Matlab HW

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Essential Engineering Toolbox

October 21, 2019

Problem 17. Solve the IVP for a pendulum with damping and forcing. Solve the pendulum problem with added nonlinear damping of the type $\nu(1-y)\dot{y}$, plot the phase plane with vector fields as well as the solution y as a function of time for three different initial conditions, one close to the center, and two somewhat away.

Solution

$$\ddot{y} + \sin y + \nu(1-y)\dot{y} = 0.$$

- pendulum oscillations with added nonlinear damping equation.

Solution:
$y_1 = y$
$y_2 = y'$,

$y_1' = y_2$
$y_2' = -\nu(1-y_1)y_2 - \sin y_1$.
These two equations are written in matrix form in the function $y_{prime} = pend(t, x)$ at the end of the code.

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%% 17. Solve the IVP for a pendulum with damping and forcing
% Solve the pendulum problem with added nonlinear damping
% of the type nu*(1-y)*\dot(y)
% plot the phase plane with vector fields as well as the solution
% y as a function of time for three different initial conditions,
% one close to the center, and two somewhat away
% you can try different ODE solvers if ode45 fails

function pendulum
%Based on Higham's pendulum solver.
% run this in the command line:
% pendulum, pause(1), end

tspan = [0 30];                        % Solve for 0 ≤ t ≤ 10.
yazero = [1; 1];                       % Initial conditions.
ybzero = [-5; 2];                       % Initial conditions.
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yczero = [5; -2];                       % Initial conditions.

[ta,ya] = ode45(@pend,tspan,yazero);
[tb,yb] = ode45(@pend,tspan,ybzero);
[tc,yc] = ode45(@pend,tspan,yczero);

[y1, y2] = meshgrid(-5:0.5:5, -3:0.5:3);
Dy1Dt = y2;
nu = 0.05
Dy2Dt = -nu.*(1 - y1).*y2 - sin(y1);
quiver(y1,y2,Dy1Dt,Dy2Dt);
hold on

plot(ya(:,1),ya(:,2),yb(:,1),yb(:,2),yc(:,1),yc(:,2))
xlabel('y_1(t)', 'Interpreter', 'latex')
ylabel('y_2(t)', 'Interpreter', 'latex')
title('Phase plane')
axis equal
axis([-5 5 -3 3])
hold off

figure(2)
subplot(1,3,1)
plot(ta, ya(:,1))
xlabel('xa', 'Interpreter', 'latex')
ylabel('ya', 'Interpreter', 'latex')
grid on

subplot(1,3,2)
plot(tb, yb(:,1))
title('Solutions')
xlabel('xb', 'Interpreter', 'latex')
ylabel('yb', 'Interpreter', 'latex')
grid on

subplot(1,3,3)
plot(tc, yc(:,1))
xlabel('xc', 'Interpreter', 'latex')
ylabel('yc', 'Interpreter', 'latex')
grid on

function yprime = pend(t,y)
%Simple pendulum
    nu = 0.05
    yprime = [y(2); -nu.*(1 - y(1)).*y(2) - sin(y(1))];

end

end
```

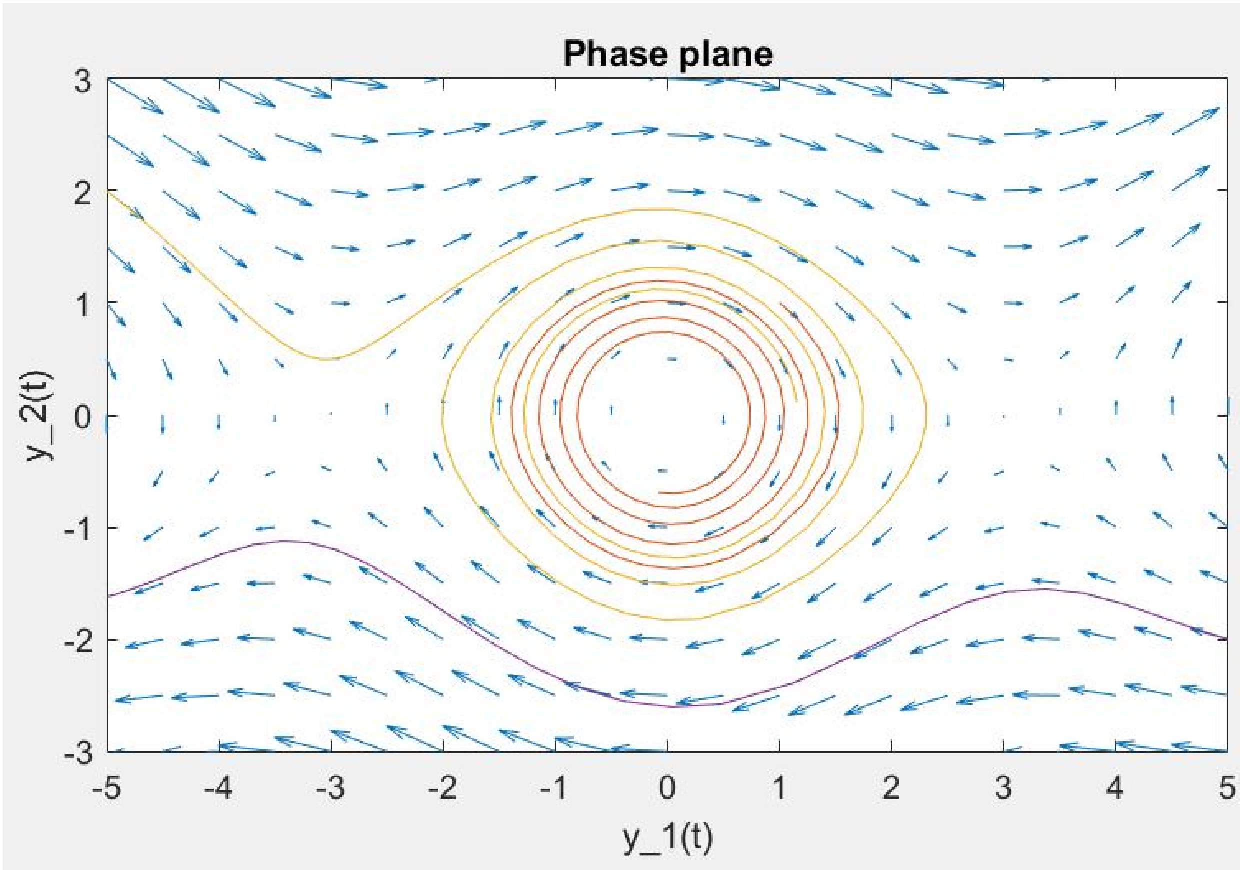

Figure 1: Phase plane, $\nu = 0.05$.

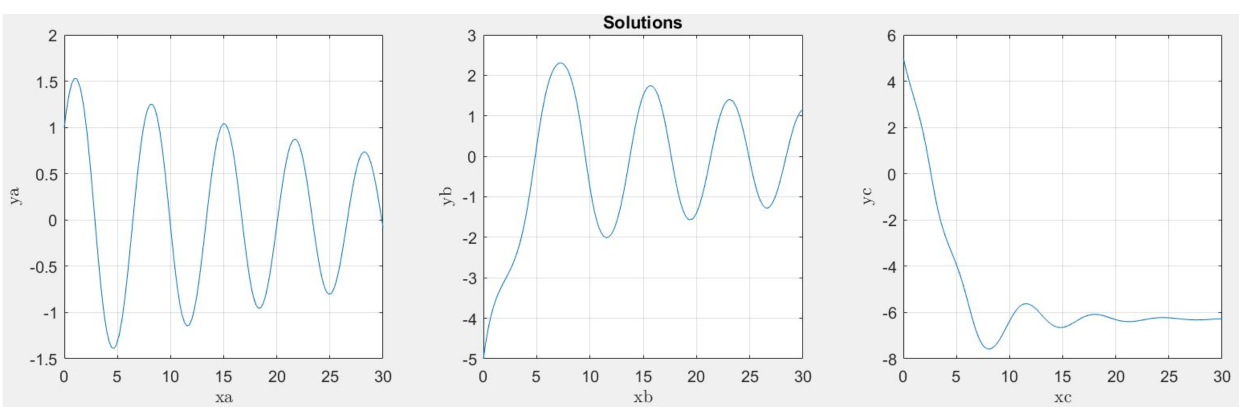

Figure 2: Solutions for three different initial conditions, $\nu = 0.05$.

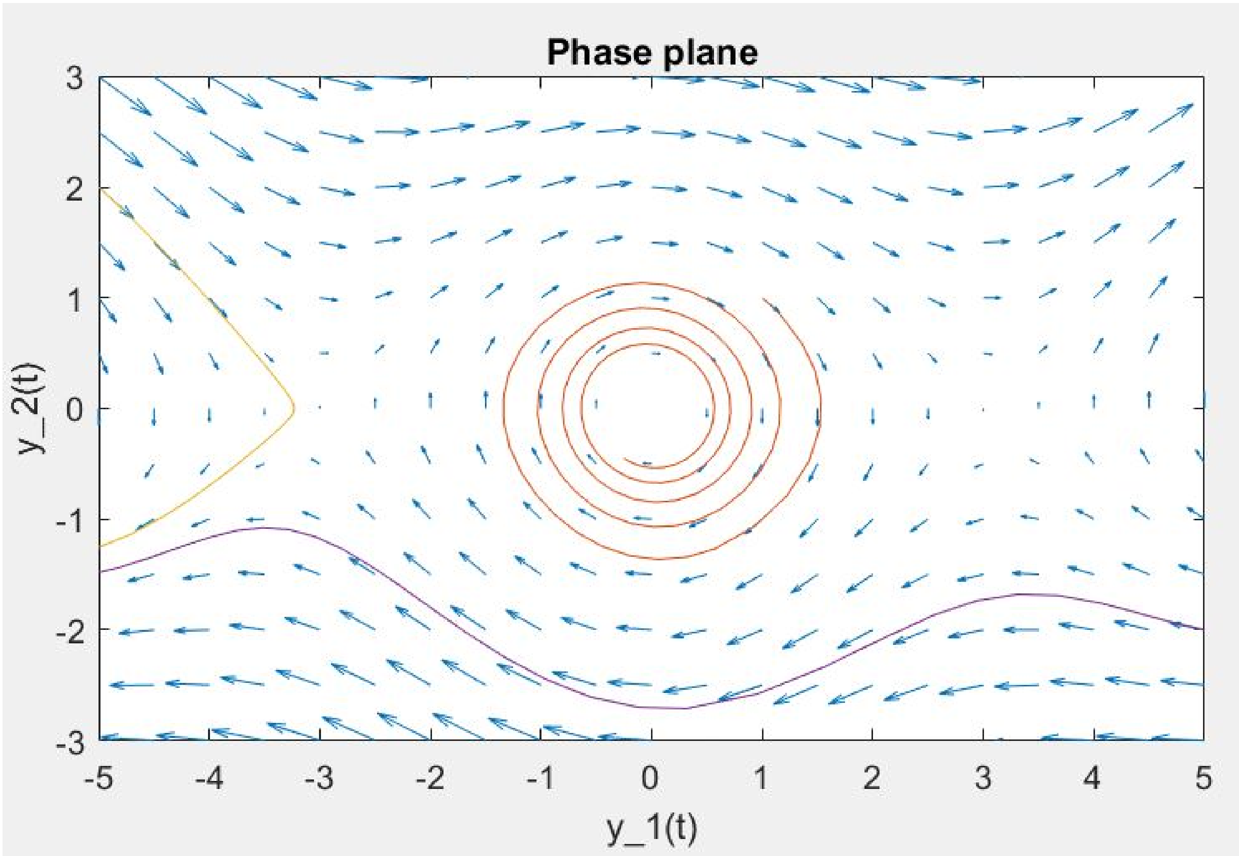

Figure 3: Phase plane, $\nu = 0.07$.

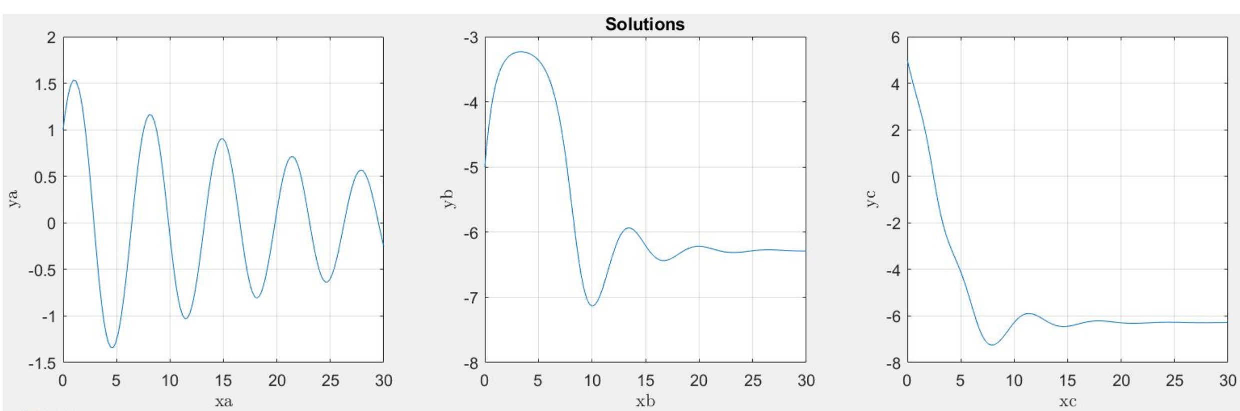

Figure 4: Solutions for three different initial conditions, $\nu = 0.05$.

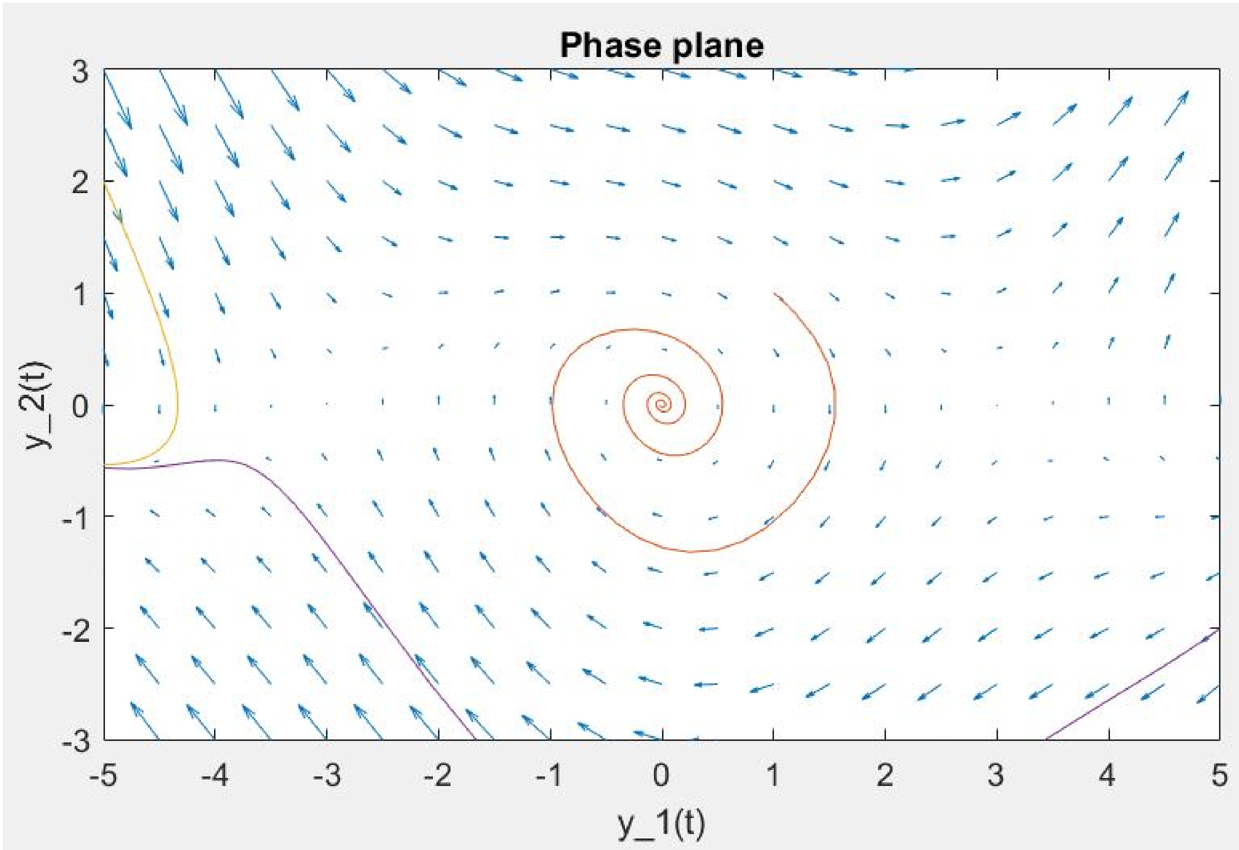

Figure 5: Phase plane, $\nu = 0.3$.

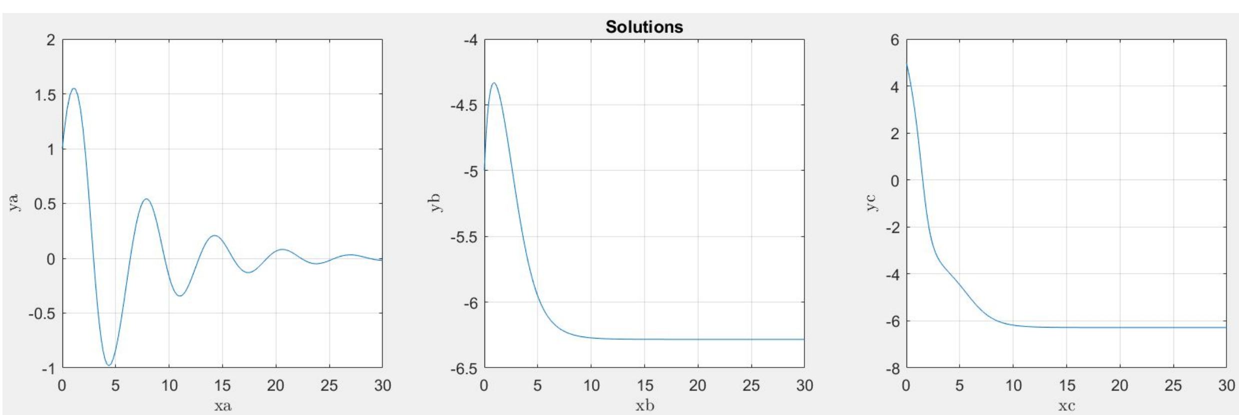

Figure 6: Solutions for three different initial conditions, $\nu = 0.3$.