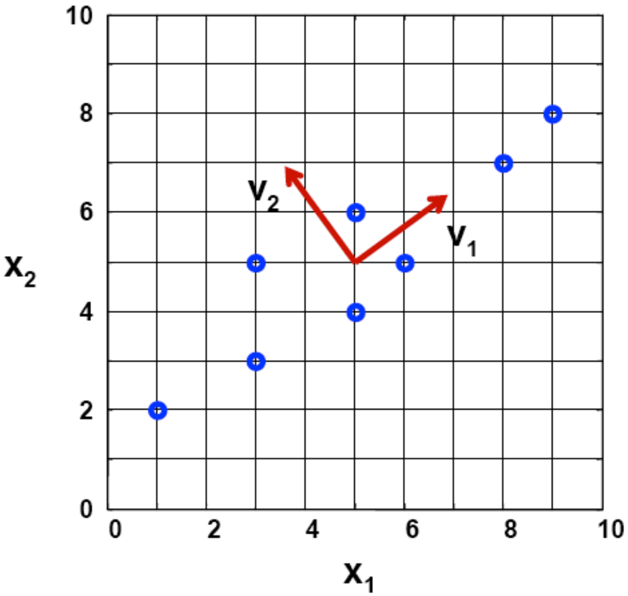
PCA example

- **Compute the principal components for the following two-dimensional dataset**
  - $X=(x_1,x_2)=\{(1,2),(3,3),(3,5),(5,4),(5,6),(6,5),(8,7),(9,8)\}$
    - Let's first plot the data to get an idea of which solution we should expect



- **SOLUTION (by hand)**
  - The (biased) covariance estimate of the data is:

$$\Sigma_x = \begin{bmatrix} 6.25 & 4.25 \\ 4.25 & 3.5 \end{bmatrix}$$

  - The eigenvalues are the zeros of the characteristic equation

$$\Sigma_x v = \lambda v \Rightarrow |\Sigma_x - \lambda I| = 0 \Rightarrow \begin{vmatrix} 6.25-\lambda & 4.25 \\ 4.25 & 3.5-\lambda \end{vmatrix} = 0 \Rightarrow \lambda_1 = 9.34;\ \lambda_2 = 0.41;$$

  - The eigenvectors are the solutions of the system

$$\begin{bmatrix} 6.25 & 4.25 \\ 4.25 & 3.5 \end{bmatrix}\begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} = \begin{bmatrix} \lambda_1 v_{11} \\ \lambda_1 v_{12} \end{bmatrix} \Rightarrow \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} = \begin{bmatrix} 0.81 \\ 0.59 \end{bmatrix}$$

$$\begin{bmatrix} 6.25 & 4.25 \\ 4.25 & 3.5 \end{bmatrix}\begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} = \begin{bmatrix} \lambda_2 v_{21} \\ \lambda_2 v_{22} \end{bmatrix} \Rightarrow \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} = \begin{bmatrix} -0.59 \\ 0.81 \end{bmatrix}$$

    - HINT: To solve each system manually, first assume that one of the variables is equal to one (i.e. $v_{i1}=1$), then find the other one and finally normalize the vector to make it unit-length