13. Ponyo’s Heat Tube

Yuezhen (Lily) Dong

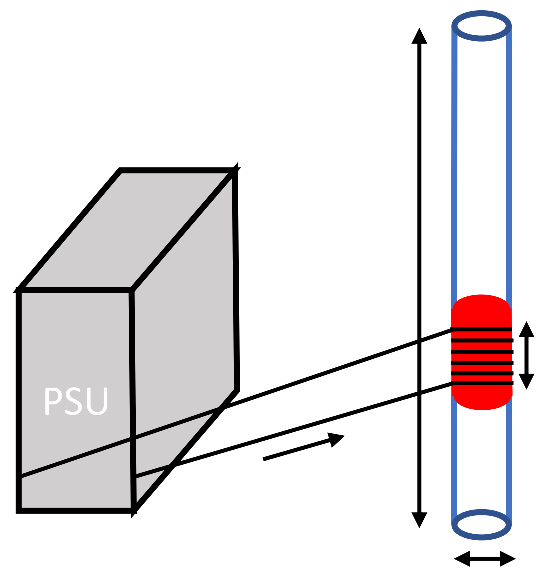
"A glass tube with a sealed top is ***filled with water*** *and* ***mounted vertically****. The bottom end of the tube is* ***immersed in a beaker of water*** *and a short segment of the tube is* ***heated****. Investigate and explain the* ***periodic motion of the water*** *and any* ***vapour bubbles*** *observed."*

Problem Statement

*"A glass tube with a sealed top is **filled with water** and **mounted vertically**. The bottom end of the tube is **immersed in a beaker of water** and a short segment of the tube is **heated**. Investigate and explain the **periodic motion of the water** and any **vapour bubbles** observed."*

Parameters:

1. *Heated segment Length*
2. *Length of tube*
3. *Diameter of tube*
4. *Initial temperature of water bath*
5. *Power input for heating*



Overview

1 **_Phenomenon_**
Reproduction of Phenomenon, Qualitative Explanation

2 **_Experimental Setup_**
Measurement Techniques, Camera Views

3 **_Theoretical Model_**
Qualitative Explanation, Quantitative Models

4 **_Key Parameter Interactions_**
Effects of Changing Physical Parameters

5 **_Conclusion_**
Further Insights and General Investigations

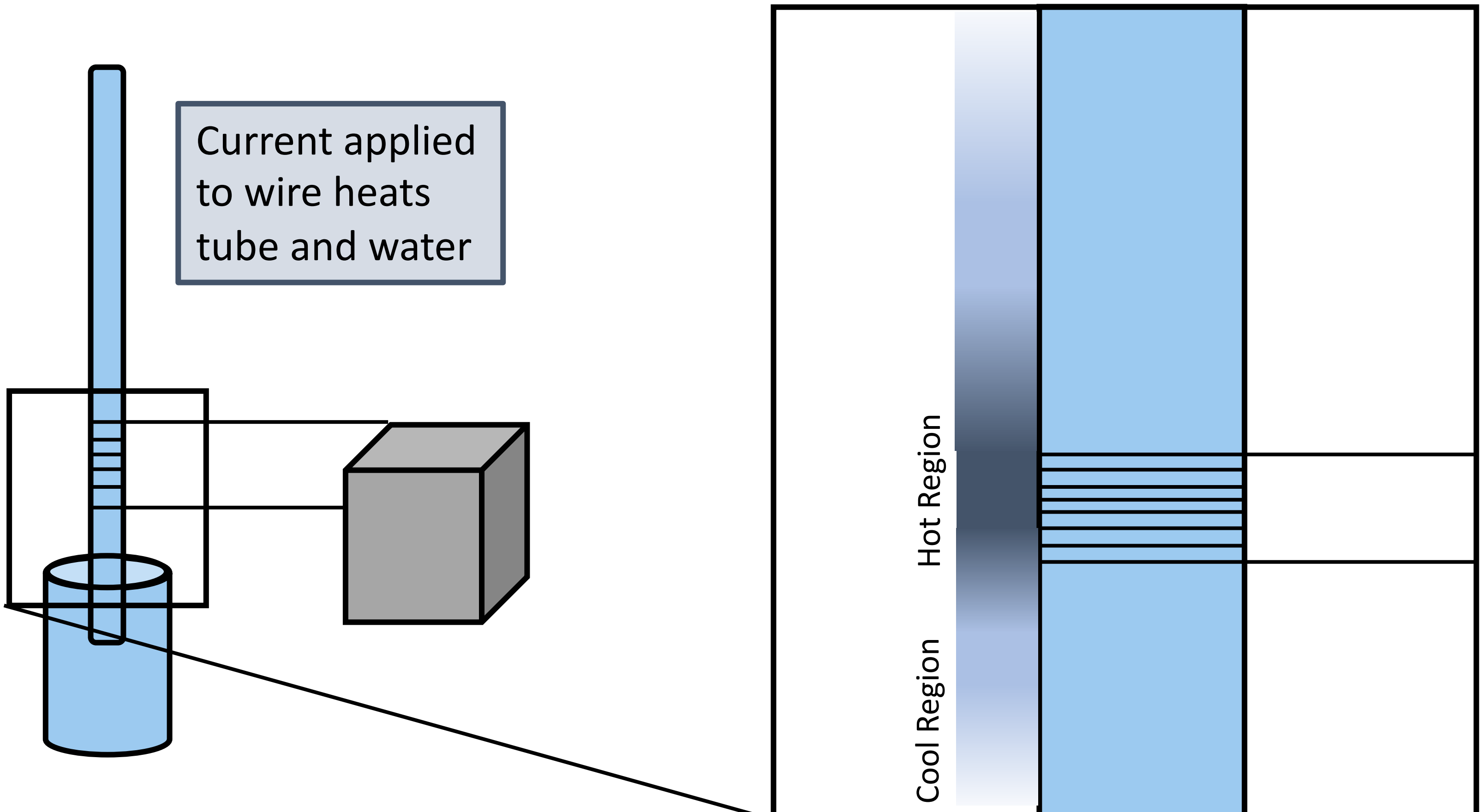
Phenomenon

Bubble formed

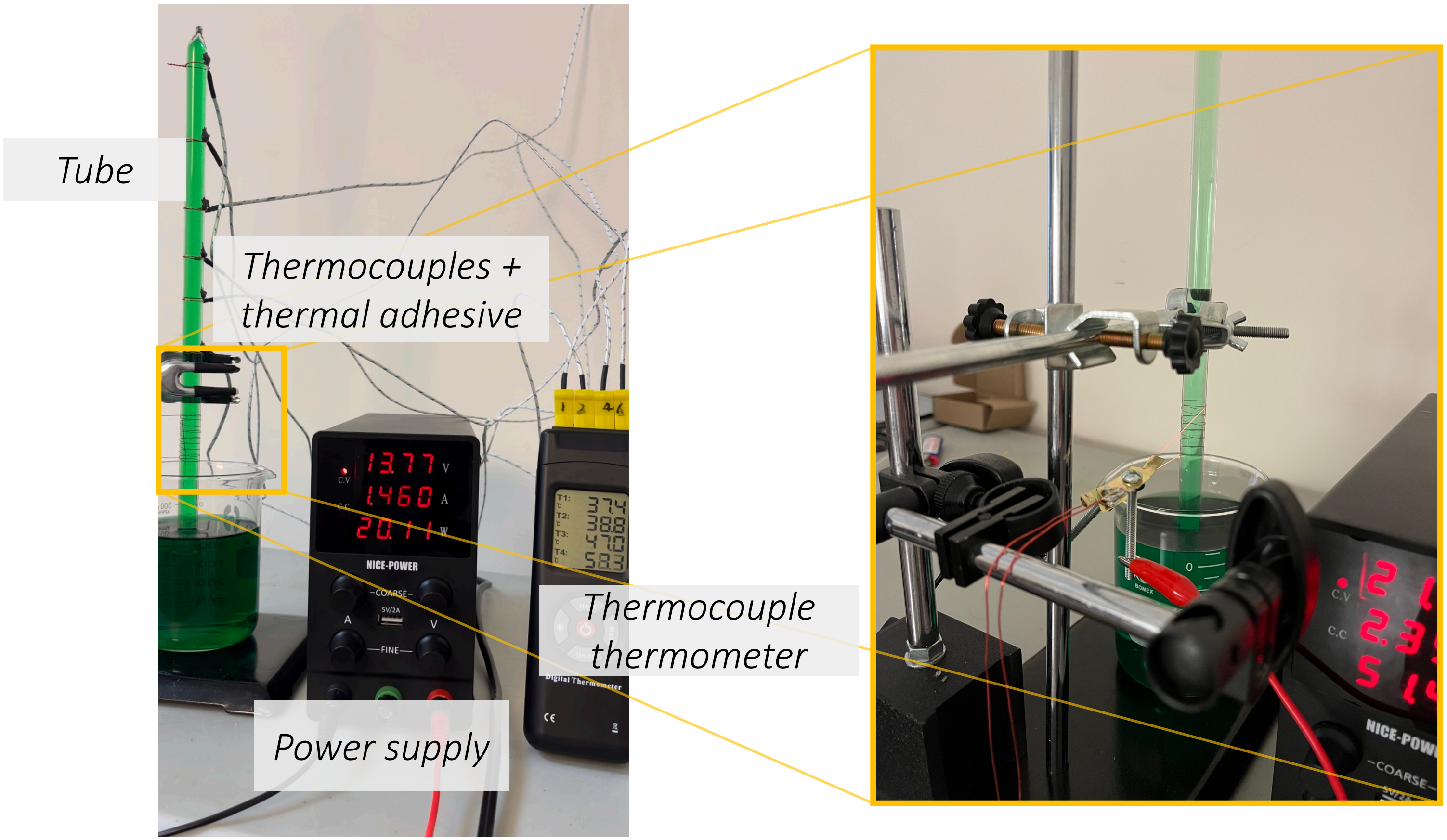
$$heat\ -boil\ -condense\&buoyancy\ -bubble\ disappear$$

Qualitative Explanation

Qualitative Explanation



Experimental Setup



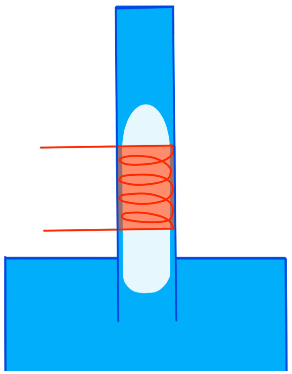
Experimental Measurement

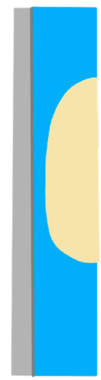
Thermocouple Thermometer ($\pm 0.3\%$)

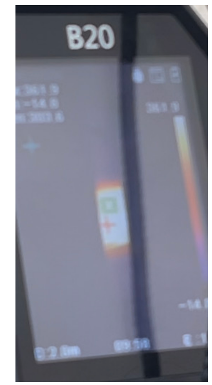
Beaker ($\pm 5\%$)

Theoretical Model

Theoretical Model

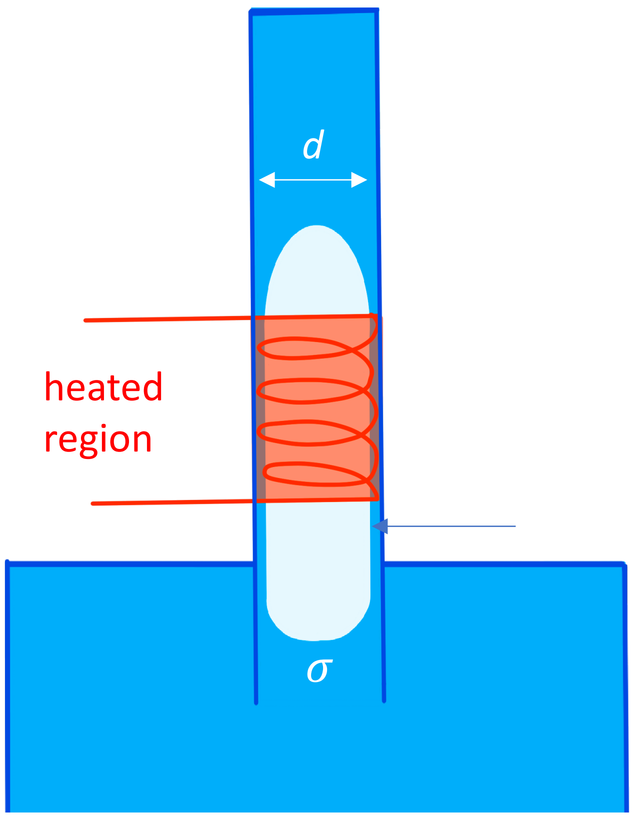






Dynamics → Heat Exchange → Superheated Vapour

Geometry



$d = diameter\ of\ the\ tube$
$\sigma = surface\ tension$

Assumptions

- *The impact of water based dye is negligible*
- *Water evaporation from the beaker is negligible*
- *Heat loss to the environment through water bath and beaker is negligible*
- *Inertial Effects are negligible*
- *Bubble is radially symmetrical*

Simple Rising Dynamics

Volume expansion coefficient justifies the variation of density with temperature

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$$

The change in density can then be approximated as

$$\beta \approx -\frac{1}{\rho}\frac{\Delta \rho}{\Delta T} \to \Delta \rho \approx -\rho \boxed{\beta \Delta T}$$

fraction of volume change that corresponds to temperature change

Balanced Rising Dynamics

Considering buoyancy and viscous forces, the Grashof number can be expressed as

$$\boxed{Gr = \frac{g\beta(T_s - T_\infty)\pi r^2 h}{v^2}}$$

Balanced buoyancy and viscous forces

Where
g = gravitational acceleration
β = coefficient of volume expansion
r = radius of bubble
h = height of the bubble
v = kinematic viscosity of the fluid

Primary force balance on a rising vapour bubble

Natural Convection

Rayleigh number that drives the natural convection can be expressed as:

$$\boxed{Ra = \frac{\rho_l g \beta_l (T^* - T_0)(L-h)^3}{D_v \nu_l} \approx 10^9}$$

Balanced buoyancy and viscous forces

$Ra > 10^8$ implies strong presence of convection

Ratio of conductive to convective heat transfer can be expressed using Nusselt number,

$$Nu = \frac{hL}{k} \approx 685 \gg 1$$

where
h = convective thermal heat transfer coefficient
k = thermal conductivity

Nu > 100 *implies convection dominance over conduction*

Capillary Force

Considering the capillary force from the tube

$$\vec{F}_{drag} \approx \frac{(\mu \cdot U)}{r} \left(\frac{l_s}{R}\right) (\pi r^2)$$

where l_s is the length of liquid section

$$\vec{F}_{cap} \approx \sigma \cdot (2\pi r)$$

The scaling ratio of drag and capillary force can then be deduced as

$$\left(\frac{\vec{F}_{drag}}{\vec{F}_{cap}}\right) \approx \underbrace{\boxed{\frac{(\mu \cdot U)}{\sigma}}}_{\text{Capillary number}} \left(\frac{l_s}{2r}\right) \approx Ca \left(\frac{l_s}{d}\right)$$

Pressure Drop

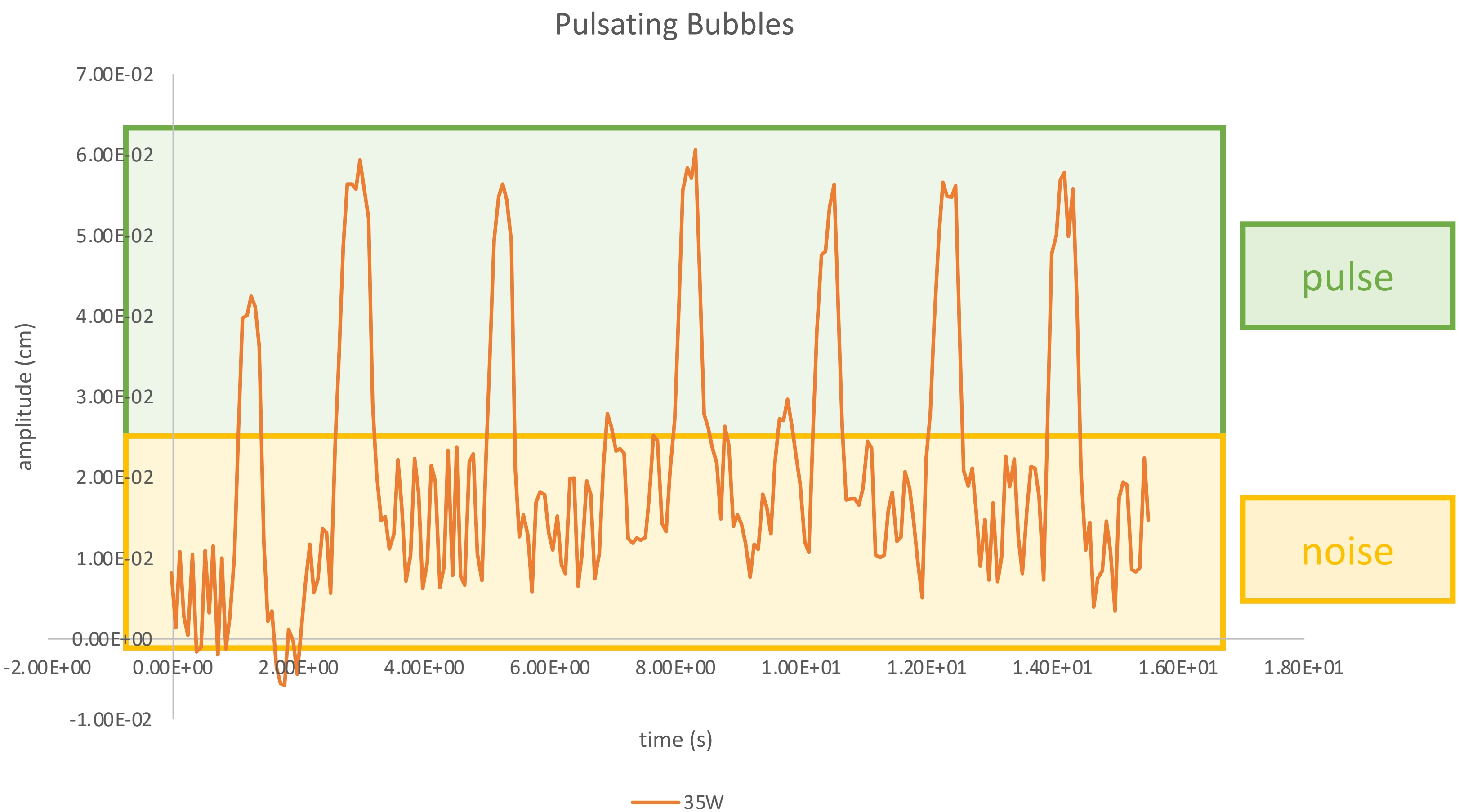
Due to the phase change, the pressure inside the tube changes,

$$\Delta P_f = k(\rho_l \cdot U_s \cdot \mu_l)^{-0.25} \cdot (\rho_l \cdot U_s^2) \cdot (\frac{2l_s}{d})$$

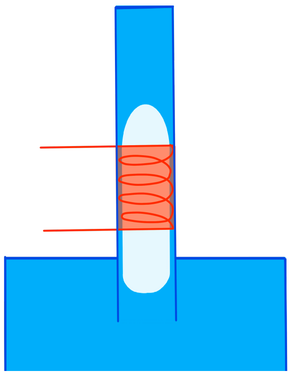
where k is a dimensionless constant

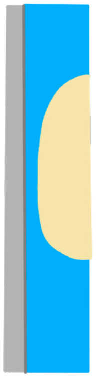
$With\ the\ formation\ \&\ Fading\ of\ bubbles, the\ pressure\ constantly\ changes$

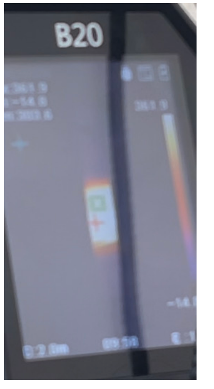
Preliminary Experiment Results



Theoretical Model

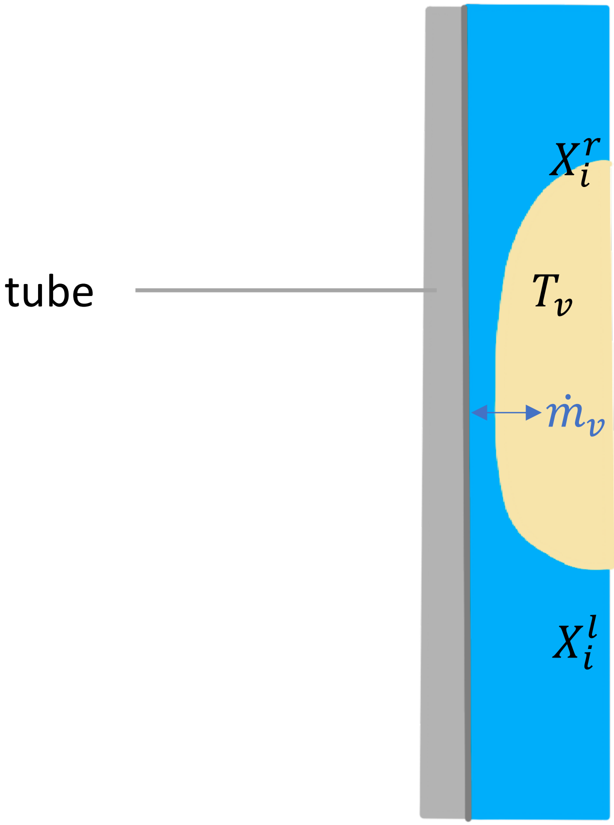






Dynamics → Heat Exchange → Superheated Vapour

Geometry



T_v = *Temperature of vapor bubble*
$\dot{m}_v$ = *mass exchange rate in vapor bubble*
Ω = *volume of vapor*
m = *mass of vapor*

Assumptions

- *The impact of water based dye is negligible*
- *Water evaporation from the beaker is negligible*
- *Heat loss to the environment through water bath and beaker is negligible*
- *Liquid-gas interface has the saturation temperature*

System Characterization

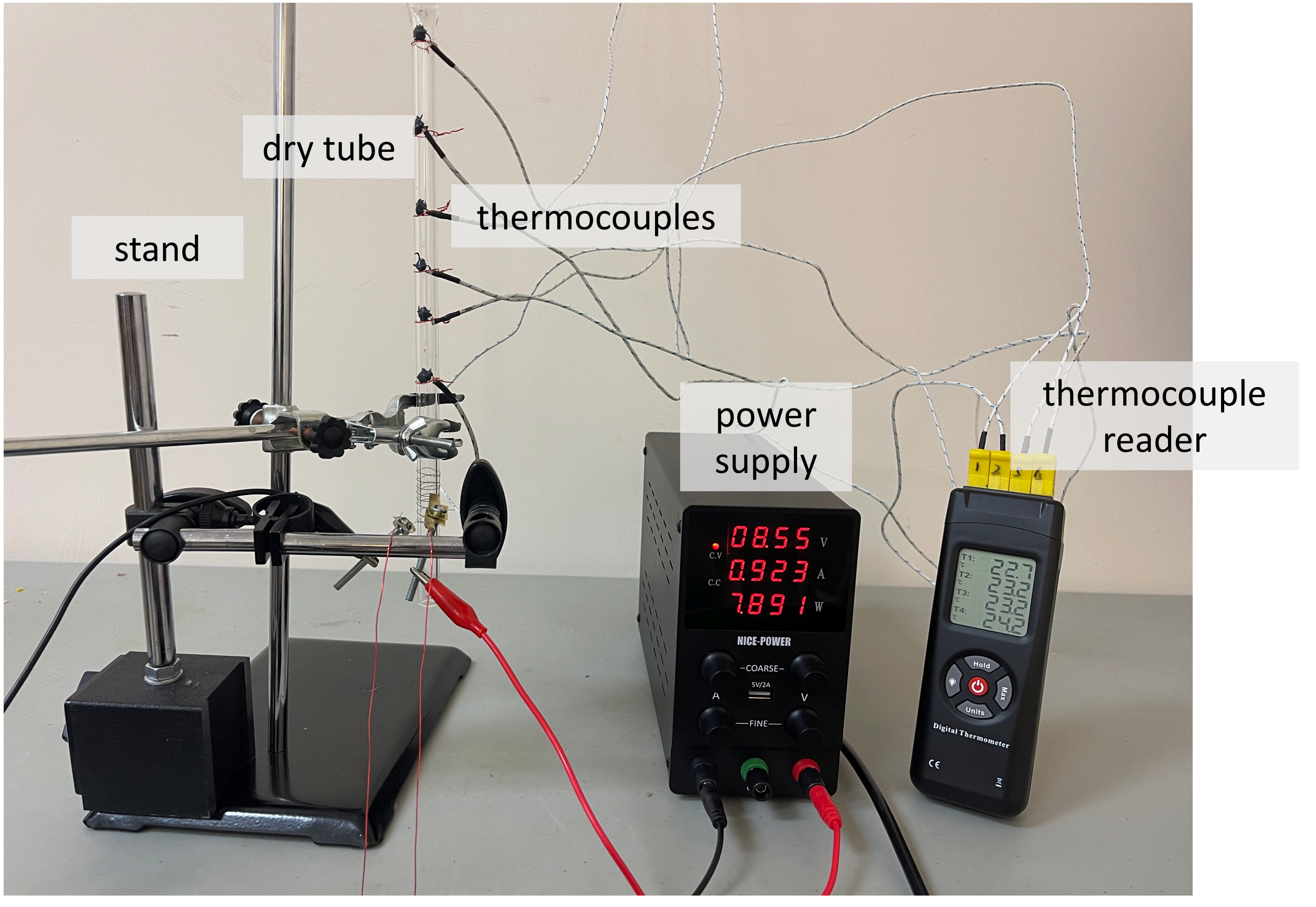
The rate of heat lost after the empty tube reaches thermal equilibrium can be expressed as:

$$P_{in} = \dot{Q}_{lost} = \int_0^L d\dot{Q} = \boxed{\int_0^L k(T(x) - T_\infty)dx}$$

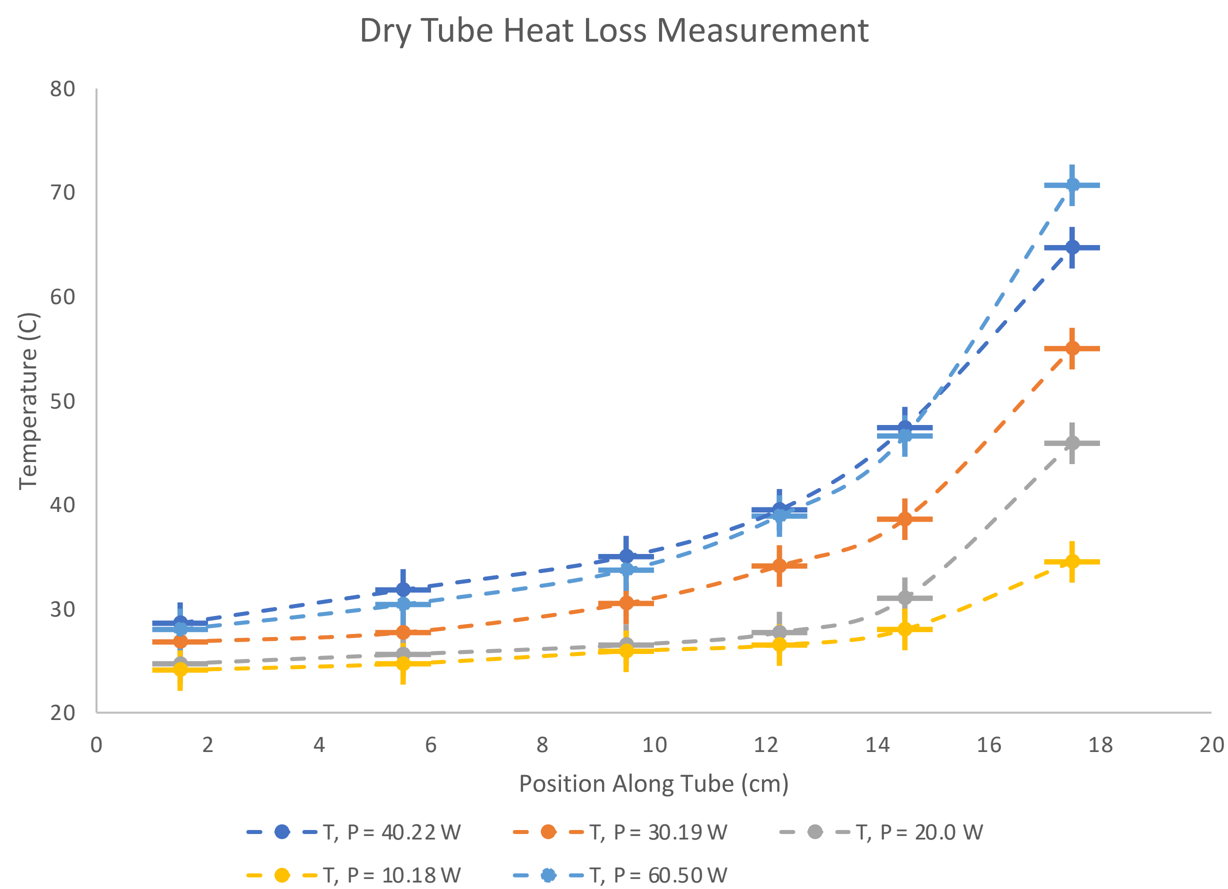
heat transfer coefficient

$$\text{where } k = \frac{d\dot{Q}}{dT}$$

Dry Tube Setup



System Characterization



Heat Exchange

The heat exchange in the liquid can be expressed as

$$\frac{\partial T_l}{\partial t} = D_l \frac{\partial^2 T_l}{\partial x'^2} + \frac{\pi d U_l}{\rho_l c_l S} [T_w(x' - T_l(x')]$$

The temperature distribution along the inner tube wall can be described as:

$$\boxed{\frac{\partial T_w}{\partial t} = D_w \frac{\partial^2 T_w}{\partial x^2} + \frac{j_w}{\rho_w c_w}}$$

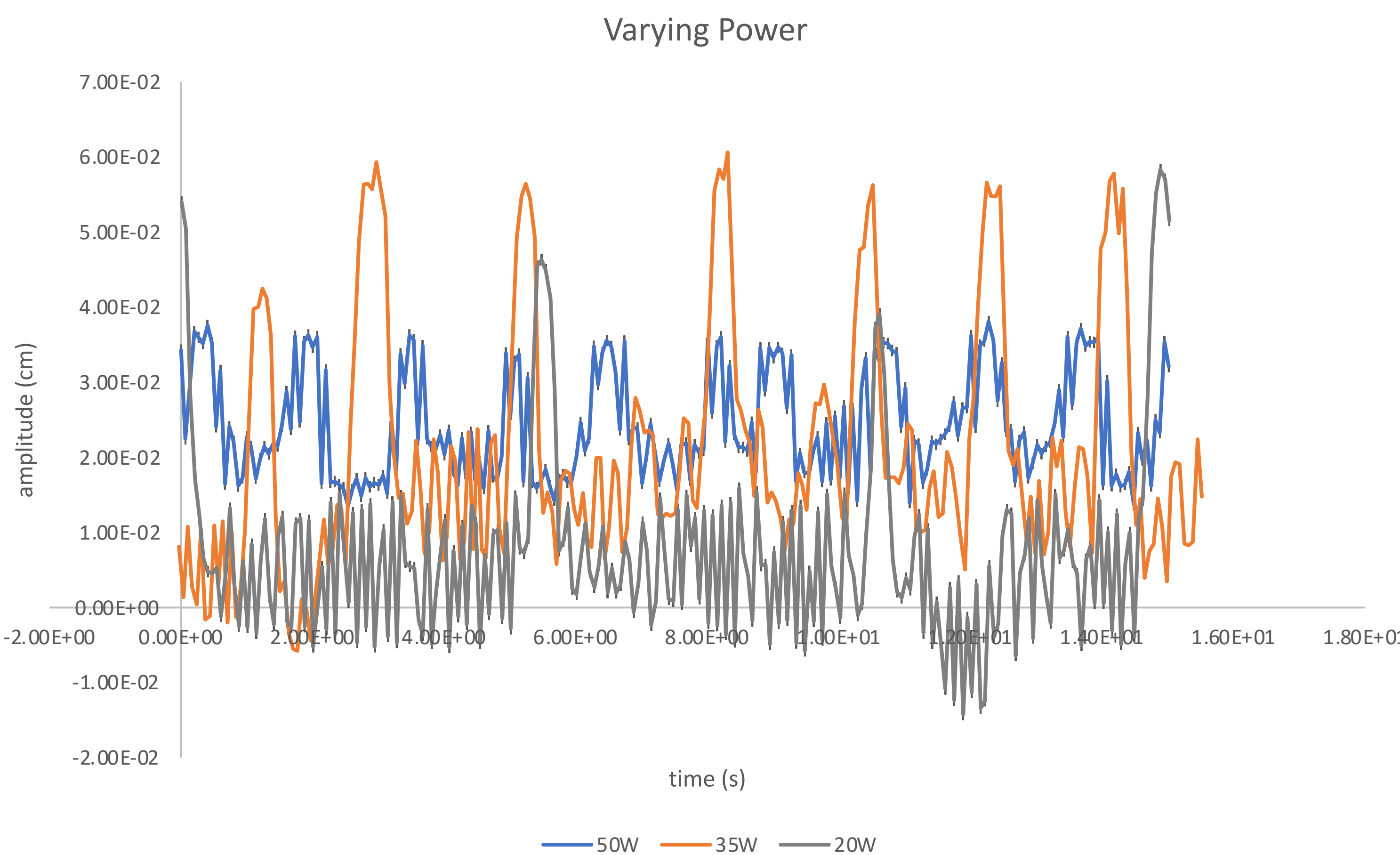
1D thermal diffusion equation

Phase Change

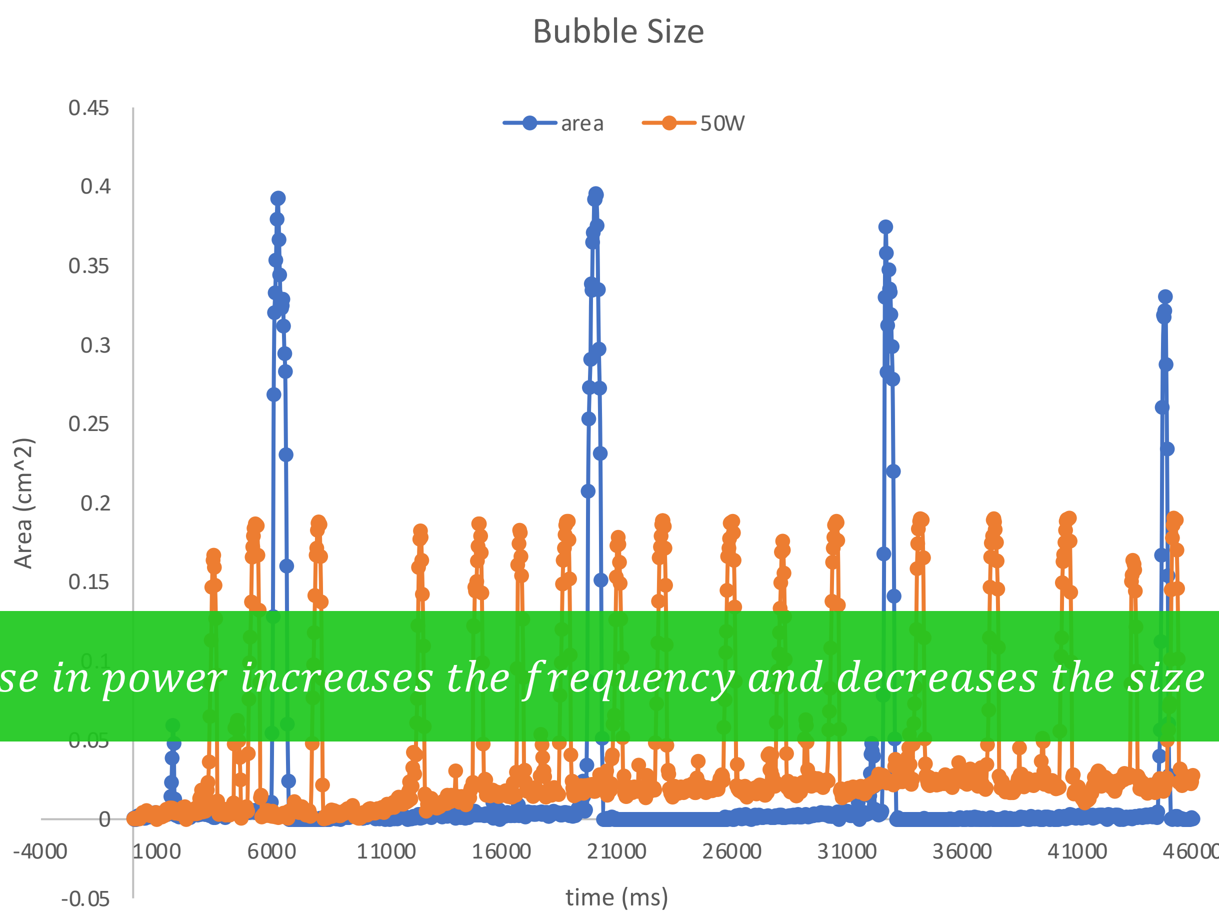
The phase change rate can be expressed as

$$J_{f,i}^{n} = \frac{2\pi U_i^n r_i^n}{L} = \int_{X_i^l}^{X_i^r} \left[T_{\mathrm{w}}(x) - T_{\mathrm{sat},i}\right] dx$$

Experimental Result

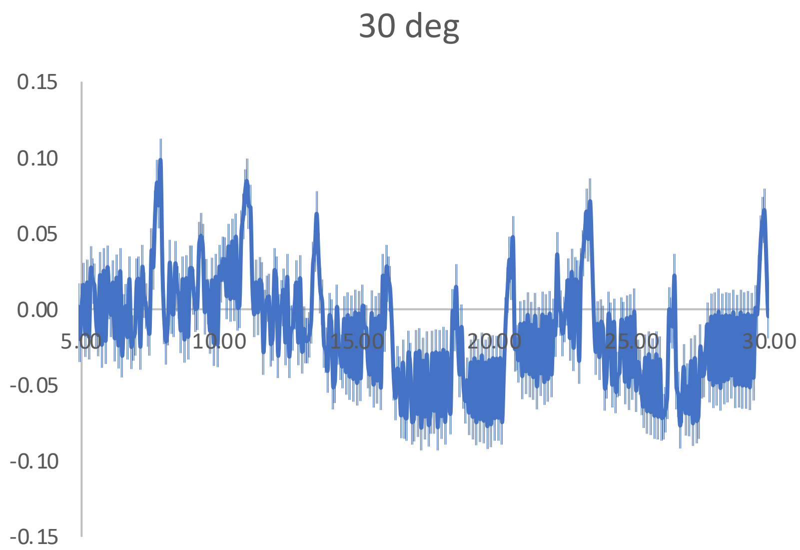


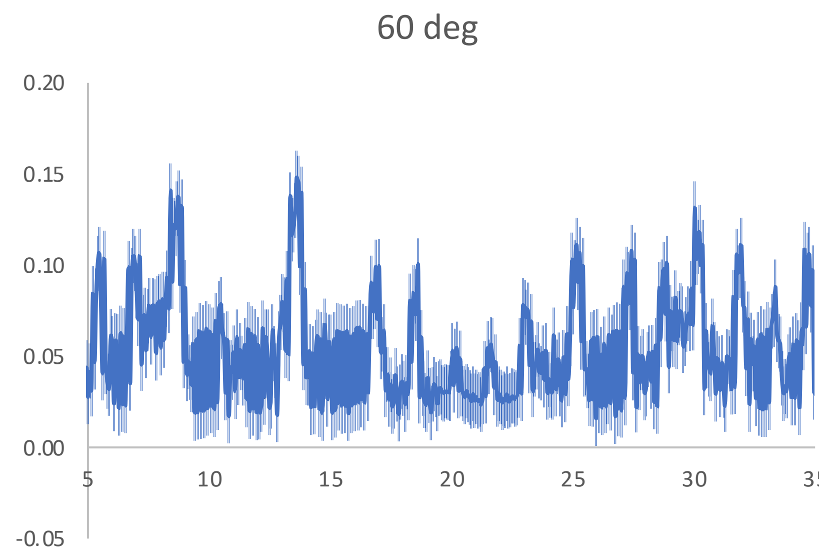
Experimental Result



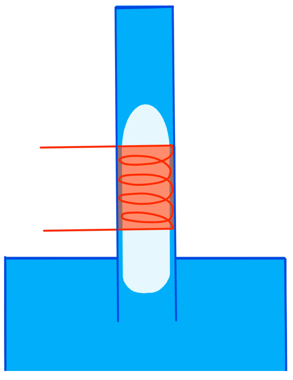
Increase in power increases the frequency and decreases the size of bubbles

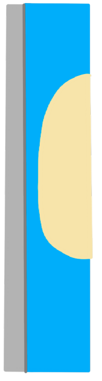
Experimental Result

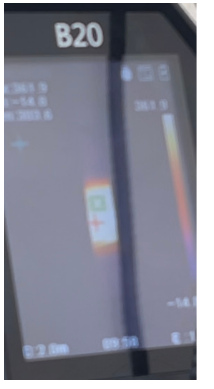




Theoretical Model







Dynamics → Heat Exchange → Super Heated Vapour

Assumptions

- *The impact of water based dye is negligible*
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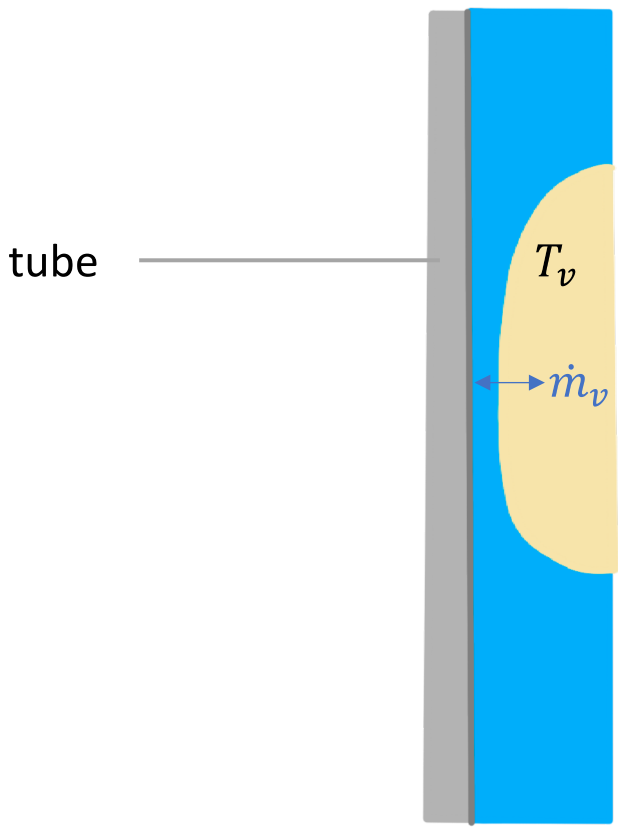
- *Heat loss to the environment through water bath and beaker is negligible*

Geometry



T_v = *Temperature of vapor bubble*
$\dot{m}_v$ = *mass exchange rate in vapor bubble*
Ω = *volume of vapor*
m = *mass of vapor*

Super Heated Vapor

The thermal energy of the vapor can be described as:

$$c_{vv}m_v\dot{T}_v = R_g T_v \dot{m}_v - p_v \dot{\Omega}_v$$

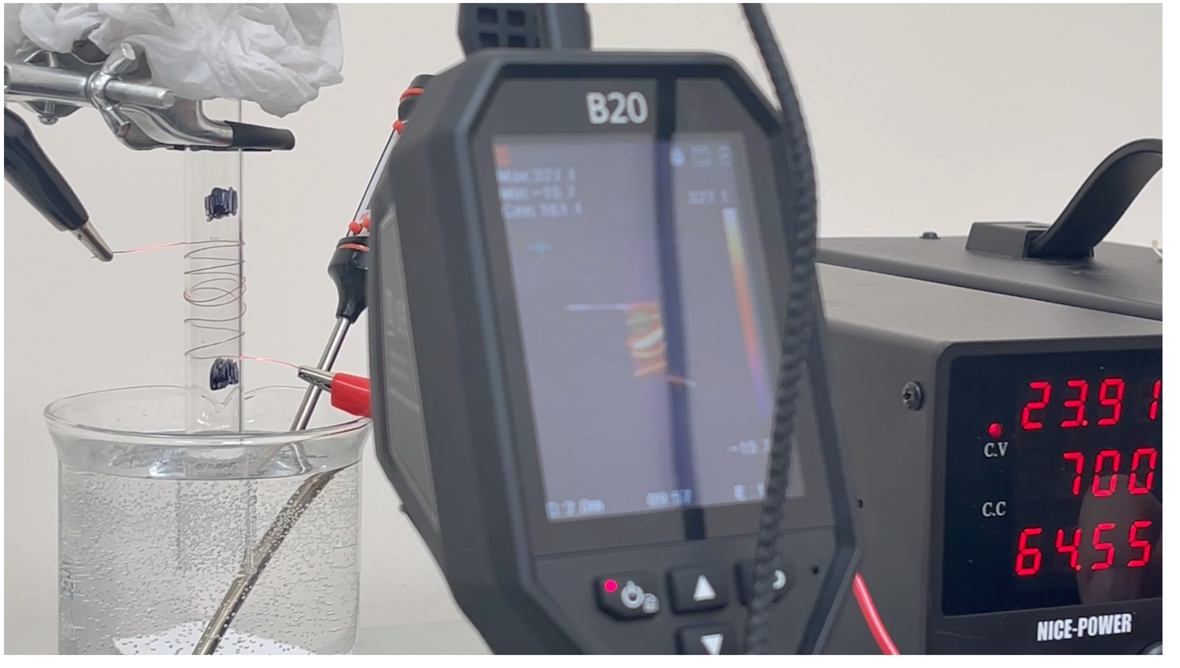
where c_{vv} is the vapor specific heat at constant volume

The energy equation of the system can be expressed as:

$$c_{vv}m_v\dot{T}_v = \boxed{\pi d[U_e L_{ve}(T_e - T_v) + U_c L_{vc}(T_c - T_v)]} - p_v \dot{\Omega}_v$$

where
U_e is the heat transfer coefficient
L_{ve} is the length of bubble in the evaporator

Experiment Demonstration

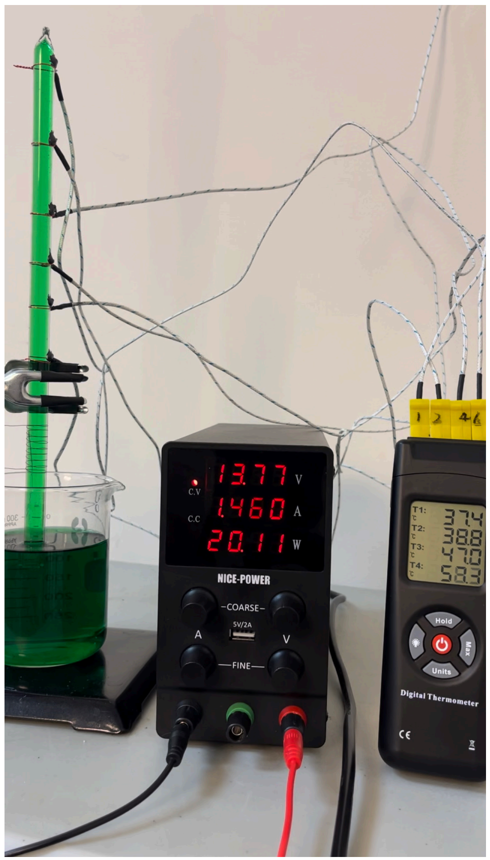


Dynamics

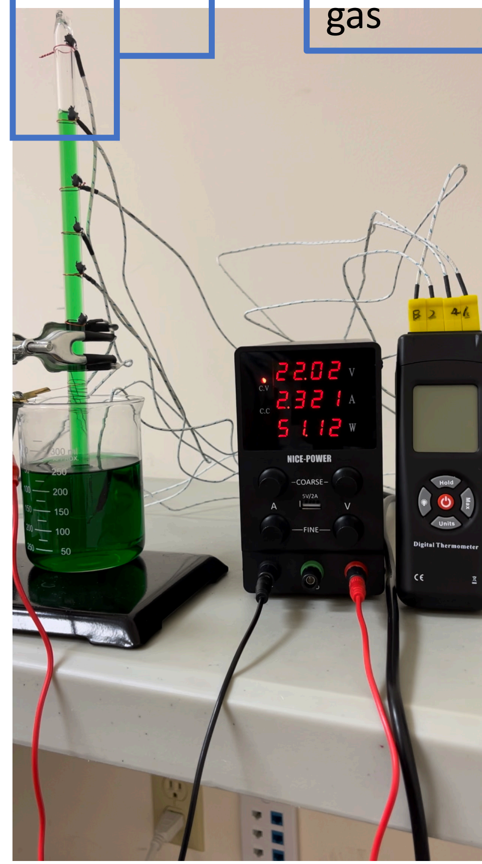
Heat Exchange

Superheated Vapour

Further Insights



t = 0 min

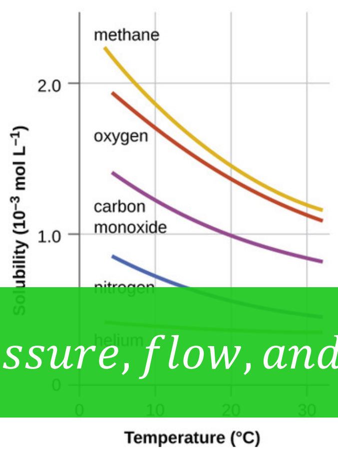


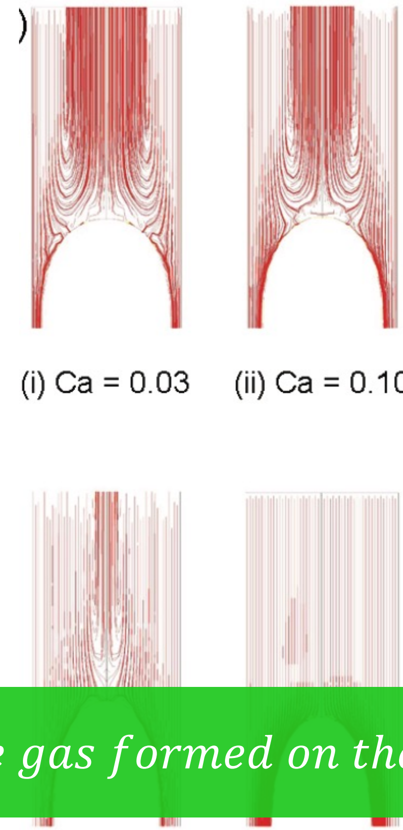
accumulated gas

t = 40 min

Further Insights

$$F_{\tau,\text{sing}} = \begin{cases} \frac{1}{2}A\rho u^2 \ \text{if} \ u > 0 \quad \text{(enlargement)} \\ \frac{1}{4}A\rho u^2 \ \text{if} \ u < 0 \quad \text{(contraction)} \end{cases}$$



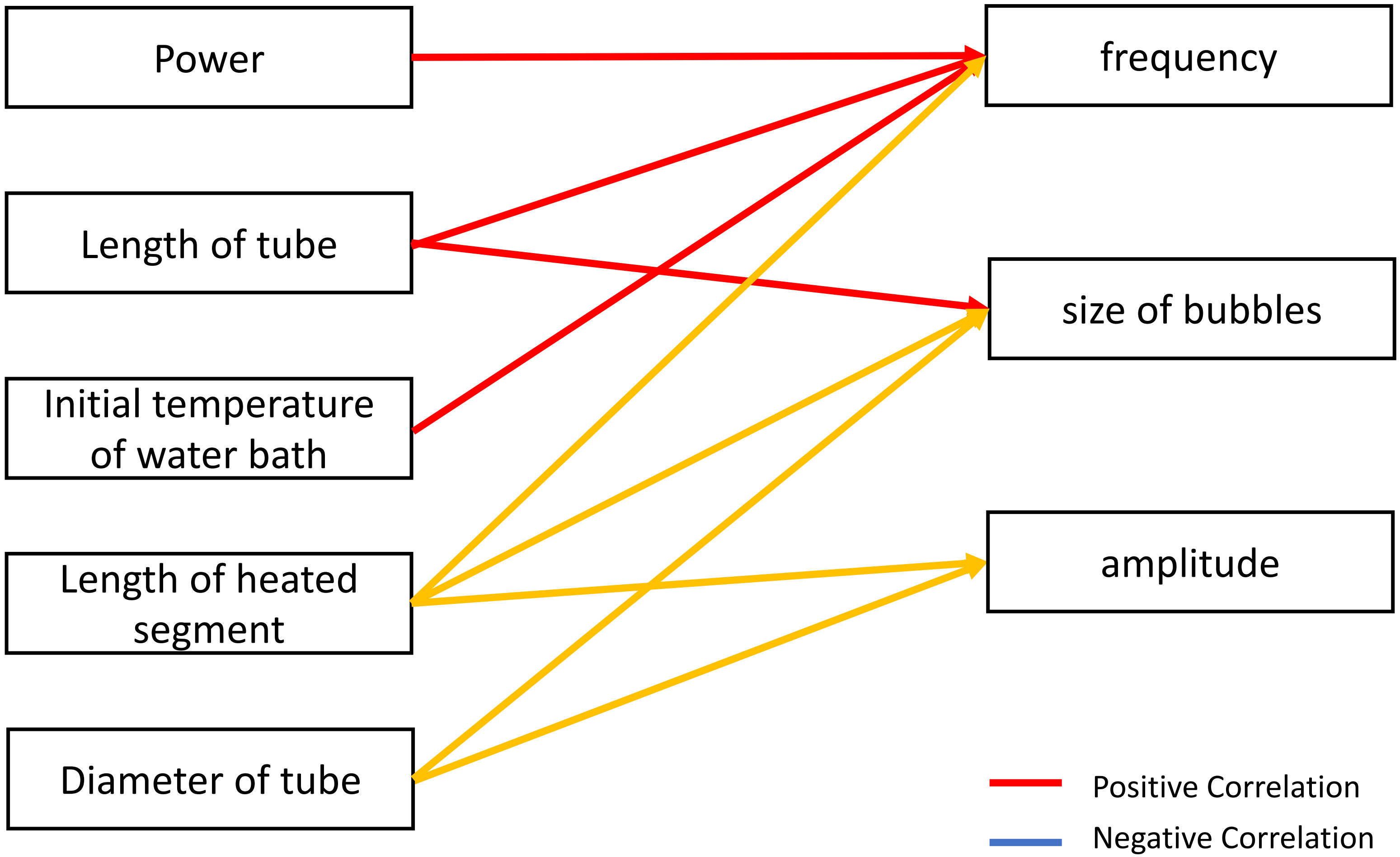


(i) Ca = 0.03 (ii) Ca = 0.10

(iii) Ca = 0.3 (iv) Ca = 1.34

The pressure, flow, and dissolved gases may cause gas formed on the top

Key Parameters

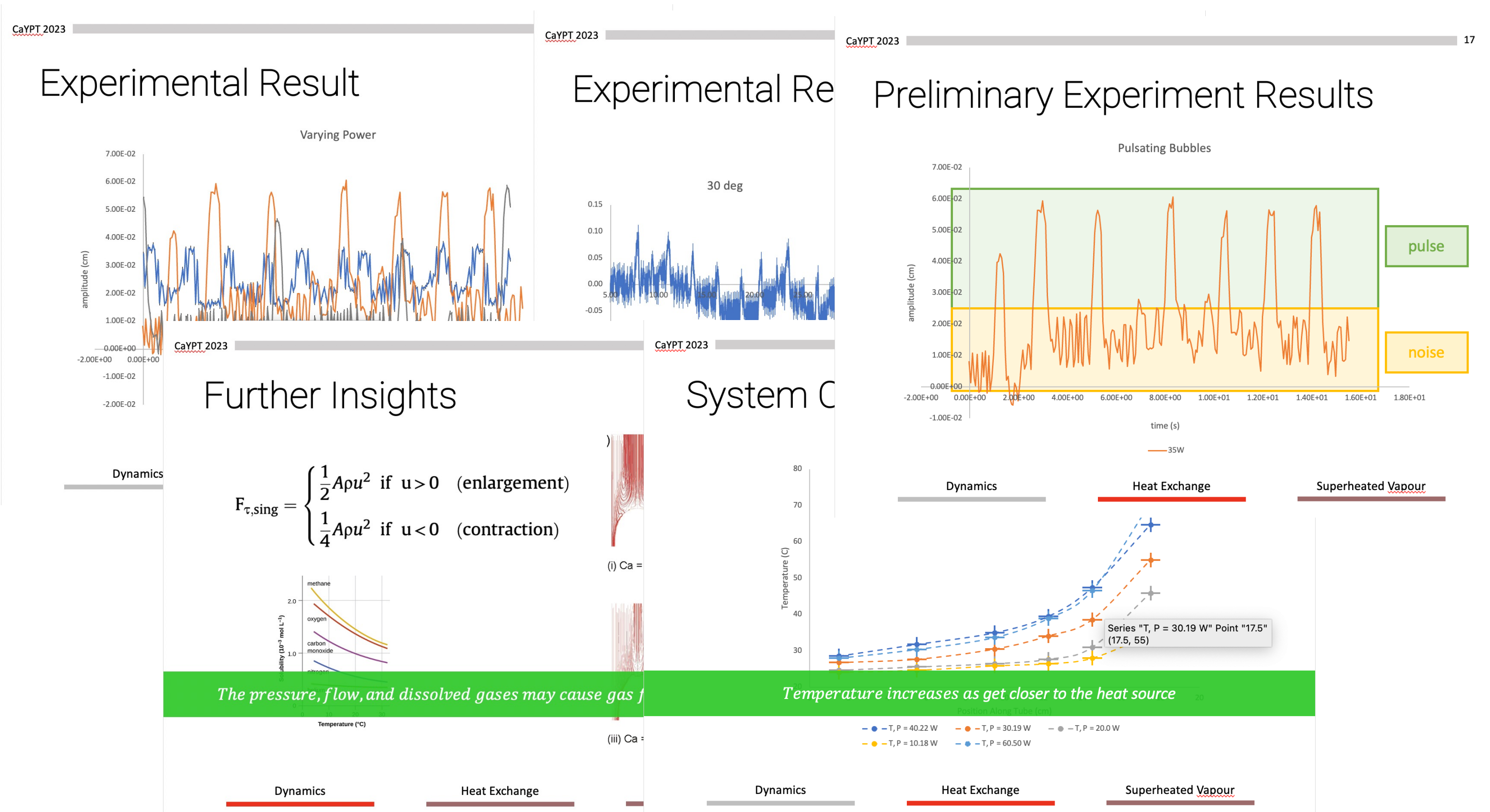

Power
Length of tube
Initial temperature of water bath
Length of heated segment
Diameter of tube
frequency
size of bubbles
amplitude
Positive Correlation
Negative Correlation

Conclusion

*"A glass tube with a sealed top is **filled with water** and **mounted vertically**. The bottom end of the tube is **immersed in a beaker of water** and a short segment of the tube is **heated**. Investigate and explain the **periodic motion of the water** and any **vapour bubbles** observed."*

Input power is the **most relevant** parameter to the problem, while other parameters like **dimensions of tube, initial temperature of water bath** and **heat segment length** have an impact as well. **Periodic vapor bubble** occurs through the **boiling of water** caused by **segment heating,** and **higher heat exchange** leads to **greater frequency.**

Conclusion


CaYPT 2023
Experimental Result
Varying Power
CaYPT 2023
Experimental Re
30 deg
CaYPT 2023 17
Preliminary Experiment Results
Pulsating Bubbles
pulse
noise
time (s)
amplitude (cm)
35W
Dynamics
Heat Exchange
Superheated Vapour
CaYPT 2023
Further Insights
$F_{\tau,sing} = \begin{cases} \frac{1}{2}A\rho u^2 \text{ if } u>0 \text{ (enlargement)} \\ \frac{1}{4}A\rho u^2 \text{ if } u<0 \text{ (contraction)} \end{cases}$
The pressure, flow, and dissolved gases may cause gas f
CaYPT 2023
System C
Temperature (C)
Series "T, P = 30.19 W" Point "17.5"
(17.5, 55)
Temperature increases as get closer to the heat source
T, P = 40.22 W
T, P = 30.19 W
T, P = 20.0 W
T, P = 10.18 W
T, P = 60.50 W
Dynamics
Heat Exchange
Superheated Vapour

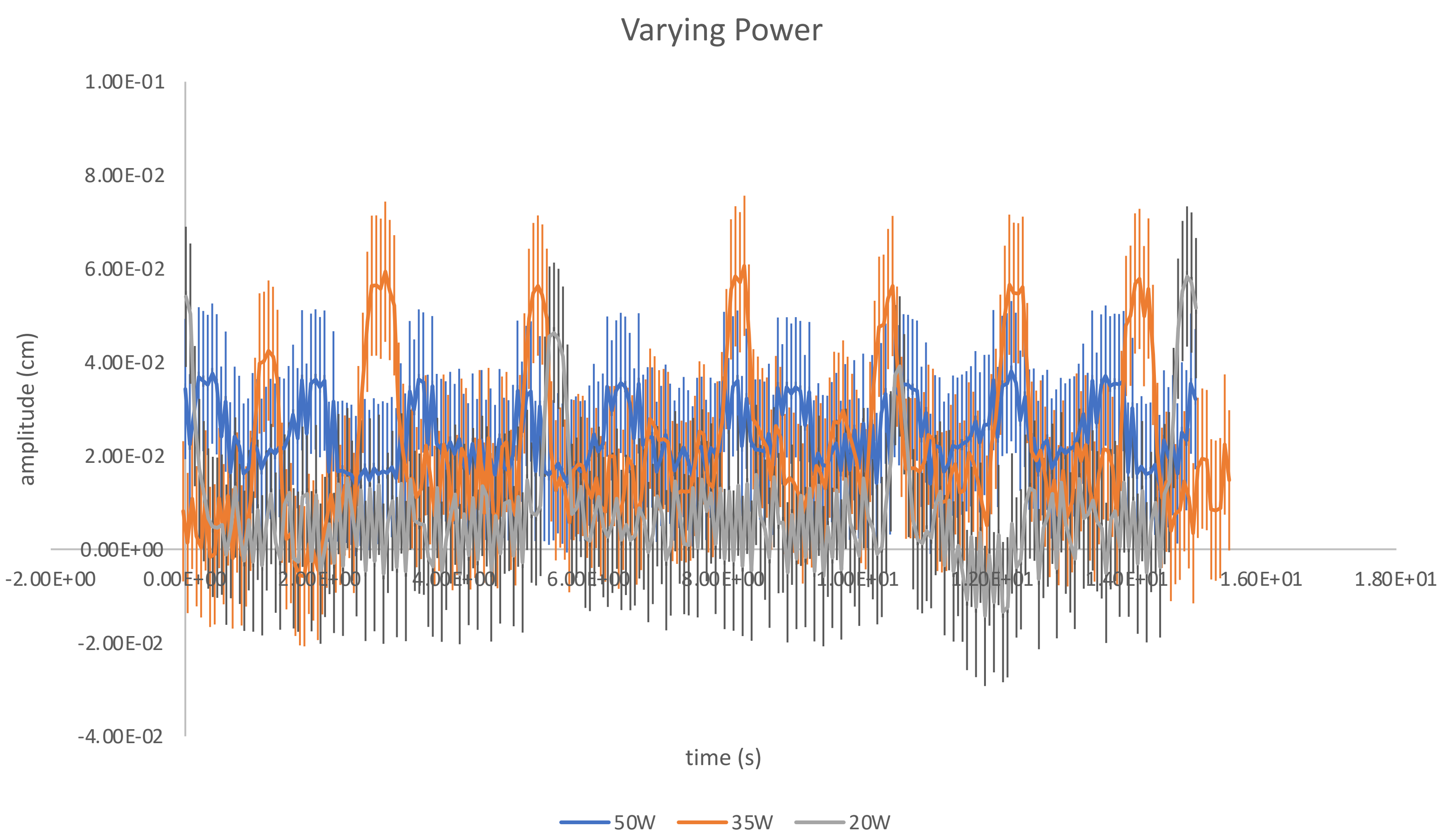
Thank you for listening

References

[1] *https://theses.hal.science/tel-03953103/document*
[2] International Journal of Thermal Sciences 115 (2017) 29e42
[3] *https://www.sfu.ca/~mbahrami/ENSC%20388/Notes/Natural%20Convection.pdf*
[4] https://www.researchgate.net/publication/48412257
[5]On the departure behaviors of bubble at nucleate pool boiling

Appendix

Varying Power (with error bars)



Triple Contact Line

forces and to fluid dynamics. The condition for static equilibrium of a triple contact line involving an ideal solid (perfectly smooth and chemically homogeneous), liquid and a gas/vapor is stated as the Young's equation [Carey, 2007]:

$$\sigma_{SV} - \sigma_{SL} - \sigma_{LV} \cos(\theta_{eq}) = 0 \tag{4}$$

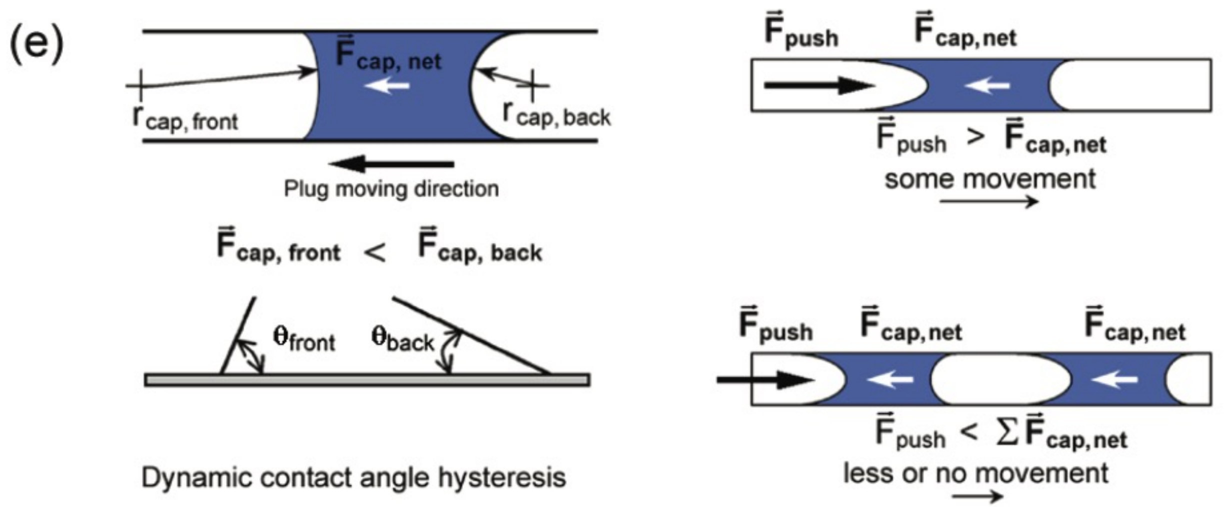
Equalibrium Spreading Coefficient

$$S_{eq} = \sigma_{SV} - (\sigma_{SL} + \sigma_{LV}) = \sigma_{LV}(\cos(\theta_{eq}) - 1) \tag{5}$$

Initial Spreading Coefficient is defined as follows,

$$S_i = \sigma_{S0} - (\sigma_{SL} + \sigma_{LV}) \tag{6}$$

Triple Contact Line



With Laplace pressure

$$\Delta P_{cap} = (2\sigma / R)\left(-\cos\theta_a + \cos\theta_r\right)$$

Triple Contact Line

$$f^* = \frac{16}{Re^*}\left[1 + A\left(\frac{D}{L_s}\right)(Re^* / Ca^*)^B\right] = \frac{-(D/4)(dP/dz)}{\left(\frac{1}{2}\rho_l (j_l + j_g)^2\right)}$$

diameter tube used in the experiments. Besides, the frictional pressure drop for the liquid slug is calculated as:

$$\Delta P_f = 0.078\left(\rho_l \cdot U_s \cdot D/\mu_l\right)^{-0.25} \cdot (\rho_l \cdot U_s^2) \cdot (2L_s/D) \qquad (26)$$

$$\Delta P_f = k(\rho_l \cdot U_s \cdot \mu_l)^{-0.25} \cdot (\rho_l \cdot U_s^2) \cdot \left(\frac{2L_s}{D}\right)$$