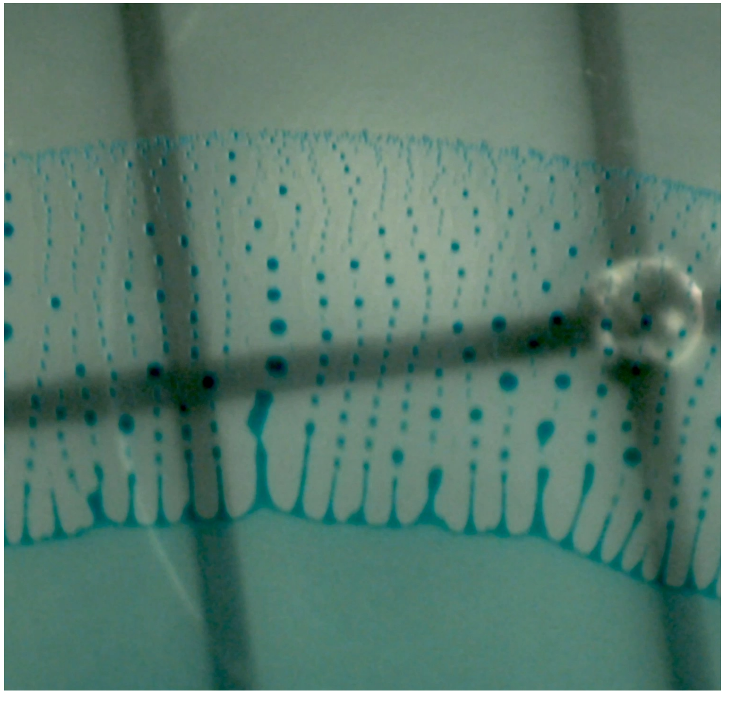
10. Droplet Explosion

Yuezhen (Lily) Dong

*"When a drop of a water mixture **(e.g. water-alcohol)** is deposited on the surface of a **hydrophobic liquid** (e.g. vegetable oil), the resulting drop may sometimes **fragment** into smaller droplets. Investigate the parameters that affect the **fragmentation** and the size of the final droplets."*

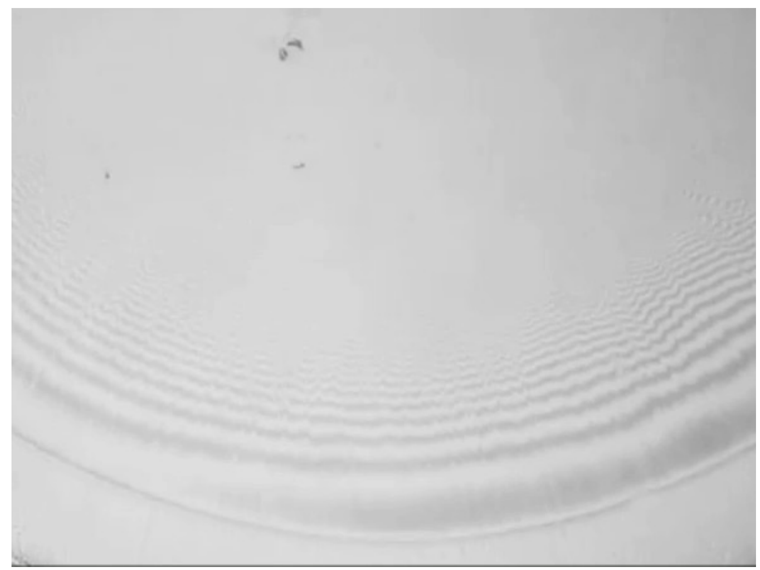


Problem Statement

"When a drop of a water mixture ***(e.g. water-alcohol)*** *is deposited on the surface of a* ***hydrophobic liquid*** *(e.g. vegetable oil), the resulting drop may sometimes* ***fragment*** *into smaller droplets. Investigate the parameters that affect the* ***fragmentation*** *and the size of the final droplets."*

Parameters:

1. *Initial concentration of alcohol*
2. *Initial volume of droplets*
3. *Depth of the hydrophobic liquid*
4. *Types of solvent*

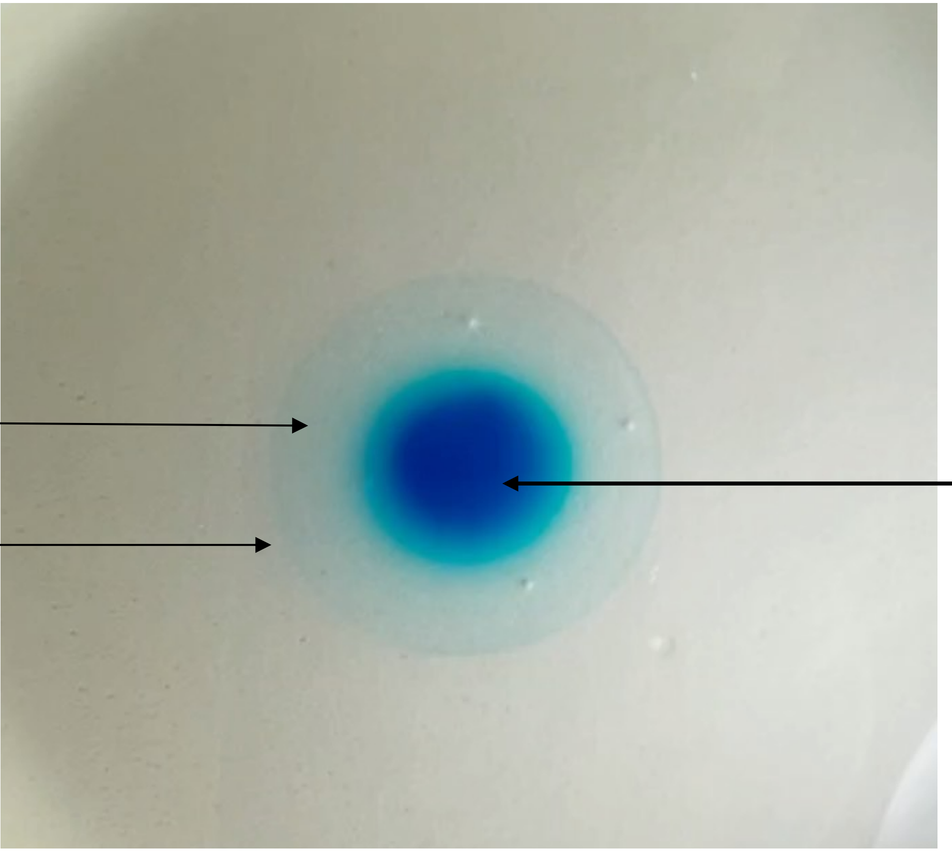


Overview

Phenomenon

Outermost rim

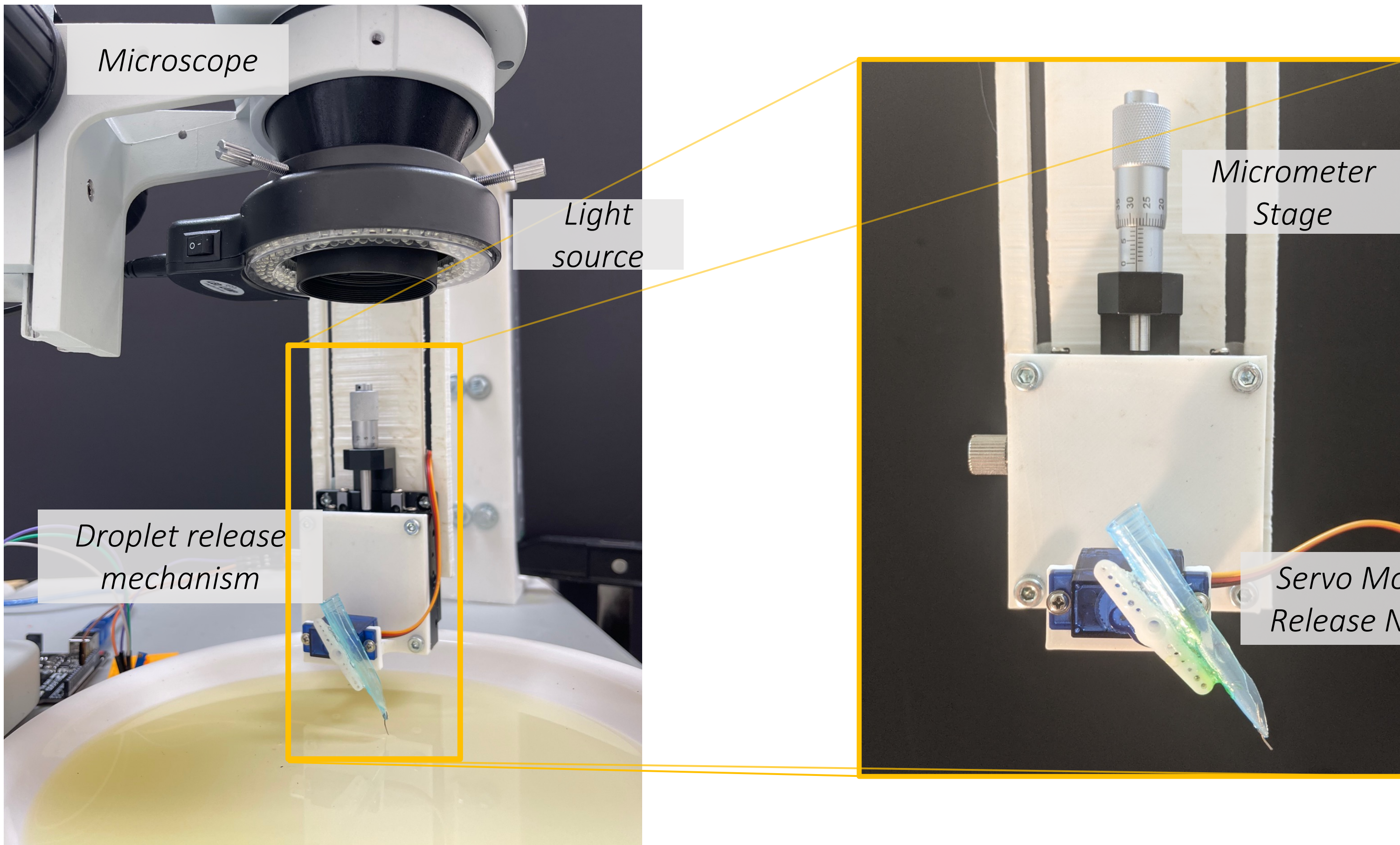
Droplets emitted



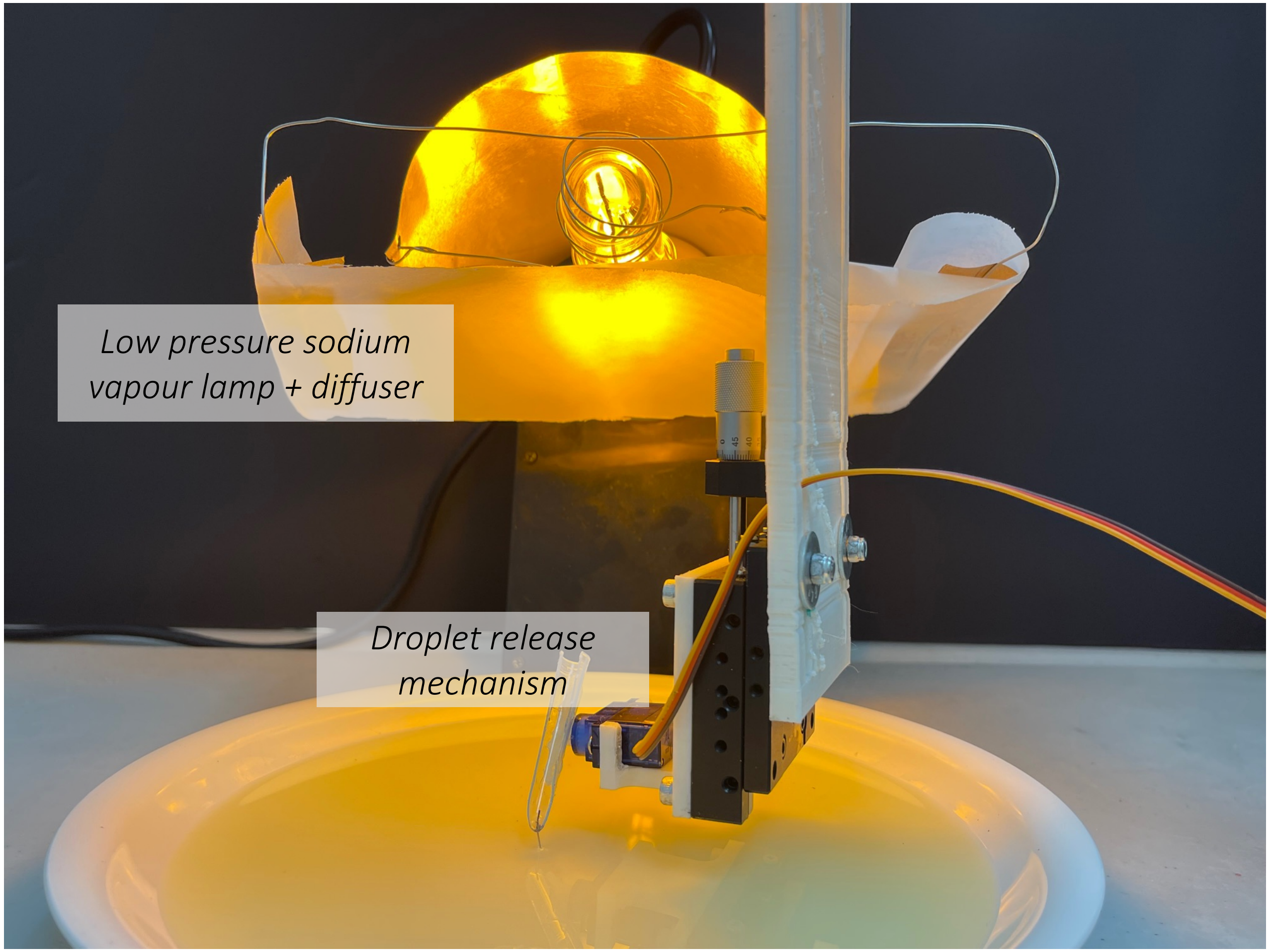
Alcohol-water mixture

Experimental Setup

Microscopy Setup



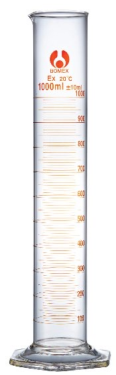
Interferometry Setup



Experimental Measurement

Syringes ($\pm 0.5mL$)

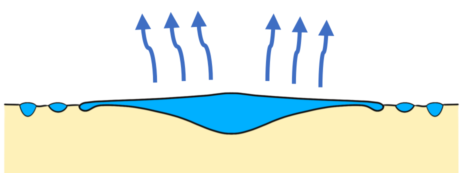
Micropipette
($\pm 0.6\%$)



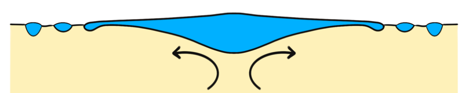
200 mL graduated cylinder
($\pm 2mL$)

Theoretical Model

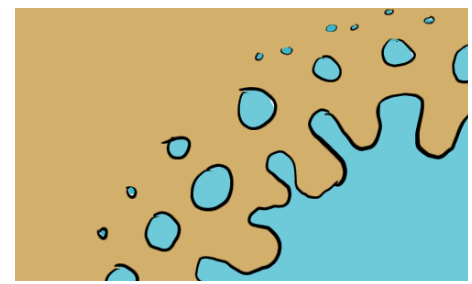
Theoretical Model



Alcohol Evaporation



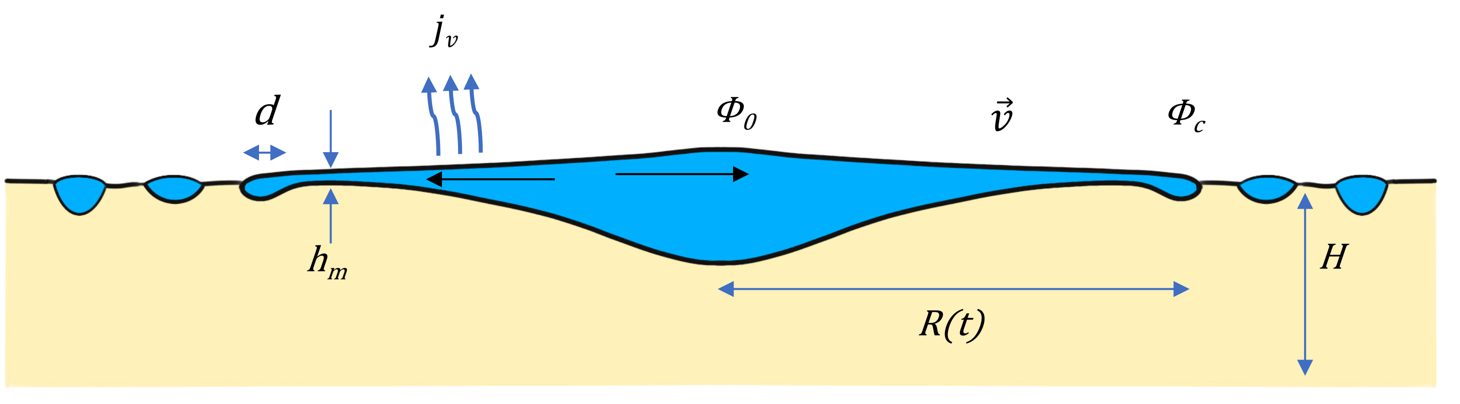
Interfacial Tension & Shear Stress



Droplet Fragmentation

Geometry

j_v = *evaporation rate of alcohol*
d = *width of the rim*
h_m = *minimum thickness of the droplet expansion*
Φ_0 = *initial mass fraction of alcohol in the center of the drop*
Φ_c = *mass fraction of alcohol at the rim of the expansion*
$\vec{v}$ = *velocity of the Marangoni flow*
$R(t)$ = *radius of droplet*
H = *depth of the oil layer*

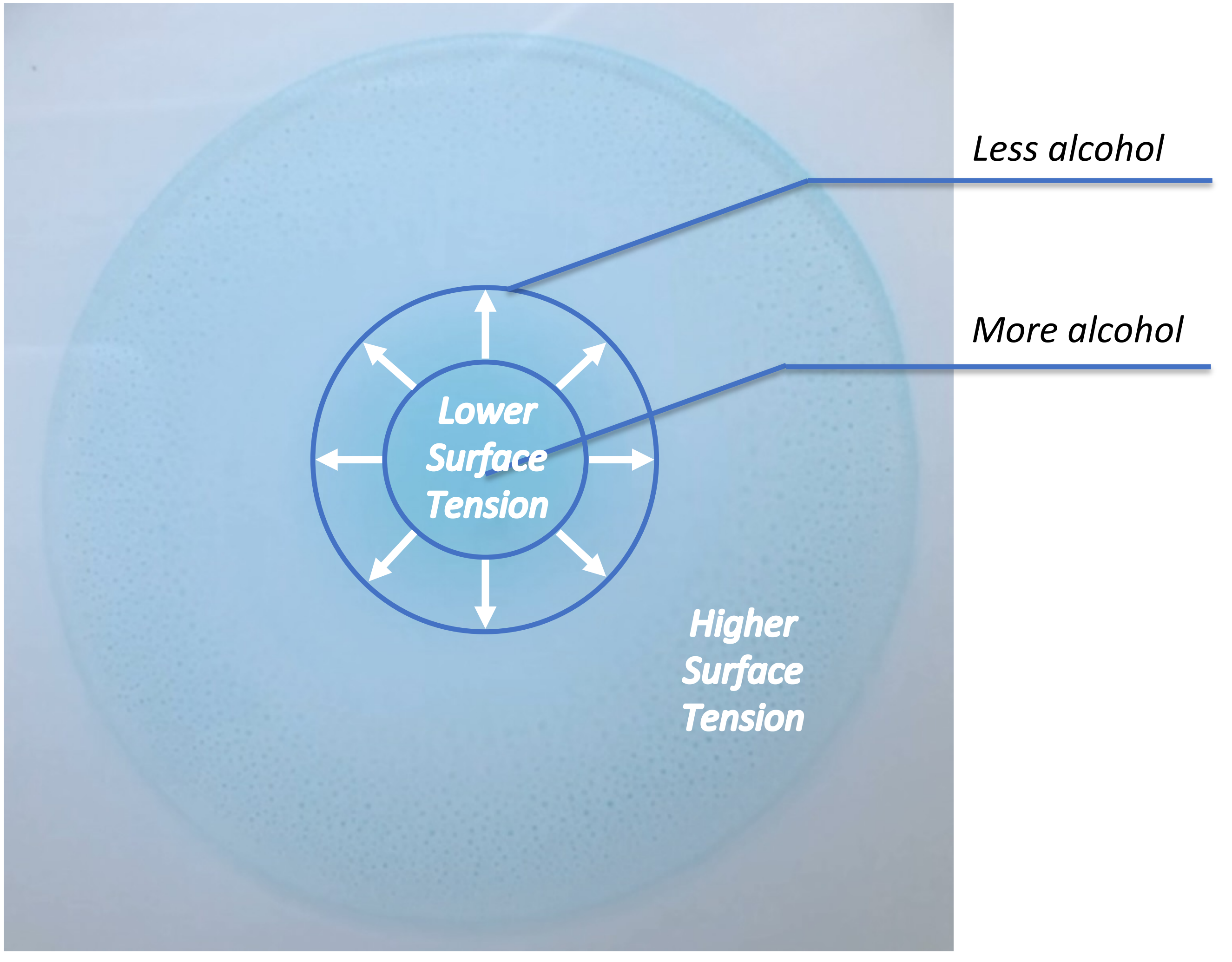


Alcohol Evaporation
Interfacial Tension
Droplet Fragmentation

Assumptions

- *The thinner film of the droplet is caused by alcohol evaporation*
- *The evaporation of water is negligible*
- *Ambient room temperature remains constant*
- *The impact of water based dye is negligible*

Marangoni Flow



Alcohol Evaporation

Interfacial Tension

Droplet Fragmentation

Evaporation Flux [8]

To calculate the evaporation flux, use Fick's law,

$$j_m = -D\nabla c \cdot \boldsymbol{n}$$

j_m is the evaporation flux
$\boldsymbol{n}$ is the unit normal vector
c is the molar concentration of IPA

Considering the mass conservation across the interface, this leads to

$$\rho_l(\boldsymbol{u}_l \cdot \boldsymbol{n} - u_\Gamma) = j_m$$

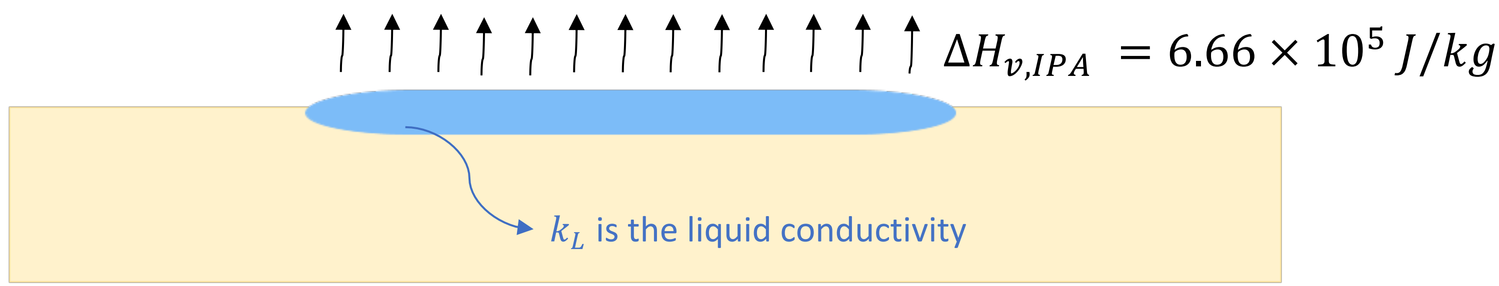
u_Γ is the normal velocity of the interface
$\boldsymbol{n}$ is the unit normal vector
c is the molar concentration of IPA

Interfacial Energy Balance

For negligible heat conduction and convection in the air, the interfacial energy balance is,

$$-k_L \boldsymbol{n} \cdot \nabla T = \Delta H_v j(x)$$

ΔH_v *is the specific latent heat of evaporation, IPA* $= 6.66 \times 10^5\ J/kg$

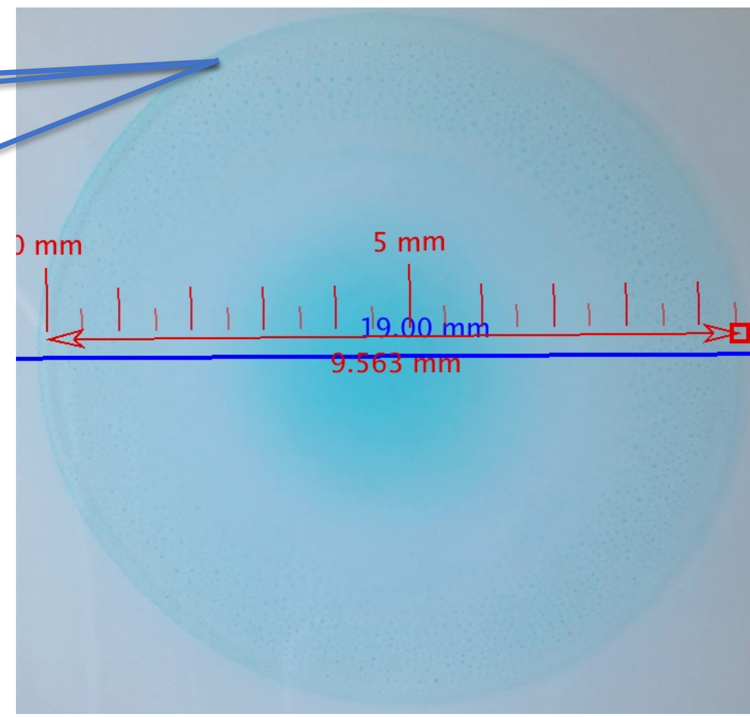


Alcohol Evaporation

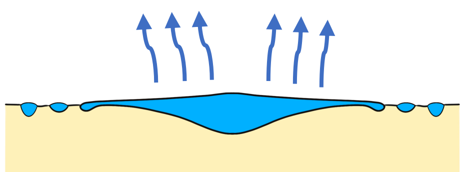
The spreading parameter, $S = \gamma_{oa} - \gamma_{ma} - \gamma_{mo}$

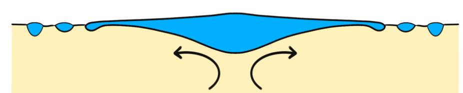
Evaporation and total Mass flux, $J_m = \int j_m d\Gamma$

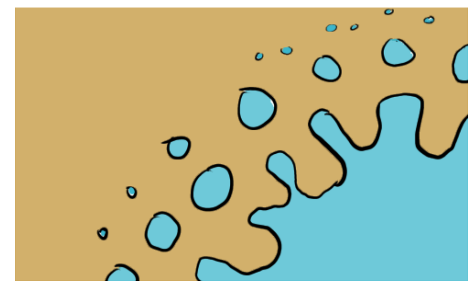
J_m *at the rim* $< J_{m,initial}$



Theoretical Model





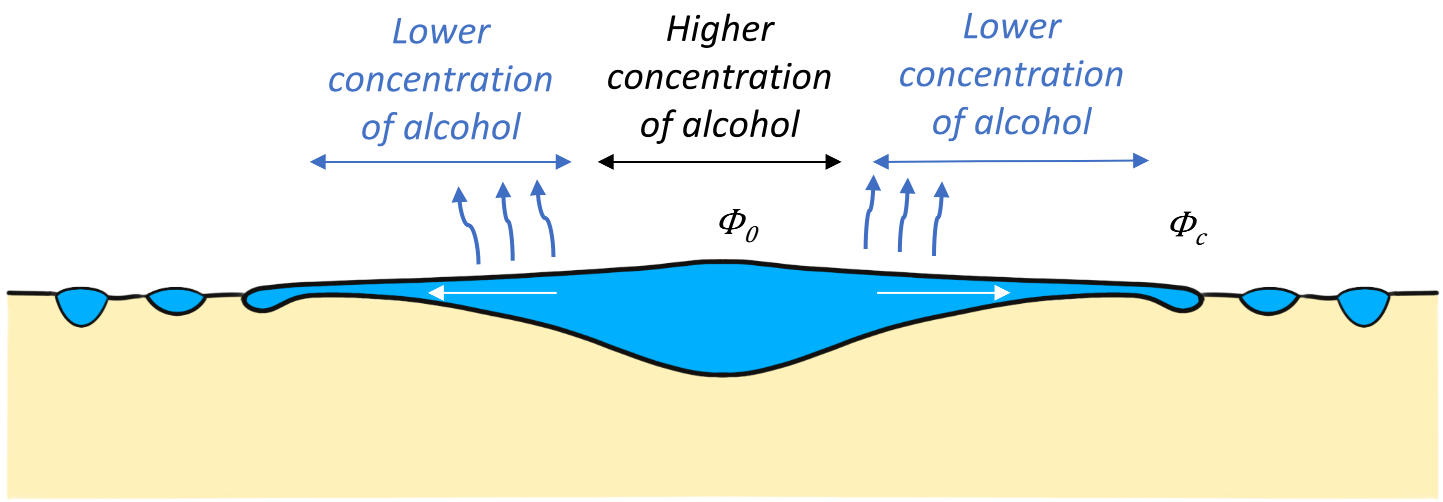


Alcohol Evaporation

Interfacial Tension & Shear Stress

Droplet Fragmentation

Geometry


Lower concentration of alcohol
Higher concentration of alcohol
Lower concentration of alcohol
Φ_0
Φ_c

Assumptions

1. *Ambient room temperature is constant.*
2. *The dropping height remains constant throughout the experiment.*
3. *The dimensions of the oil plate remains constant.*
4. *The effects caused by the ingredients of water-based food dye are negligible.*
5. *The alcohol evaporation during the dropping process is negligible.*
6. *The evaporation of water in this experiment is negligible.*

Marangoni Flow

Surface tension gradient is the same magnitude of $\frac{\Delta\gamma}{R^*}$

$$\boxed{\tau \sim \frac{R^*}{V}}$$

Time scale of the experiment

$\Delta\gamma = \gamma_c - \gamma_0$

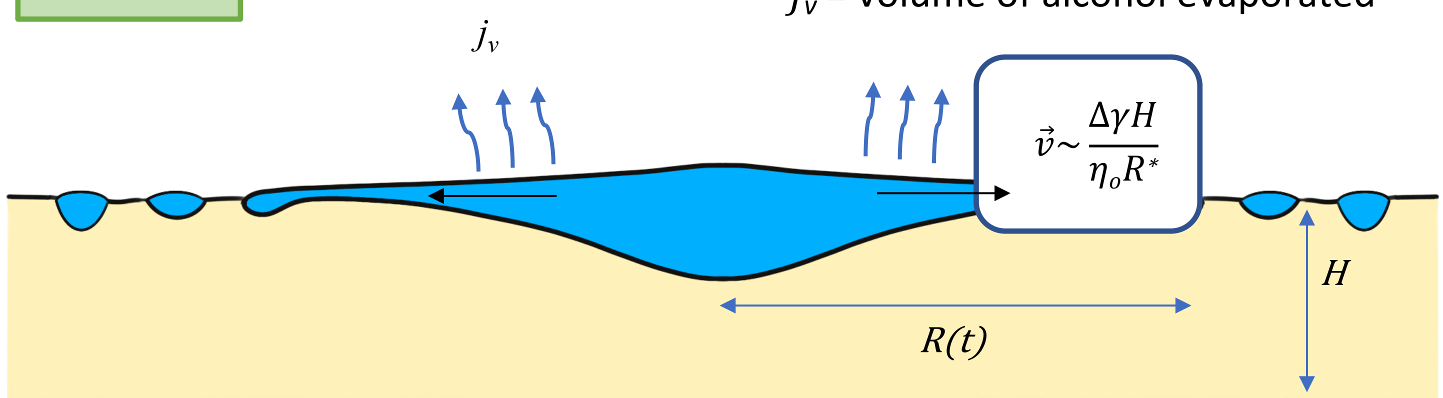
γ_c = *interfacial tension corresponding to* Φ_c

γ_0 = *interfacial tension corresponding to* Φ_0

$R*$ = characteristic radius of the drop

η_o = viscosity of oil

j_v = volume of alcohol evaporated



Marangoni Flow [7]

The evolution equation for the height, h, as a function of radial coordinate r and time t,

Pressure Gradient

$$\frac{\partial h}{\partial t} = \frac{1}{r\mu}\frac{\partial}{\partial r}\left(\boxed{\frac{rh^3}{3}\frac{\partial p}{\partial r}} - \boxed{\frac{rh^2}{2}\frac{\partial \sigma}{\partial r}}\right) + \omega_e$$

Surface Tension Gradient

p *is the pressure in the droplet*

ω_e *is the evaporative volume flux*

μ *is viscosity of droplet*

σ *is surface tension*

Marangoni Flow

The pressure in the droplet is dominated by surface tension. Therefore, the pressure p can be given by the Laplace pressure:

$$p = -\frac{1}{r}\frac{\partial}{\partial r}(\sigma \frac{r\partial_r h}{\sqrt{1+(\partial_r h)^2}})$$

The boundary and initial conditions of h(r,t) are,

$$(\frac{\partial h}{\partial r})_{r=0} = 0$$

$$h(r,t) = 0$$

$$h(r,0) = h_0 r$$

Marangoni Flow [1]

Solved for characteristic radius of the droplet expansion

$$R^* \sim \left(\frac{(\Phi_0 - \Phi_c)\Delta\gamma H \Omega_0}{(1 - \Phi_c)\eta_o j_v} \right)^{\frac{1}{4}}$$

$$\tau \sim \left(\frac{(\Phi_0 - \Phi_c)\eta_o \Omega_0}{(1 - \Phi_0)\Delta r H j_v} \right)^{\frac{1}{2}}$$

Solved time scale based on the convection of the liquid from the center to the edge

$$(0.4 < \Phi_0 < 0.8$$

$$1mm < H < 20\ mm$$

$$2mm^3 < \Omega_0 < 500\ mm^3)$$

Restrictions applied

$$R_{max} \sim 0.28(\pm 0.02)R^*$$

$$t_{exp} \sim 1.5(\pm 0.15)\tau$$

Estimated maximum radius and correlated time

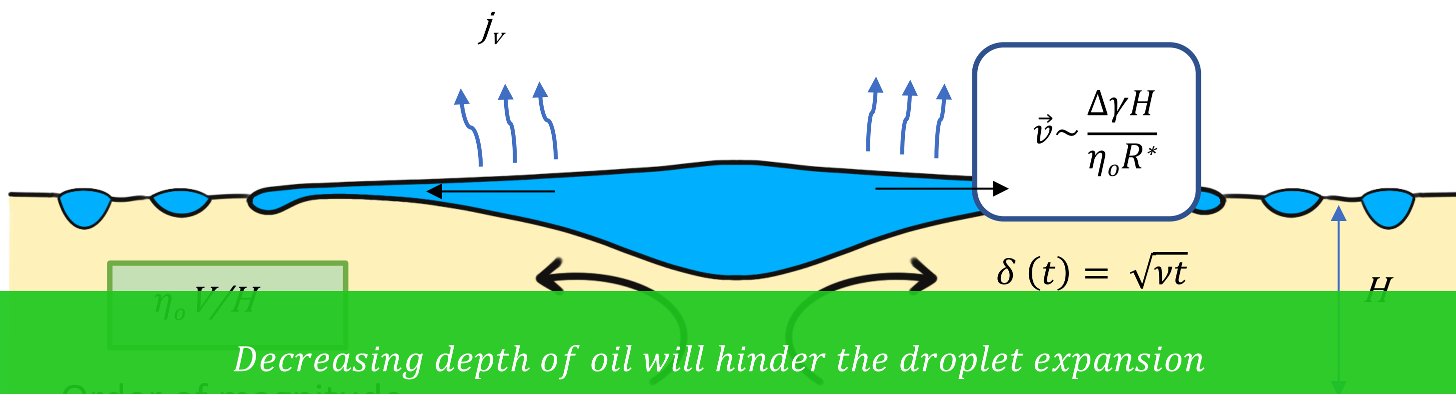
[1] Keiser, L., Bense, H., Colinet, P., Bico, J., Reyssat, E., 2017.

Viscous stress

Kinematic viscosity of oil

$$\nu = 5 \times 10^{-5}\ \mathrm{m^2/s}$$

Velocity of Marangoni flow



$$\vec{v} \sim \frac{\Delta\gamma H}{\eta_o R^*}$$

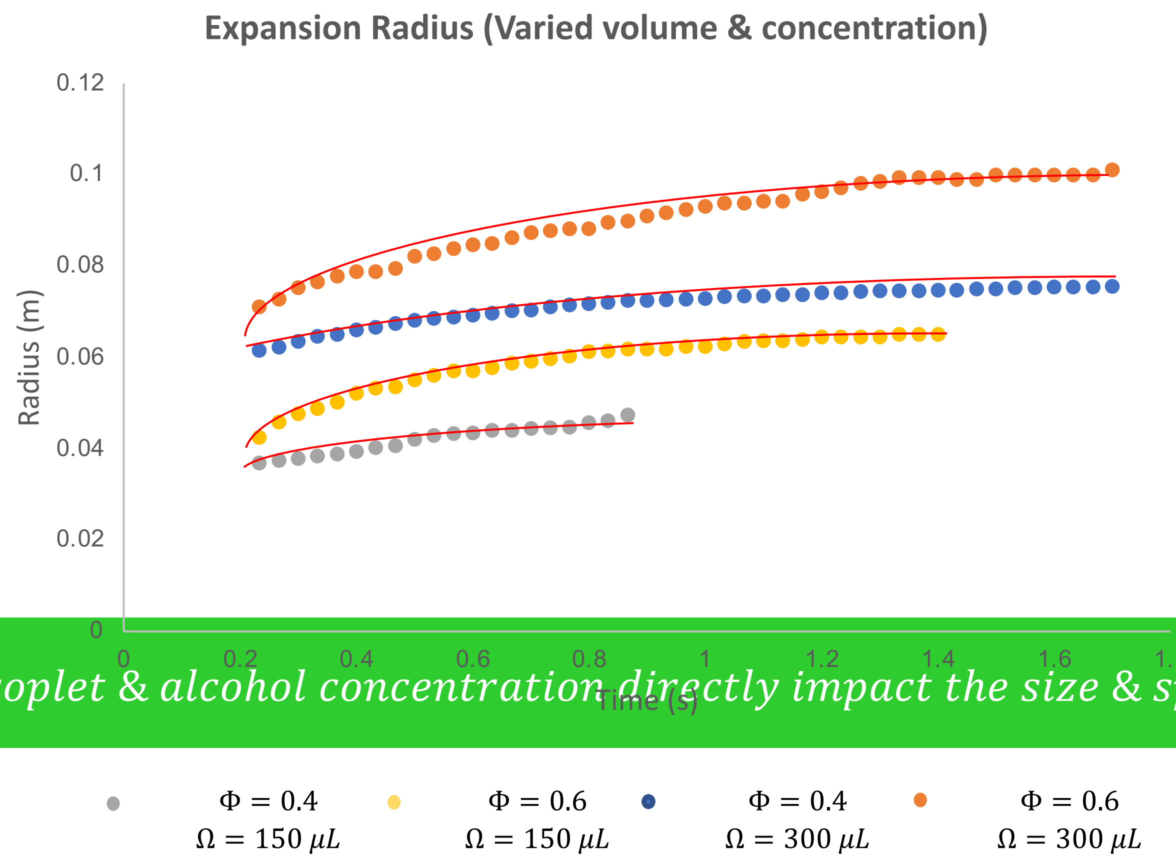
$$\delta\,(t) = \sqrt{\nu t}$$

$$\eta_o V/H$$

Decreasing depth of oil will hinder the droplet expansion

Order of magnitude of the viscous stress

Preliminary Experiment Results



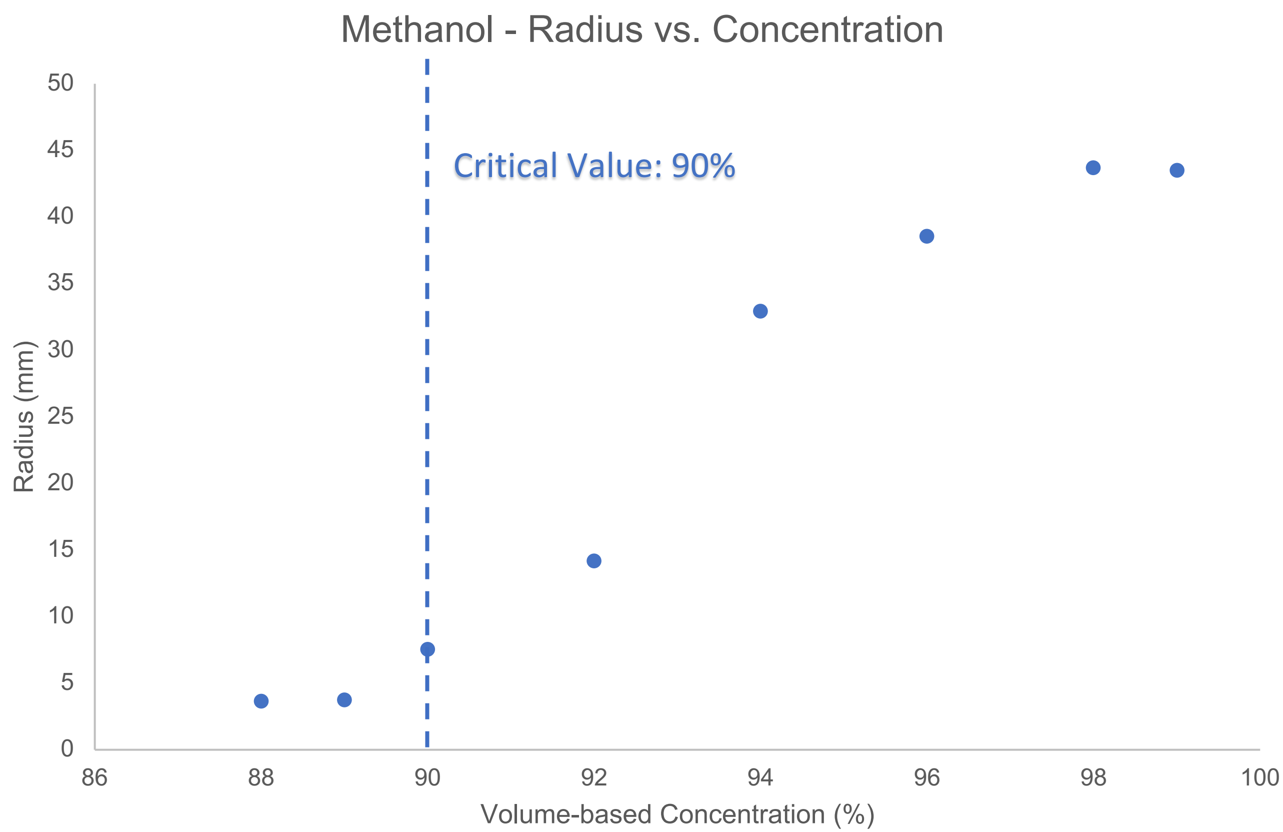
Volume of droplet & alcohol concentration directly impact the size & speed of expansion

Critical Point of the Spreading Parameter - Ethanol

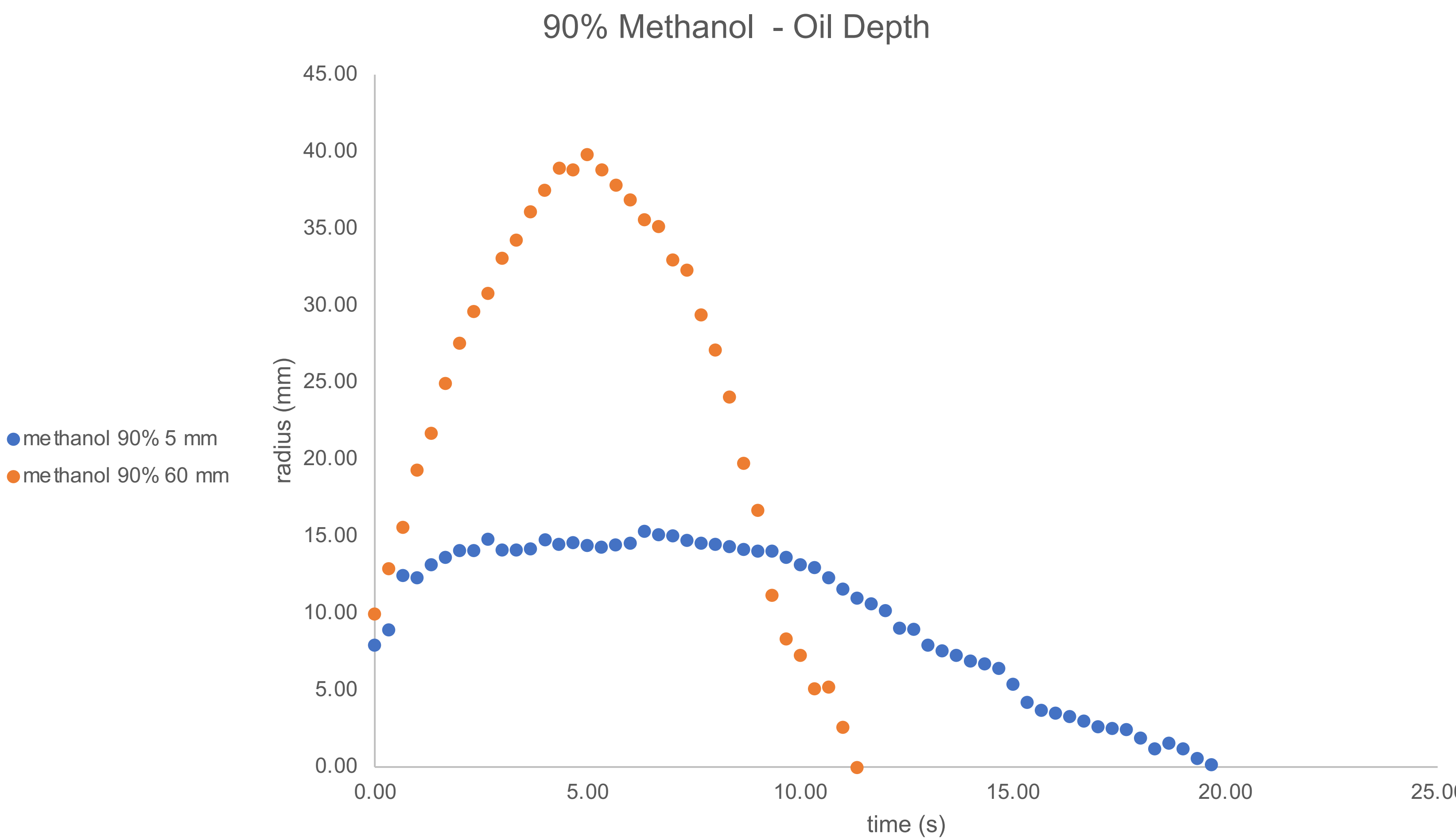
$\Phi = 0.5$

$\Phi = 0.9$

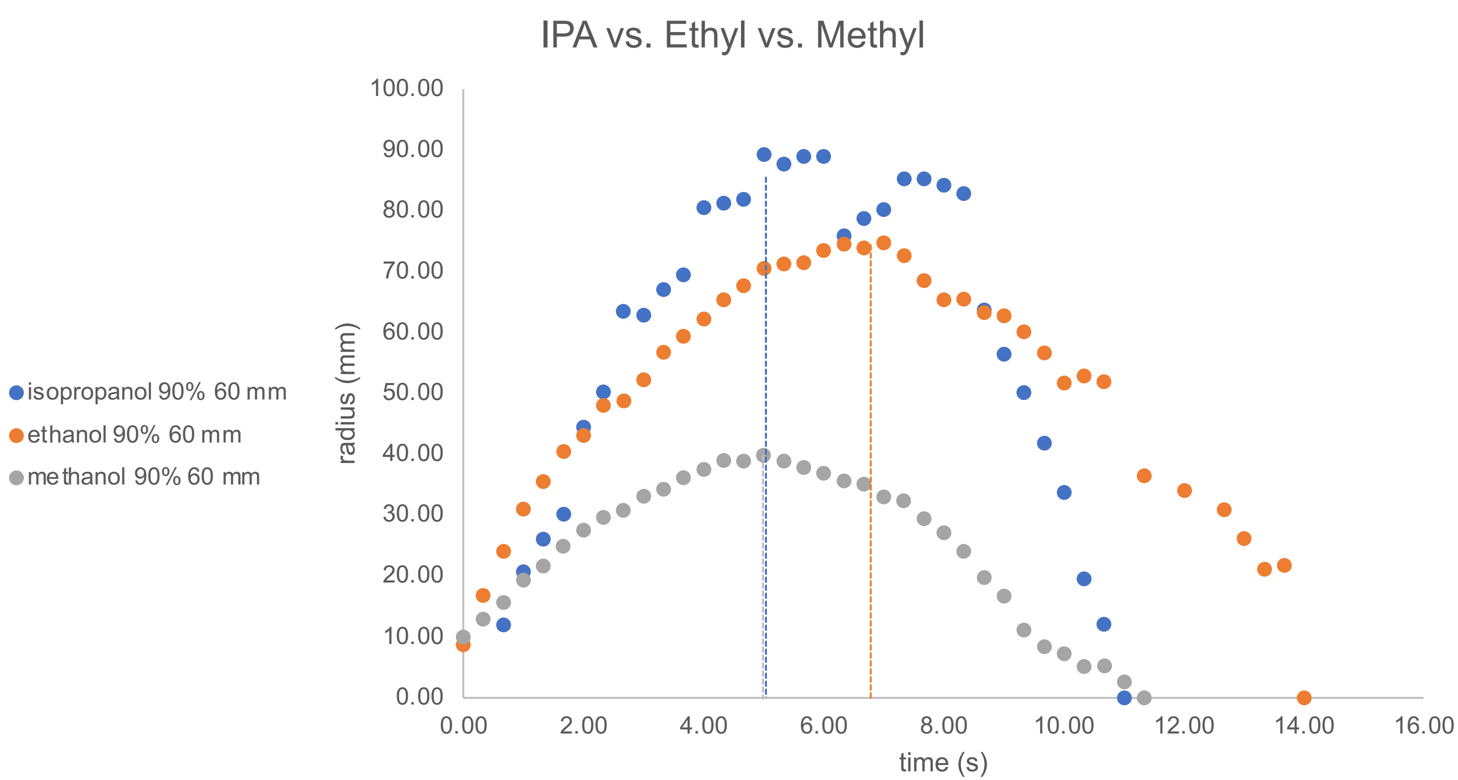
Critical Value - Vary Concentration



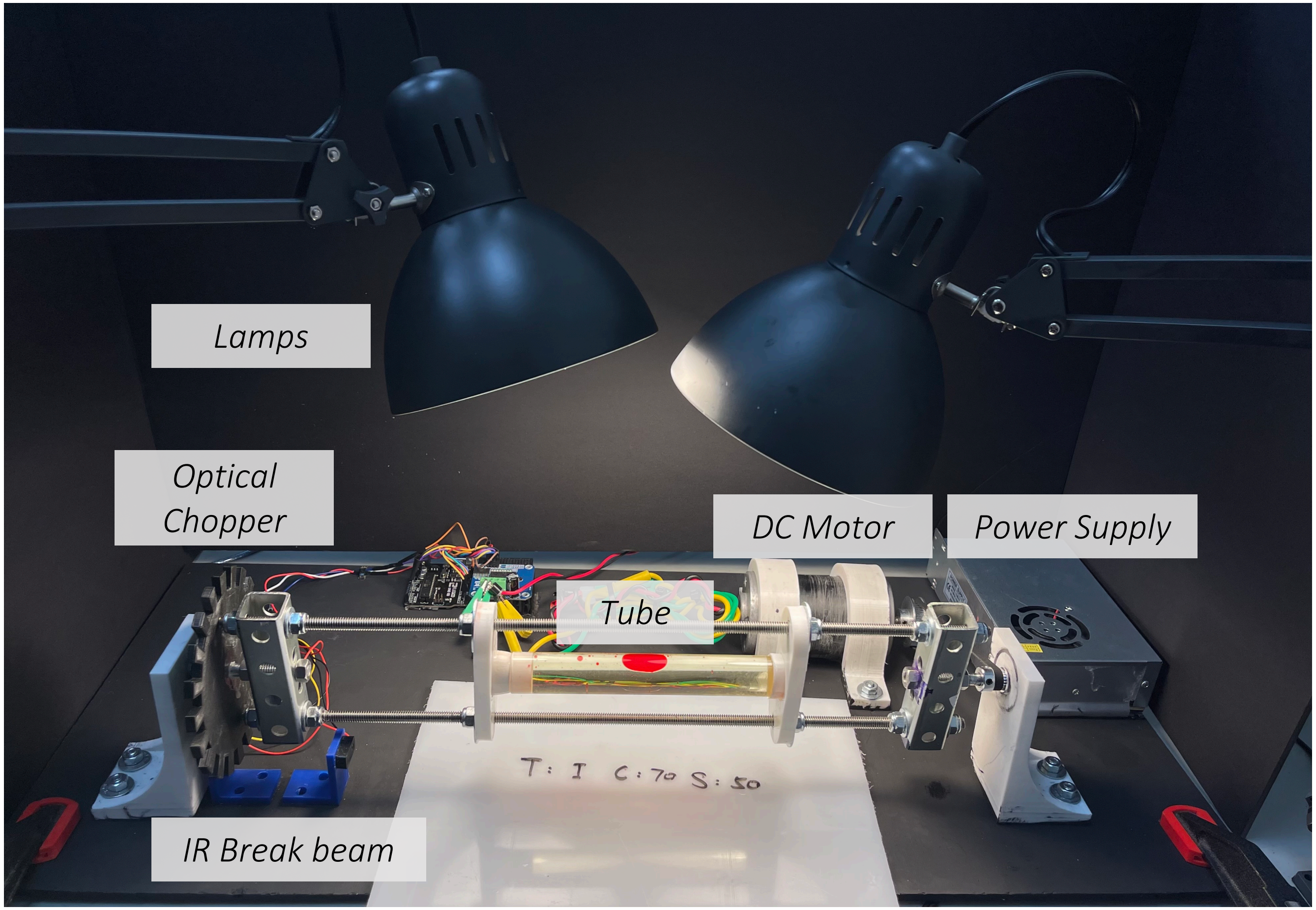
Vary Oil Depth



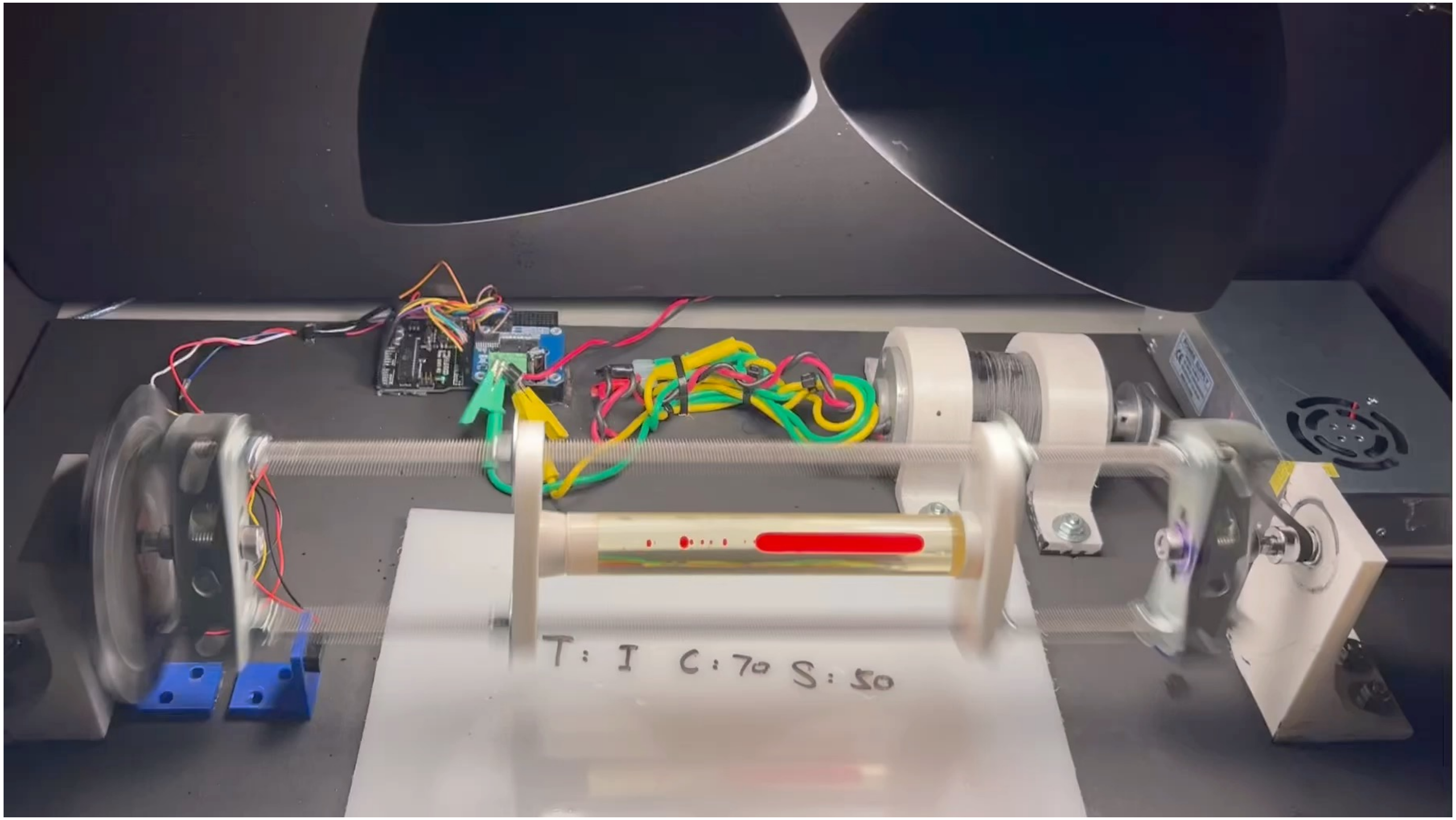
Vary Solvent



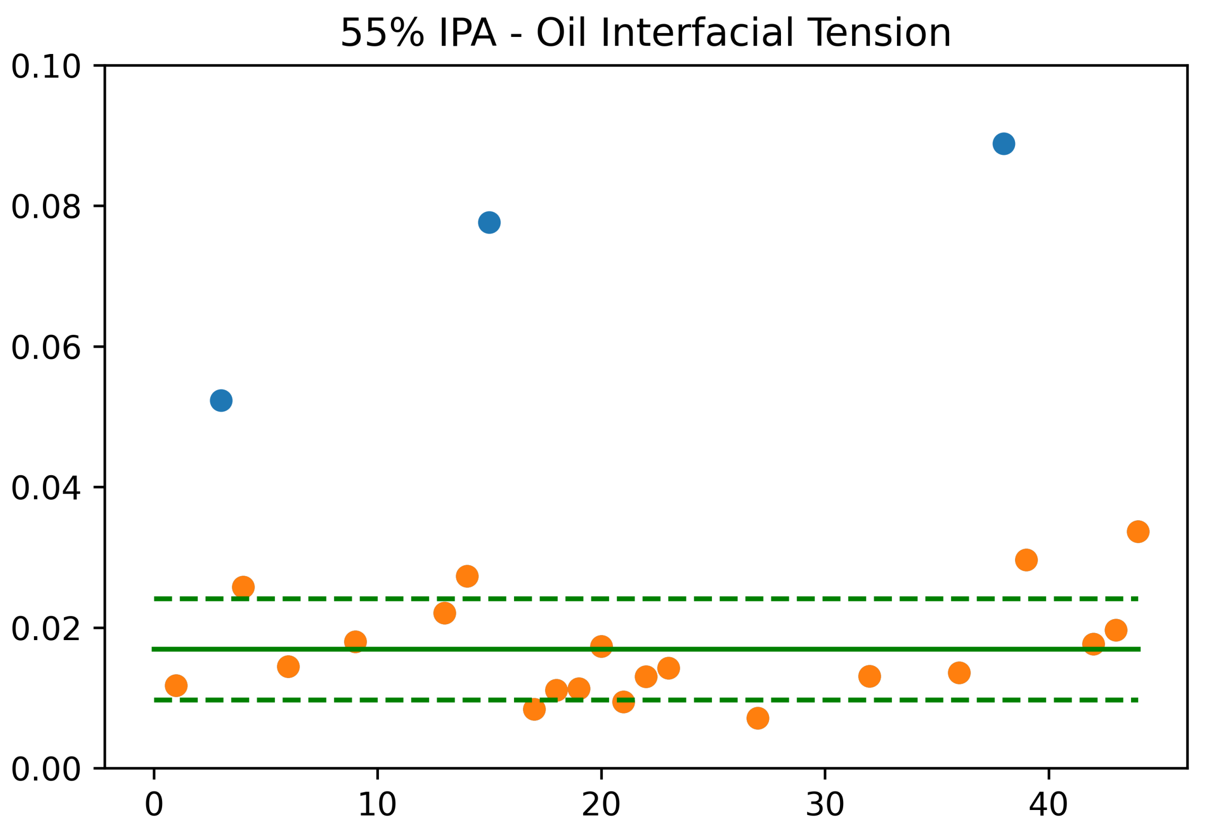
Interfacial Tension Measurement



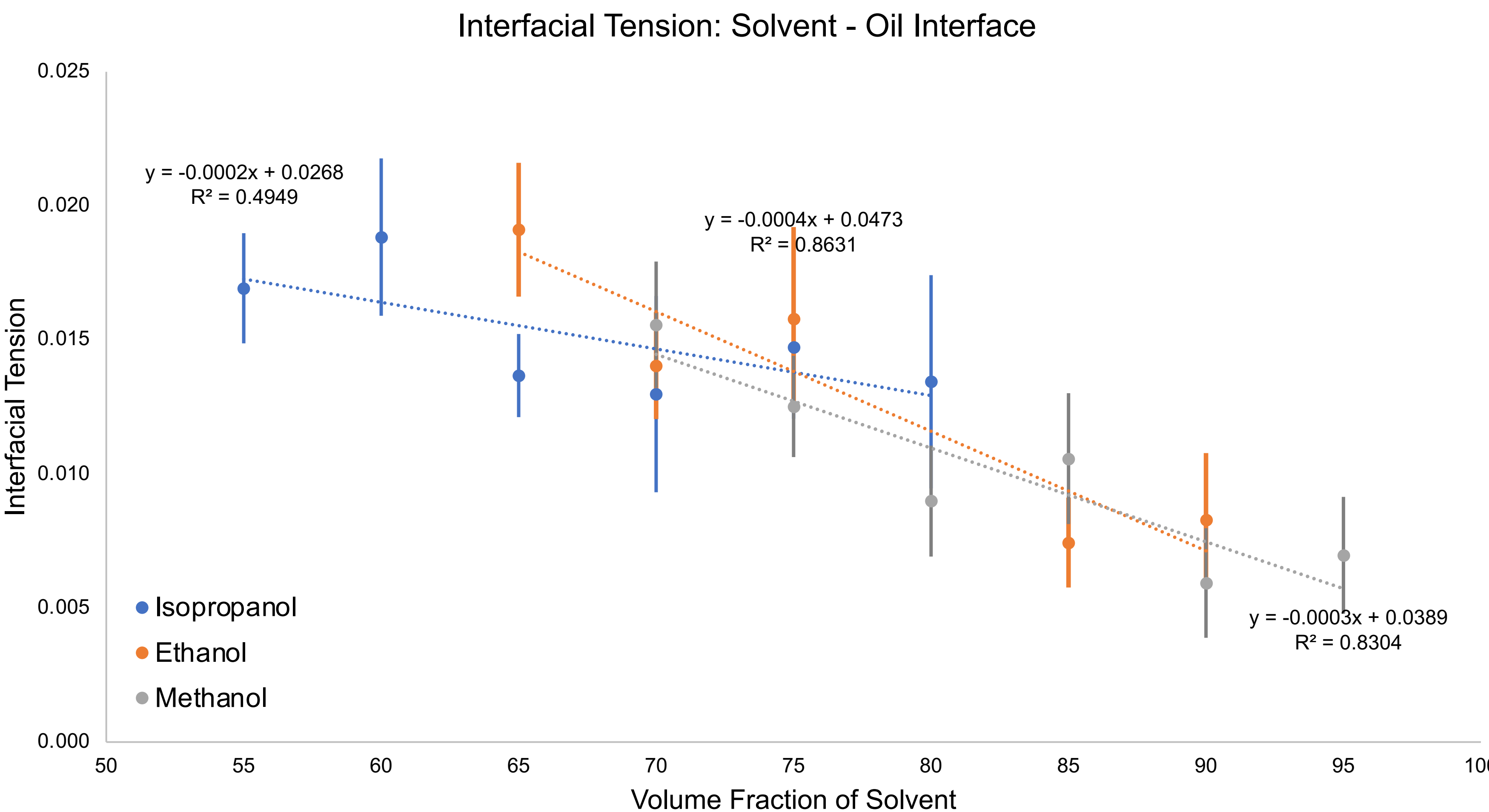
Interfacial Tension Measurement



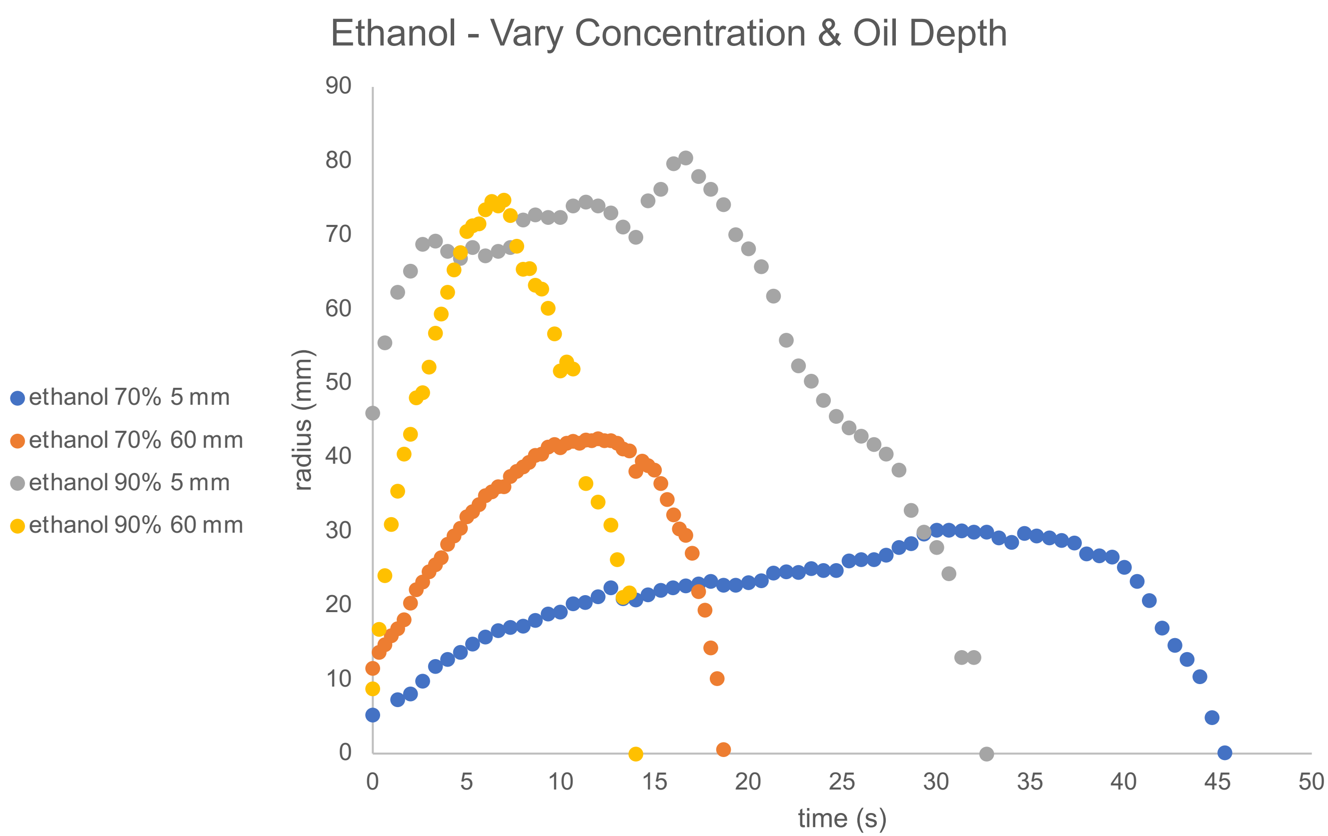
Interfacial Tension



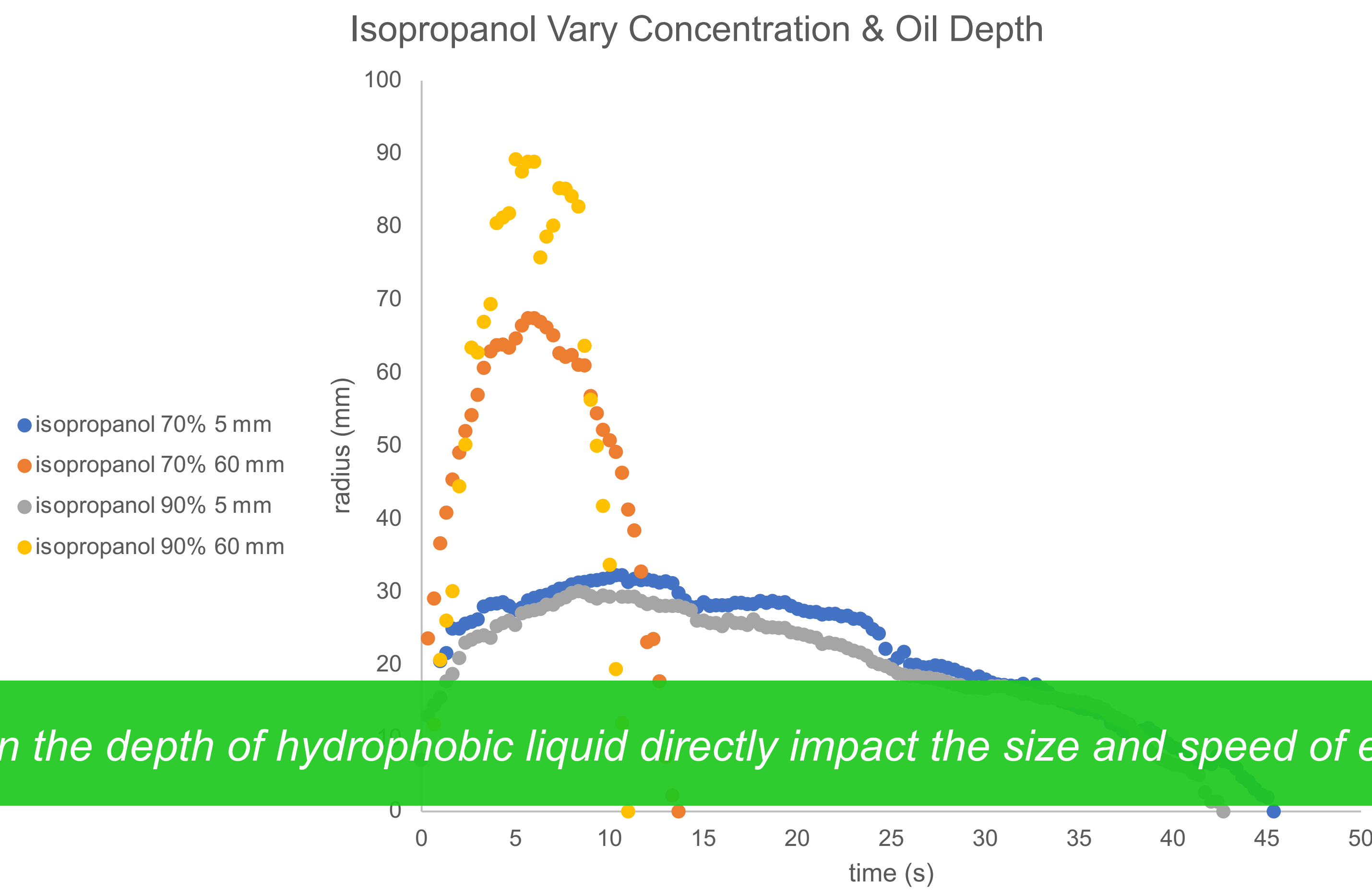
Interfacial Tension



Experimental Verification

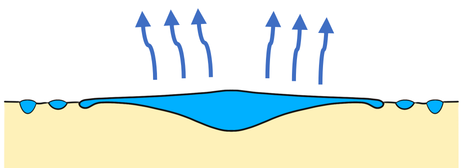


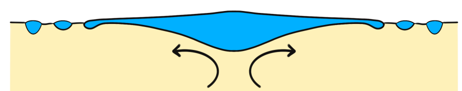
Experimental Verification

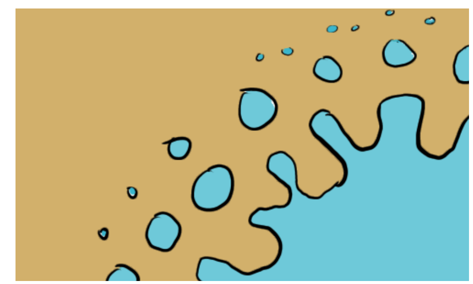


Varies in the depth of hydrophobic liquid directly impact the size and speed of expansion

Theoretical Model





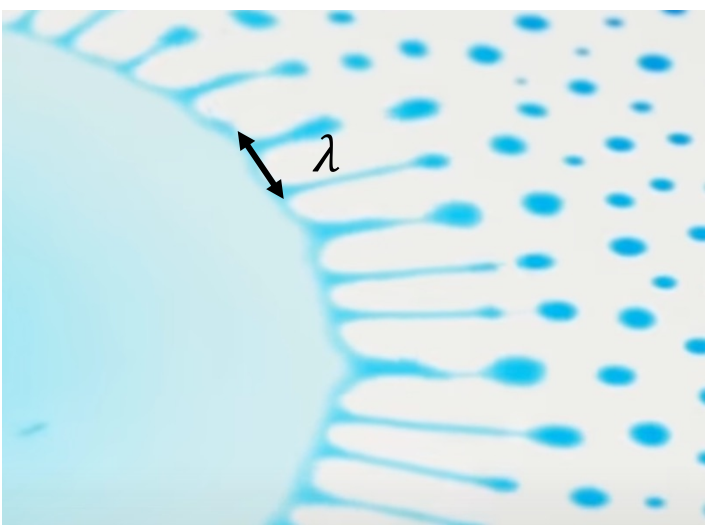


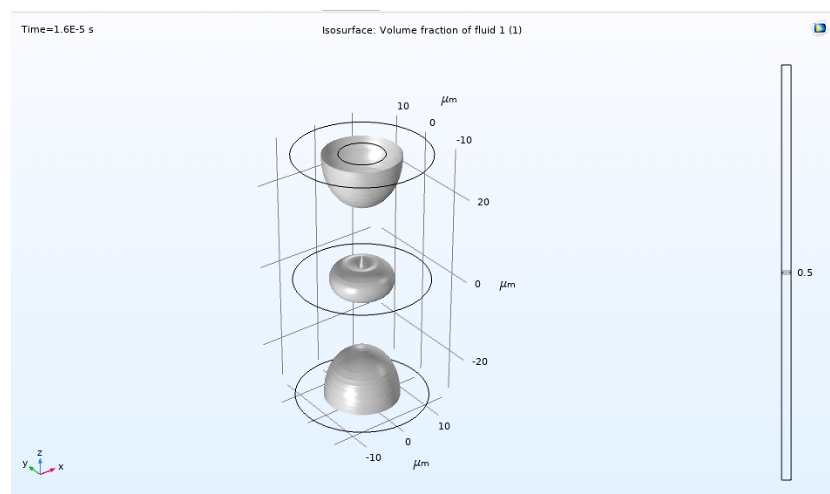
Alcohol Evaporation → Interfacial Tension & Shear Stress → Droplet Fragmentation

Plateau-Rayleigh Instability

The drop is bounded by a circular rim which destabilizes into small droplets through a process reminiscent of the Plateau-Rayleigh instability of liquid cylinders. Both the wavelength of the instability λ and the size of the resulting droplets strongly depend on the concentration of IPA,

$$\lambda \sim 9.02 R_0$$





Plateau-Rayleigh Instability

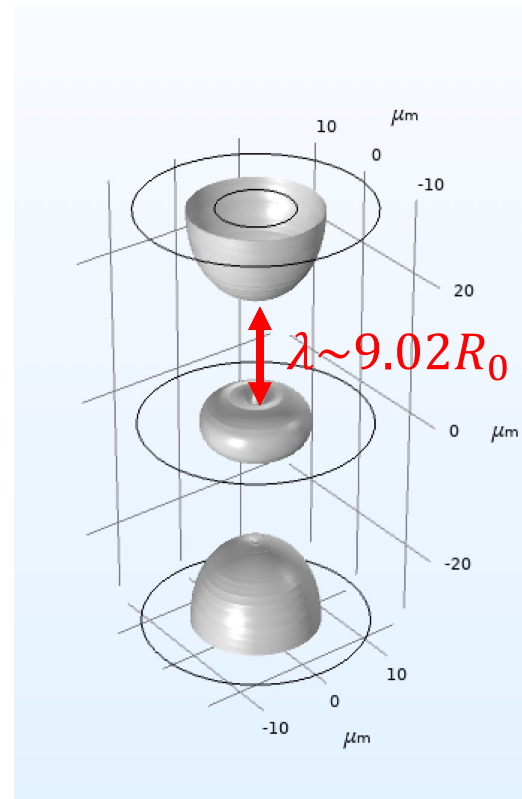
The dependence of the growth rate of the liquid column ω, on wave number k, is:

$$\omega^2 = \frac{\sigma}{\rho R_0^3} k R_0 \frac{I_1 k(R_o)}{I_0(kR_0)}(1 - k^2 R_0^2)$$

For a liquid column (assumed the rim) where σ is the surface tension, ρ is the density, and radius R_o. I_o and I_1 correspond to the solutions of the modified first order Bessel function

And the break up time for the liquid column is,

$$t_{breakup} \sim 2.91 \sqrt{\frac{\rho R_0^3}{\sigma}}$$



Scaling of Droplet Size

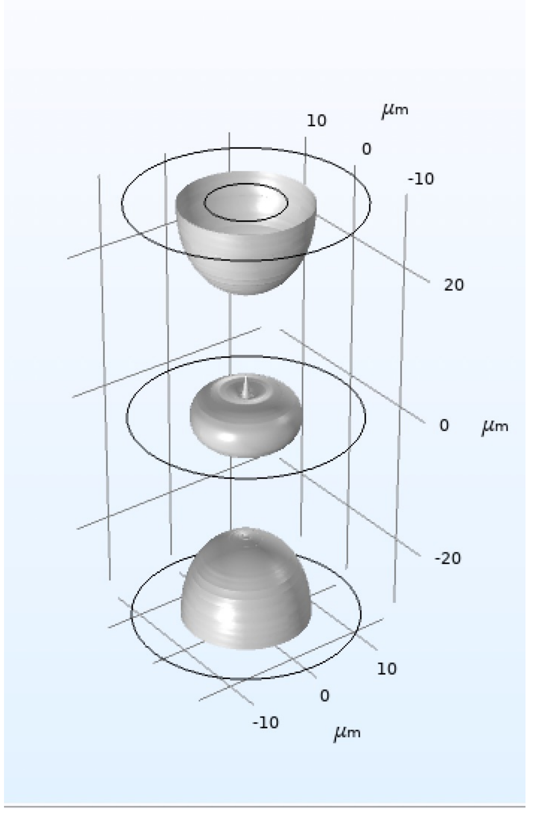
The volume of the main drop,

$$\frac{4}{3}\pi\left[r_{mother\ drop}{}^{3} + \sum r_{daughter\ drop}{}^{3}\right] = \pi r^{2}\frac{\omega}{f}$$

While Area is simply, $area = \pi r^2$

Substituting the equation derived for the Plateau-Rayleigh instability,

$$V = \pi r^{2}\frac{\left(\frac{\sigma}{\rho R_0^3} k R_0 \frac{I_1 k(R_o)}{I_0(kR_0)}(1 - k^2 R_0^2)\right)^{\frac{1}{2}}}{f}$$

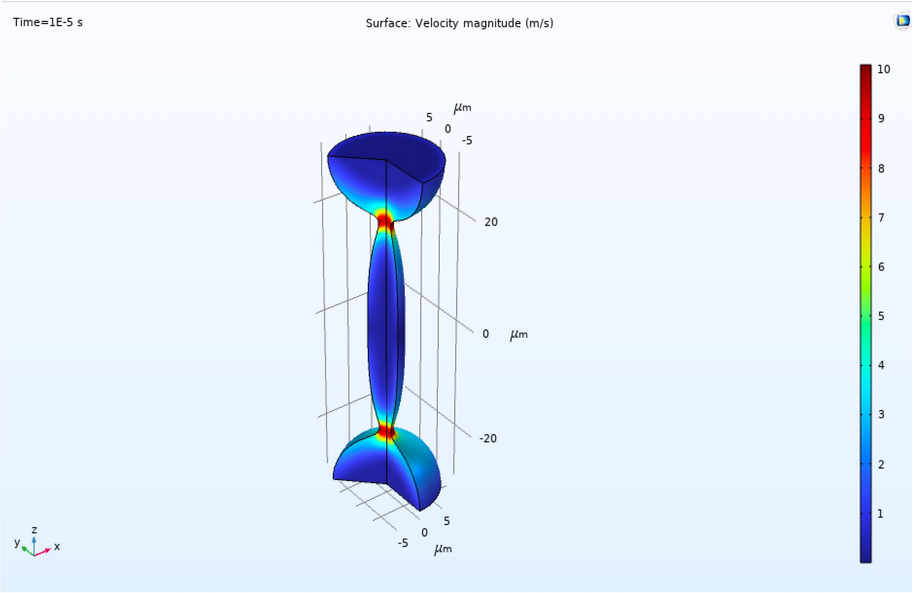


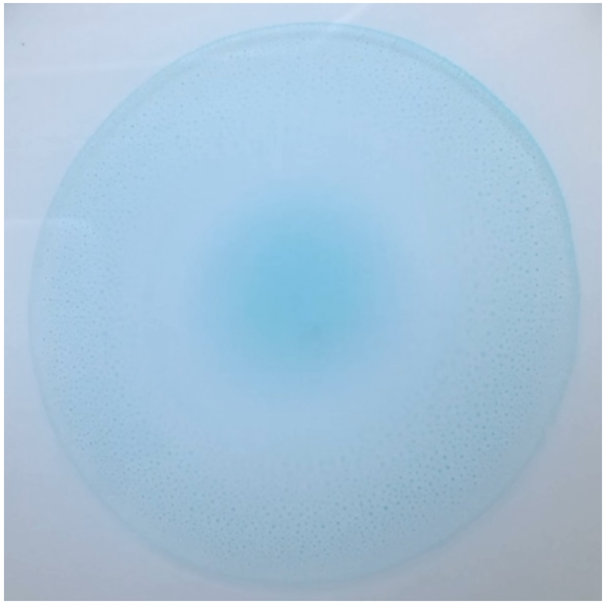
As shown above, diameter of ejected droplets, $\sim 20\mu m$ – 0.5mm

Droplet Fragmentation

Surface tension balances the Marangoni velocity at the rim, and fragmentation occurs,

In which a bulge is created due to this effect

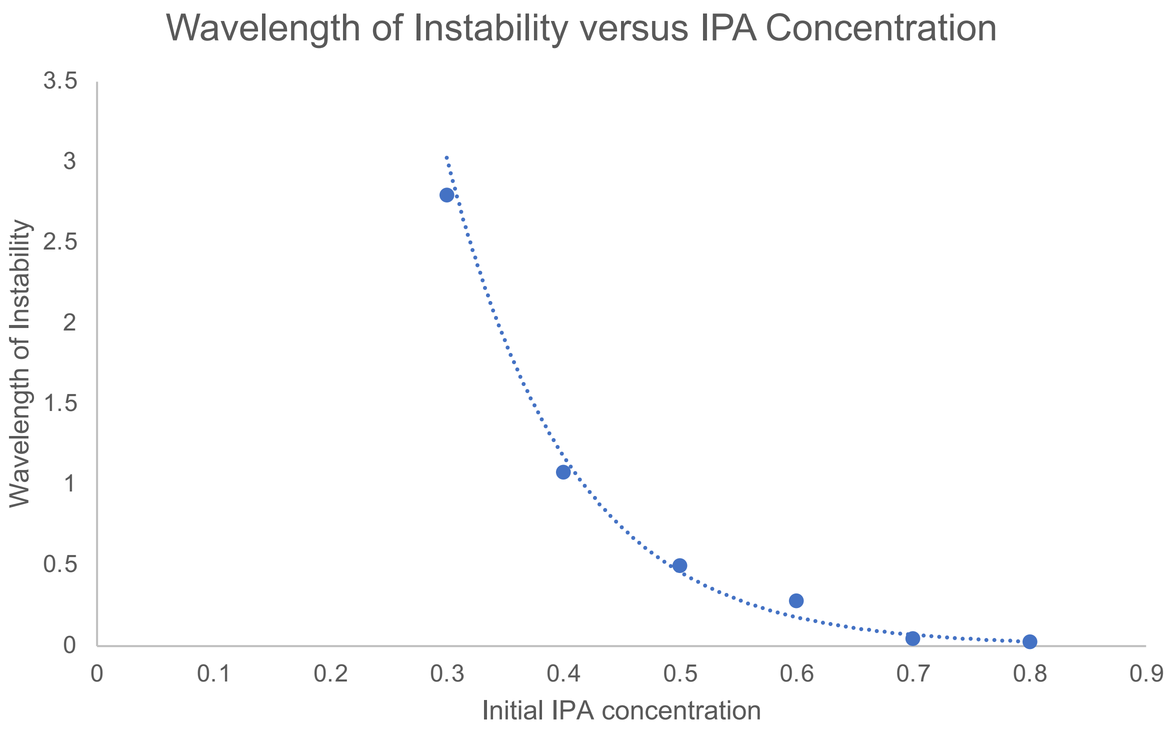




Simulation Results

COMSOL microfluidics plateau-Rayleigh instability simulation results

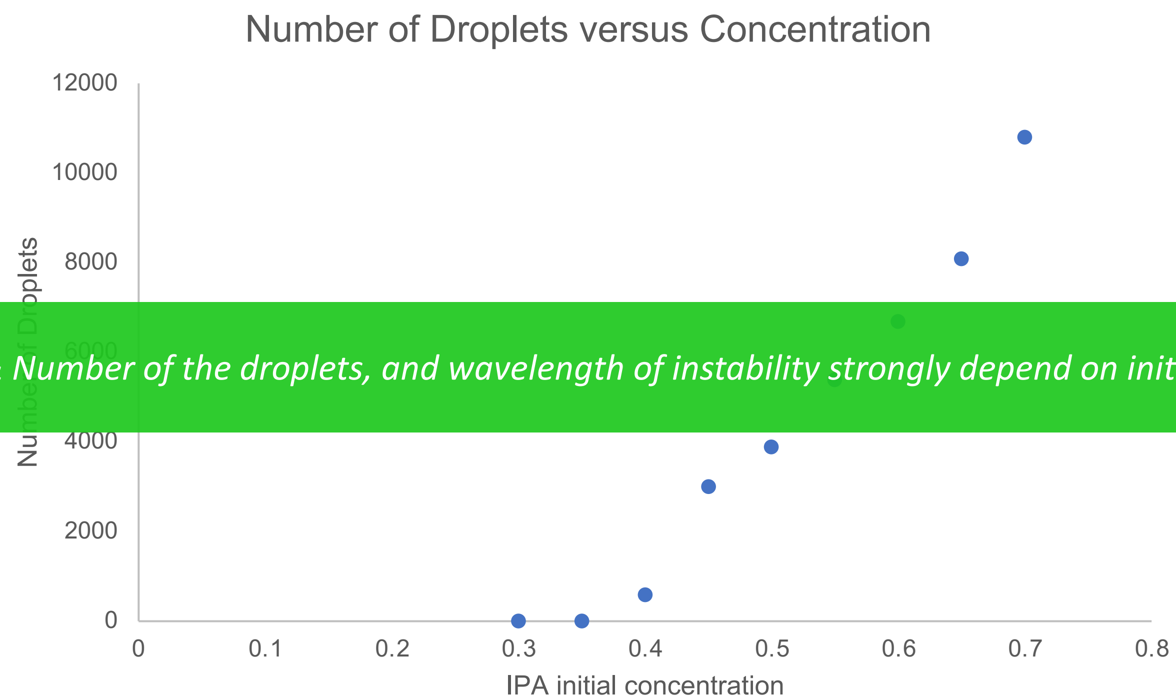
For the wavelength of instability as initial concentration is inputted,



Simulation Results

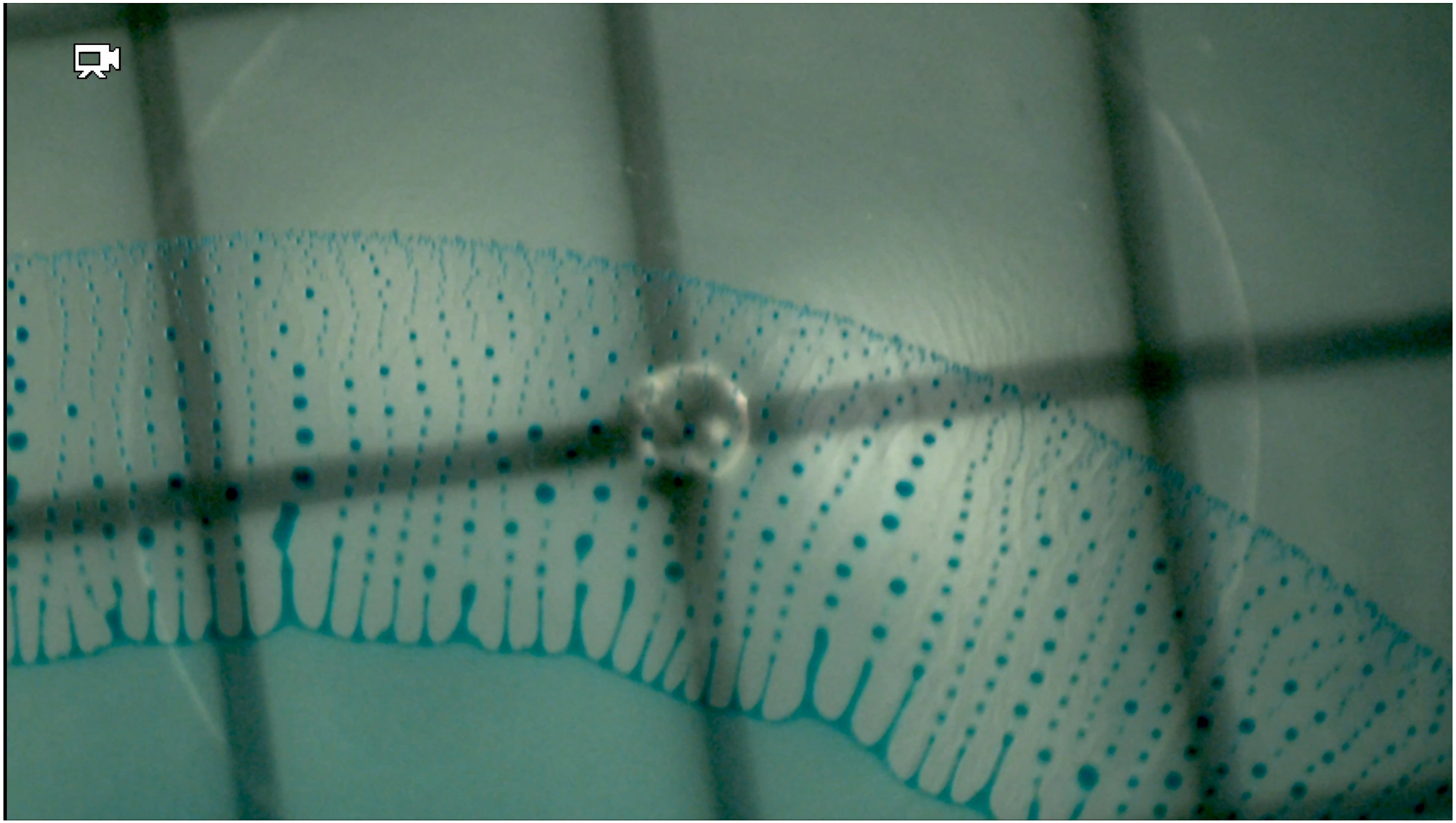
COMSOL microfluidics plateau-Rayleigh instability simulation results

For the number of droplets as initial concentration is inputted,

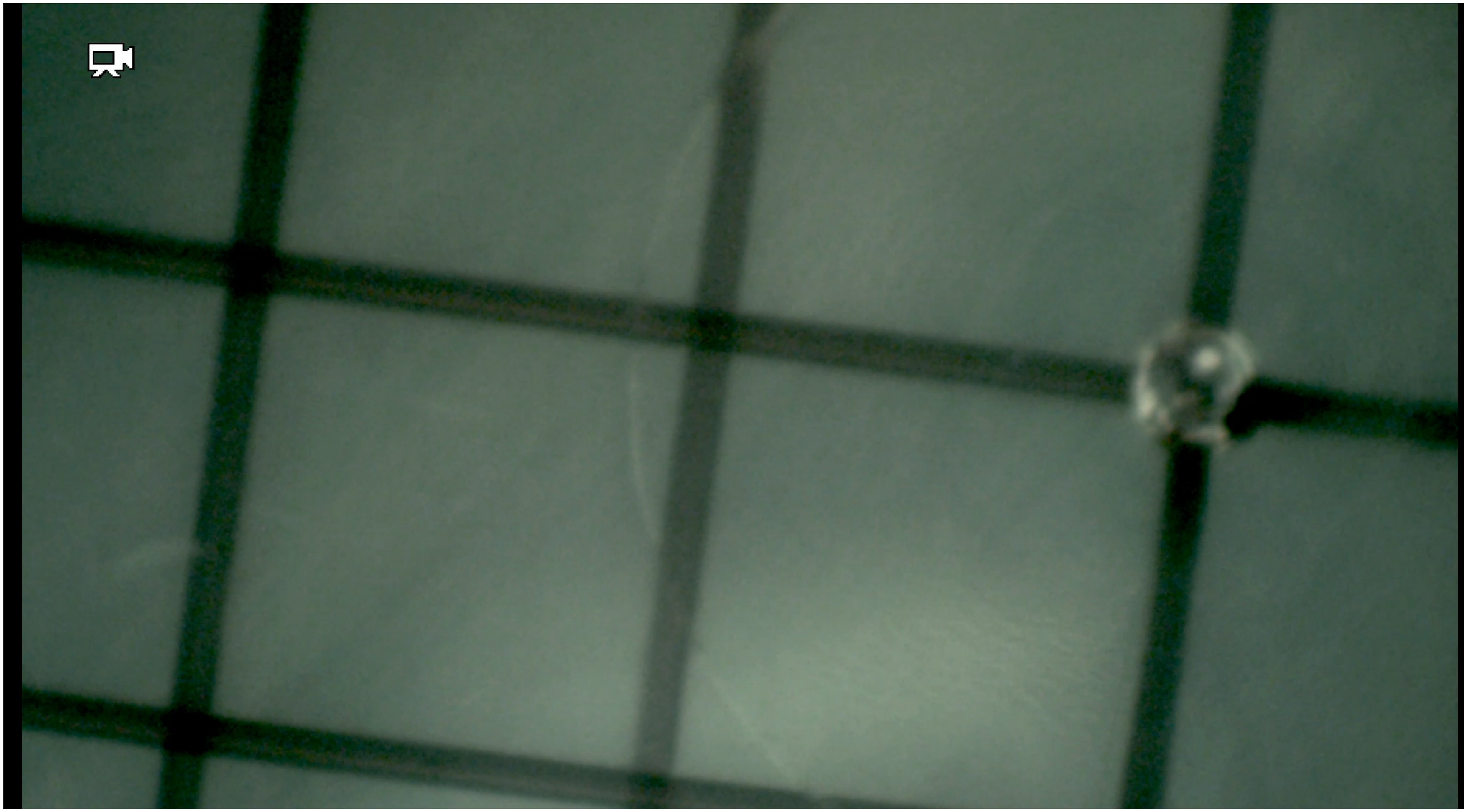


Size & Number of the droplets, and wavelength of instability strongly depend on initial IPA concentrationz

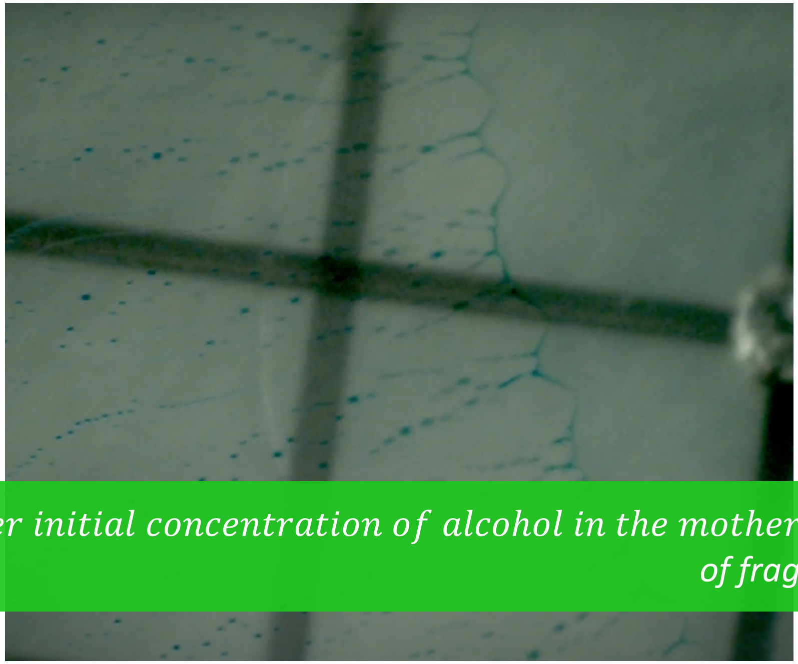
Microscopy – 50% Isopropanol



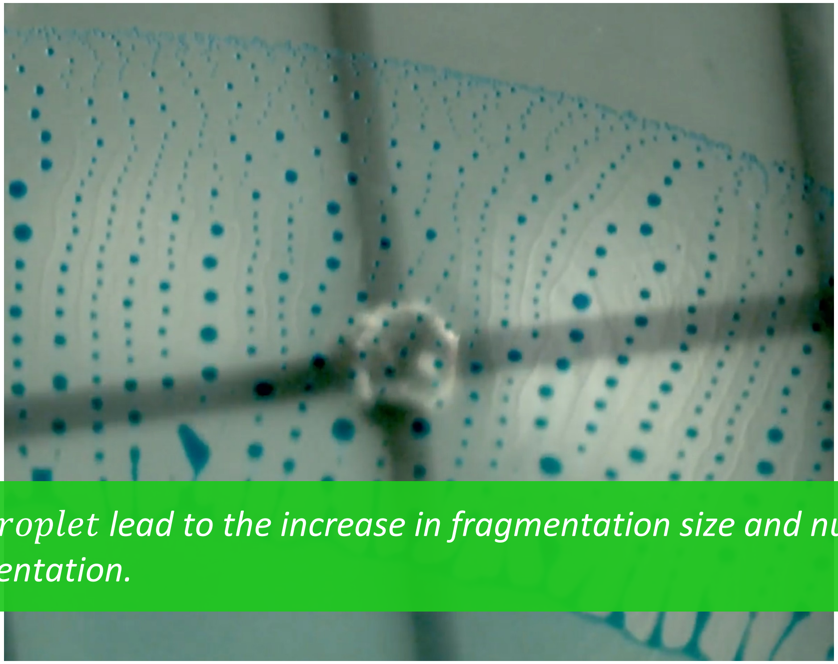
Microscopy – 70% Isopropanol



Droplet Fragmentations



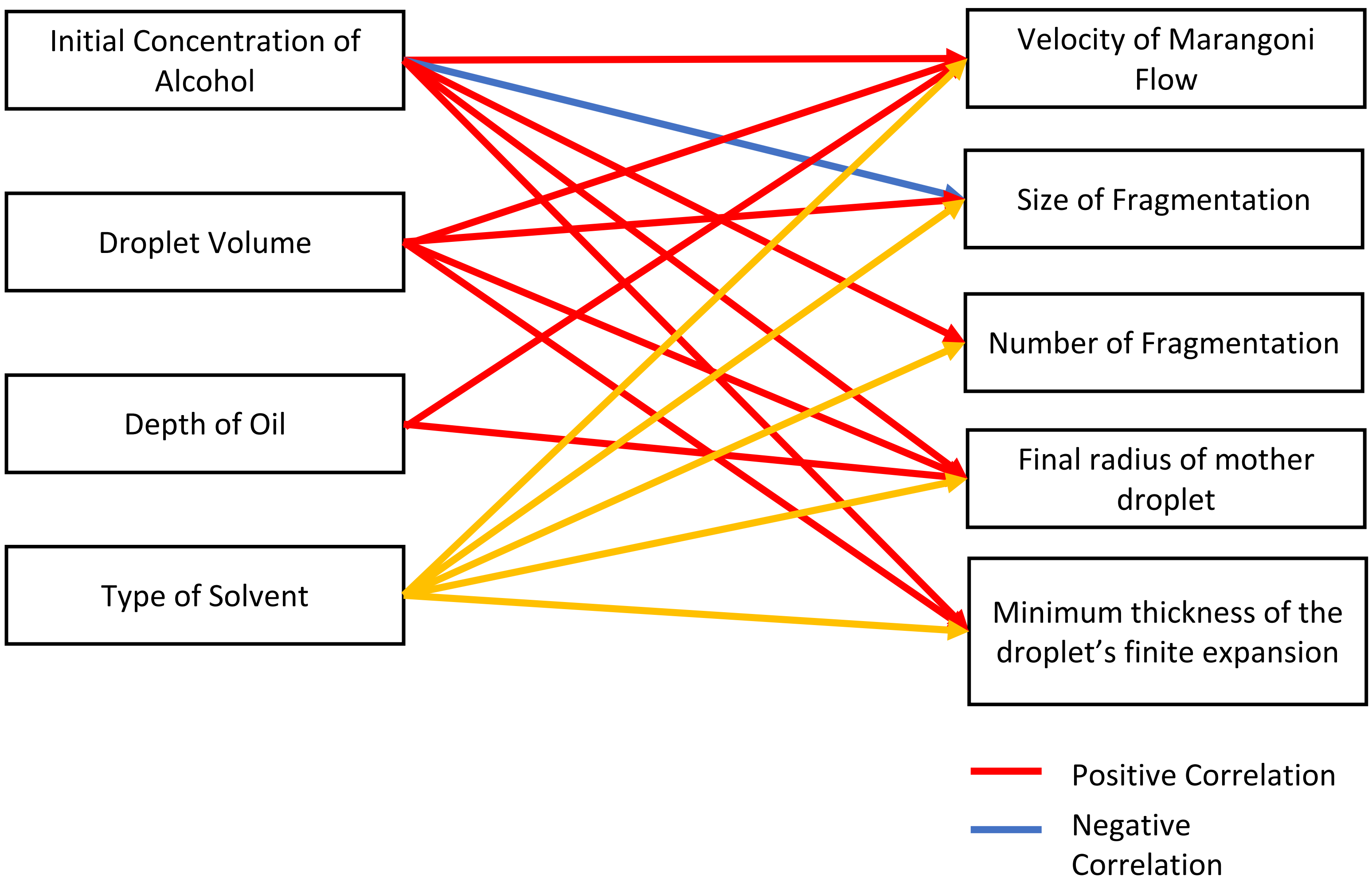
70% isopropanol



50% isopropanol

Higher initial concentration of alcohol in the mother droplet lead to the increase in fragmentation size and number of fragmentation.

Key Parameters



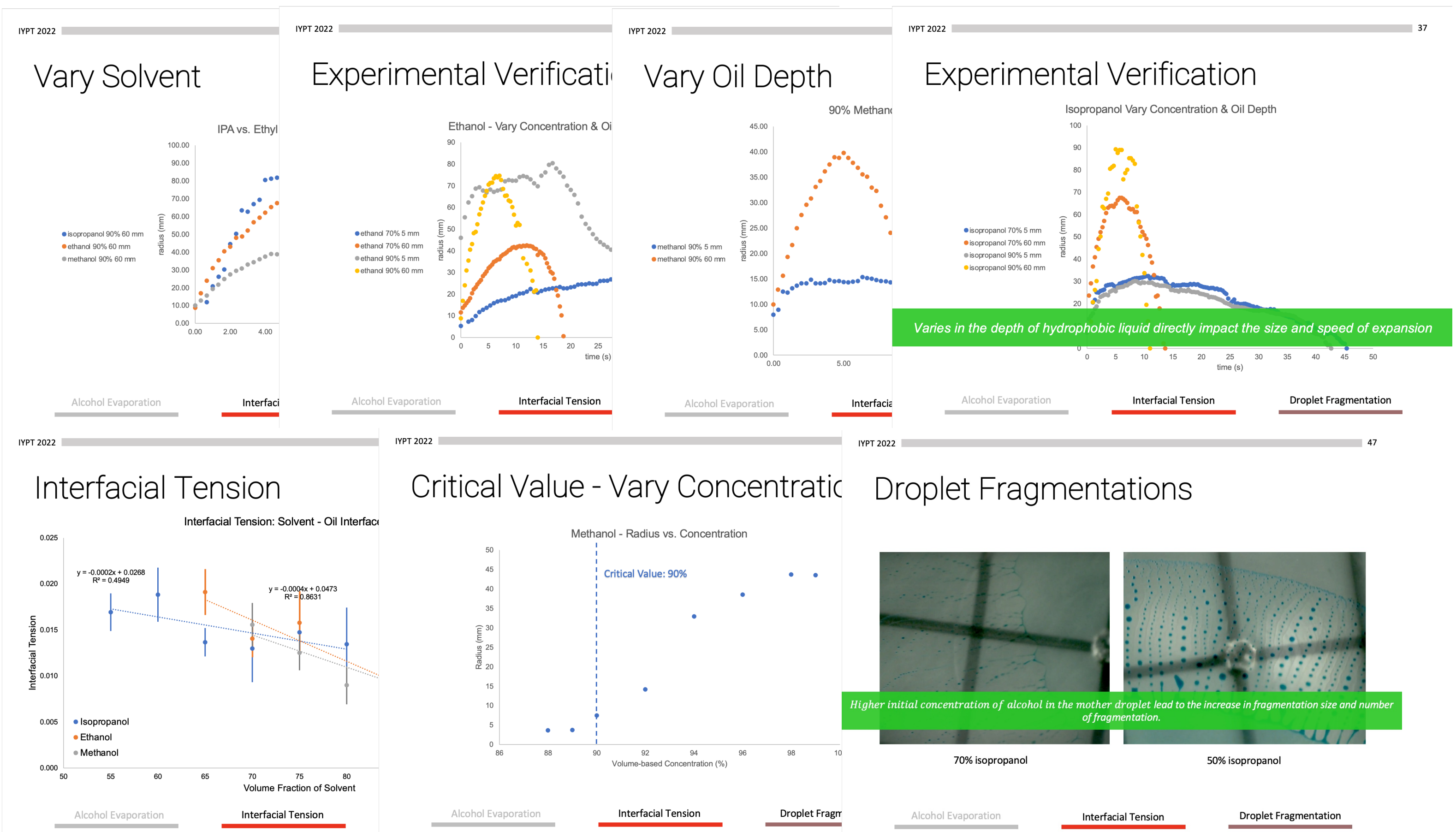
Initial Concentration of Alcohol
Droplet Volume
Depth of Oil
Type of Solvent
Velocity of Marangoni Flow
Size of Fragmentation
Number of Fragmentation
Final radius of mother droplet
Minimum thickness of the droplet's finite expansion
Positive Correlation
Negative Correlation

Conclusion

*"When a drop of a water mixture **(e.g. water-alcohol)** is deposited on the surface of a **hydrophobic liquid** (e.g. vegetable oil), the resulting drop may sometimes **fragment** into smaller droplets. Investigate the parameters that affect the **fragmentation** and the size of the final droplets."*

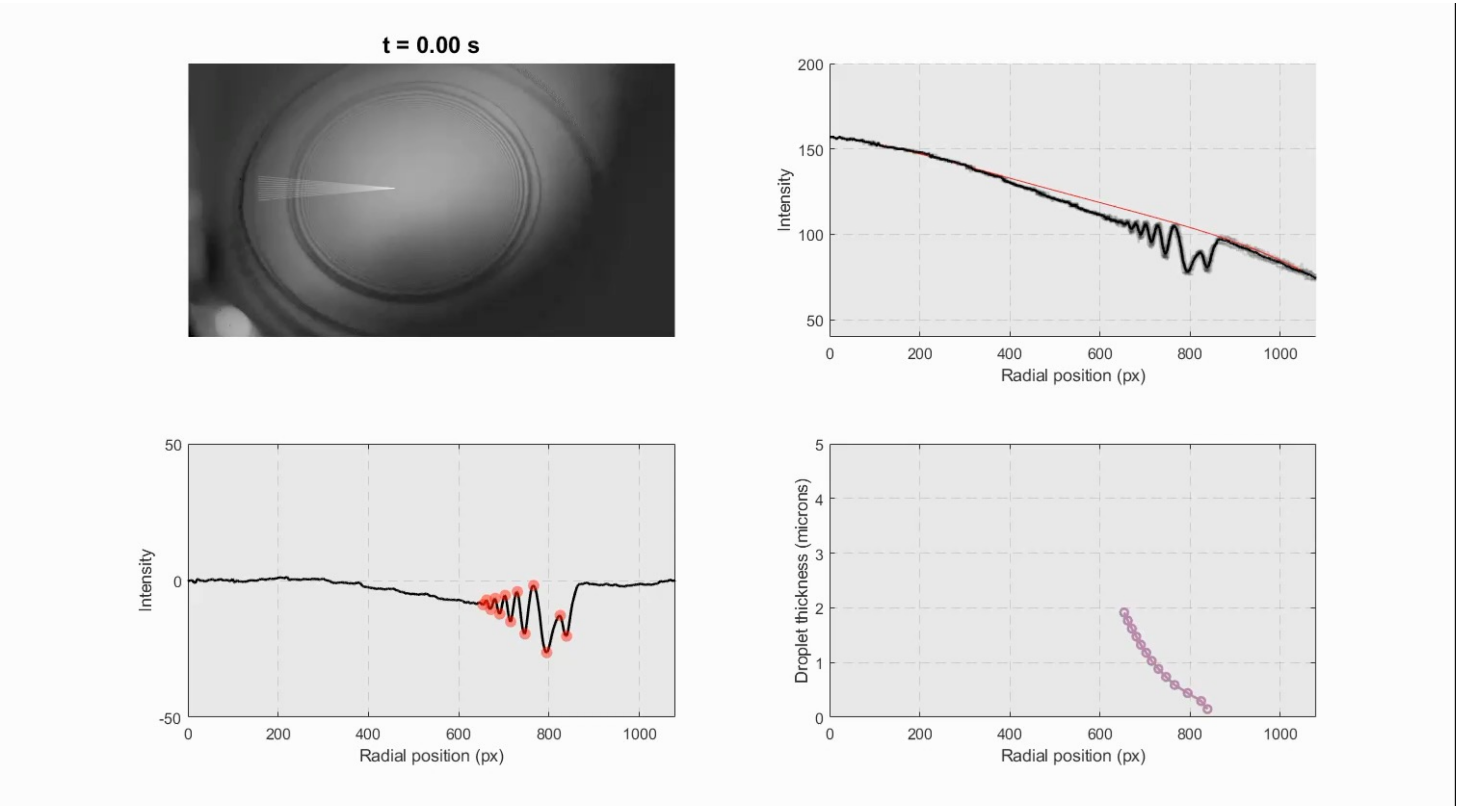
Solvent concentration is the **most relevant** parameter to the problem, while other parameters like **hydrophobic liquid surface tension** and **droplet size** have an impact as well. **Fragmentation** occurs through the **surface tension's** forces overcoming **intermolecular bonds,** and **more destabilization** leads to **more and smaller droplets.**

Conclusion



Vary Solvent
IPA vs. Ethyl
Experimental Verificati
Ethanol - Vary Concentration & Oi
Vary Oil Depth
90% Methan
Experimental Verification
Isopropanol Vary Concentration & Oil Depth
Varies in the depth of hydrophobic liquid directly impact the size and speed of expansion
Interfacial Tension
Interfacial Tension: Solvent - Oil Interfac
y = -0.0002x + 0.0268
R² = 0.4949
y = -0.0004x + 0.0473
R² = 0.8631
Isopropanol
Ethanol
Methanol
Volume Fraction of Solvent
Critical Value - Vary Concentratic
Methanol - Radius vs. Concentration
Critical Value: 90%
Volume-based Concentration (%)
Droplet Fragmentations
Higher initial concentration of alcohol in the mother droplet lead to the increase in fragmentation size and number of fragmentation.
70% isopropanol
50% isopropanol

Further Insights



References

[1] Keiser, L., Bense, H., Colinet, P., Bico, J., Reyssat, E., 2017. Marangoni Bursting: Evaporation-Induced Emulsification of Binary Mixtures on a Liquid Layer. Physical Review Letters 118.. doi:10.1103/physrevlett.118.074504

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Thank you for listening

Appendix

Interfacial Tension Calculation

```python
import numpy as np
import matplotlib.pyplot as plt

import glob
print("Using glob.glob()")
files = glob.glob("//Users/dongyuezhen/Desktop/data_cal copy/55 IPA/*.csv",
                   recursive = True)

speed = np.zeros(0)
for file in files:
    #print(float(str.split(file, '_')[4]))
    speed =np.append(speed, [float(str.split(file, '_')[4])])
    #print(file)


d0 = 0.92
dm = 0.91340
dd = d0-dm
omega = speed

#initialize H,G arrays
H = np.zeros(len(speed))
G = np.zeros(len(speed))
x = np.zeros(len(speed))

for i in range(len(files)):
    x, y = np.loadtxt(files[i], delimiter = ',', unpack = True)
    x = x/1000
    x0 = (np.max(x) - np.min(x))/2
    x[i] = x0
    H[i] = (dd*omega[i]**2)/4
    G[i] = (3*x0/2)*((dd*omega[i]**2)/4)**(1/3)
    #plt.show()
#print(x , speed)
print('Data File Number = '+str(len(speed)+1))


#initialize array to compare every combination of two points
combination = len(speed)*(len(speed)-1)/2
index = np.zeros((int(combination), 2))


```

Interfacial Tension Calculation

```
counter = 0
for i in range(len(speed)):
    for j in range(len(speed)):
        if(i!=j and i<j):
            index[counter] = [i,j]
            counter += 1

gamma = np.zeros(int(combination))

for i in range(int(combination)):
    i1 = int(index[i][0])
    i2 = int(index[i][1])
    gamma[i] = G[i1] - H[i1]*((G[i1]-G[i2])/(H[i1]-H[i2]))


#gamma = filter_until(gamma,0.1)

m = np.arange(0,len(gamma), 1)
print(gamma)

gamma_clip = gamma
m_clip = m


m_clip = m_clip[gamma_clip > 0]
gamma_clip = gamma_clip[gamma_clip > 0]
m_clip = m_clip[gamma_clip <= 4*10**(-2)]


gamma_clip = gamma_clip[gamma_clip <= 4*10**(-2)]



print(gamma_clip)

gammabar = np.average(gamma_clip)
gammasd = np.std(gamma_clip)
print(gammabar, gammasd)
plt.plot(m,gamma, 'o')
plt.plot(m_clip, gamma_clip, 'o')

plt.plot([m.min(), m.max()], [gammabar, gammabar], 'g')
```

Interfacial Tension Calculation

```
    gamma[i] = G[i1] - h[i1]*((G[i1]-G[i2])/(h[i1]-h[i2]))


#gamma = filter_until(gamma,0.1)

m = np.arange(0,len(gamma), 1)
print(gamma)

gamma_clip = gamma
m_clip = m


m_clip = m_clip[gamma_clip > 0]
gamma_clip = gamma_clip[gamma_clip > 0]
m_clip = m_clip[gamma_clip <= 4*10**(-2)]


gamma_clip = gamma_clip[gamma_clip <= 4*10**(-2)]



print(gamma_clip)

gammabar = np.average(gamma_clip)
gammasd = np.std(gamma_clip)
print(gammabar, gammasd)
plt.plot(m,gamma, 'o')
plt.plot(m_clip, gamma_clip, 'o')

plt.plot([m.min(), m.max()], [gammabar, gammabar], 'g')
plt.plot([m.min(), m.max()], [gammabar+gammasd, gammabar+gammasd], 'g--')
plt.plot([m.min(), m.max()], [gammabar-gammasd, gammabar-gammasd], 'g--')
plt.ylim(0,1e-1)
plt.title('55% IPA - Oil Interfacial Tension')
plt.savefig('55%_IPA_interfacial', facecolor = "white", dpi = 600)
plt.show()
```