Exel

$$W(x) = \begin{cases} 1 & |x| < R \\ 0 & \text{else} \end{cases}$$

The ~~Fourier~~ transformed of this window function from definition:

$$\tilde{W}(k) = \int_{-\infty}^{\infty} W(x) e^{-ikx} dx$$, piecewise function, split in three regions

$$= \int_{-\infty}^{-R} 0 \cdot e^{-ikx} dx + \int_{-R}^{R} 1 \cdot e^{-ikx} dx + \int_{R}^{\infty} 0 \cdot e^{-ikx} dx$$

$$= \int_{-R}^{R} e^{-ikx} dx = \frac{i e^{-ikx}}{k} \Bigg|_{-R}^{R} = \frac{i}{k}\left[e^{-ikR} - e^{ikR}\right]$$

~~or~~ : $$= -\frac{1}{ki}\left[e^{ikR} - e^{-ikR}\right]$$ | $\sin x = \frac{e^{ix} - e^{-ix}}{2i}$

$$\tilde{W}(k) = \frac{2\sin(kR)}{k}$$

Problem when $k \to 0$?
↳ Test with L'Hôpitals:

$$\lim_{k\to 0} \tilde{W}(k) = \lim_{k\to 0} \frac{\frac{d}{dk} 2\sin kR}{\frac{d}{dk} k} = \lim_{k\to 0} \frac{2R\cos(kR)}{1}$$

$$= 2R$$

↳ Well behaved on hole real line.

Finding the FWHM numerically is quite easy.

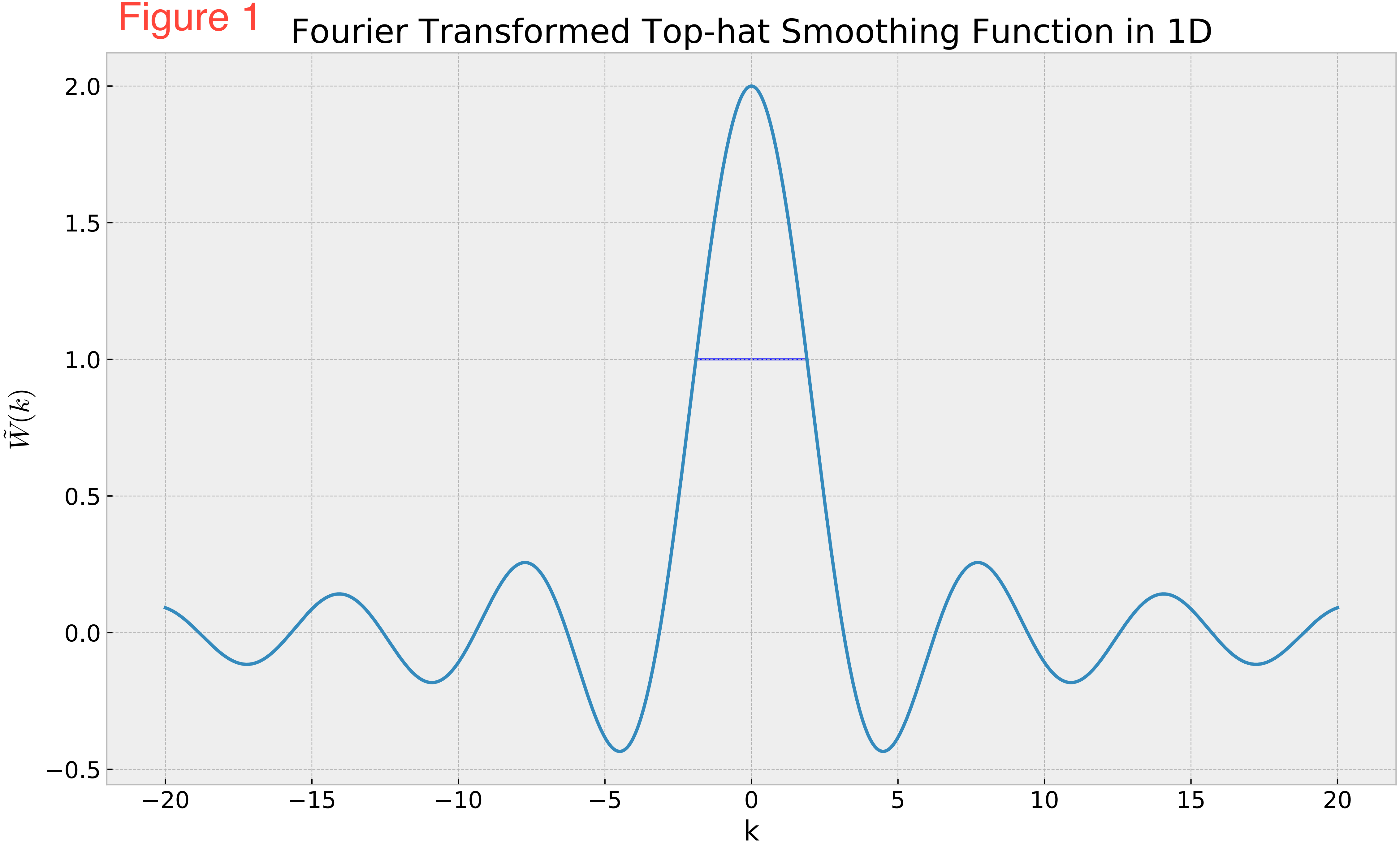
As the maximum is at $k=0$, we find the index of k where $\hat{w}(k) = \frac{2R}{2} = R$.

$(\hat{w}(0) = \hat{w}_{max} = 2R)$

Then the full width here is twice the value for k at $\hat{w}(k) = R$.

Numerically we find $\text{FWHM} = \underline{3,79}$

Figure 1



Fourier Transformed Top-hat Smoothing Function in 1D
2.0
1.5
1.0
0.5
0.0
−0.5
$\tilde{W}(k)$
−20
−15
−10
−5
0
5
10
15
20
k

Exe2

- $$\sigma^2 = \frac{\pi}{S_c^4} < 10^{-4}, \quad S_c = \left(\frac{\pi}{\sigma^2}\right)^{1/4} = \frac{2\pi}{k}$$

$$\hookrightarrow k = 2\pi\left(\frac{\pi}{\sigma^2}\right)^{-1/4}$$

I start by choosing $\sigma^2_{init} = 0{,}95 \cdot 10^{-4}$

$$\varepsilon = \begin{cases} 0{,}1 & \text{for crossing} \\ 0{,}05 & \text{for non-crossing} \end{cases}$$

Using the provided steps I get, using script exe2.py, the random walk results included in fig 2 and 3.

- In fig. 2, the distribution for δ clearly match the analytical expression for the GRF with mean zero and $\sigma^2 = \frac{\pi}{S_c^4}$, where the last obtained value for the smoothing scale was used.

- When only keeping walks with $\delta_{nc} < \delta_{crit} = 1$, we find the distribution seen in fig 3. Here the analytical expression doesn't fit so well, and I use a fudge factor of 2.2 to force it to fit better. We see that the calculated distribution is a bit screwed to the right, with a bit too sharp cut of at $\delta_{nc} = 1$. This cut of is expected from the filtred restrictions we implement in the code, but should idealy be a bit smoother. I've reduced the ε-value for this run, which helps but also slows down the code significantly. With even lower ε the results should look better.

Scanned with CamScanner

Figure 2

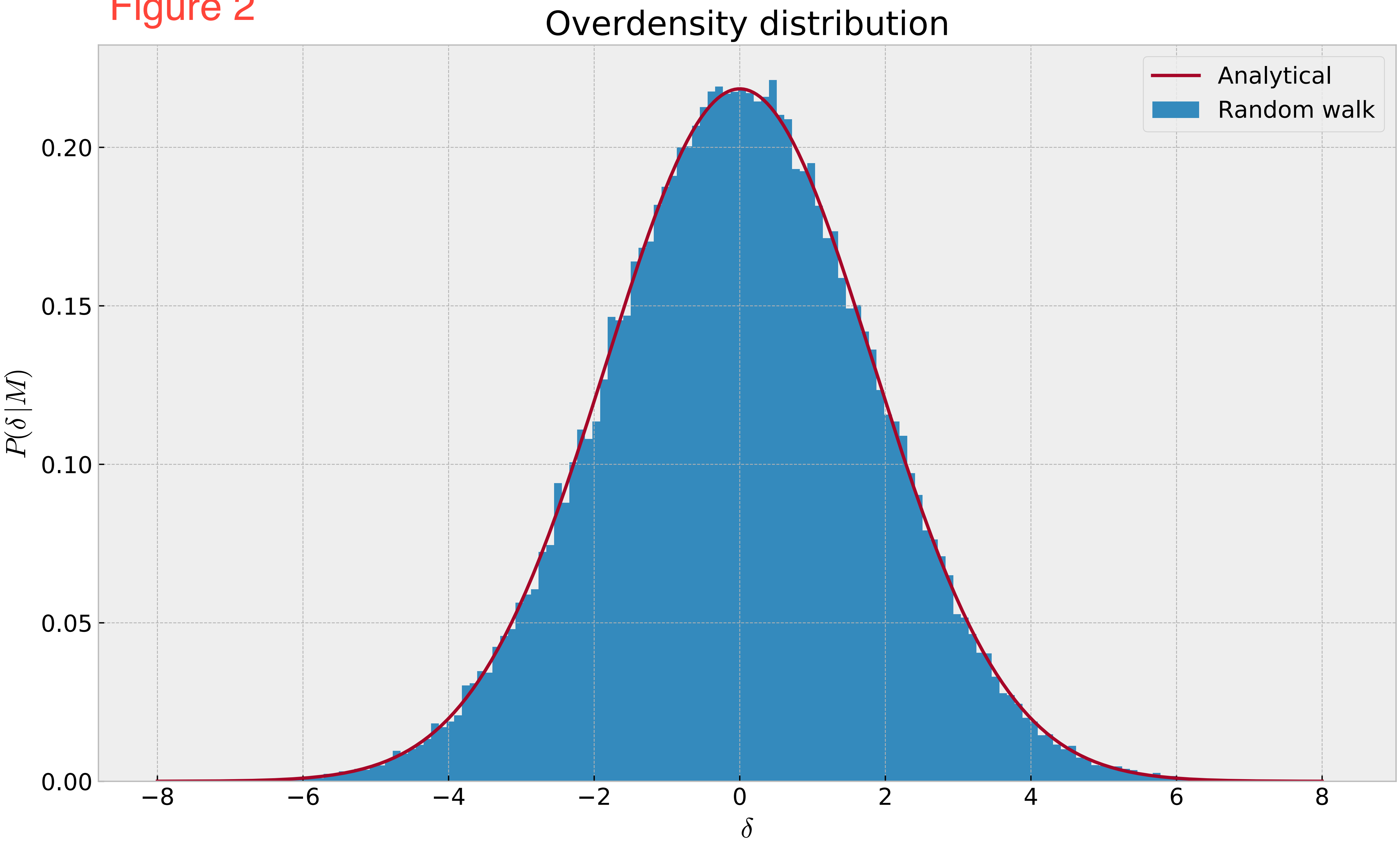
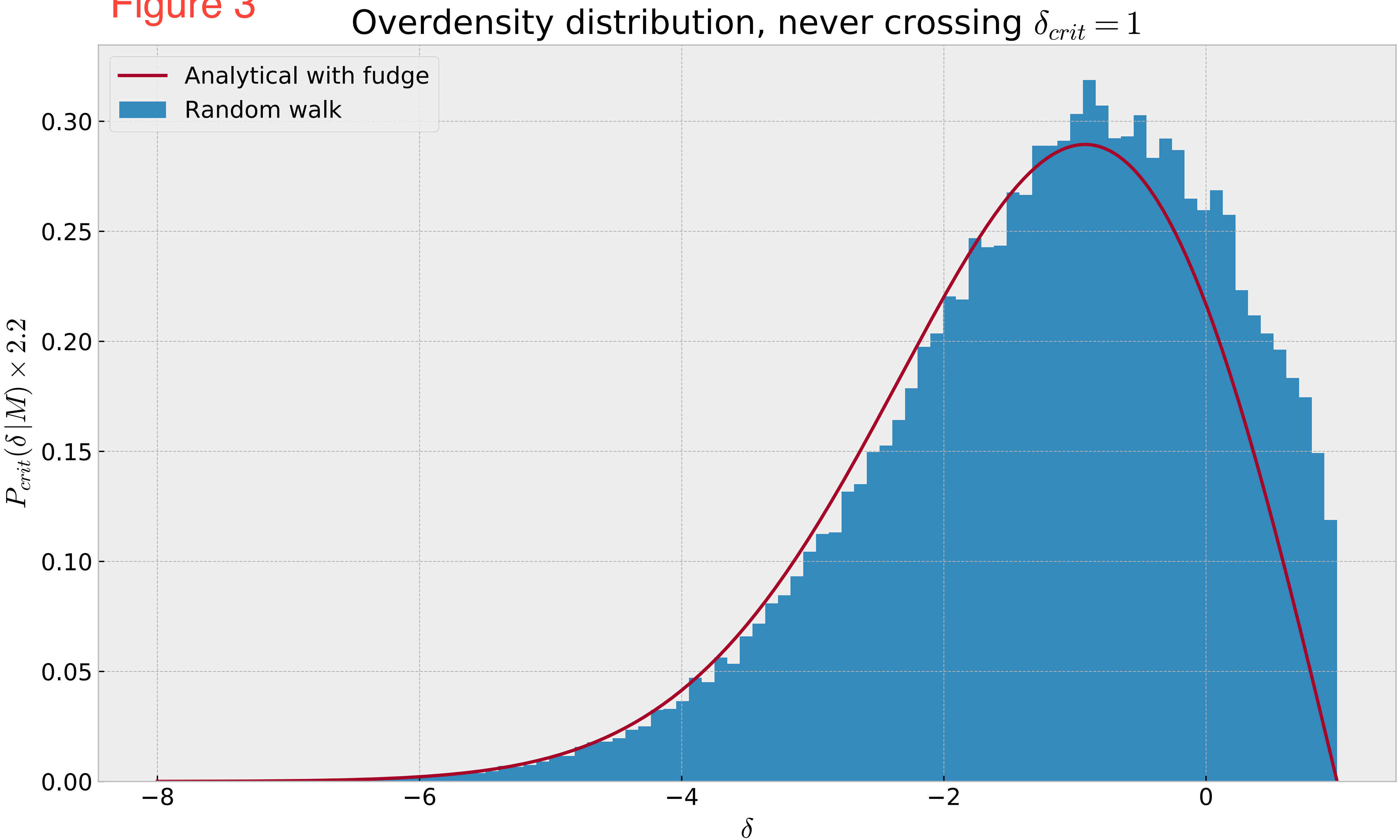

Overdensity distribution
Analytical
Random walk
0.00
0.05
0.10
0.15
0.20
$P(\delta|M)$
−8
−6
−4
−2
0
2
4
6
8
δ

Figure 3



Overdensity distribution, never crossing $\delta_{crit} = 1$
Analytical with fudge
Random walk
0.00
0.05
0.10
0.15
0.20
0.25
0.30
$P_{crit}(\delta|M) \times 2.2$
−8
−6
−4
−2
0
δ

Exe 3

a) From exercise 2 we had distribution $P_{nc}(\delta|M)$, which gives the probability of finding/having $\delta < \delta_{crit}$. These δs are not over the critical density, which means the mass at position $\vec{x}$ with $\delta < \delta_{crit}$ should probably not collapse, with probability

$$\int_{-\infty}^{\delta_{crit}} P_{nc}(\delta|M)\, d\delta$$

The mass at $\vec{x}$ does either collapse, or doesn't collapse, so the probability for collapse must be 1 minus the probability of not collapsing

This gives us the probability

$$P(>M) = 1 - \int_{-\infty}^{\delta_{crit}} P_{nc}(\delta|M)\, d\delta$$