AST4320 COSMOLOGY AND EXTRAGALACTIC ASTRONOMY
ASSIGNMENT 3

Deadline: Friday, November 29

Exercise 1 The Lyα forest Use cosmological parameter $\Omega_\Lambda = 0.692$, $\Omega_m = 0.308$, $\Omega_b = 0.048$ and $\Omega_r = 0$ for the following questions.

- Assuming the intergalactic medium (IGM) contains only hydrogen and helium, and the hydrogen and helium mass fractions are $X = 0.76$ and $Y = 0.24$, respectively, compute the mean molecular weight of the IGM, μ.

- Compute the Jeans length of the IGM as a function of redshift. You can assume that the IGM temperature is $T = 10^4$ K. What value for k does this correspond to?

- In redshift-space, the Jeans length would correspond to a velocity width. What is this velocity width?

- What does the finite width of the Lyα extinction profile do to absorption features in the Lyα forest?

- In lectures we discussed that the intrinsic Lyα extinction profile function (i.e. the Lorentz profile) is broadened by gas thermal motions, and the thermal broadening scale is $v_{th} = \sqrt{(2k_bT/m)}$, where T is the temperature of the gas and m is the mass of the particle. Compare the thermal broadening scale to the Jeans length.

Exercise 2 Optical depth of the ionised IGM
In the lectures we derived that the total optical depth of the ionised IGM due to electron scattering is given by

$$\tau_{\rm e}(z) = c\int_0^z \frac{n_e(z)\sigma_{\rm T}dz}{(1+z)H(z)},$$

where $\sigma_{\rm T} = \frac{8\pi}{3}r_e^2 = 6.65 \times 10^{-25}$ cm^2 denotes the cross-section for electron scattering. Assuming the IGM is highly ionised, and the average number density of hydrogen evolves as $\bar{n}_{\rm H}(z) \sim 1.9 \times 10^{-7}(1+z)^3$ cm $^{-3}$, compute the optical depth as a function of z under cosmological parameter $\Omega_\Lambda = 0.692$, $\Omega_m = 0.308$ and $\Omega_r = 0$). Plot $\tau_{\rm e}(z)$ as function of z for z = 0-10.

Exercise 3 Isothermal density profile

In the lectures we obtained a second order differential equation for $\rho(r)$ if assuming the halo is "isothermal" (i.e. dark matter particles have the same velocity dispersion everywhere in the halo):

$$-\frac{k_{\rm b}T}{m_{\rm DM}r^2}\frac{d}{dr}r^2\frac{d}{dr}\ln\rho = 4\pi G\rho(r),$$

- Show that

  $$\rho(r) = \frac{A}{r^2}, \quad A = \frac{k_{\rm b}T}{2\pi G m_{\rm DM}}.$$

  provides a solution to this equation.

- For gas in hydrostatic equilibrium with gravity we have

  $$\frac{dp}{dr} = -\frac{GM(<r)\rho}{r^2}.$$

  Show that isothermal gas settles into a similar state.

Exercise 4 The Cusp-Core problem

In the lectures, we discussed the small-scale "crisis" for cold dark matter. One of them is the "Cusp-Core Problem". One possible solution to reconcile simulations and observations (without introducing alternative dark matter models) is that supernova feedback can eject gaseous material out of galaxies in a repeated way so that the dark matter responded dynamically and redistribute into a cored profile. In this exercise, we investigate whether the energies provided in supernova are actually enough to create cores of observed sizes.

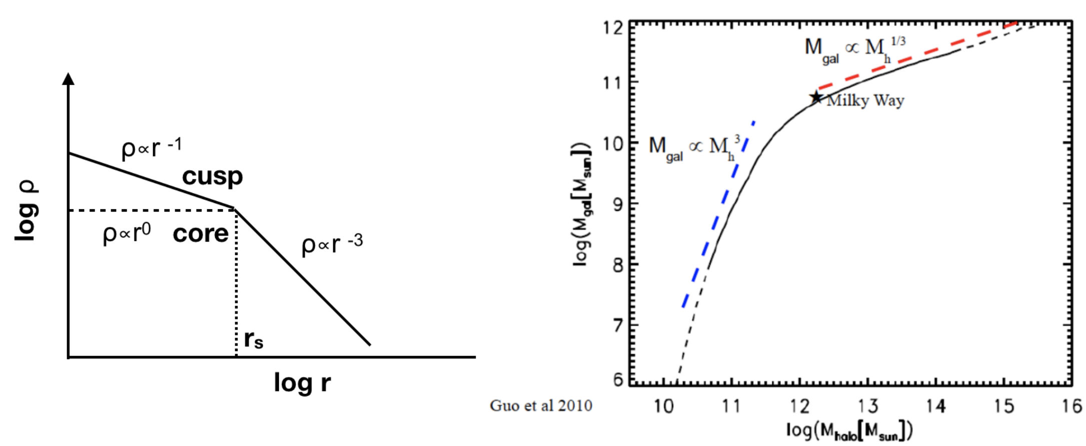

FIGURE 1

a Assume a dark matter halo has a cuspy density profile which is described by:

$$\rho^{\text{cusp}}(r) = \begin{cases} \rho_0 \left(\frac{r}{r_s}\right)^{-1}, & \text{if } r < r_s \\ \rho_0 \left(\frac{r}{r_s}\right)^{-3}, & \text{if } r \geq r_s \end{cases}$$

Here, ρ_0 is a normalization parameter and r_s is the scale length. In lectures we discussed how baryonic processes such as supernova feedback can result in the formation of a dark matter core. If we assume the resultant "cored" profile is described by:

$$\rho^{\text{core}}(r) = \begin{cases} \rho_0 & \text{if } r < r_c \\ \rho_0 \left(\frac{r}{r_c}\right)^{-3}, & \text{if } r \geq r_c \end{cases}$$

Here, r_c is the size of the core. For simplicity, we adopt $r_c = r_s$ for this exercise and assume ρ_0 is the same in both profiles. The profiles are illustrated in the left panel of Figure 1. Show that the total mass enclosed within radius r is

$$M^{\text{cusp}}(< r) = \begin{cases} 2\pi\rho_0 r_s r^2, & \text{if } r < r_s \\ 2\pi\rho_0 r_s^3 + 4\pi\rho_0 r_s^3(\mathrm{lnr} - \mathrm{lnr_s}), & \text{if } r \geq r_s \end{cases}$$

for the cuspy profile, and

$$M^{\text{core}}(< r) = \begin{cases} \frac{4}{3}\pi\rho_0 r^3, & \text{if } r < r_s \\ \frac{4}{3}\pi\rho_0 r_s^3 + 4\pi\rho_0 r_s^3(\mathrm{lnr} - \mathrm{lnr_s}), & \text{if } r \geq r_s \end{cases}$$

for the cored profile.

b From Virial theorem we can derive that the minimum total energy needed to create a cored profile from a cuspy profile is given by $\Delta E = (W^{core} - W^{cusp})/2$, where W is the total gravitational potential energy. Show that if the density distribution is spherical symmetric, the gravitational potential energy at the virial radius r_{vir} is

$$W = -4\pi G \int_0^{r_{vir}} \rho(r) M(< r) r dr \tag{1}$$

c Using the density and mass profile given in the previous questions, show that for $r_s \ll r_{vir}$, the minimum energy needed to create a cored profile is

$$\Delta E = \frac{32}{15}\pi^2 G \rho_0^2 r_s^5 \tag{2}$$

Please expand all the terms before you do the integrations because you do not need to do all of them!

d Consider a dwarf galaxy at z = 0 with $M_{vir} = 3 \times 10^{10}$ $M_\odot$, $R_{vir} = 45$ kpc and scale length $r_s = 1$kpc, compute the energy needed to create a dark matter core with $r_c = r_s = 1$ kpc.

e The right panel of Figure 1 shows a relationship between the total stellar mass of a galaxy and the virial mass (M_{gal} in this figure is the same as total stellar mass). Assuming the number of supernova explosions per solar mass formed is $\xi = 0.004$ and energy injection per supernova is $E_{SN} = 10^{51}$ ergs, use the information in the figure to compute the total energy available from supernova feedback. How does it compare to the energy required to generate 1 kpc core in the dark matter profile?

f Consider a galaxy with 10 times smaller virial mass (i.e. $M_{vir} = 3 \times 10^9 M_\odot$), but **the same virial radius and scale length**, use the information given in the right panel of Figure 1 (especially the scaling relations), comment on whether supernova energy is sufficient to generate a 1 kpc core in this halo. (*You do not need to compute any numbers here, simply observe the dependencies on M_{vir} for both the energy required and the energy provided by supernova.*)