Project assignment AST2210 –
Analysis of four-year COBE-DMR data

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1 Introduction

We will in this project assignment repeat the analysis of the four-year COBE-DMR observations that were published in 1994, and we will focus on the two important cosmological parameters, Q and n, corresponding to the *amplitude* and *tilt* of the CMB spectrum. The analysis is to be described in the form of an Astrophyiscal Journal Letters (ApJL) paper, and it is this paper that will be the final product of the project.

Before starting the work, it might be a good idea to read the paper by Górski et al. (1994), which is available in the directory "∼hke/AST2210/cobe_project" – you will more or less plagiarize this paper :-) All data and program files are also available in the same directory.

It might also be worth reading the paper by Eriksen et al. (2007) that is available in the same directory, since that has a structure that is closer to the one you will use. While Górski et al. (1994) had a companion paper that described the methods, you will describe both method, data and results in the same paper. Eriksen et al. (2007) is an example of this structure, with a topic that is closely related to the current project.

1.1 The Cosmic Microwave Background (CMB)

In this project we will consider observations of the so-called cosmic microwave background (CMB). Before we start the analysis of these data, it is a good idea to understand what this is, and we therefore provide a quick summary here:

The CMB consists of the oldest photons in the universe. These photons were released about 380 000 years after the Big Bang. Before that time, the universe was too hot and dense for electrons and protons to form neutral hydrogen, and all space was therefore filled with a plasma of free electrons and protons. In such a gas, photons are effectively trapped within the plasma, due to frequent scattering. However, once the temperature fell below 3000K, the photons were no longer sufficiently energetic to ionize hydrogen. At this particular time in the history of the universe, suddenly all free electrons effectively disappeared, merging with protons into hydrogen, and the photons were free to move throughout the universe without additional scattering.

Originally, these photons had a temperature of about 3000K. However, because of the expansion of the universe, the wavelength of these photons is also

stretched. Furthermore, since the universe has expanded by a factor of about 1100 since that time, the effective CMB photon temperature is today about 2.7K. The wavelength of such cool radiation corresponds to microwaves, and it today is observable at radio frequencies.

Today we can detect this radiation essentially using a very fancy TV antenna, and measure its intensity in different directions on the sky. The stronger the intensity, the hotter the photons. And the hotter the photons, the higher the density was at the time when they were released. Therefore, a CMB picture corresponds to a "baby picture" of the universe: It shows where the matter density was high and low shortly after the Big Bang.

1.1.1 The spherical harmonic decomposition and angular power spectrum

To analyze this radiation, we need a few useful mathematical tools. As outlined above, our main concern is the variation in the CMB temperature as a function of sky position. In order to understand the statistical properties of these fluctuations, we will need the spherical harmonic decomposition. Spherical harmonics, $Y_{\ell m}$, are spherical wave functions. They form a complete basis on the sphere, and are therefore widely used in all fields of physics. For instance, you may already have encountered these in quantum physics, when modelling the hydrogen atom.

The spherical harmonic decomposition corresponds to a spherical equivalent of the Fourier transform,

$$\Delta T(\hat{n}) = \sum_{\ell=0}^{\ell_{\max}} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\hat{n}), \tag{1}$$

where the $a_{\ell m}$-s are wave mode amplitudes describing the strength of each individual wave length. Each mode is described by two characteristic numbers, ℓ and m, where the first quantifies the effective wavelength of the mode and the latter describes the phase (or morphology) of the mode.

One can show that for a statistically isotropic and homogeneous field (ie., a field that is supposed to look statistically the same in all directions and from all places), there is no important information carried in the m quantity: The field is essentially just noise. The important thing is instead simply the *amplitude* of the signal as a function of wavelength, or equivalently, it's variance. This is called the *angular power spectrum*, and is defined as

$$\langle a_{\ell m} a_{\ell' m'}^* \rangle = \delta_{\ell \ell'} \delta_{m m'} C_\ell. \tag{2}$$

You will learn much more about this in Cosmology 2, if you choose to continue with cosmology, where you will learn how to predict this from first principles.

It is the power spectrum that ties together theory and observations, in that different models for the evolution of the universe predicts different shapes and amplitudes for the function C_ℓ. By comparing those predictions to that measured from the real CMB sky, we can determine which models fit best with the real universe.

2 Maximum likelihood analysis of COBE-DMR

2.1 Data description

COBE (*COsmic Background Explorer*) was launched by NASA in 1989 to measure the CMB with unprecedented detail. The two most striking results from this experiment were, first, its measurements of the CMB intensity spectrum, which followed a virtually perfect blackbody spectrum shape, and, second, the first detection of spatial variations in the CMB temperature. Two of the leaders of the experiment, John Mather and George Smoot, won the Nobel Prize in Physics for these discoveries in 2006.

In this project we will repeat their analysis of the DMR observations. In particular, we will consider the observations taken at 53 and 90GHz, corresponding to the two cleanest frequency channels, ie., those with the lowest noise and least astrophysical foreground contamination. The 90 GHz map is shown in the top panel of Figure 1.

Next, in order to estimate the variance of the CMB fluctuations, we first need to know something about the variance of the instrumental noise. After all, what we will be looking for is essentially an *excess* variance in the real data above that what is expected from the instrument. For this, we will assume that the noise per pixel is Gaussian distributed with zero mean and standard deviation equal to σ_p, where $\sigma_p = \sigma_0/\sqrt{N_{obs,p}}$; σ_0 is the noise per observation, while $N_{obs,p}$ is simply how many times the instrument has looked at pixel p. The values of σ_p are tabulated for you. Furthermore, we assume that the noise is independent between any two pixels, such that the noise covariance matrix can be written as

$$N_{pp'} = \sigma_p^2 \delta_{pp'}. \tag{3}$$

As seen in the 90 GHz map, there are some regions of the sky that are strongly dominated by *foregrounds*. These are strong sources of radio emission from for instance free electrons or dust grains in the Milky Way, and they tend to swamp out the CMB signal near the Galactic plane. To make sure our analysis is not contaminated by this radiation, we choose to exclude the most contaminated pixels through the use of a *mask*. This is essentially simply a map consisting of either 0 or 1's, indicating whether a pixel should be excluded or not. Figure 2 shows the mask we will use in this project.

Next, we have to account for what is called the instrumental *beam*. This is a function that tells us how large area of the sky is seen by the instrument at any given time. In optical astronomy this function is often called the "point spread function". At any given point, the CMB photon bucket that is our CMB telescope will collect photons from more than one point on the sky. In fact, the COBE-DMR beam extends 7 degrees (!) on the sky! This beam essentially corresponds to smoothing the true sky by 7 degrees, thereby effectively suppressing small angular scales. Mathematically, we can describe this operation as a convolution in pixel space, which turn in to a multiplication in harmonic space due to the Fourier convolution theorem. The net result is that Equation
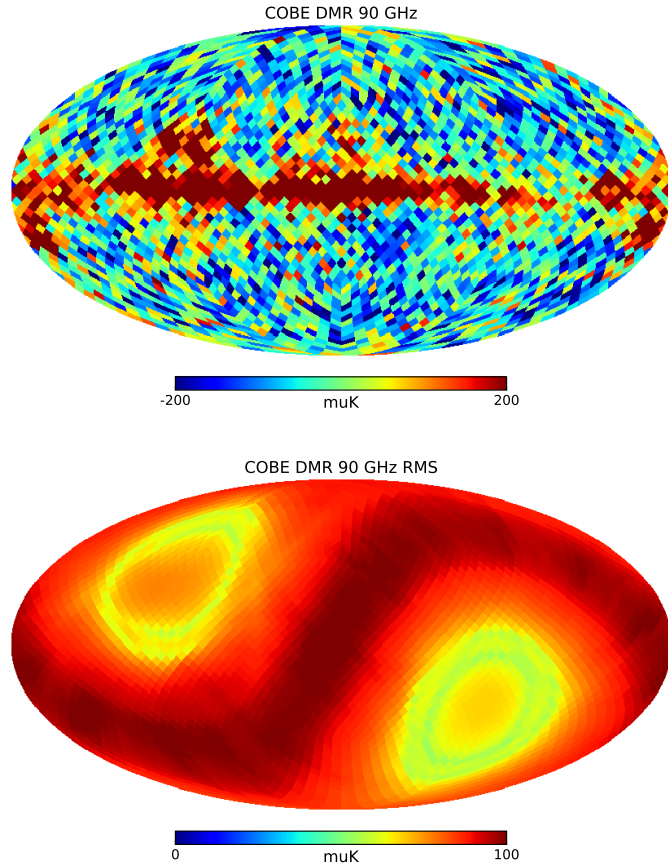
3

COBE DMR 90 GHz

COBE DMR 90 GHz RMS

Figure 1: Map and corresponding RMS distribution for the COBE-DMR 90 GHz channel. Note that these maps show the whole sky projected onto a plane, using a so-called Mollweide projection. They are shown in Galactic coordinates, and the red "band" across the top panel corresponds to excess foreground radiation from the Milky Way. In comparison, the bottom panel shows the effective scanning strategy of the COBE satellite, in the sense that where the map is red, the noise is high, and the satellite has spent little observation time here; where it is yellow, the noise is low, and the satellite has spent much time here. The dark red band corresponds to the Ecliptic plane, ie., the path of the Sun across the sky during a year. The satellite is designed to avoid looking at the Sun, and therefore spends less time in this region.
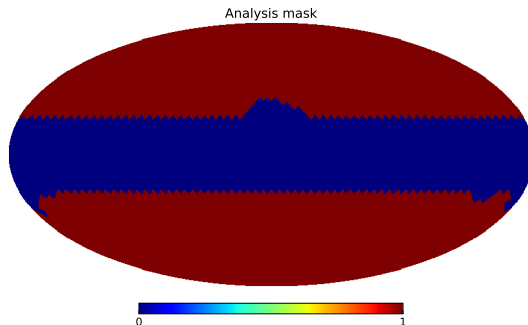
Figure 2: Analysis mask. The red pixels are included in the analysis, whereas the blue pixels hear the Galactic plane are excluded to minimize foreground contamination.

?? is replaced by

$$\tilde{T}(\hat{n}) = \sum_{\ell=0}^{\ell_{\max}} \sum_{m=-\ell}^{\ell} b_\ell a_{\ell m} Y_{\ell m}(\hat{n}), \qquad \text{beam convolved map.} \tag{4}$$

The DMR beam is well known, and we will describe later how to account for this in the actual analysis. You should also plot it to gain some intuition for what it means.

All in all, the DMR data set consists of the following items:

- Two files with CMB measurements at 53 and 90 GHz, each consisting of a total of 1941 data points with associated direction vectors. 1131 pixels out of a total of 3072 have been removed by masking. The linear size of each pixel is about 220' ($\sim 3.7°$)

- Two associated files containing the variance of each pixel, σ_p^2, where σ_p is given in the files.

- the instrumental beam, b_ℓ.

These are all available in the directory mentioned above.

2.2 Data model

2.2.1 On Gaussian fields

Perhaps the most important probability distribution in all of science is the normal distribution, also called the Gaussian distribution. In one dimension this looks as follows,

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \tag{5}$$

where μ is the average of the distribution, and σ^2 is its variance. σ alone is called the standard deviation (or root-mean-square; RMS), and quantifies intrinsic uncertainty; a given observation lies within a distance of 1σ from the true answer with 68% probability.

When a distribution has $\mu = 0$, it is called centered. This applies to many commonly encountered distributions, for instance those describing noise. For us, this applies to both the instrumental noise and the cosmological noise that is the CMB itself. From now on, we will therefore disregard this parameter.

If the data set in question has multiple Gaussian distributed variables, one has to consider the multi-variate Gaussian distribution,

$$p(\mathbf{x}) \propto \frac{1}{\sqrt{|\mathbf{C}|}} e^{-\frac{1}{2}\mathbf{x}^t \mathbf{C}^{-1} \mathbf{x}}, \tag{6}$$

where $\mathbf{x}$ is a vector consisting of all data points, and $\mathbf{C} \equiv \langle \mathbf{x}\mathbf{x}^t \rangle$ is its covariance matrix[1]. The covariance matrix has the same task as the variance in the one-dimensional case, but in addition to measuring the variance in each one-dimensional variate (which are listed on the diagonal of the matrix), it also measures how strongly two different elements are *correlated*. This is stored in the off-diagonal elements C_{ij}.

A main point, however, is that if you know the covariance matrix for a centered Gaussian distribution, you know *everything* about the distribution. This quantity is therefore the prime target to measure whenever one is considering data analysis with Gaussian variables.

2.3 Specialization to CMB observations

As mentioned above, CMB observations are extremely well approximated as Gaussian distributed, and we therefore want to find an expression for their covariance matrix. To do so, we model our data as a sum of a CMB component and an instrumental noise component,

$$d(\hat{n}) = s(\hat{n}) + n(\hat{n}) + f(\hat{n}). \tag{7}$$

Here d is the observed signal in direction $\hat{n}$, s is the true CMB signal, n is noise, and f is possible non-cosmological foreground signals. We assume that none of these are correlated with each other, such that all cross-products have zero mean $\langle sn \rangle = \langle sf \rangle = \langle nf \rangle = 0$. (For instance, the CMB radiation do not affect the properties of radiation from the Milky Way or the noise in our own instrument.)

The covariance matrix of d is therefore given by

$$\mathbf{C} \equiv \langle \mathbf{d}\mathbf{d}^t \rangle = \langle (\mathbf{s} + \mathbf{n} + \mathbf{f})(\mathbf{s} + \mathbf{n} + \mathbf{f})^t \rangle = \langle \mathbf{s}\mathbf{s}^t \rangle + \langle \mathbf{n}\mathbf{n}^t \rangle + \langle \mathbf{f}\mathbf{f}^t \rangle \equiv \mathbf{S} + \mathbf{N} + \mathbf{F}, \tag{8}$$

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[1]We disregard normalization constants such as 2π here, since they won't affect the final answer.

where $\mathbf{S}$ is the covariance matrix for the CMB signal alone, $\mathbf{N}$ is the covariance matrix of the noise, and $\mathbf{F}$ is the covariance matrix of the foregrounds. Each of these is an $N_{\mathrm{pix}} \times N_{\mathrm{pix}}$ matrix in our case.

We don't know the exact values of either the CMB signal or the noise, but only their statistical properties. To start with the most simple case, let us assume that the noise is Gaussian and uncorrelated between pixels, with standard deviation given by σ_p. The noise covariance matrix is therefore

$$N_{ij} = \langle n_i n_j \rangle = \sigma_i^2 \delta_{ij}, \tag{9}$$

where i and j are two pixel indices. In other words, the noise covariance matrix is diagonal, and containing the usual variance along the diagonal; the multivariate noise distribution is plain and simply a product of usual one-dimensional Gaussian distribution for each pixel. This is a very good approximation for DMR.

Next, we assume that [2] the CMB field is Gaussian and isotropic, but correlated between pixels. In this case it is possible to show that the pixel-pixel covariance matrix reads

$$S_{ij} = \frac{1}{4\pi} \sum_{\ell=0} (2\ell + 1)(b_\ell p_\ell)^2 C_\ell P_\ell(\cos \theta_{ij}), \tag{10}$$

where b_ℓ is the instrumental beam described above, p_ℓ is called the *pixel window*, and quantifies the effect of finite pixelization in our maps (which behaves in principle just like the beam: The sky has small-scale variations that we cannot detect because of the finite size of our pixels. Plot this function together with the instrumental beam!), $P_\ell(x)$ are the Legendre polynomials, and θ_{ij} is the angle between pixels i and j.

This is where the cosmological theories enter the picture: The power spectrum C_ℓ is a theoretical function that depends critically on cosmological parameters. Therefore, by changing those cosmological parameters different correlation structures are predicted in the CMB field. In this particular project we will consider a special class of models, namely those parametrized by an amplitude Q and a spectral index n $(P(k) \propto k^n$; Bond and Efstathiou 1987) on the form

$$C_\ell = \frac{4\pi}{5} Q^2 \frac{\Gamma(\ell + \frac{n-1}{2})\Gamma(\frac{9-n}{2})}{\Gamma(\ell + \frac{5-n}{2})\Gamma(\frac{3+n}{2})}. \tag{11}$$

(The choice of the somewhat old-fashioned Q normalization is done in order to be able to compare our results to those presented in 1994; today other parametrizations are more common. The use of Γ functions makes this expression look quite complicated. However, this function is described in mathematical tables or online. Its most important property for our purposes, though, is its recursive nature, $\Gamma(1 + x) = x\Gamma(x)$. Additionally, the expression for $\ell = 2$ may be simplified to $C_2 = 4\pi/5\,Q^2$. One may therefore evaluate $C_{\ell+1}$ recursively, given C_2 as

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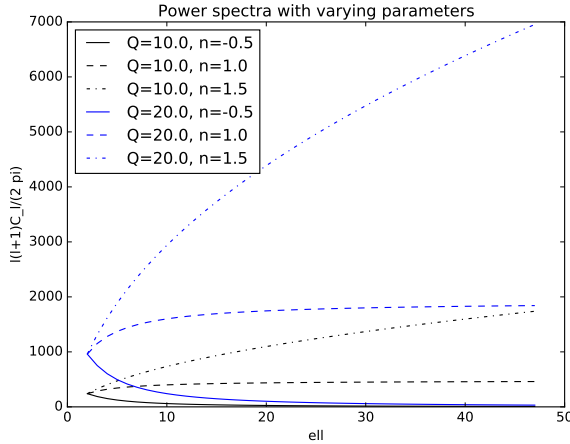[2]This is a very good and physically motivated assumption that you will learn much more about in AST3220/5220.

Figure 3: Angular power spectra for different values of Q and n.

a function of Q. Note also that one has to set $C_0 = C_1 = 0$ manually, because we remove the monopole and dipole (ie., contributions to Equation 1 which has $\ell = 0$ and $\ell = 1$) as described below. Figure 3 shows how the power spectrum within this class varies with different parameter values.

With this definition we have connected the CMB covariance matrix – and thereby our observed data – to two simply inflationary parameters Q and n.

Finally, we have to include a term that removes the effect of a possible monopole or dipole in our maps. First of all, DMR is a so-called differential experiments, which means that it only measures the *difference* between two points on the sky, never their absolute values. It is therefore impossible to say what the true monopole is. (On the other hand, this is what COBE-FIRAS did, by comparing the sky to an internal controlled load on the satellite.) Second, it is also impossible to measure the true cosmological CMB dipole: Earth's motion through space results in a Doppler shift of the CMB monopole, which takes the form of a dipole. The observed dipole is therefore only a reflection of our motion, rather than a cosmologically interesting quantity.

To account for these terms, we should *marginalize* over the amplitude of these two modes, such that the final answers are independent of their true values. One way to do this is to add another term to the covariance matrix with "infinite" variance,

$$\mathbf{F} = \lambda \mathbf{f}\mathbf{f}^t. \tag{12}$$

where λ is a numerically large constant (e.g., 10^3), and $\mathbf{f}$ is the known structure (or template) on the sky one wants to be insensitive to. $\mathbf{f}\mathbf{f}^t$ is the outer-product o this template with itself. Thus, if you want to marginalize[3] over the monopole,

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[3]Marginalization means to remove one variable from a common distribution by integrating over it. If we have a joint distribution $f(x, y)$, and wants to find the distribution for x

you simply add a large constant to the total covariance matrix.

To summarize, the final covariance matrix is given by

$$\mathbf{C}(Q, n) = \mathbf{S}(Q, n) + \mathbf{N} + \mathbf{F}, \tag{13}$$

where the three individual covariance matrices are given as above.

2.4   Maximum likelihood analysis

Our main goal in this project is to determine the best-fit values of the parameters , Q and n, given the COBE-DMR observations. There are several methods for doing so, but we will adopt the so-called *maximum-likelihood* framework.

The likelihood function is defined simply as,

$$\mathcal{L}(Q, n) = p(\mathbf{d}|Q, n).$$

This function quantifies the probability of obtaining some data set, given the true cosmological parameters. Of course, the data are given once and for all, so our task is not to change the data as such, but rather ask which parameters are most likely given those data.

The joint distribution above is given by the multi-variate Gaussian described in Equation 6, with the data vector $\mathbf{d}$ as its variable. Having constructed the covariance matrix, the rest of the analysis is fairly straightforward. Based on this distribution, the *log-likelihood* reads

$$-2 \log \mathcal{L}(Q, n) = \mathbf{d}^T \mathbf{C}^{-1} \mathbf{d} + \log |\mathbf{C}| + \text{constant}, \tag{14}$$

and our task is to evaluate this for different values of (Q, n).

Note that it is usual to consider the log-likelihood rather than the likelihood itself, since this is a numerically more well-behaved number. With 1000 pixels, the log-likelihood will typically have values of order 1000, while the likelihood itself will be $\mathcal{O}(e^{-1000})$. Numerical errors quickly become difficult to handle. It is therefore a very good tip to perform all calculations in log units, and only exponentiate at the very end. You can also disregard any constants in the log-likelihood, since this corresponds just to a normalization factor, which you can reconstruct in the end by requiring that the final function should have integral equal to one.

The main product in this project is thus a two-dimensional contour plot of the likelihood evaluated for (Q, n). In our particular case, the data set is small enough that brute-force grid evaluation is feasible even on a laptop. However, for larger data sets or more sophisticated models, Markov Chain Monte Carlo methods may be preferable. If you want to, you can of course solve the current problem using this method, which is both fun and instructive.

After calculating the two-dimensional likelihood, you must also compute the one-dimensional marginal best-fit values for each parameter individually, with

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alone, we can marginalize over y, which means to integrate over all possible values of y: $f(x) = \int f(x, y)\, dy$

associated uncertainties. Mathematically, you obtain the distribution for one parameter by integrating over the other,

$$\mathcal{L}(Q) = \int \mathcal{L}(Q, n) dn. \tag{15}$$

If you choose MCMC, this task is trivial, since you then simply can make a one-dimensional histogram from the samples directly. If you perform the mapping by 2D gridding, you actually have to perform the integral numerically. In both cases, you want to derive an answer on the form $n = 1.1 \pm 0.2$.

2.5 Implementation

In principle, you are completely free to implement this project in whatever computer language you want. However, the simplest solution is clearly to use the Python template that is made available in the standard directory. This template is a close to complete version of the final program, with only a few critical lines removed.

The program consists of two files, namely the main program ("cmb_likelihood.py") and a utility module ("cmb_likelihood_utils.py"). The main program is complete, and you don't have to do anything with this, unless you want to. But you definitely need to read through it, and make sure you understand what happens at the various places. Furthermore, if you want to output the final results in another format than the default, you have to do this in the main program.

The utility module is however not complete. All subroutines are in place, but the most critical lines are missing. What actually is missing is listed in the beginning of the module. This is also marked in each routine, although not necessarily how it should be done.

2.5.1 Programming tips

There are two operations in this project that may turn out very time-consuming, at least if you use Python. For these, it is important to implement the codes in a clever way. This concerns the evaluation of the signal covariance matrix $\mathbf{S}$ (Equation 10) and the final log-likelihood (equation 14).

Covariance matrix: Given the expression for $\mathbf{S}$ it is easy to set up a triple for-loop to evaluate this matrix. However, this is very slow in Python, and it is always a good idea to vectorize Python code whenever possible. Fortunately, we have NumPy, which is a C-based library, that is optimized for array operations. You therefore want to put on your "vector glasses" when implementing this expression: It may be considered as a series of element-wise vector multiplications followed by one inner product, corresponding to the sum over ℓ. You should try both methods, just to check how much faster the code runs with vectorization, and to make sure they give the same answer, for debugging purposes.

Log-likelihood: The expression for $\log \mathcal{L}$ also suggests an obvious solution strategy: Invert $\mathbf{C}$ with, say, scipy.linalg.inv, and then compute the determinant with for instance scipy.linalg.det or numpy.linalg.slogdet. Matrix inversion and determinant evaluation are however expensive operations, and fortunately people have worked on this before us, and there are today fast libraries for such operations, such as LAPACK. However, it is actually quite rare that one needs to actually invert a covariance matrix, as one can often use what is called a Cholesky decomposition instead.

The Cholesky decomposition is a product of a triangular matrix with its own transpose

$$\mathbf{A} = \mathbf{L}\mathbf{L}^{\mathrm{T}} \quad \Rightarrow \quad \mathbf{A}^{-1} = (\mathbf{L}^{-1})^{\mathrm{T}}\mathbf{L}^{-1}.$$

With this decomposition, the determinant is essentially free, since one has

$$\det \mathbf{A} = \det \mathbf{L} \, \det(\mathbf{L}^{\mathrm{T}}) = (\det \mathbf{L})^2,$$

since the determinant of a triangular matrix is just to product over all diagonal elements.

Solving an equation set on the form

$$\mathbf{L}\mathbf{x} = \mathbf{y}$$

for $\mathbf{x}$ is very simple, and it is much faster than inverting a matrix, even with use of LAPACK. The trick is therefore to rewrite

$$\mathbf{d}^{T}\mathbf{C}^{-1}\mathbf{d}, \quad \text{der} \quad \mathbf{C}^{-1} = (\mathbf{L}^{-1})^{\mathrm{T}}\mathbf{L}^{-1},$$

to something like

$$\mathbf{x}^{T}\mathbf{x},$$

where $\mathbf{x}$ is the solution of the triangular equation set as shown above.

SciPy-rutinene scipy.linalg.cholesky and scipy.linalg.solve_triangular will be useful for this.

If you are stuck, don't hesitate to ask Hans Kristian either in person or by email (h.k.k.eriksen@astro.uio.no): Do not spend massive amounts of time to figure out a problem if it takes two minutes to ask somebody else! :-)

2.6   Data

The required data are stored in the standard directory given in the beginning of the project description:

- cobe_dmr_53GHz_n16.npy(.dat)– DMR map and RMS values for the 53GHz channel, given in form of a list of unmasked pixel values and corresponding directional vectors.

- cobe_dmr_90GHz_n16.npy(.dat)– DMR map and RMS values for 90GHz channel, as above

- cobe_dmr_beam.npy(.dat)– DMR beam function b_ℓ, given for each multi-pole ℓ.

- pixwin_n16.npy(.dat)– Healpix pixel window at $N_{\mathrm{side}} = 16$.

- params.py – parameter file for the cmb_likelihood program

Results should be generated and shown for both the 53 and 90 GHz channels.

Note that the data files are given in two equivalent formats, either binary NumPy array files or ordinary ASCII text files. For those who choose to do the project in Python, the first format is definitely preferable, while the .dat files are available for those who prefer a different language.

2.7   Plotting the results

As always, how to make the contour plots is up to you. We do however provide a very simple template that works as an example, called "plot_contours.py" in the 'work' directory. This script takes the output from the cmb_likelihood program, and shows the corresponding contour plot on the screen. Writing this to file, and make the figure prettier, is your task. You can of course ask your fellow students – or do it in a completely different way, if you prefer that.

3   Part 2 – "Publishing" in Astrophysical Journal Letters

The programming in this project is quite simple, and consists of adding a few simple commands in a nearly finished program. The run time of the code will take a few hours, perhaps a day, depending on how many grid points you want.

The main work load is associated with writing the paper. You should do this in the standard format that has already been provided, using all the tips that were given in the first few projects.

Regarding style, imagine that you write this paper for the first time in 1994, and not in 2016. In other words, pretend that this is the first discovery of the CMB fluctuations. So, don't be too modest – this will result in a Nobel prize in 15 years. Yet, use a scientific language and style: You know that you have something really cool on your hands, but you don't want to be annoying :-)

The paper should contain (at least) the following points:

1. An abstract that briefly summarizes the problem, the methods and the results.

2. An introduction that provides a setting for the analysis – why do we do this, what does the theory tell us, and what is the current status of the field?

3. A method section that describes the model (including definition of the covariance matrix), the likelihood, and the algorithm used to map the likelihood

4. Data summary: Which data are used, which frequencies, what angular resolution etc.

5. Results

   (a) Both 53 and 90GHz maps must be analyzed

   (b) 2D contour plots for each case, preferably over-plotted

   (c) Best-fit one-dimensional values listed in a table, including uncertainties

   (d) Details regarding the analysis (e.g., run time) may also be included.

6. Conclusion: What has been found, and what does it imply? What should be done next, in order to bring the CMB field to its full fruition?

Finally, if something is unclear, wrong or needs improvement, please don't hesitate to say so. All feedback is very much welcome :-)