FYS3120 Classical Mechanics and Electrodynamics

Problem set 4

February 10, 2019

Problem 1 The figure shows a rod of length b and mass m, with the mass evenly distributed along the rod. One endpoint of the rod is constrained to move along a horisontal line and the other endpoint along a vertical line. The two lines are in the same plane. There is no friction and the acceleration due to gravity is g. The set-up is illustrated in Fig. 1.

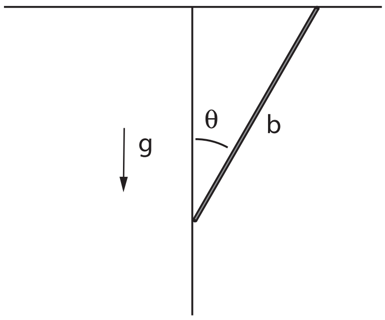

Figure 1: Constrained rod.

a) Find the Lagrangian L with the angle θ as coordinate, and show that Lagrange's equation gives

$$\ddot{\theta} + \frac{3g}{2b}\sin\theta = 0. \tag{1}$$

Hint: For the moment of inertia of the rod, see Problem 2 in Set 2.

b) What is the stable equilibrium position of the rod? Find the period T_0 for small oscillations about equilibrium.

c) Since L has no explicit time dependence, there is a corresponding constant of motion. What is the expression for this constant and what is the physical interpretation? Comment on how the expression is related to the equation of motion.

d) Assume the rod oscillates about the equilibrium position with a maximum angle θ_0, with $0 < \theta_0 \leq \pi/2$. Show that the period T of the oscillations is generally expressed by the integral

$$T = T_0\frac{\sqrt{2}}{\pi}\int_0^{\theta_0}\frac{d\theta}{\sqrt{\cos\theta - \cos\theta_0}}. \tag{2}$$

Determine the ratio T/T_0 for the maximum amplitude $\theta_0 = \pi/2$. *Hint:* In Rottman you will find a general formula, which can be used to express the integral in terms of the Euler gamma-functions. Give the numerical value of the ratio.

Problem 2 Reduced mass

Let us look at the generic two-body problem with two objects of mass m_1 and m_2. Assume that the potential is only dependent on the distance between the two objects, as would be the case for both gravitational and electrostatic potentials.

a) Write down the Lagrangian L in terms if the coordinates $\vec{r}_1$ and $\vec{r}_2$ of the two objects.

b) Make the change of variables to the coordinates $(\vec{r}, \vec{R})$, where

$$\begin{aligned} \vec{r} &= \vec{r}_1 - \vec{r}_2, && (3) \\ \vec{R} &= \frac{m_1}{m_1+m_2}\vec{r}_1 + \frac{m_2}{m_1+m_2}\vec{r}_2, && (4) \end{aligned}$$

and show that the resulting Lagrangian is

$$L = \frac{1}{2}(m_1+m_2)\dot{\vec{R}}^2 + \frac{1}{2}\mu\dot{\vec{r}}^2 - V(r), \qquad (5)$$

where

$$\mu = \frac{m_1 m_2}{m_1+m_2}, \qquad (6)$$

is the **reduced mass**.

c) Find Lagrange's equations in the new coordinates and solve for the motion of $\vec{R}$.

d) What is the physical interpretation of these new coordinates and the change of variables in the Lagrangian?