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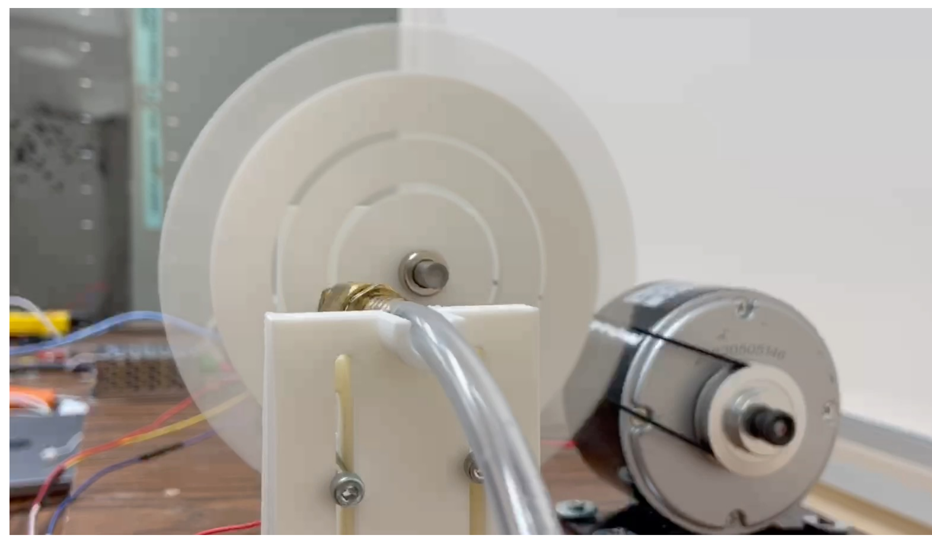
"If you direct an air flow onto a rotating disk with holes, a sound may be heard. Explain this phenomenon and investigate how the sound characteristics depend on the relevant parameters."

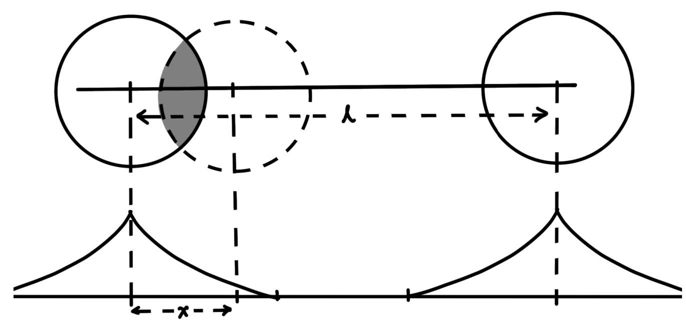


Problem Statement

*"If you direct an **air flow** onto a **rotating disk** with holes, a **sound** may be heard. Explain this phenomenon and investigate how the sound characteristics depend on the relevant parameters."*

Parameters:

1. *Flowrate of the air flow*
2. *Rotational speed of disk*
3. *Overlapping area between airflow and holes*
4. *Number of holes*



Overview

1 **_Phenomenon_**
Reproduction and Explaination of the Phenomenon

2 **_Experimental Setup_**
Measurement Techniques, Camera Views

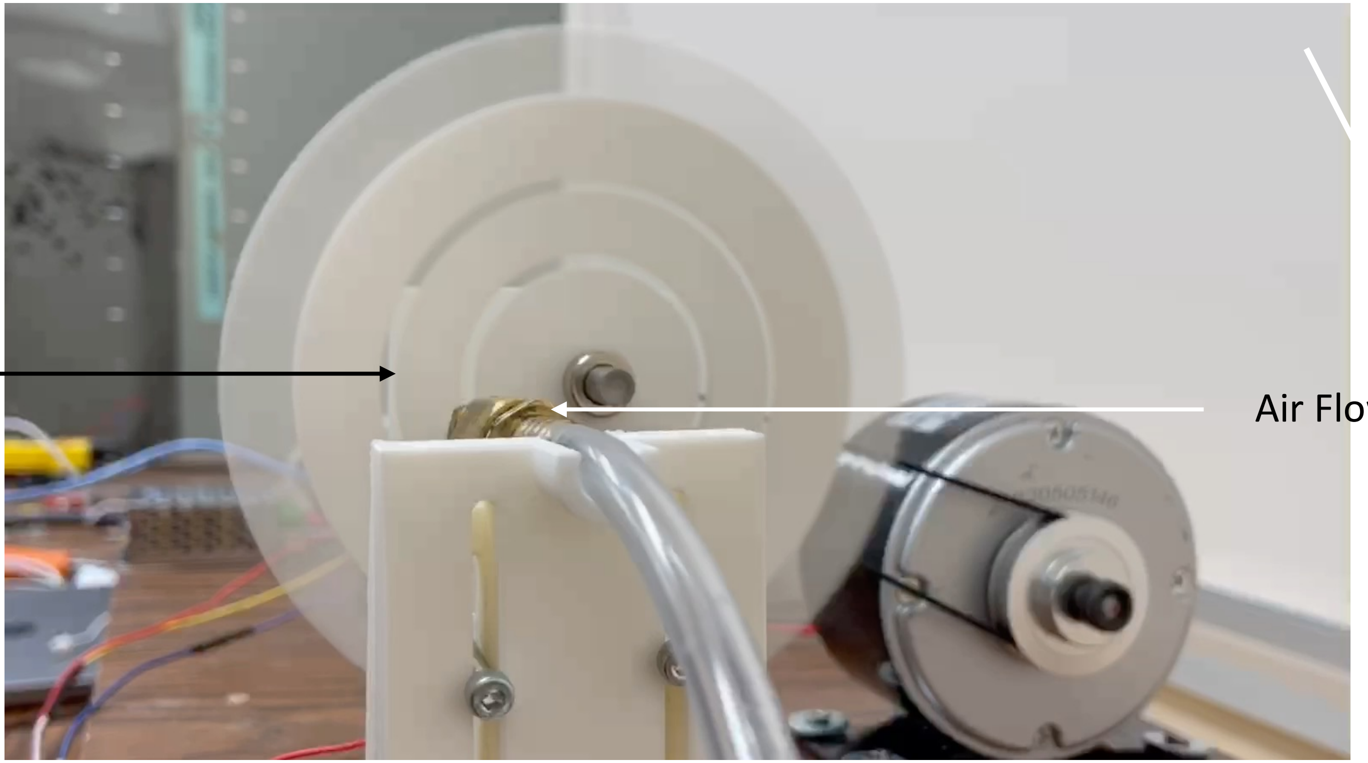
3 **_Theoretical Model_**
Qualitative Explanation, Quantitative Models

4 **_Key Parameter Interactions_**
Effects of Changing Physical Parameters

5 **_Conclusion_**
Further Insights and General Investigations

Phenomenon

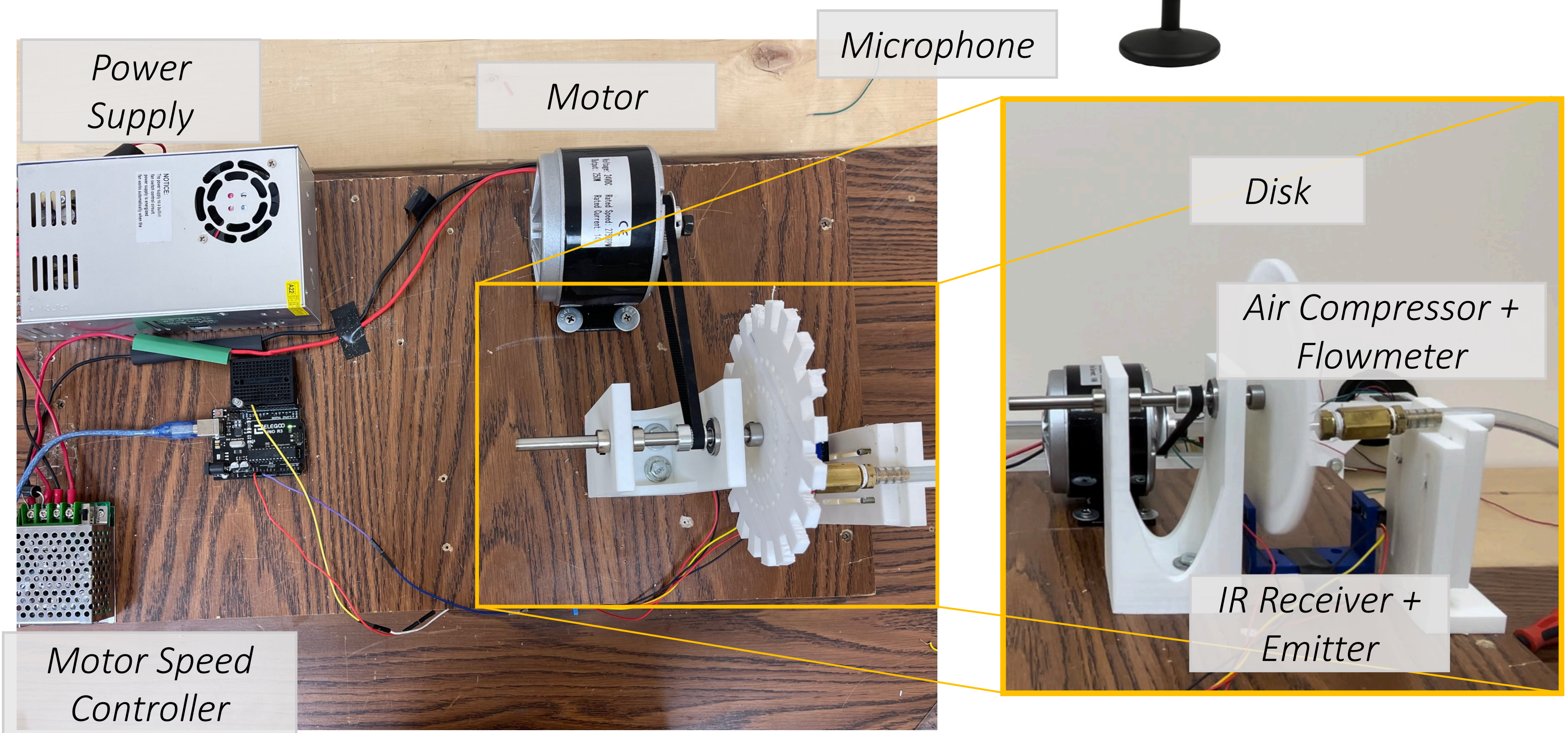
Rotating Disk



Air Flow

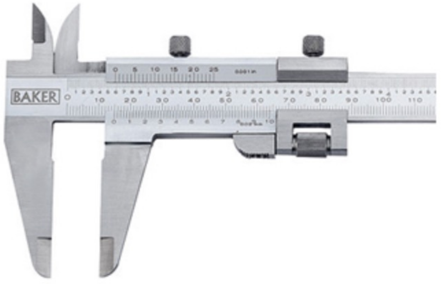
Experimental Setup

Microscopy Setup



Experimental Measurement

Flowmeter
($\pm 0.5\ L/min$)

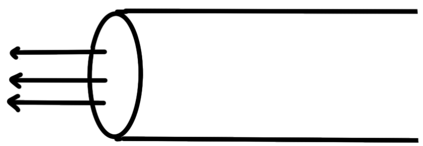


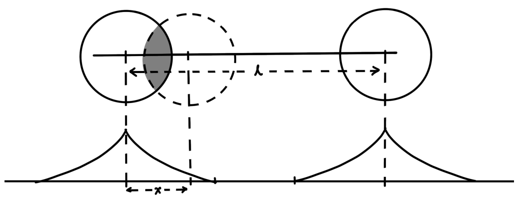
Caliper
($\pm 0.05\ mm$)

Microphone

Theoretical Model

Theoretical Model

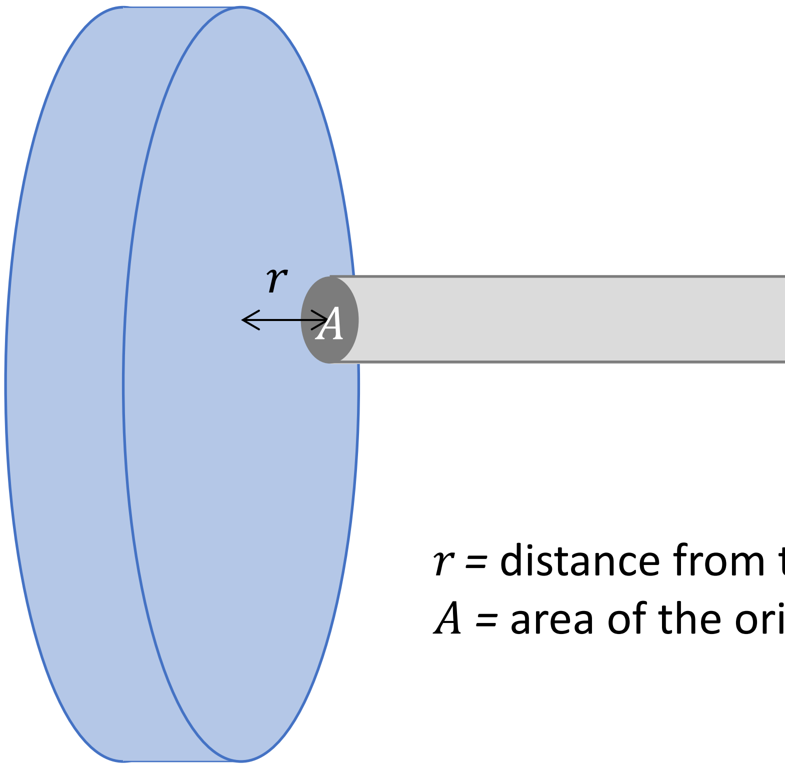




Air Flow

Sound Characteristics

Geometry



r = distance from the orifice of pipe to the disk
A = area of the orifice exposed

Assumptions

- *The air flow is a steady stream with super imposed oscillations*
- *The actual flux of air at any moment is proportional to the area of the orifice exposed*
- *Supply of air is at constant pressure*

Flux

The velocity potential of air flow, ϕ, can be expressed as

$$\phi = \frac{f(t-\frac{r}{c})}{4\pi r} \quad [1]$$

where t is time and c is speed of sound

Omitting the density factor, the actual flux at the orifice is

$$\operatorname*{Lt}_{r\to 0}\left(-4\pi r^2\frac{\partial\phi}{\partial r}\right) = \boxed{f(t)} \quad [2]$$

Must be proportional to the area of orifice exposed at time t

$f(t)$ can be rewritten in

$$f(t) = A_0 + A_1\cos 2\pi \boxed{p}t + \cdots + A_n\cos 2\pi \boxed{p}nt + \cdots \quad [3]$$

where p is the fundamental frequency obtained

Flux

The radial velocity of the air with respect to time is

$$-\frac{\partial\phi}{\partial r}=\frac{f\left(t-\frac{r}{c}\right)}{4\pi r^{2}}+\frac{f'\left(t-\frac{r}{c}\right)}{4\pi rc} \qquad [4]$$

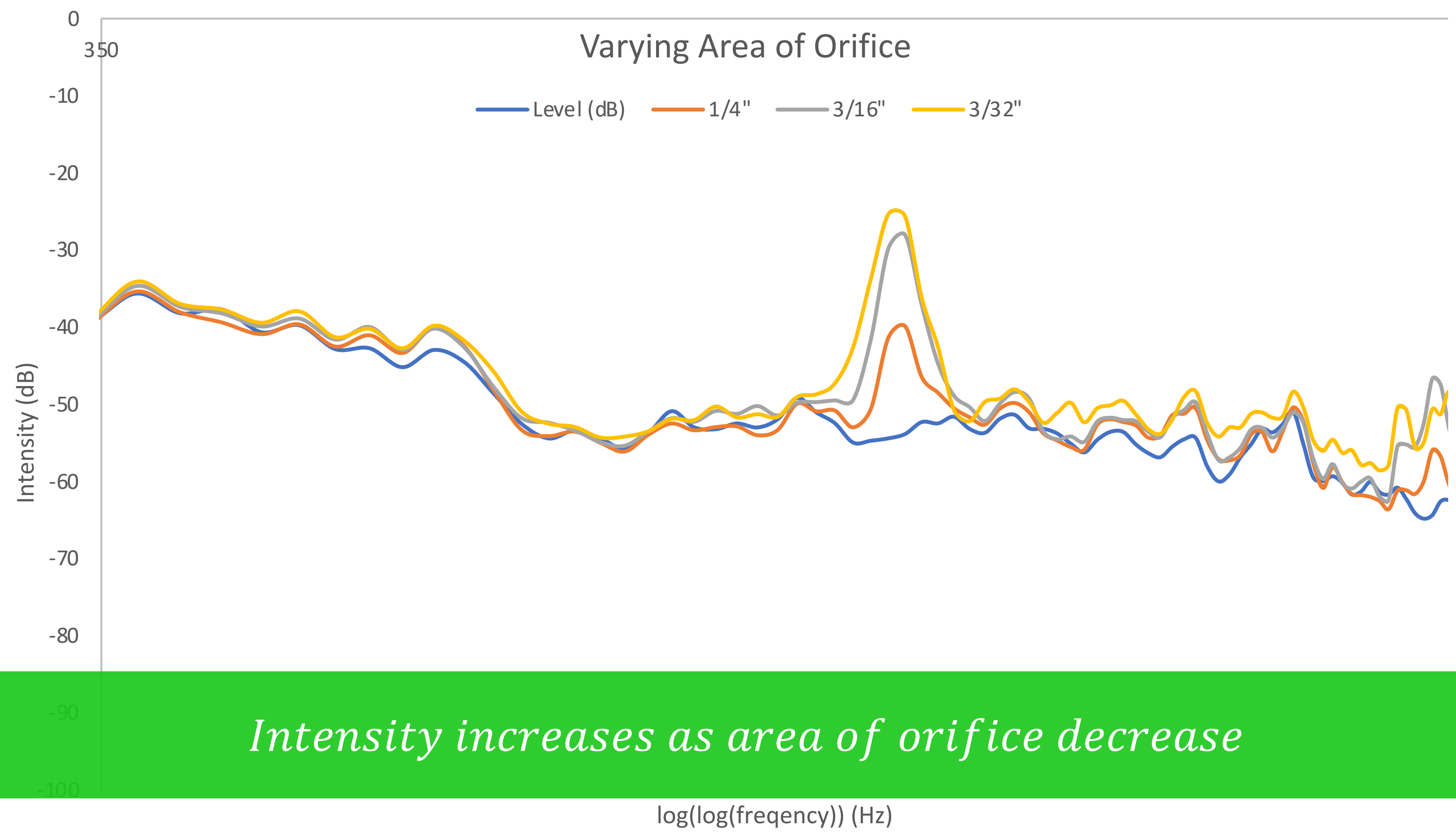
Recall equation [3], if r is much greater than the fundamental frequency, the first term may be neglected, which can be rewritten as

$$-\frac{p}{2rc}\sum nA_n \sin 2\pi pn\left(t-\frac{r}{c}\right) \qquad [5]$$

The intensity of nth harmonic is proportional to $n^2A_n^2$

Vary Area of Orifice

Vary Area of Orifice

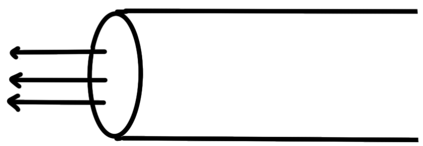


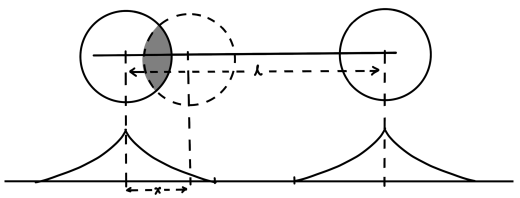
Intensity increases as area of orifice decrease

Air Flow

Sound Characteristics

Theoretical Model



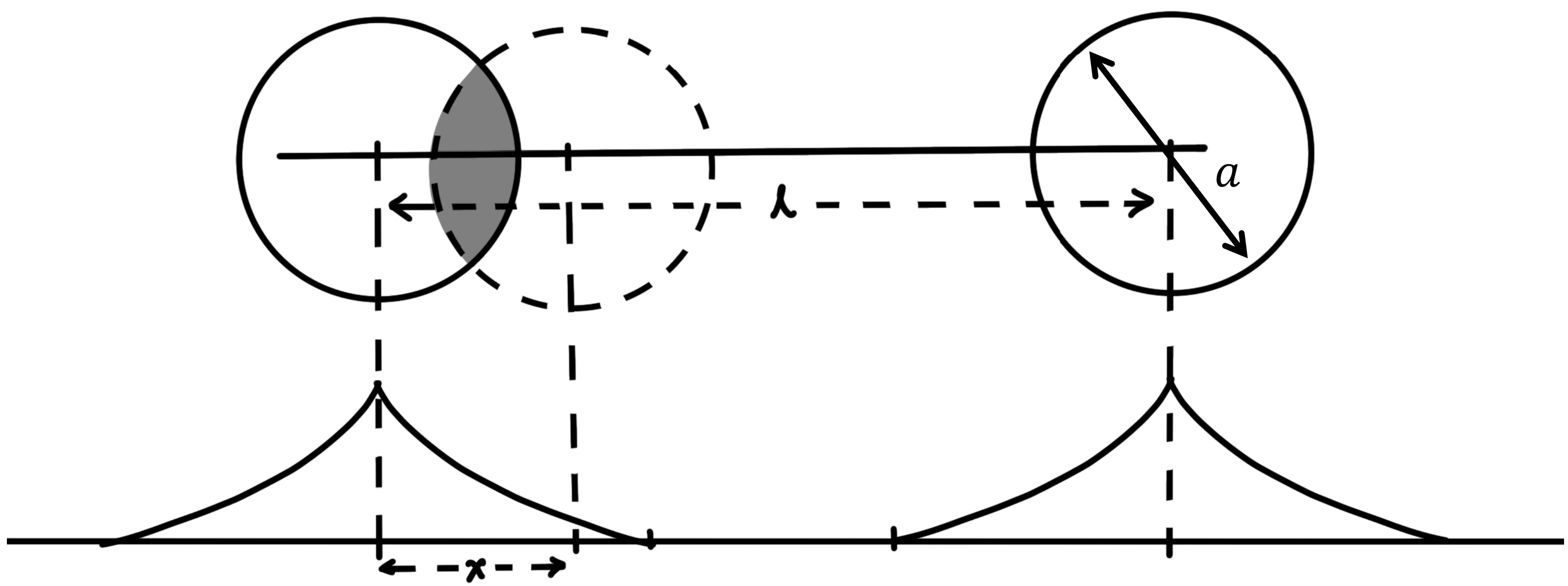


Air Flow

Harmonics

Geometry

l = distance between the center of the adjacent disk holes
x= distance between the center of the pipe and the disk hole
a= diameter of the disk hole



Assumptions

- *The air flow is a steady stream with super imposed oscillations*
- *The actual flux of air at any moment is proportional to the area of the orifice exposed*
- *Supply of air is at constant pressure*

Frequency

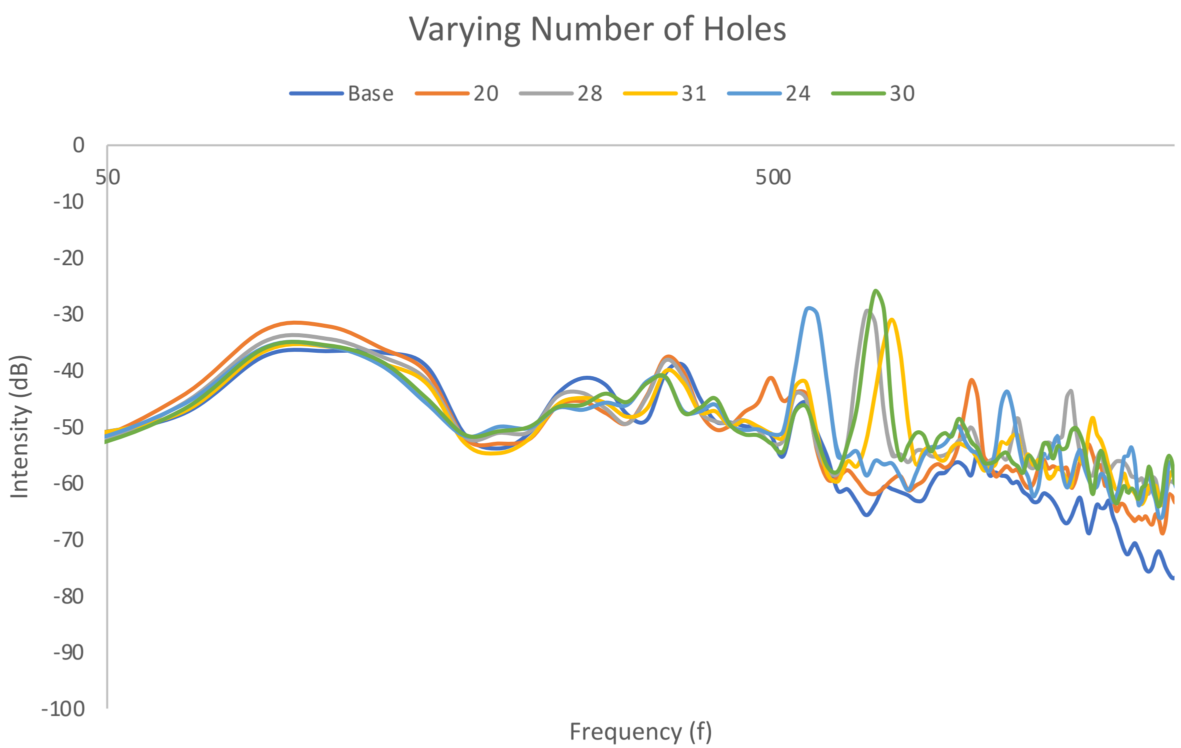
Qualitatively, the rotating disk continuously chops off the steady air flow from the pipe, resulting in the sound of a particular frequency.

The frequency can be calculated as

$$\boxed{f = v \cdot N}$$

where v is the rotational speed of disk

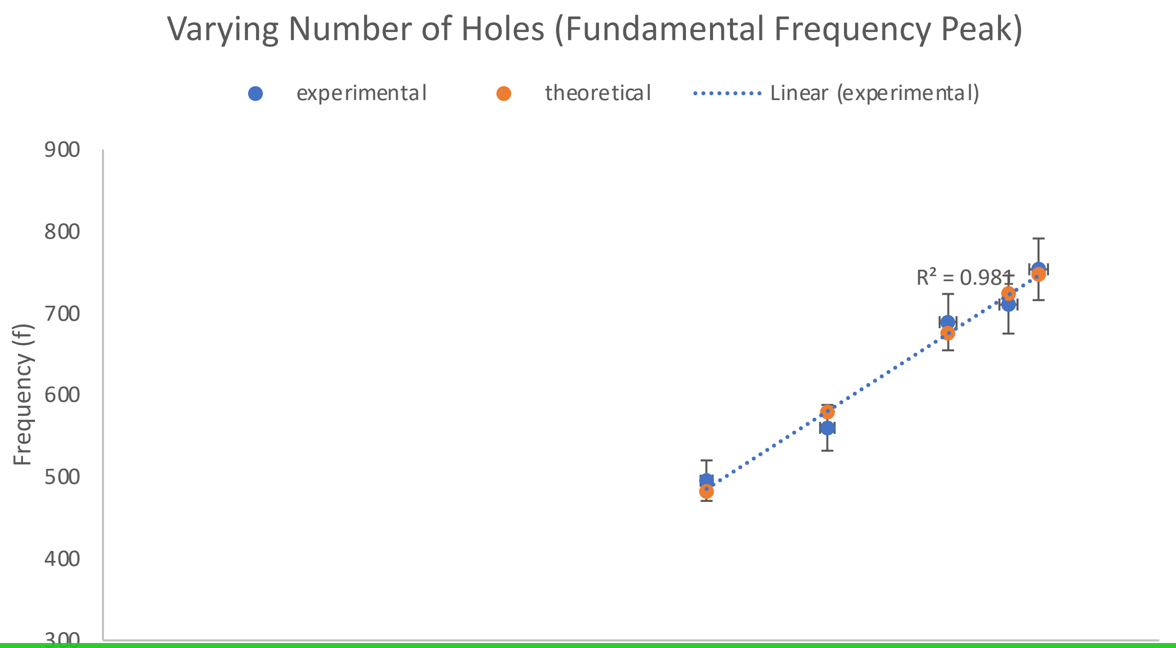
Vary Number of Holes



Air Flow

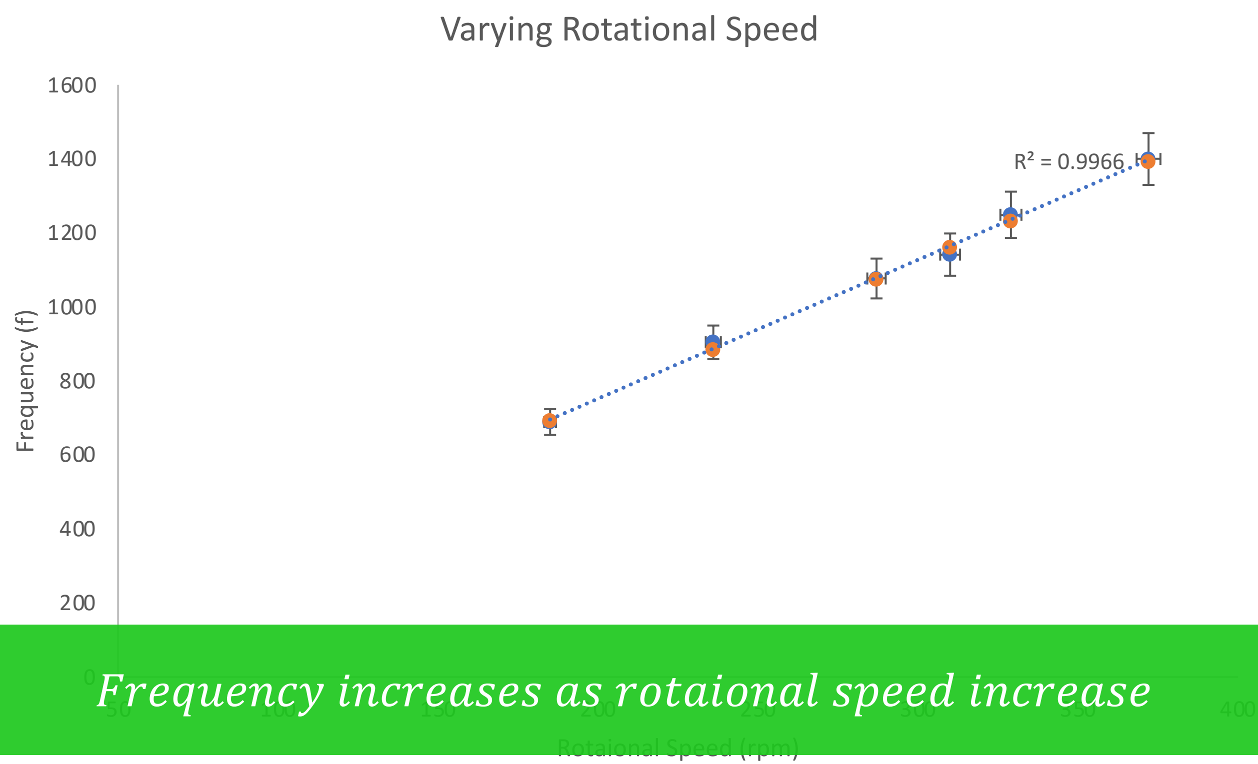
Sound Characteristics

Vary Number of Holes



Frequency increases as number of holes increase

Vary Rotational Speed



Frequency increases as rotaional speed increase

Air Flow

Sound Characteristics

Area Overlap ($l \geq 2a$)

Suppose the orifice never covers two holes at the same time,

$$A(x) = A(-x) = A(l - x) \qquad [5]$$

$$A(x) = 0, \qquad \boxed{(a \leq x \leq l - a)} \qquad \boxed{\text{Suppose } l \geq 2a} \qquad [5']$$

By expanding equation [5] into a Fourier's cosine series

$$A(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{2\pi n x}{l} \qquad [6]$$

Area Overlap $(l \geq 2a)$

Where coefficients a_n is defined as

$$a_n = \frac{2}{l}\int_0^l A(x)\cos\frac{2\pi nx}{l}\,dx$$

$$= \frac{4}{l}\int_0^a A(x)\cos\frac{2\pi nx}{l}\,dx \qquad [7]$$

$$\Downarrow$$

$$a_n = -\frac{2}{\pi n}\int_0^a \sin\frac{2\pi nx}{l}\frac{dA(x)}{dx}\,dx \qquad \text{for } n \geq 1 \qquad [8]$$

Area Overlap ($l \geq 2a$)

$$dA(x) = -2\left(\frac{1}{4}a^2 - \frac{1}{4}x^2\right)^{\frac{1}{2}} dx$$ [9]

Overlapping area between the orifice and the disk holes

$$a_n = \frac{2}{\pi n}\int_0^a \sin\frac{2\pi n x}{l}\,(a^2 - x^2)^{\frac{1}{2}}\,dx$$

$$= \frac{4a^3}{l}\frac{1}{z}\int_0^1 \sin zt(1 - t^2)^{\frac{1}{2}}\,dt$$ [10]

where $$z = \frac{2\pi n a}{l}$$ [11]

Area Overlap $(l \geq 2a)$

To resolve for all converging values, z, using Lommel's function

$$S(z) = \frac{1}{z}\int_0^1 \sin zt(1-t^2)^{\frac{1}{2}}\,dt \quad [12]$$

$$S(z) = \frac{\pi}{8}\sum_0^\infty \frac{(-)^n\left(\frac{1}{2}z\right)^{2n}}{\Gamma\left(n+\frac{3}{2}\right)\Gamma(n+\frac{5}{2})}; \quad [13]$$

$$S(z) = \frac{1}{1^2.3} - \frac{z^2}{1^2.3^2.5} + \frac{z^4}{1^2.3^2.5^2.7} \quad [14]$$

Area Overlap ($\frac{1}{2}l \leq a \leq l$)

Suppose the orifice sometimes covers two holes at the same time,

$$A(x), \qquad (0 \leq x \leq l - a)$$

$$A(x) + A(l - x), \qquad \left(l - a \leq x \leq \frac{1}{2}l\right);$$

Coefficients a_n now defines as

$$a_n = \frac{4}{l}\left(\int_0^{l-a} A(x) + \int_{l-a}^{\frac{1}{2}l} \{A(x) + A(l - x)\}\right)\cos\frac{2\pi nx}{l}\, dx$$

$$= \frac{4}{l}\int_0^{a} A(x)\cos\frac{2\pi nx}{l}\, dx \qquad [13]$$

Intensity of Harmonics

Since equation [7] and equation [13] are the same,

$$\frac{4a^3}{l}\left[\frac{1}{6}+\sum_{n=1}^{\infty} S\left(\frac{2\pi na}{l}\right)\cos\frac{2\pi nx}{l}\right], \qquad [14]$$

The intensities of harmonics is

$$I_n \propto \left\{nS\left(\boxed{\frac{2\pi na}{l}}\right)\right\}^2 \qquad [15]$$

Coefficient z

Intensity of Fundamental

Based on equation [15],

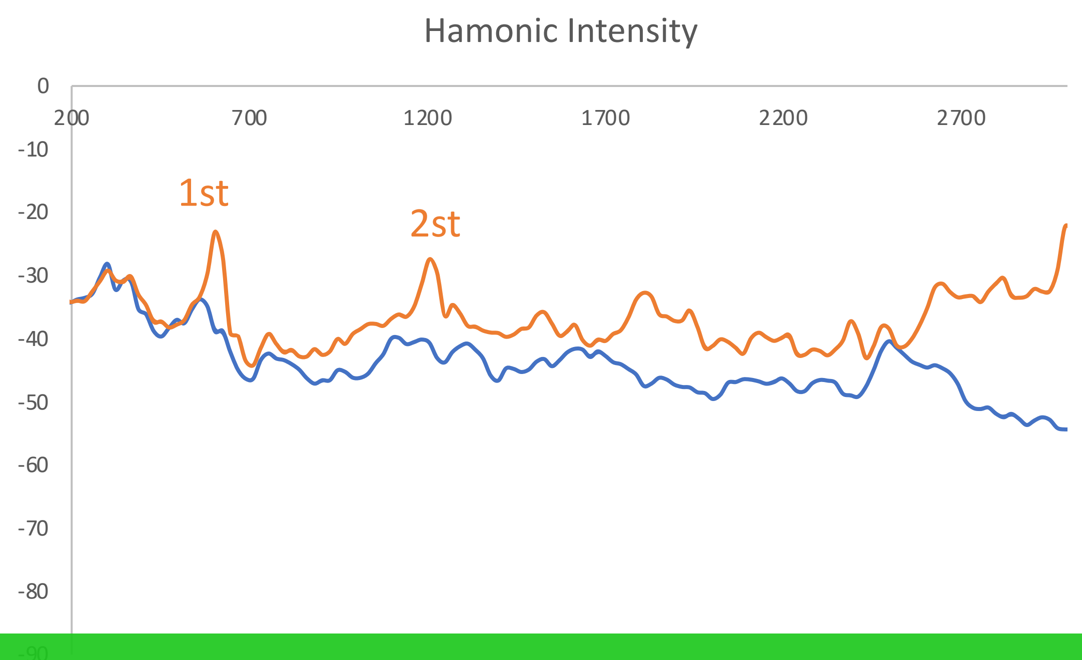
$$\frac{l}{2}\cdot\left(\frac{8\pi a^3}{L^2}\right)^2\sum_1^\infty n^2\left[S\left(\frac{2\pi n a}{l}\right)\right]^2=\int_0^l\left(\frac{dA(x)}{dx}\right)^2\,dx$$

$$=2\int_0^a(a^2-x^2)\,dx=4a^3/3 \qquad [16]$$

Thus the intensity in the fundamental can be expressed as

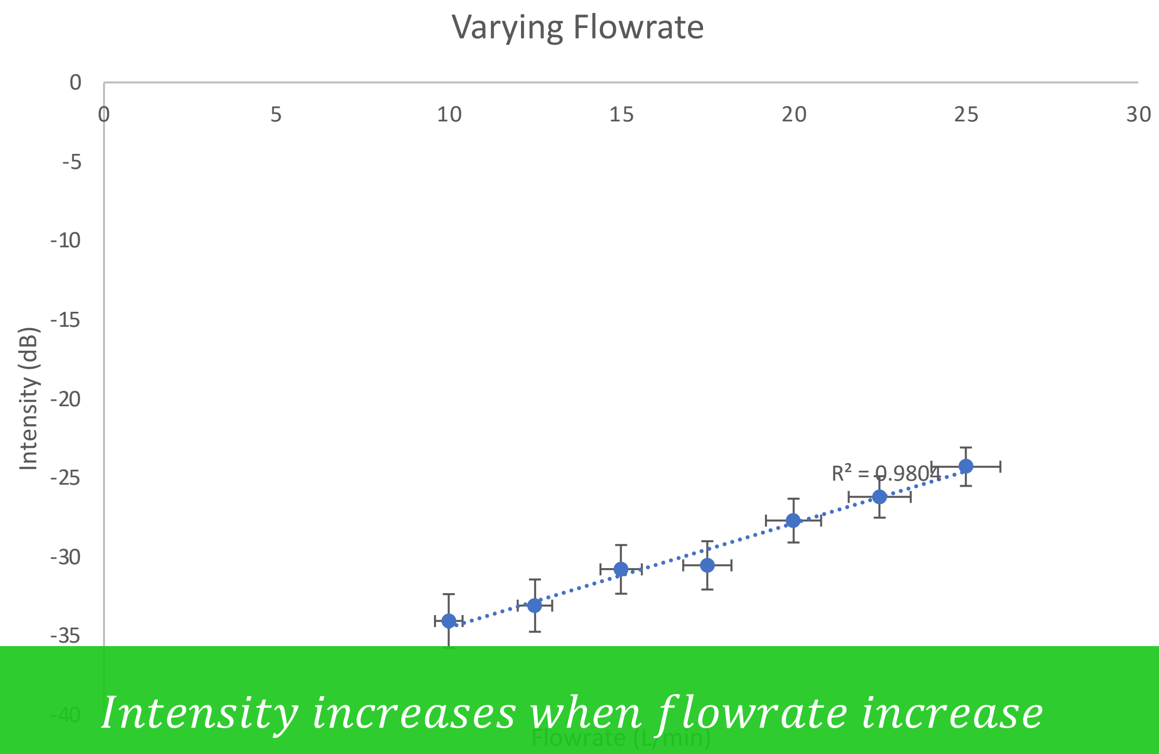
$$\frac{[S(y)]^2}{\sum_1^\infty[nS(ny)\}^2}=\frac{3y^3}{\pi}[S(y)]^2 \qquad \text{where } y=\frac{2\pi a}{l} \qquad [17]$$

Harmonic Intensity



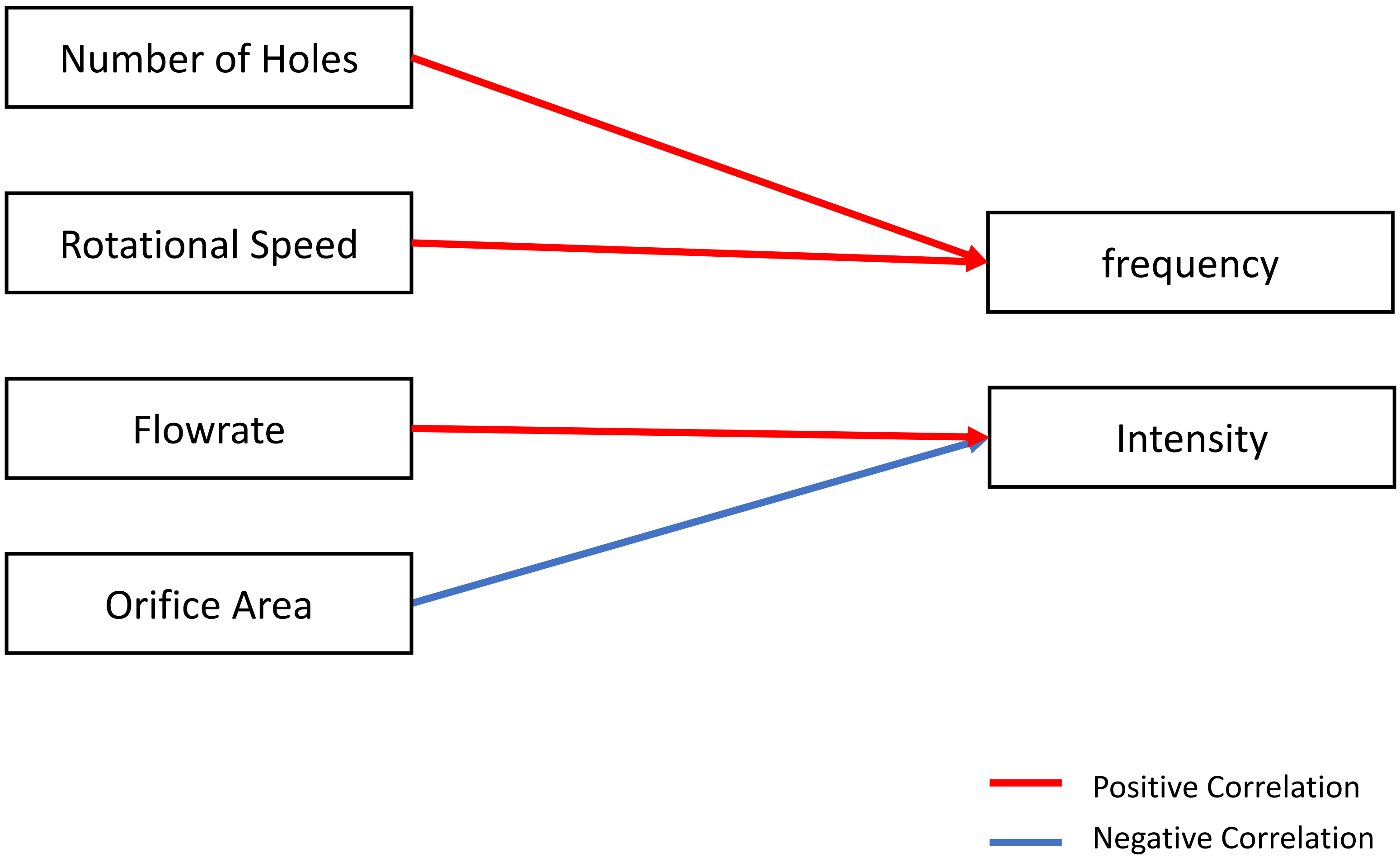
Concentrated energy decreases for each harmonics

Varying Flowrate



Intensity increases when flowrate increase

Key Parameters

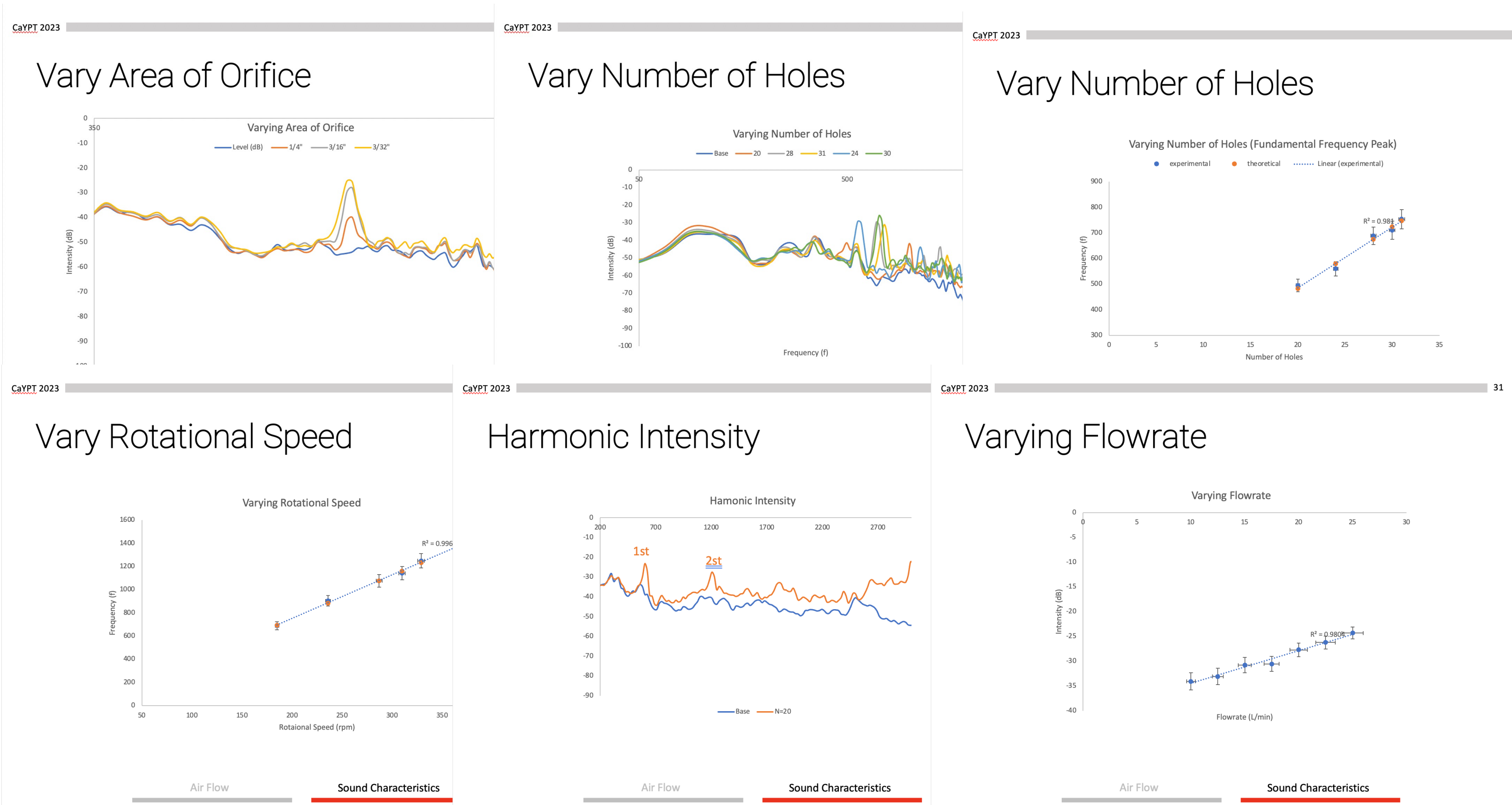

Number of Holes
Rotational Speed
Flowrate
Orifice Area
frequency
Intensity
Positive Correlation
Negative Correlation

Conclusion

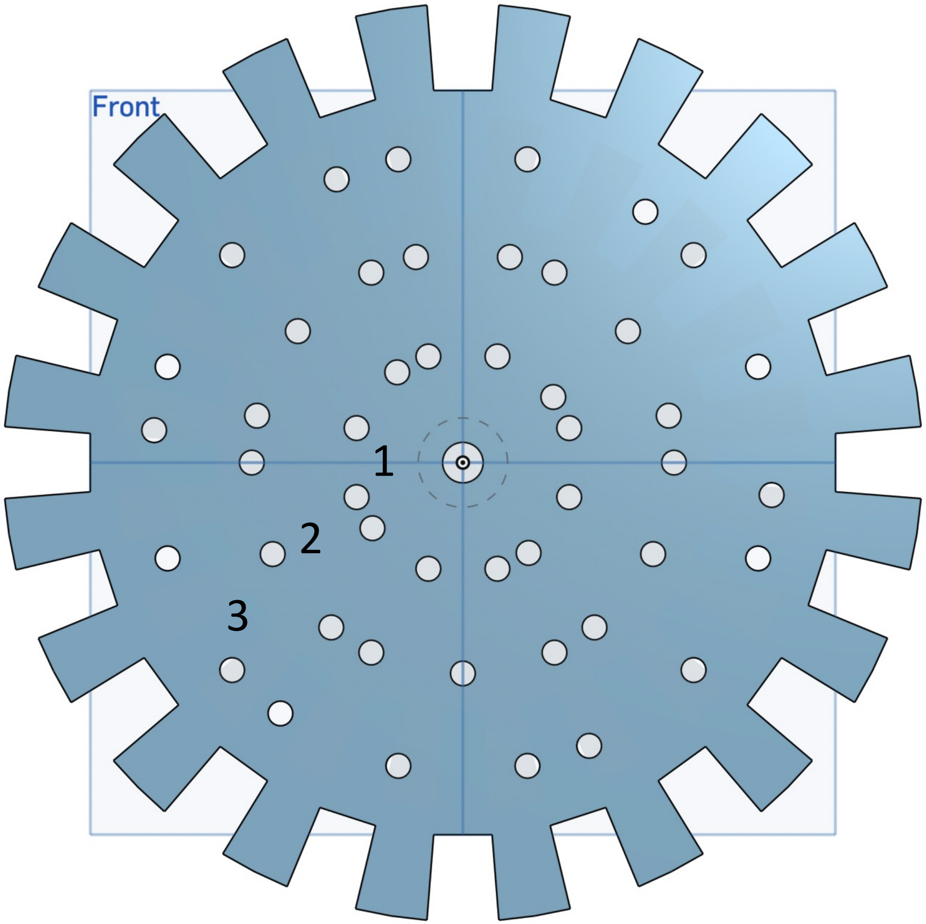
*“When a drop of a water mixture **(e.g. water-alcohol)** is deposited on the surface of a **hydrophobic liquid** (e.g. vegetable oil), the resulting drop may sometimes **fragment** into smaller droplets. Investigate the parameters that affect the **fragmentation** and the size of the final droplets."*

Number of Holes and **Rotational Speed** are the **most relevant** parameters to the intensity of the produced sound, while other parameters like **Flowrate** and **Orifice Area** have an impact as well. **Sound** occurs through the **chop** of air flow through the **rotating holes,** and a decrease in **intensity** is resulted from **loss of concentrated energy** for harmonics.

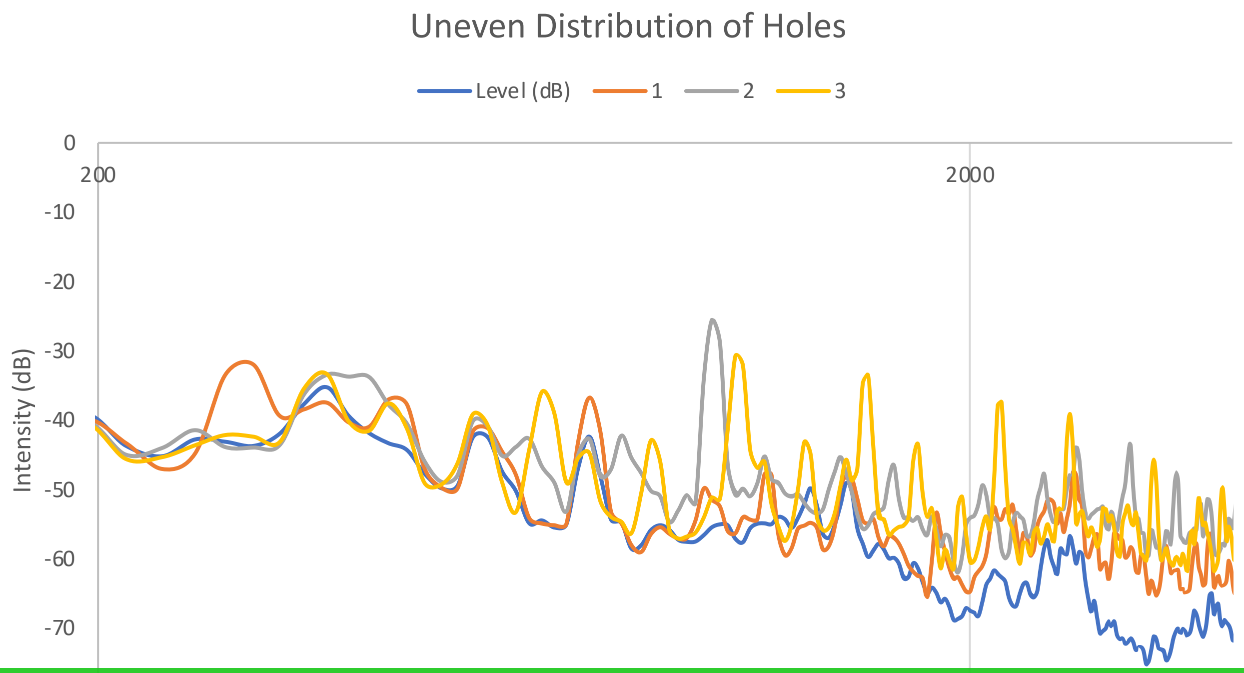
Conclusion


CaYPT 2023
Vary Area of Orifice
Varying Area of Orifice
CaYPT 2023
Vary Number of Holes
Varying Number of Holes
Frequency (f)
CaYPT 2023 21
Vary Number of Holes
Varying Number of Holes (Fundamental Frequency Peak)
Number of Holes
CaYPT 2023
Vary Rotational Speed
Varying Rotational Speed
Rotaional Speed (rpm)
CaYPT 2023
Harmonic Intensity
Hamonic Intensity
1st
2st
CaYPT 2023 31
Varying Flowrate
Varying Flowrate
Flowrate (L/min)
Air Flow
Sound Characteristics
Air Flow
Sound Characteristics
Air Flow
Sound Characteristics

Further Insights



Further Insights



Multiple frequency (fundamental + harmonic) observed

References

[1] *https://iypt.ru/wp-content/uploads/2022/09/Siren_Harmonics_and_a_Pure_Tone_Siren.pdf*

[2] *https://digital.kenyon.edu/cgi/viewcontent.cgi?article=1044&context=physics_publications*

Thank you for listening

Appendix