t) = 2 + 3\cos(3t) - 5\sin(6t - \frac{\pi}{6}) + 4\cos(13t - \frac{\pi}{9})$

(b) $v(t) = 1 + \sum_{k=1}^{\infty} \frac{1}{k}\cos(2kt)$

Determine whether the system is stable. (BIBO)

Since the system only apply scaling to the input $\Rightarrow$ Stable

(a) $H(jw) = 1$, when $2 \le w \le 7$ and $-7 \le w \le -2$

$$v(t) = \underset{\downarrow \atop w=0}{2} + \underset{\downarrow \atop w=3}{3\cos(3t)} - \underset{\downarrow \atop w=6}{5\cdot\sin(6t - \tfrac{\pi}{6})} + \underset{\downarrow \atop w=13}{4\cos(13t - \tfrac{\pi}{9})}$$

$$y(t) = 3\cos(3t) - 5\cdot\sin(6t - \tfrac{\pi}{6})$$

(b) $v(t) = 1 + \sum_{k=1}^{\infty} \frac{1}{k}\cos(2kt)$

$$= 1 + \tfrac{1}{1}\cos(2t) + \tfrac{1}{2}\cos(4t) + \tfrac{1}{3}\cos(6t) + \tfrac{1}{4}\cos(8t) + \cdots$$

$$\Rightarrow y(t) = \cos(2t) + \tfrac{1}{2}\cos(4t) + \tfrac{1}{3}\cos(6t)$$

3. A linear time invariant continuous-time system has the frequency response

$$H(j\omega) = \frac{1}{j\omega + 1}.$$

Compute the output response y when the input v is

(a) $v(t) = \cos(t)$, $-\infty < t < \infty$.

(b) $v(t) = \cos(t + \frac{\pi}{4})$, $-\infty < t < \infty$.

Step 1: whether the system is stable

$$H(s) = \frac{1}{s+1} \quad \Rightarrow P = -1 \quad \Rightarrow \text{stable}$$

(b) Show that the output of the system from the input $v(t) = \cos(\omega_0 t)$ under zero initial conditions is

$$y(t) = |H(j\omega_0)| \cos(\omega_0 t + \angle H(j\omega_0))$$

(a) $w_0 = 1$ $\quad H(j1) = \frac{1}{j+1} = \frac{\sqrt{2}}{2} \angle -\frac{\pi}{4} = |H(j1)| \angle H(j1)$

$$y(t) = \frac{\sqrt{2}}{2} \cos(t - \frac{\pi}{4}) = 0.707 \cos(t - \frac{\pi}{4})$$

(b) $w_0 = 1$

$$y(t) = \frac{\sqrt{2}}{2} \cos(t + \frac{\pi}{4} - \frac{\pi}{4}) = \frac{\sqrt{2}}{2} \cos(t)$$

5. A periodic signal v with period T has complex Fourier series

$$v(t) = \sum_{k=-\infty}^{\infty} c_k^v e^{jk\omega_0 t}$$

This signal is applied to the LTI continuous-time system with frequency response function

$$H(j\omega) = \begin{cases} 10e^{-j5\omega}, & |\omega| > \frac{\pi}{T}, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Let the resulting output response y have complex Fourier series

$$y(t) = \sum_{k=-\infty}^{\infty} c_k^y e^{jk\omega_0 t}.$$

Express the coefficients c_k^y in terms of the coefficients c_k^v.

(b) Suppose $c_0^v = 2$. Find the constants a, b, and c such that

$$y(t) = av(t-b) + c$$

(c) Suppose the input signal is

$$v(t) = \begin{cases} 1, & -0.5 < t < 0.5, \\ 0, & 0.5 < t < 1.5, \end{cases} \quad \text{and} \quad v(t) = v(t+2)$$

Compute and plot $y(t)$ for this input signal.

(a) the system only apply time shift and scaling to the input $\Rightarrow$ Stable

$$v(t) = \sum_{k=-\infty}^{\infty} C_k^v \cdot e^{jkW_0 t} \qquad W_0 = \frac{2\pi}{T}$$

$$y(t) = v(t) \cdot H(jkW_0) = \begin{cases} \sum_{k=-\infty}^{\infty} C_k^v \cdot e^{jkW_0 t} \cdot 10 \cdot e^{-j5kW_0} & |kW_0| > \frac{\pi}{T} \\ 0 & \text{otherwise.} \end{cases}$$

since $W_0 = \frac{2\pi}{T}$, $|kW_0| = |k\frac{2\pi}{T}| > \frac{\pi}{T} \Rightarrow k \neq 0$

$$\Rightarrow C_k^y = \begin{cases} C_k^v \cdot 10 \cdot e^{-j5kW_0} & k \neq 0 \\ 0 & k = 0 \end{cases}$$

(b) $C_0^v = 2$, $v(t) = \sum_{k=-\infty}^{\infty} C_k^v \cdot e^{jkW_0 t}$

$$y(t) = \sum_{k=-\infty}^{\infty} C_k^y e^{jkW_0 t}$$

$$= \sum_{k\neq 0} C_k^v \cdot 10 \cdot e^{-j5kW_0} \cdot e^{jkW_0 t} = 10 \cdot \sum_{k \neq 0} C_k^v \cdot e^{jkW_0(t-5)}$$

$$= 10[v(t-5) - C_0^v \cdot 1 \cdot 1] = 10v(t-5) - 20$$

$\Rightarrow a = 10$, $b = 5$, $c = -20$

(c) Suppose the input signal is

$$v(t) = \begin{cases} 1, & -0.5 < t < 0.5, \\ 0, & 0.5 < t < 1.5, \end{cases} \quad \text{and} \quad v(t) = v(t+2)$$

Compute and plot $y(t)$ for this input signal.

(c) $C_0^v = ?$

$$C_0^v = \frac{1}{2}\int_{-0.5}^{0.5} 1 \cdot dt = 0.5$$

$$y(t) = 10 \cdot [v(t-5) - 0.5] = 10v(t-5) - 5$$

6. Consider the RL series circuit below:

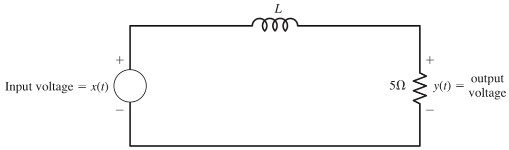

Figure 1: Diagram for Problem 6

(a) Use circuit laws to derive a differential equation for the output y in terms of R, L and the input x. Hence obtain $H(j\omega)$, the frequency response of the circuit.

(b) Suppose the input x is the periodic signal

$$x(t) = 10|\sin(377t)|, \quad x(t) = x(t+\pi/377)$$

Obtain c_0^x, the constant term in the complex Fourier series for x.

(c) Let y be the output from this input x. Use the frequency response $H(j\omega)$ to obtain c_0^y, the constant term in the complex Fourier series for y.

(a) $$\frac{Y(jw)}{X(jw)} = \frac{5\cdot I}{(5+jwL)I} = \frac{5}{5+jwL} = H(jw)$$

(b) $$C_0^x = \frac{1}{T}\int_0^T 10|\sin 377t|\,dt$$
$$= \frac{10}{T}\int_0^T \sin 377t\,dt$$
$$= \frac{10}{T}\left[\frac{\cos 377t}{-377}\right]_0^T = \frac{10}{T}\left[\frac{\cos 377T - 1}{-377}\right]$$

$T = \frac{\pi}{377}$
$$\Rightarrow C_0^x = \frac{10\times 377}{\pi}\left[\frac{-1-1}{-377}\right] = \frac{20}{\pi}$$

(c) $H(s) = \frac{5}{s+5}$ $\Rightarrow p = -5$ $\Rightarrow$ the system is stable

$$C_0^y = C_0^x \cdot H(jkw_0) = C_0^x \cdot H(0) = C_0^x \cdot 1 = \frac{20}{\pi}$$