Project 5A Equipo La 4T: Simpson's rule

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1 Introduction

1.1 Objectives

With the realization of this project, we mainly seek to understand the application of the method known as Simpson's rule both in a mathematical way by performing a practice of its use as well as in a technical way by coding the method.

1.2 Problem Statement

In this project we are going to calculate the area under the curve of a given segment using mathematical functions and applying the previously mentioned rule.

2 Application of the method

2.1 Enforcement activity

Using Matlab, write a program to numerically integrate a function using Simpson's rule. Use the t distribution as the function.

Thoroughly test the program. At a minimum, calculate the values for the t distribution integral for the values in Table 1. Expected values are also included in Table 1.

Test		Expected Value	Actual Value
x	*dof*	*p*	
0 to x= 1.1	9	0.35006	
0 to x= 1.1812	10	0.36757	
0 to x= 2.750	30	0.49500	

Table 1

This is the general statement and proposal of the problem we are going to solve using programming and to apply Simpson's rule.

3 Background and general information

3.1 Definition

In numerical integration, Simpson's rules are several approximations for definite integrals, named after Thomas Simpson (1710–1761).

The most basic of these rules, called Simpson's 1/3 rule, or just Simpson's rule, reads:

$$\int_a^b f(x)\,dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

3.2 Previous history

In German and some other languages, it is named after Johannes Kepler, who derived it in 1615 after seeing it used for wine barrels (barrel rule, Keplersche Fassregel). The approximate equality in the rule becomes exact if f is a polynomial up to and including 3rd degree.

If the 1/3 rule is applied to n equal subdivisions of the integration range [a, b], one obtains the composite Simpson's rule. Points inside the integration range are given alternating weights 4/3 and 2/3.

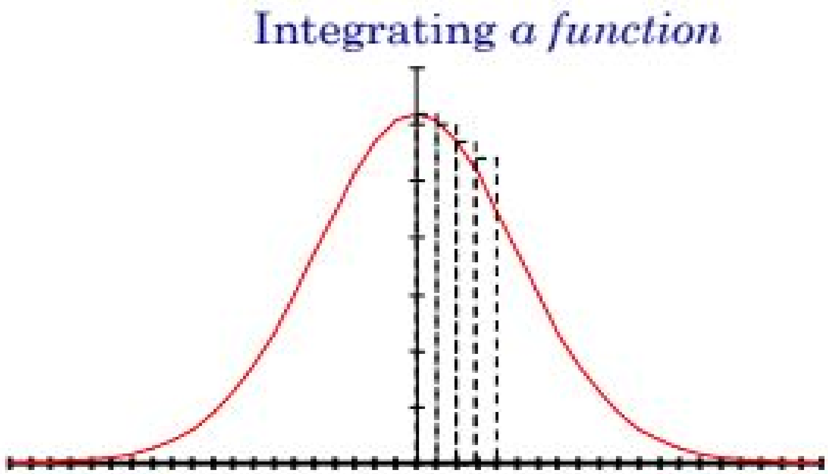
Simpson's 3/8 rule, also called Simpson's second rule, requires one more function evaluation inside the integration range and gives lower error bounds, but does not improve on order of the error.

Simpson's 1/3 and 3/8 rules are two special cases of closed Newton–Cotes formulas.

In naval architecture and ship stability estimation, there also exists Simpson's third rule, which has no special importance in general numerical analysis, see Simpson's rules (ship stability).

3.3 Overview

Numerical integration is the process of determining the area "under" some function. Numerical integration calculates this area by dividing it into vertical "strips" and summing their individual areas. The key is to minimize the error in this approximation.



Here comes the Simpson's Rule, can be used to integrate a symmetrical statistical distribution function over a specified range (example: from 0 to some value x).

1. *num_seg* = initial number of segments, an even number
2. W = *x*/*num_seg*, the segment width
3. E = the acceptable error, e.g., 0.00001
4. Compute the integral value with the following equation.

$$p = \frac{W}{3}\left[F(0) + \sum_{i=1,3,5..}^{num\_seg-1} 4F(iW) + \sum_{i=2,4,6..}^{num\_seg-2} 2F(iW) + F(x)\right]$$

5. Compute the integral value again, but this time with *num_seg* = *num_seg**2.
6. If the difference between these two results is greater than E, double *num_seg* and compute the integral value again. Continue doing this until the difference between the last two results is less than E. The latest result is the answer.

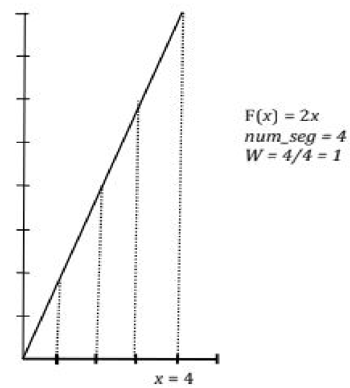
4 Graphic example

Let's look at a simple function, where F(x) = 2x.

Note: This example is a triangle. The area of a triangle is

$$\frac{1}{2}(base)(height)$$

$$\frac{1}{2}(4)(8) = \frac{32}{2} = 16$$



In this example, we can expand Simpson's rule

$$p = \frac{W}{3}\left[F(0) + \sum_{i=1,3,5..}^{num\_seg-1} 4F(iW) + \sum_{i=2,4,6..}^{num\_seg-2} 2F(iW) + F(x)\right]$$

to

$$p = \frac{1}{3}[F(0) + 4F(1) + 2F(2) + 4F(3) + F(4)]$$

and then substitute calculated values for the function F(x)= 2x

$$p = \frac{1}{3}[(0) + 4(2) + 2(4) + 4(6) + (8)] = \frac{1}{3}[0 + 8 + 8 + 24 + 8] = \frac{48}{3} = 16$$

5 Conclusion

With the completion of this report, we documented in general what the simpson rule is, its definition, history, application as well as posed a problem that addresses its usefulness and set out to solve it.

6 References

PDF file "Assignment Kit for Simpson's rule"
https://en.wikipedia.org/wiki/Simpson
https://mathworld.wolfram.com/SimpsonsRule.html