Assignment Kit for Simpson's rule

Equipo La 4T

Proyecto 5A

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Table of contents

Introduction:

1. Objectives: With the realization of this project, we mainly seek to understand the application of the method known as Simpson's rule both in a mathematical way by performing a practice of its use as well as in a technical way by coding the method.
2. Problem Statement: In this project we are going to calculate the area under the curve of a given segment using mathematical functions and applying the previously mentioned rule.

Application of the method:

Using Matlab, write a program to numerically integrate a function using Simpson's rule. Use the t distribution as the function.

Thoroughly test the program. At a minimum, calculate the values for the t distribution integral for the values in Table 1. Expected values are also included in Table 1.

Test		**Expected Value**	**Actual Value**
x	***dof***	***p***	
0 to x= 1.1	9	0.35006	
0 to x= 1.1812	10	0.36757	
0 to x= 2.750	30	0.49500	

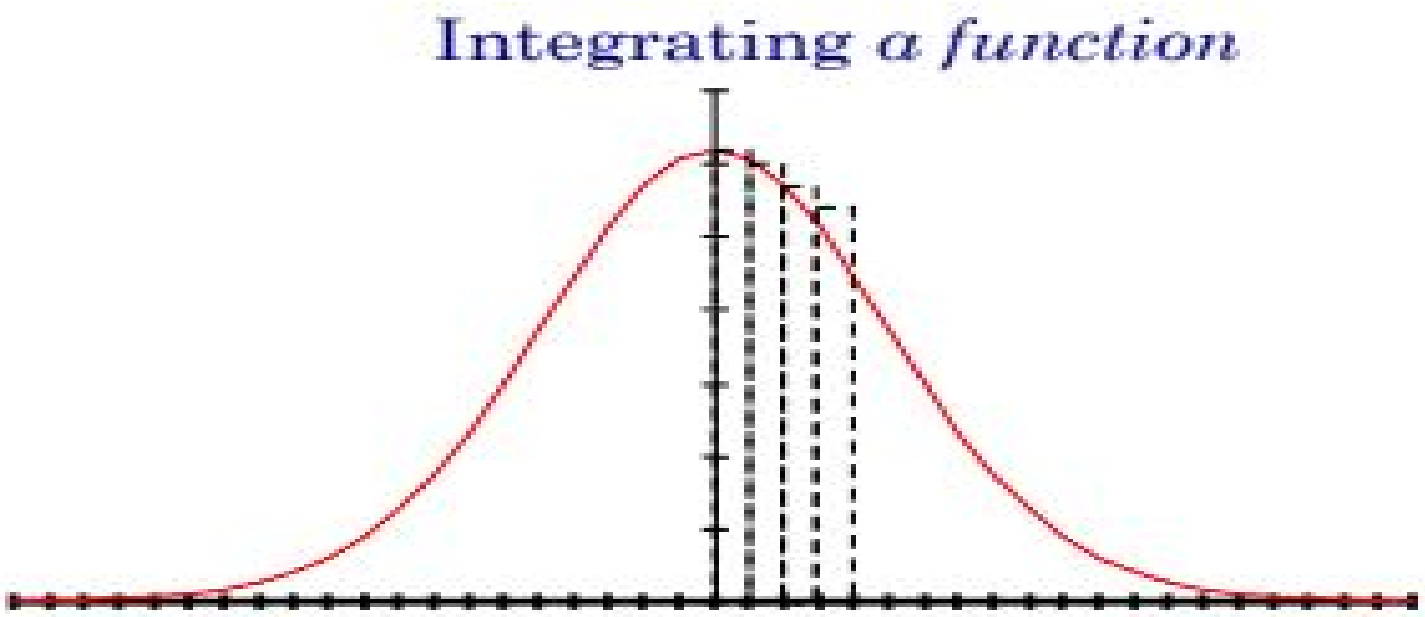
Table 1

Background and general information: Definition in numerical integration, Simpson's rules are several approximations for definite integrals, named after Thomas Simpson (1710–1761). The most basic of these rules, called Simpson's 1/3 rule, or just Simpson's rule, reads:

$$\int_a^b f(x)\,dx \approx \frac{b-a}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]$$

Previous history In German and some other languages, it is named after Johannes Kepler, who derived it in 1615 after seeing it used for wine barrels (barrel rule, Keplersche Fassregel). The approximate equality in the rule becomes exact if f is a polynomial up to and including 3rd degree. If the 1/3 rule is applied to n equal subdivisions of the integration range [a, b], one obtains the composite Simpson's rule. Points inside the integration range are given alternating weights 4/3 and 2/3. Simpson's 3/8 rule, also called Simpson's second rule, requires one more function evaluation inside the integration range and gives lower error bounds, but does not improve on order of the error.

Overview: Numerical integration is the process of determining the area "under" some function. Numerical integration calculates this area by dividing it into vertical "strips" and summing their individual areas. The key is to minimize the error in this approximation.



Here comes the Simpson's Rule, can be used to integrate a symmetrical statistical distribution function over a specified range (example: from 0 to some value x).

1. *num_seg* = initial number of segments, an even number
2. W = *x*/*num_seg*, the segment width
3. E = the acceptable error, e.g., 0.00001
4. Compute the integral value with the following equation.

$$p = \frac{W}{3}\left[F(0) + \sum_{i=1,3,5..}^{num\_seg-1} 4F(iW) + \sum_{i=2,4,6..}^{num\_seg-2} 2F(iW) + F(x)\right]$$

5. Compute the integral value again, but this time with *num_seg* = *num_seg**2.
6. If the difference between these two results is greater than E, double *num_seg* and compute the integral value again. Continue doing this until the difference between the last two results is less than E. The latest result is the answer.

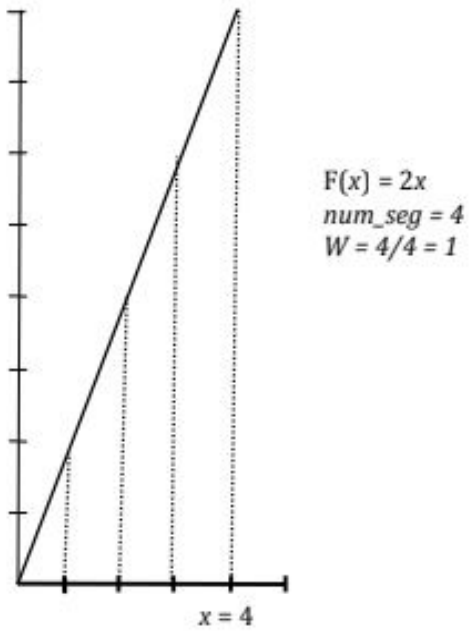
Graphic example:

Let's look at a simple function, where $F(x) = 2x$.

Note: This example is a triangle. The area of a triangle is

$$\frac{1}{2}(base)(height)$$

$$\frac{1}{2}(4)(8) = \frac{32}{2} = 16$$



In this example, we can expand Simpson's rule

$$p = \frac{W}{3}\left[F(0) + \sum_{i=1,3,5..}^{num\_seg-1} 4F(iW) + \sum_{i=2,4,6..}^{num\_seg-2} 2F(iW) + F(x)\right]$$

to

$$p = \frac{1}{3}\left[F(0) + 4F(1) + 2F(2) + 4F(3) + F(4)\right]$$

Conclusions: With the completion of this report, we documented in general what the Simpson rule is, its definition, history, application as well as posed a problem that addresses its usefulness and set out to solve it.

References: PDF file "Assignment Kit for Simpson's rule"
https://en.wikipedia.org/wiki/Simpson
https://mathworld.wolfram.com/SimpsonsRule.html